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INFLUENCE OF LIMIT STATES ON THE OPTIMIZATION OF REINFORCED CONCRETE PLANE FRAMES

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Abstract. *Most current structural design codes are based on the concept of limit states, that is, when a structure fails to meet one of its purposes, it is said that it has reached its limit state. According to the Brazilian code ABNT NBR 6118 (2014), in the design of reinforced concrete structures, the Ultimate Limit State (ULS) and the Serviceability Limit State (SLS) must be checked. Therefore, this paper presents an optimization scheme for reinforced concrete structures, based on the lowest cost, considering ULS and SLS constraints. The optimal characteristics of the structures subject to each limit state, individually, as well as to both limit states simultaneously, are discussed. The methodology is applied to a plane frame using genetic algorithms (GA) to solve the optimization problem, since this problem involves several local minima. The objective function employed corresponds to the cost of the structural elements, which includes the consumption of concrete, steel and formwork. For the case studied, it is noted that the optimal sections for the ULS are preponderant to the optimal sections for the SLS, so that when considering both limit states simultaneously the solution found is equal to the ULS.*

Keywords: *Optimization, Reinforced concrete, Limit states, Plane frames*

1 INTRODUCTION

The objectives of structural designs are to design structures that meet all the needs for which they were developed and present an acceptable cost. Such design must ensure that the structure will support in a safe, stable and without excessive deformation, all the actions and loads it will be subjected to during its lifespan. For a given design load, the verification of such requirements depends on the dimensions of the structural elements, as well as on the mechanical properties of the materials involved; and these characteristics directly affect the cost of the structure. In this scenario, the importance of studies related to the design concept of structures is valid.

Usually, the design of reinforced concrete structures involves a large number of variables. Therefore, many different design configurations, with different costs and performances, can meet the requirements. Usually, the choice of a configuration is not simple, which makes it difficult to obtain, by means of traditional methods, a design that is safe and presents the lowest cost possible.

Several studies in the field of optimization of reinforced concrete structures have been developed in the last decades. Usually, the optimization has as objective to find, among the structures that meet the constraints, the structure which presents the lowest cost. Among these studies, optimization of reinforced concrete plane frames was performed, for example, by Gerlein and Beaufait (1980), Adamu and Karihaloo (1995-a), Adamu and Karihaloo (1995-b), Vianna (2003), Kwak and Kim (2008) and Camp and Huq (2013). On the other hand, genetic algorithms (GA) have been probably the most used methods to optimize these structures, as can be seen in Coello, Christiansen and Hernández (1997), Koumousis and Arsenis (1998), Rajeev and Krishnamoorthy (1998), Rafiq and Southcombe (1998), Leps and Sejnoha (2003), Camp, Pezeshk and Hansson (2003), Govindaraj and Ramasamy (2005), Kwak and Kim (2009), Lima (2011), Mingqi and Xing (2011), Rosa Filho (2015) and Coêlho et al. (2016).

However, there is still a lack of studies that discuss the optimal characteristics found and their relationships with constraints recommended by standard codes (Juliani and Gomes, 2017). Most current codes, including the Brazilian ABNT NBR 6118 (2014), are based on the concept of limit states. According to these codes, the structure must satisfy certain limit states simultaneously. Within this context, an analysis of the influence of such limit states on what would be the optimal configuration of reinforced concrete structures needs to be better evaluated, with the purpose of assisting the studies in structural design.

Conceptually, a limit state is given by the situation from which a structural element no longer meets one of its design goals, in other words, when a structure fails to satisfy any of the purposes of its construction. The current Brazilian code establishes that the following limit states must be considered (ABNT NBR 8681, 2003):

a) Ultimate Limit State (ULS) - related to any situation that determines the interruption, in whole or in part, of the use of the structure;

b) Serviceability Limit State (SLS) - characterized by situations that, due to their occurrence, repetition or duration, generate structural effects that do not meet the conditions specified for the normal use of the structure, or indicate impairment of its durability.

In this sense, the present paper proposes to analyze the optimal configuration of reinforced concrete plane frames, through the minimization of its costs, considering two groups of constraints: the first is related to the ULS and the second to the SLS. The optimal characteristics of this structural element when subject to each group of constraints, individually, as well as to both groups, simultaneously, are discussed.

The main motivation of this paper is to expand the studies on the design of reinforced concrete structures through an analysis of the influence of the constraints related to the limit states on the optimal characteristics of these structures, as well as to develop a computational tool capable of optimizing such structures. This paper focuses on plane frames because there are many practical applications of this kind of structure. Furthermore, the analysis of plane frames requires consideration of the global structural behavior and the interaction between the two different types of structural elements involved: beams and columns.

The paper is organized as follows: section 2 introduces the problem that is addressed in this paper; section 3 describes the optimization method used; section 4 presents a numerical example in order to verify the efficiency of the proposed approach; section 5 presents the conclusions obtained in this paper.

2 OPTIMIZATION OF REINFORCED CONCRETE PLANE FRAMES

The problem addressed herein consists of minimize the cost of reinforced concrete plane frames. The cross sections of the beams and columns are rectangular and the structure is subjected to vertical and horizontal loads. Limit states constraints, formulated in accordance with ABNT NBR 6118 (2014), are imposed. For the structural analysis of the plane frame, the stiffness matrix method is used, which initially determines the structural response in terms of the displacements and from these displacements the internal forces are obtained (McGuire, Gallagher and Ziemian, 2014). Linear elastic structural analyses are performed, and all possible nonlinear effects are disregarded.

In the case of limit state verifications of the elements, a simplified rectangular stress distribution is assumed for the concrete. Also, for the sake of simplicity, in all cases the moments of inertia are computed considering the entire cross-section, i.e., concrete cracking is disregarded, and the effect of concrete creep is not taken into account despite its importance for SLS verifications.

In this paper, codes for checking beam and column limit states are developed, which are coupled with a structural analysis routine of plane frames and an optimization scheme. All algorithms are developed in MATLAB software (MathWorks, 2017).

To declare the optimization problem, the following items must be defined: design variables; objective function; constraints.

2.1 Design variables

The design variables to be determined during the optimization process are presented in Table 1. They are represented by a vector $\mathbf{x}$. Figure 1 illustrates a typical section of a beam and a column considered in this paper. It is emphasized that the longitudinal reinforcement of the column is said to be symmetrical.

To vary the amount of reinforcement along the beam, the structure is discretized in a predefined number of segments. Thus, for each segment, values of n_s^b, ϕ_s^b, n_s^t, ϕ_s^t, n_{sw} and ϕ_{sw}

Table 1: Design variables

	Beam
b	Cross section width
h	Cross section height
n_s^b	Number of bottom longitudinal reinforcement bars
ϕ_s^b	Diameter of the bottom longitudinal reinforcement bars
n_s^t	Number of top longitudinal reinforcement bars
ϕ_s^t	Diameter of the top longitudinal reinforcement bars
n_{sw}	Number of transverse reinforcement bars
ϕ_{sw}	Diameter of the transverse reinforcement bars
	Column
b	Cross section width
h	Cross section height
n_s	Number of longitudinal reinforcement bars
ϕ_s	Diameter of the longitudinal reinforcement bars
n_{sw}	Number of transverse reinforcement bars
ϕ_{sw}	Diameter of the transverse reinforcement bars

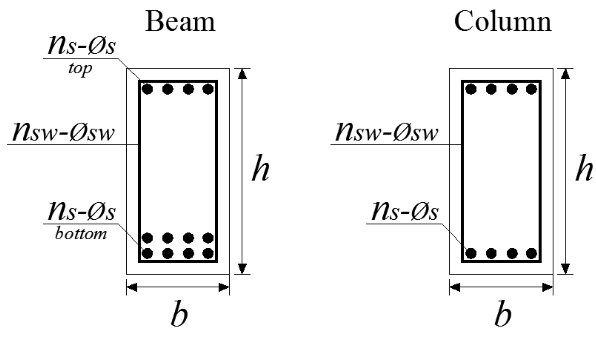

Figure 1: Typical section of a beam and a column

are determined, based on the maximum values of bending moment and shear force on the segment. The bars extend over the entire length of the segment and the anchorage of longitudinal bars is not considered. For example, a beam discretized into three elements would have a total of twenty variables: $\mathbf{x} = [b; h; n_{s_1}^b; \phi_{s_1}^b; n_{s_1}^t; \phi_{s_1}^t; n_{sw_1}; \phi_{sw_1}; n_{s_2}^b; \phi_{s_2}^b; n_{s_2}^t; \phi_{s_2}^t; n_{sw_2}; \phi_{sw_2}; n_{s_3}^b; \phi_{s_3}^b; n_{s_3}^t; \phi_{s_3}^t; n_{sw_3}; \phi_{sw_3}]$. Structural elements with different sections along their length result in lower material costs, although in general they have a negative impact in the labor cost.

2.2 Objective function

The objective function employed corresponds to the cost of the structure, that is, the sum of the costs of the beams and columns (Eq. (1)). It is based on the cost of concrete volume, longitudinal and transverse reinforcement mass and formwork area. The unit costs adopted include materials and labor to construct the structure.

$$f(\mathbf{x}) = V_c C_c + M_s C_s + M_{sw} C_{sw} + A_f C_f \tag{1}$$

Where:

$\mathbf{x}$ = Vector of design variables;

V_c = Concrete volume (m^3);

M_s = Longitudinal reinforcement mass (kg);

M_{sw} = Transverse reinforcement mass (kg);

A_f = Formwork area (m^2);

C_c = Unit cost of concrete ($R\$/m^3$);

C_s = Unit cost of longitudinal reinforcement ($R\$/kg$);

C_{sw} = Unit cost of transverse reinforcement ($R\$/kg$);

C_f = Unit cost of formwork ($R\$/m^2$).

The formwork area of a beam and of a column are calculated from Eq. (2) and Eq. (3), respectively, where l is the length of the structural element.

$$A_f^{beam} = (2 \cdot h + b) \cdot l \tag{2}$$

$$A_f^{column} = (h + b) \cdot 2 \cdot l \tag{3}$$

2.3 Constraints

The constraints imposed on the problem are represented by the vector $\mathbf{g}$ described by Eq. (4), where η is the number of constraints $g_i(\mathbf{x})$ of the problem, with $i = 1, ..., \eta$. The constraints of the beams and columns are divided into constraints of the ULS and SLS, as shown in the Fig. 2. The ULS constraints are related to the strength capacity of the structural element section and the SLS constraints are represented by the limit state of excessive deformation. Besides the constraints related to the limit states, in all cases, constructive constraints are also considered; for example: maximum and minimum limits of dimensions and reinforcement rates; ductility conditions; and others.

$$\mathbf{g} = [g_1(\mathbf{x}), ..., g_\eta(\mathbf{x})] \tag{4}$$

Beam constraints

- ULS constraints

Given the applied load, the structure must withstand the design bending moment (M_{Sd}) and the design shear force (V_{Sd}). The internal forces are obtained from the normal ultimate

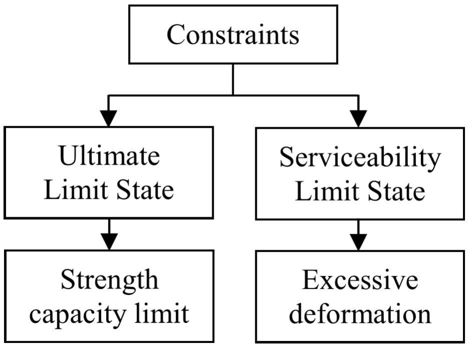

Figure 2: Constraint scheme of the problem

combination of applied loads.

- Load-bearing capacity of a reinforced beam cross-section in bending:

The section of the element is safe with respect to the bending moment if it satisfies the constraint given by Eq. (5), where M_{Rd} is the ultimate (design) moment.

$$g_1(\mathbf{x}) = M_{Sd} - M_{Rd} \leq 0 \tag{5}$$

M_{Rd} is calculated from Eq. (6), obtained from the equilibrium of forces acting on the cross section, where A_s is the cross sectional area of longitudinal tension reinforcement, f_{yd} is the design yield strength of longitudinal steel reinforcement, d is the effective depth (the distance from the extreme compression fibers of the concrete to the centroid of the reinforcement), λ is a parameter depending on the characteristic compressive strength of concrete (f_{ck}) and z defines the position of the neutral axis.

$$M_{Rd} = A_s \cdot f_{yd} \cdot (d - 0.5 \cdot \lambda \cdot z) \tag{6}$$

The value of z is obtained from Eq. (7), where α_c is a parameter of reduction of the compressive strength of concrete and f_{cd} is the design compressive strength of concrete.

$$z = \frac{A_s \cdot f_{yd}}{\lambda \cdot \alpha_c \cdot f_{cd} \cdot b} \tag{7}$$

- Load-bearing capacity of a reinforced beam cross-section in shear:

The strength of the section with respect to the shear force is guaranteed if Eq. (8) and Eq. (9) are checked, where V_{Rd2} and V_{Rd3} are, respectively, the design shear strength of compressive diagonals of concrete and the design shear strength of tension diagonals (supplied by concrete and transverse reinforcement). Model I of the code is used to obtain shear strengths.

$$g_2(\mathbf{x}) = V_{Sd} - V_{Rd2} \leq 0 \tag{8}$$

$$g_3(\mathbf{x}) = V_{Sd} - V_{Rd3} \leq 0 \tag{9}$$

The value of V_{Rd2} is obtained from the Eq. (10), where α_{v2} is a parameter which depends on the f_{ck}.

$$V_{Rd2} = 0.27 \cdot \alpha_{v2} \cdot f_{cd} \cdot b \cdot d \tag{10}$$

V_{Rd3} is obtained from Eq. (11), where f_{ctd} is the design tensile strength of concrete, A_{sw} is the cross-sectional area of transverse reinforcement bar, s is the spacing between the transverse reinforcements and f_{ywd} is the design tension in the transverse reinforcement.

$$V_{Rd3} = 0.6 \cdot f_{ctd} \cdot b \cdot d + \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \tag{11}$$

- SLS constraints

The serviceability limit state is represented by Eq. (12), where a_v is the calculated vertical deflection and a_v^{lim} is the maximum vertical deflection allowed.

$$g_4(\mathbf{x}) = a_v - a_v^{lim} \leq 0 \tag{12}$$

Column constraints

- ULS constraints

Due to the applied load, the columns must have sufficient structural capacity to withstand combined effects of axial force and bending moment while meeting specifications of the ABNT NBR 6118 (2014). To ensure safety, a load-moment interaction diagram is constructed for each column. If the combination of the axial load and the bending moment on a column fall inside the load-moment interaction diagram, the capacity of the designed column is adequate (constraint $g_5(\mathbf{x})$). Figure 3 shows an example of a diagram constructed for a column whose width and height are both 20cm, reinforced by two 10mm diameter bars at each side. The f_{ck} of the concrete is 40MPa.

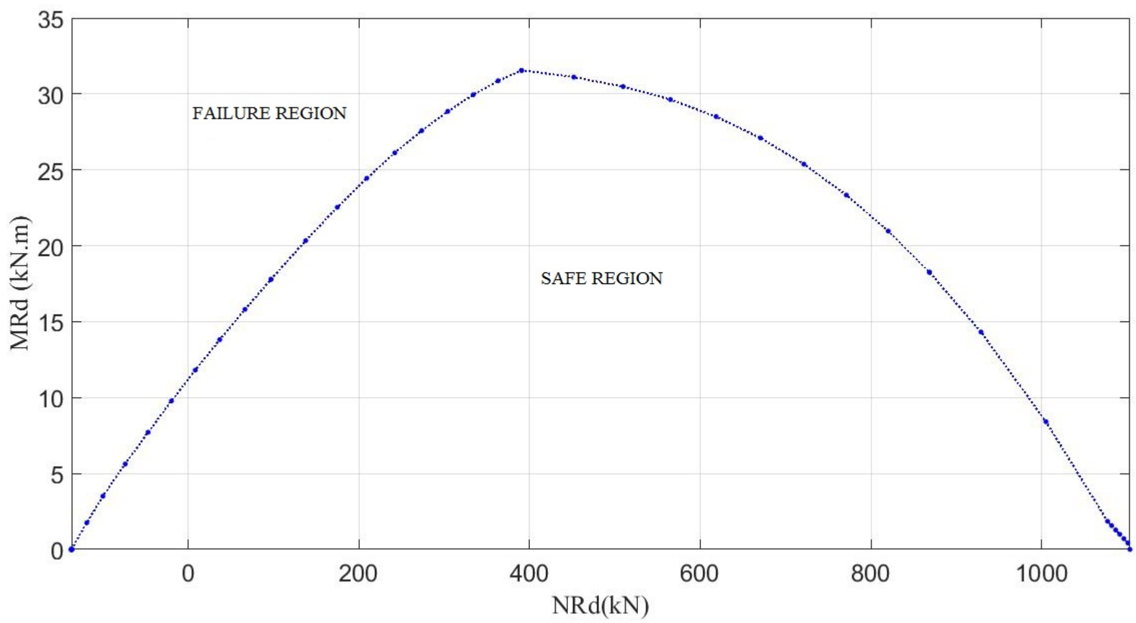

Figure 3: Load-moment interaction diagram

- SLS constraints

In the serviceability limit state, the calculated horizontal displacement between two adjacent floors, a_h, must comply with the maximum limit allowed by the code a_h^{lim} (Eq. (13)). Also, the calculated horizontal displacement at the top of the frame a_h^{top} must comply the maximum allowable limit $a_h^{top_{lim}}$ (Eq. (14)).

$$g_6(\mathbf{x}) = a_h - a_h^{lim} \leq 0 \tag{13}$$

$$g_7(\mathbf{x}) = a_h^{top} - a_h^{top_{lim}} \leq 0 \tag{14}$$

3 GENETIC ALGORITHMS

Genetic algorithms are search algorithms developed in the 1970s by John Hollad at the University of Michigan. They are based on the theory of evolution of species, declared by Charles Darwin in the 19th century (Goldberg, 1989). The algorithm can be applied to solve problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable or highly nonlinear.

These algorithms use some terms in analogy to natural genetics (Fig. 4): the individual is a solution, which may or may not be viable; the population is the set of solutions; the chromosome is the coding that represents the individual; the gene is the coding that represents the variable.

$$\begin{array}{ccl} \mathbf{x} & = & \overbrace{[x_1,\ \ldots\ , x_n]}^{\text{Chromosome}} \\ \downarrow & & \downarrow \\ \text{Individual} & & \text{Gene} \end{array}$$

Figure 4: Analogy between natural genetics and genetic algorithms

In general, the genetic algorithm method creates a random initial population and then performs the following steps: initially, the algorithm evaluates the fitness of each individual of the current population; then selects some of these individuals based on the value of their fitness, naming them as parents; later, children are produced from changes in the characteristics of a single parent (mutation) or by combining characteristics of a parent pair (crossover); after, individuals with the best fitness of the current population are chosen as "elite members" (elitism); finally, the current population is replaced by the individuals generated during the mutation, the crossover and the elitism phases, forming the new generation of the population. The steps are repeated until some stop criteria are satisfied. Figure 5 presents a flowchart of the GA. In this paper, a preprogrammed genetic algorithm routine, contained in the MATLAB software, is used (MathWorks, 2016).

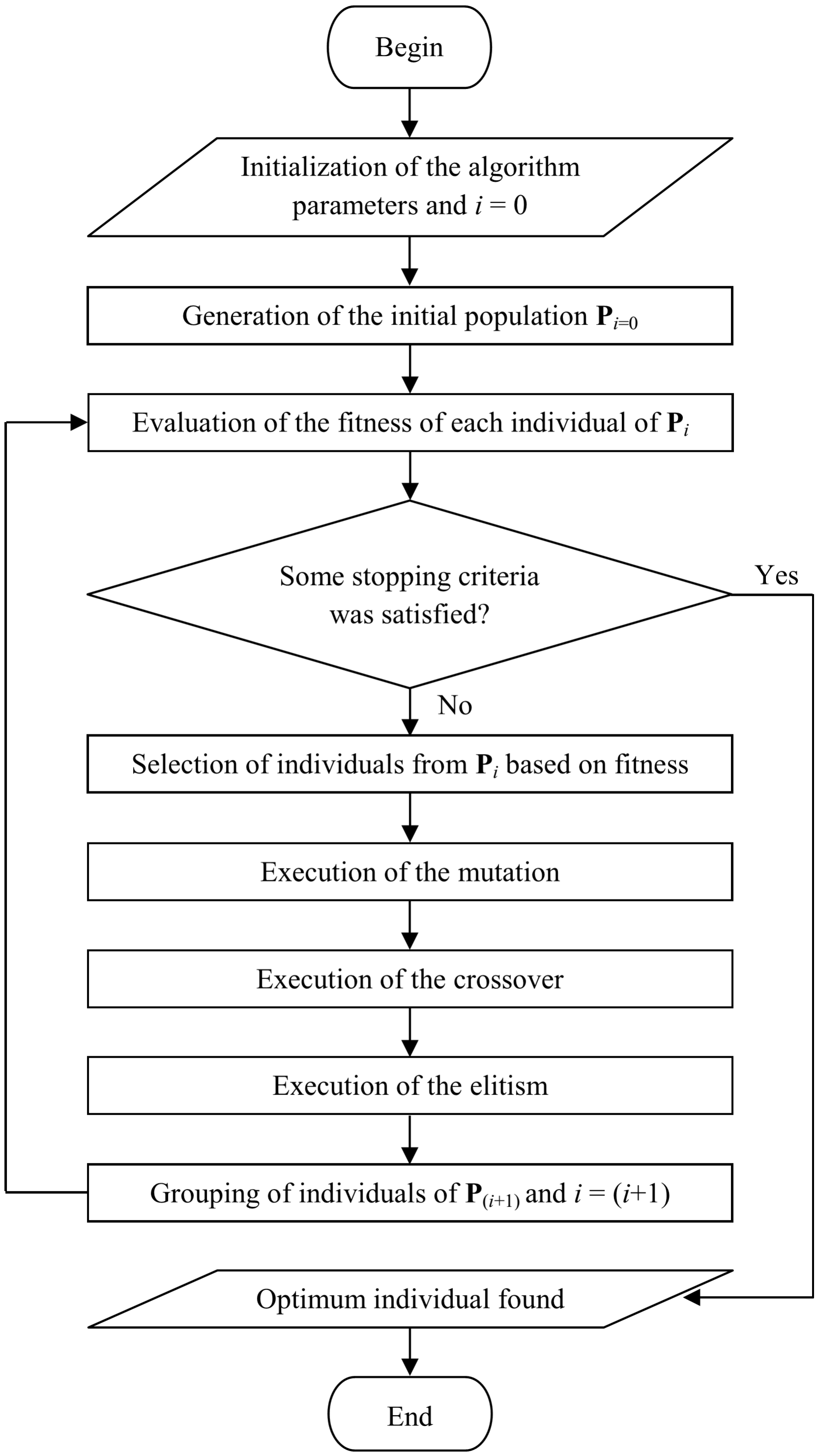

Figure 5: Basic scheme of a genetic algorithm

4 NUMERICAL EXAMPLE

The example consists of a reinforced concrete plane frame whose geometry was presented by Adamu and Karihaloo (1995-a) (Fig. 6). The input data used in the example is shown in

Table 2 and the values of unit costs are taken from SCO (2017). The following loads values were adopted for ULS: f_{x1} = 10kN; f_{y1} = 50kN; q_1 = 60kN/m; f_{x2} = 5kN; f_{y2} = 30kN; q_2 = 40kN/m. For SLS, loads equal to 80% of ULS loads are adopted. The load due to the weight of the beam and column are also considered, that is, in addition to the loads already mentioned, at each iteration the weight of the structure is updated. For the variables of the beams, the following intervals are adopted, in increments of 1: b = [12,25]cm; h = [30,60]cm; n_s = [2,10]; n_{sw} = [6,13]. The columns variables had the following intervals, in increments of 1: b = [19,40]cm; h = [19,40]cm; n_s = [2,10]; n_{sw} = [15,35]. Diameter of 10mm is adopted for the longitudinal reinforcement and of 6.3mm for the transverse reinforcement.

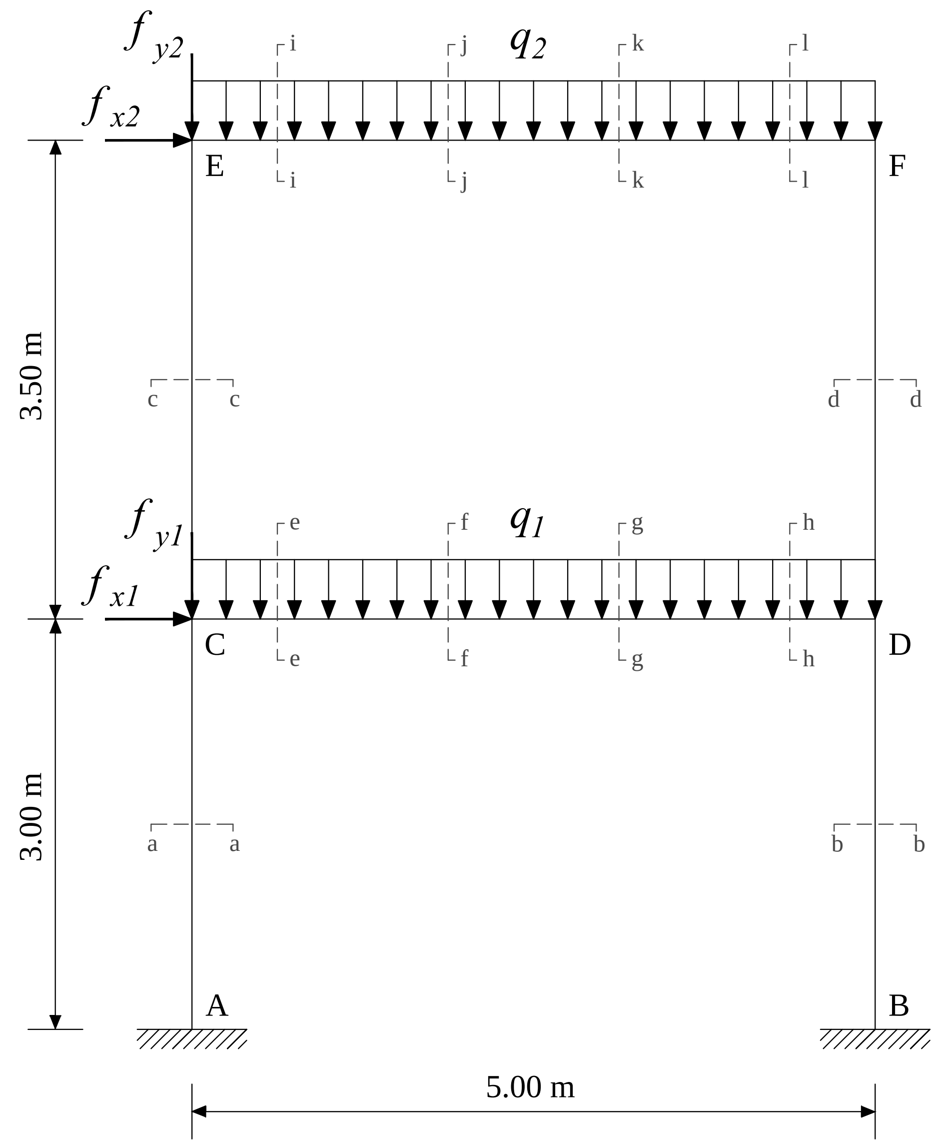

Figure 6: Geometry of the plane frame

Table 2: Input data

	Data	Value	Unit
E_s	Modulus of elasticity of longitudinal steel reinforcement	210,000	MPa
E_{cs}	Modulus of secant deformation of concrete	24,150	MPa
ρ_s	Unit mass of steel of the longitudinal reinforcement	7,850	kg/m^3
ρ_c	Unit mass of reinforced concrete	2,500	kg/m^3
f_{yk}	Characteristic yield strength of longitudinal steel reinforcement	500	MPa
f_{ck}	Characteristic compressive strength of concrete	25	MPa
c	Cover to reinforcement	2.5	cm
C_c	Unit cost of concrete	401	$\text{R\\$/m}^3$
C_s	Unit cost of longitudinal reinforcement	5.51	$\text{R\\$/kg}$
C_{sw}	Unit cost of transverse reinforcement	5.98	$\text{R\\$/kg}$
C_f	Unit cost of formwork	54.24	$\text{R\\$/m}^2$

Note: The properties of the steel of the transverse reinforcement are the same as those of the steel of the longitudinal reinforcement.

4.1 Optimization results

The optimal sections for ULS and its costs are presented in Fig. 7 and in Table 3, respectively. For SLS, such information is shown in Fig. 8 and in Table 4.

Concerning the optimization that considers only the constraints of the ULS, it is noticed, as expected, that the higher the load acting on the element the higher its dimensions and amounts of longitudinal and transverse reinforcement. Concerning the optimization that considers only the constraints of the SLS, it can be seen that the dimensions and the amount of transverse reinforcement are influenced by the load applied to the structural element, however, all sections presented the minimum amount of longitudinal reinforcement.

In general, the optimal sections of the beams for the ULS are more reinforced in both directions than for the SLS. Also, the beams heights for the ULS are greater than those obtained for the the SLS. However, the height of the beams is also preponderant in SLS. It is observed that the height of the section and the amount of longitudinal and transverse reinforcement are the most influential variables in the strength to the bending moment and the shear force. At the same time, increasing the height of the cross section is also responsible for the increased inertia of the structure and, consequently, results in smaller displacement of the structural elements. The optimal width of the beam is the same in all cases, 12 cm. It does not have as much influence on the strength and inertia of the cross section as the height, and the increase of its value increases the cost of formwork and concrete, which justifies its minimum value.

Regarding the columns, the optimal sections for the ULS have larger dimensions and amounts of longitudinal reinforcement than the optimal sections for the SLS. However, the

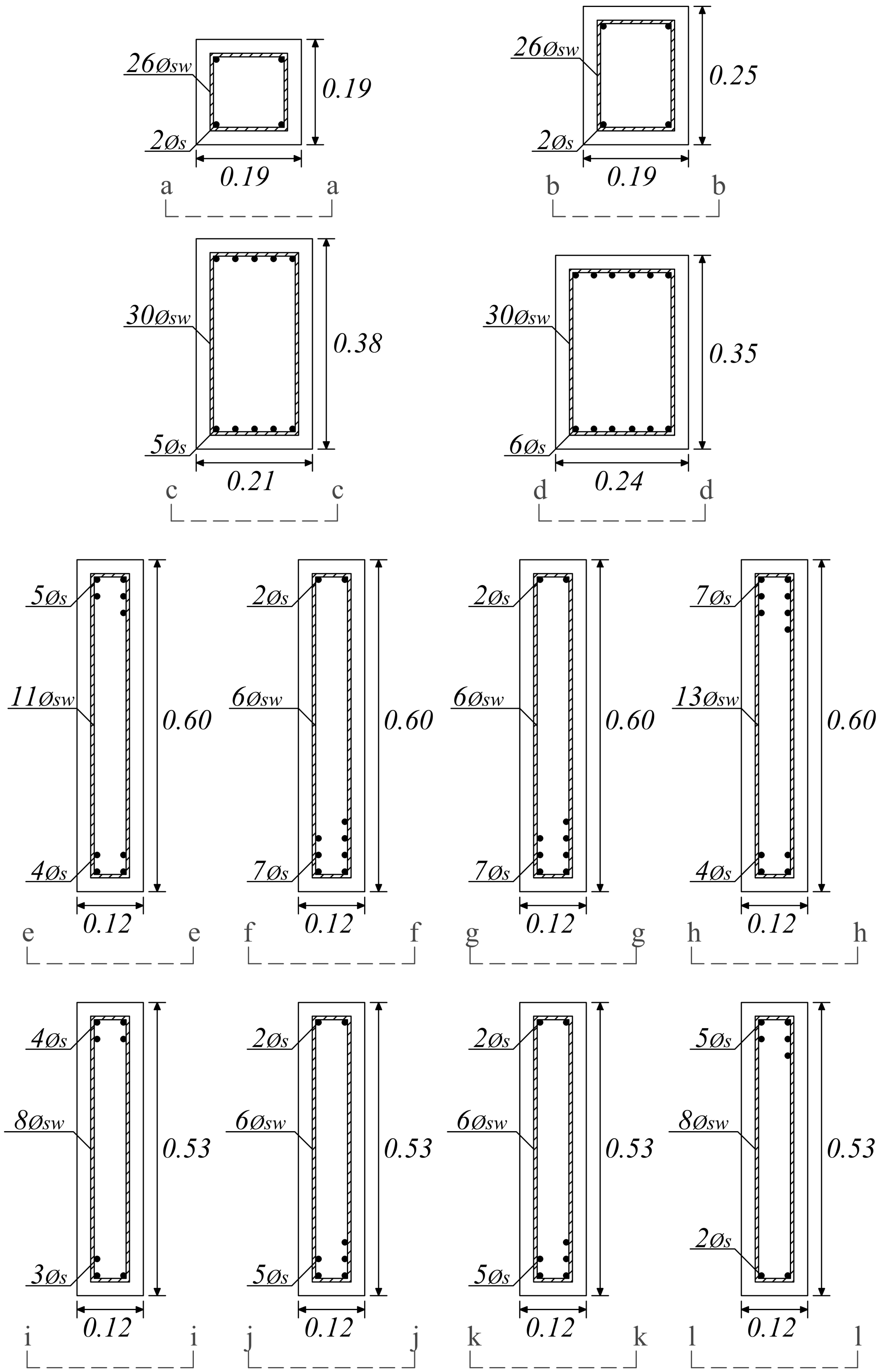

Figure 7: Detailing the optimal sections for ULS

Table 3: Optimal costs for the ULS

Element	Cost (R$)				
	Concrete	Longitudinal reinforcement	Transverse reinforcement	Formwork	Total
CD	144.36 (19.80%)	161.36 (22.13%)	65.32 (8.96%)	357.98 (49.10%)	729.03 (100%)
EF	127.52 (20.85%)	118.90 (19.44%)	45.07 (7.37%)	320.02 (52.33%)	611.50 (100%)
AC	43.43 (18.95%)	40.77 (17.79%)	21.31 (9.30%)	123.67 (53.96%)	229.17 (100%)
BD	57.14 (21.40%)	40.77 (15.27%)	25.87 (9.69%)	143.19 (53.64%)	266.97 (100%)
CE	112.00 (22.49%)	118.90 (23.88%)	43.02 (8.64%)	224.01 (44.99%)	497.93 (100%)
DF	117.89 (22.35%)	142.68 (27.04%)	43.02 (8.15%)	224.01 (42.46%)	527.61 (100%)
				Total	2,862.21

dimensions of the section are also preponderant for SLS. The amount of longitudinal reinforcement and the dimensions of the columns are related to the resistance to axial forces and to the bending moment, which shows great influence of such variables in the ULS. The dimensions obtained for the columns of the upper storey were larger than those for the columns of the lower storey, which is not an usual practical solution. However, other researchers from the literature (Adamu and Karihaloo, 1995-a) found similar solutions when dealing with the optimization of the frame considered herein. For SLS, the main influential variables are the dimensions of the section, given its impact on the displacement of the structure.

When considering the constraints of both limit states, the result is the same as found for the ULS, that is, the optimal section for the ULS also checks the SLS constraints. This is in agreement with the design practice, in which the structure is generally designed for ULS and only checked for SLS. In view of this, the cost of the optimal structure is higher for the ULS.

In terms of costs, the predominant input in all cases is the formwork, which reaches approximately 57% of the total optimum cost. It is emphasized that the reuse of the formwork is not considered, which explains the great impact of this input on the total cost of the structure.

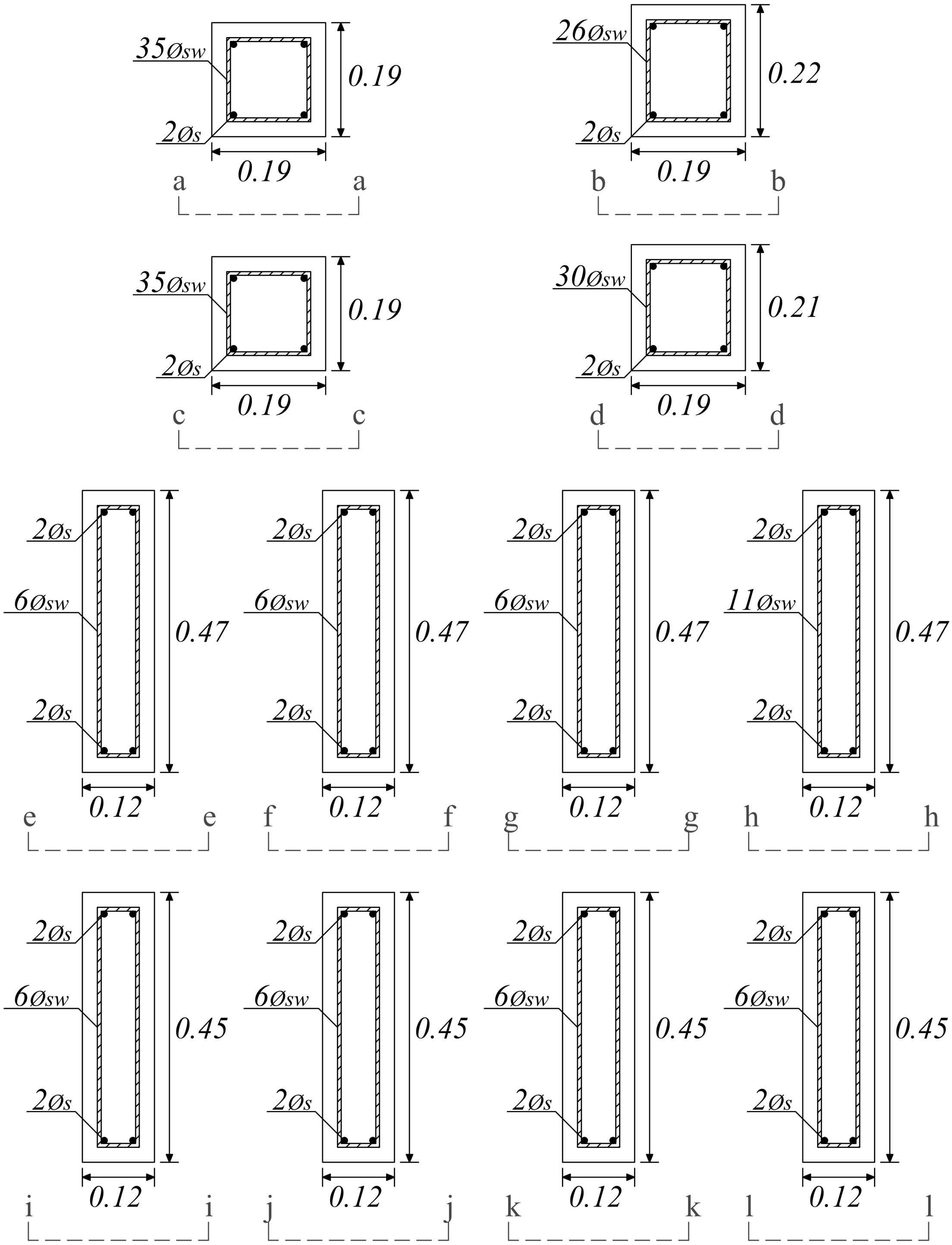

Figure 8: Detailing the optimal sections for SLS

Table 4: Optimal costs for the SLS

Element	Cost (R$)				
	Concrete	Longitudinal reinforcement	Transverse reinforcement	Formwork	Total
CD	113.08 (22.17%)	67.94 (13.32%)	41.59 (8.15%)	287.47 (56.36%)	510.08 (100%)
EF	108.27 (22.28%)	67.94 (13.98%)	33.01 (6.79%)	276.62 (56.94%)	485.85 (100%)
AC	43.43 (18.36%)	40.77 (17.23%)	28.68 (12.13%)	123.67 (52.28%)	236.54 (100%)
BD	50.29 (20.27%)	40.77 (16.43%)	23.59 (9.51%)	133.43 (53.79%)	248.07 (100%)
CE	50.67 (18.68%)	47.56 (17.54%)	28.68 (10.58%)	144.28 (53.20%)	271.19 (100%)
DF	56.00 (19.87%)	47.56 (16.88%)	26.34 (9.35%)	151.87 (53.90%)	281.77 (100%)
				Total	2,033.50

5 CONCLUSIONS

This paper presented an optimization scheme for reinforced concrete plane frames, based on the ultimate and serviceability limit states. The purpose of the paper was to verify the optimal characteristics of the structure for each limit state individually and for both limit states simultaneously. An example of a plane frame with one bay and two storeys was presented and genetic algorithms were used to optimize the frame. For the case studied, it is noted that the ULS constraints are more restrictive than the SLS constraints. Thus, the optimal section for the ULS also checks the SLS constraints, so that when considering both limit states simultaneously the solution found is equal to the ULS: the optimum sections for the ULS are more reinforced and with larger cross-sectional dimensions than the optimum sections for the SLS. This fact is in agreement with the design practice, in which the structure is generally designed for ULS and only checked for SLS.

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