THE GENERALIZED FINITE ELEMENT METHOD APPLIED TO DYNAMIC TRANSIENT ANALYSIS OF EULER-BERNOULLI BEAMS

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Abstract.

The finite element method (FEM), although widely used as an approximate solution method, has some limitations when applied in dynamic analysis. To improve the dynamic response of the structure, the generalized finite element method (GFEM) can be used to enrich the approach space with appropriate functions according to the problem under study. Since the literature presents many works showing that GFEM is an effective method in dynamic analysis, the present work aims to analyze a procedure capable of increasing the computational efficiency of the method. For this, the modal matrix, responsible for decoupling the system from dynamic equilibrium equations, is used with different percentages of modes. The problem was particularized for Euler Bernoulli beams, and in three different examples, displacements as a function of time and computational efficiency were analyzed. The results show the efficiency of the GFEM, which is able to obtain good results with few degrees of freedom, and using the modal matrix with a very reduced number of modes.

Keywords: *dynamic analysis, FEM, GFEM.*

1 INTRODUCTION

There is an increasingly effort among the academic engineering community for the design of structures that allow the efficient use of resources and construction procedures. In this context, the dynamic behavior of structures requires particular attention since most methods available for this kind of analysis need significant computational effort. Consequently, the development of more accurate methods and strategies can reduce the amount of computational effort needed in order to solve a given problem for the same accuracy, allowing the engineer to study a larger range of structural solutions and thus conceive better structures.

Several numerical methods can be applied for problems of dynamic analysis of structures. One of the most widespread approaches is the use of the Finite Element Method (FEM). The FEM, although widely used as an approximate solution method, has some limitations when applied in dynamic analysis. To improve the structure analysis, we can use the Generalized Finite Element Method (GFEM) (Melenk and Babuska, 1996; Duarte et al., 2000; Duarte and Oden, 1996;) to enrich the approach space with appropriate functions according to the problem under study.

Many applications of the GFEM can be found in literature, mostly when some information about the solution is known a priori, as in cracking analysis and dynamic problems in general. In Duarte et al. (2001) and Duarte and Kim (2008), the GFEM proved to be efficient in the cracking problem. Barros (2002) confirmed the effectiveness of GFEM in nonlinear analysis of structures, as well as Alves and Junior (2016) in composite materials.

Arndt et al. (2009, 2010 and 2012) proposed an approach of the GFEM to free vibration analysis of framed structures. In this method, trigonometric and exponential enrichment functions are added to the conventional FEM shape functions. In Arndt et al. (2011) the GFEM applied to free vibration of a uniform bar was compared with conventional *h* and *p* refinements of FEM and other enriched methods. The GFEM presented better results and convergence than the *h* version of FEM.

Torii and Machado (2012) applied the formulation proposed by Arndt (2009) for the transient analysis of bars and trusses. The results indicate that the GFEM can obtain more accurate results than the polynomial FEM in cases where the participation of the modes with high frequencies of vibration are important for the analysis. Shang, Machado e Abdalla Filho (2016) made a dynamic elastoplastic analysis of Euler Bernoulli beams using GFEM.

In this work, the behavior of the GFEM in the transient dynamic analysis of Euler Bernoulli beams is analyzed. The accuracy of the method is verified when the modal matrix is used in modal superposition with the presence of only a range of the vibration modes. The response obtained with GFEM regarding displacement is compared with reference solutions, and both, precision and computational efficiency, are analyzed.

2 EULER BERNOULLI BEAM TRANSIENT ANALYSIS

Consider an Euler Bernoulli beam of length L, cross-sectional area A, with an applied force p variable on the axis and time, according to Fig. 1. The constituent material is considered of linear behavior with modulus of elasticity E and specific mass ρ.

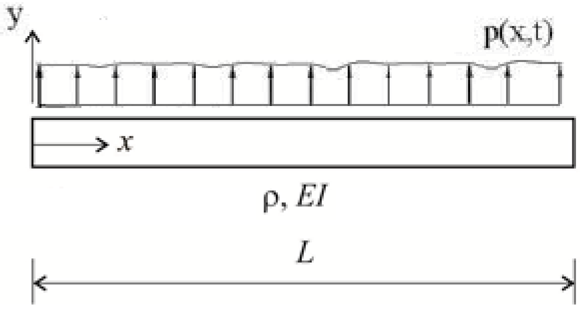

Figure 1. Euler Bernoulli beam

Applying the dynamic equilibrium, the partial differential equation governing the displacements in an Euler Bernoulli beam with uniform section is of the form (Clough; Pienzien, 1975):

$$EI\frac{\partial^4 v}{\partial x^4}+\rho A\frac{\partial^2 v}{\partial t^2}=p(x,t) \tag{1}$$

The application of the standard Finite Element Method (FEM) procedure (i.e. multiplying the partial differential equation by some trial function, integrating inside the domain and then applying Green's Theorem) (Bathe, 1996; Hughes, 1987; Zienkiewicz and Taylor, 2000) results in the following system of differential equations:

$$Ku+M\ddot{u}=F, \tag{2}$$

where K is the stiffness matrix, u is the vector of displacements, M is the mass matrix, $\ddot{u}$ is the vector of accelerations and F is the vector of applied forces. In matrix form from the contributions of each finite element is given by:

$$K_{ij}^{e}=EI\int_{\Omega^e}\left(\frac{d^2\phi_i}{\partial x^2}\frac{d^2\phi_j}{\partial x^2}d\Omega^e\right) \tag{3}$$

$$M_{ij}^{e}=\rho A\int_{\Omega^e}\phi_i\phi_j d\Omega^e \tag{4}$$

$$F_i^e=\int_{\Omega^e}p(x)\phi_i d\Omega+\bar{M}\left.\frac{d\phi_i}{\partial x}\right|_{\partial\Omega^e}-\bar{Q}\left.\phi_i\right|_{\partial\Omega^e} \tag{5}$$

The computation of Eq. (2) in displacement terms, can be done using a direct time integration method for each time step, although it is easily understood that for structural systems with a large number of degrees of freedom the computational costs starts to be very significant or even unsustainable. The modal superposition technique is computationally more efficient than the direct time integration (Bathe, 1996) once the global dynamic behavior of the structure can be properly reproduced considering the superposition of a limited number of vibration modes, being this possible if the structure is subjected to a linear analysis, as in this work. The decoupling of the system is done through the modal matrix, pre and post multiplying the system of Eq. (2). Using the modal superposition method, the system of *n x n* simultaneous equations is converted in *n* uncoupled equations:

$$\omega_i^2 u_i+\ddot{u}_i=F_i,\quad i\ =\ 1,2,...,n, \tag{6}$$

and that can be solved independently by a direct integration method, such as Newmark Method (Newmark, 1959), which was used in this work.

3 THE FEM AND THE GFEM

The FEM uses polynomials shape functions (Bathe, 1996; Hughes, 2000) in the approximated solution which can be expressed in matrix form as:

$$v_h^{e}(\xi) = N^T q, \tag{7}$$

where N is the matrix of shape functions and q is the displacement vector. The polynomial functions may be of any order. For Euler Bernoulli beam element with two degrees of freedom per node, the terms of the approximated solution (Eq. 7) are defined in the master domain as:

$$N^T = [\psi_1 \quad \psi_2 \quad \psi_3 \quad \psi_4], \tag{8}$$

$$q^T = [v_1 \quad \theta_1 \quad v_2 \quad \theta_2], \tag{9}$$

where v_1 and v_2 are the nodal displacements, θ_1 and θ_2 are the nodal rotations, ψ are the local shape functions.

The basic concepts of FEM can be extended to the GFEM (Babuska et al., 2004; Oden et al., 1998) which is a method based on the Partition Method of the Unit, proposed by Melenk and Babuska (1996). It is a Galerkin method that aims to enrich the finite element by constructing a subspace of pre-established solution approximation functions. This subspace aims to improve local and global results when compared to conventional FEM.

The approximated solution proposed by the GFEM in the master element domain may be written as the sum of two components:

$$v_h^{e} = v_{FEM} + v_{ENRICHED} \tag{10}$$

where v_{FEM} is the Finite Element Method component based on nodal degrees of freedom and $v_{ENRICHED}$ is the enriched component by the partition of unity approach based on field degrees of freedom. In this sense, the Euler Bernoulli beam approximated solution on a master element is (Arndt, 2009):

$$v_{FEM}^{e}(\xi) = \sum_{i=1}^{2} \eta_i(\xi)(\phi_{1\mathrm{i}}(\xi) v_{\mathrm{i}} + \phi_{2\mathrm{i}}(\xi)\theta_{\mathrm{i}}), \tag{11}$$

$$v_{ENRICH}^{e}(\xi) = \sum_{i=1}^{2} \eta_i \left[\sum_{j=1}^{n_l} \gamma_j(\xi) a_{ij} \right], \tag{12}$$

where η_i are the partition of unity (PU) functions, $\phi_{1\mathrm{i}}$ and $\phi_{2\mathrm{i}}$ are the functions associated with nodal degrees of freedom, γ_j are the enrichment functions, n_l is the number of enrichment levels, v_{i} and θ_{i} are the nodal displacement and rotation (nodal degrees of freedom), and a_{ij} are the field degrees of freedom related to the enrichment functions. Linear Lagrangian partition of unity (Eq. 13) were used by the works of Arndt (2009) and Torii (2012) as the basis for trigonometric GFEM and therefore will be considered here as well.

$$\begin{cases} \eta_1 = \dfrac{1-\xi}{2} \\ \eta_2 = \dfrac{1+\xi}{2} \end{cases} \tag{13}$$

It was chosen to work with the enrichment of PU with only two functions for simplicity in the construction of the approximation spaces and because this is the simplest case, allowing to focus on the study of the other factors on transient dynamic analysis using GFEM.

The functions associated with nodal degrees of freedom are

$$\begin{cases} \phi_{11} = 1 - \dfrac{\xi+\xi^2}{2} \\ \phi_{12} = \dfrac{\xi-\xi^2}{2} + 1 \\ \phi_{21} = \dfrac{1-\xi^2}{4} \\ \phi_{22} = \dfrac{\xi^2-1}{4} \end{cases} \tag{14}$$

which multiplied by the partition of unity functions (Eq. 13) results in the Hermite family of interpolation functions.

In this work, the enrichment functions for the Euler-Bernoulli beam element are the trigonometric functions proposed by Arndt (2009), which, in the domain of the master element $\Omega = (-1,1)$ are

$$\gamma_j(\xi) = \cos\left[\frac{(j-1)\pi(\xi+1)}{2}\right] - \cos\left[\frac{(j+1)\pi(\xi+1)}{2}\right] \tag{15}$$

for $j=1,2,\ldots,n_l$.

Arndt (2009) also proposed other enrichment functions for the Euler Bernoulli beams: the same analytical functions employed in the Composite Method, and the shape functions of the normal modes used in the Assumed Mode Method, which will be analyzed in future researches.

4 GFEM FREQUENCY SPECTRUM BEHAVIOR

Transient analysis can be performed initially by decoupling the system of dynamic equilibrium equations of the structure. For this, the technique of modal superposition is used, where the transformation matrix that decouples the system is the modal matrix itself. With the equations decoupled, a direct time integration method can be used to solve the problem. In this work, the Newmark Method is used.

At first, it is sought to analyze the already consolidated application of the GFEM in the free vibration analysis of structures. Afterwards, the results of the frequency spectrum of the analyzed structures are evaluated in order to observe the amount of vibration modes with good approximation in comparison with the analytical solution. With these results, analyzes regarding this percentage can be performed.

In this context, figures 2 and 3 show the frequency spectrum graphs where the approximate frequencies are related to the analytic ones (ω/ω_h) versus the ratio of the nth vibration mode to the total number of modes (n/N).

In the work of Shang, Machado and Abdalla Filho (2016) (Fig. 2a), an elastoplastic dynamic analysis was performed using trigonometric enrichment and the accuracy of GFEM with 2, 3 and 4 enrichment levels was compared to the linear FEM. In the work of Weinhardt (2015), (Fig. 2b) the application was particularized for free vibration analysis of bar element. The author applied the GFEM with trigonometric enrichment, and compared it with GFEM with Partition of the Unit defined from the Shepard functions, and also with the linear FEM.

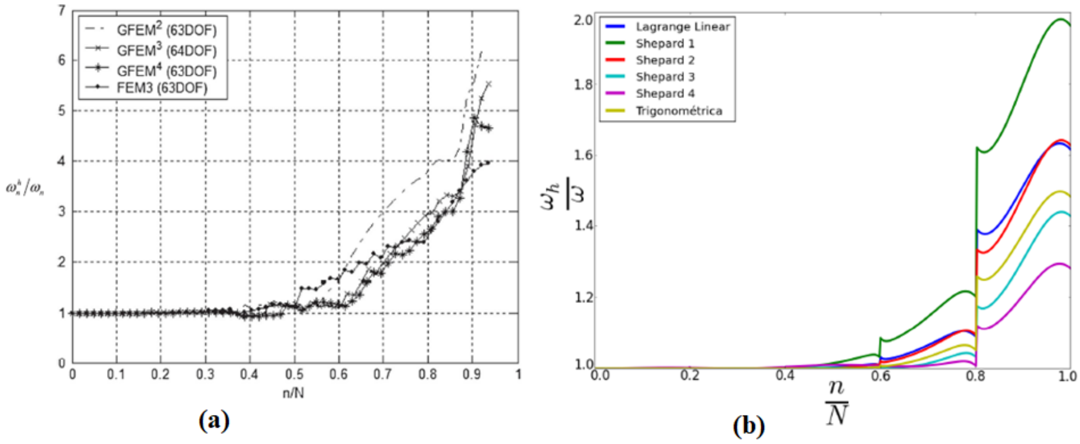

Figure 2: Frequency spectrum of Shang, Machado and Abdalla Filho (2016), and Weinhardt (2015).

The work of Garcia, Rossi and Linzmaier (2010) (Fig. 3), presented also an example of free vibration analysis of bar, where GFEM was applied with Partition of Unit defined from rational functions and polynomial enrichment, with 3 levels of enrichment, and then compared with the linear FEM.

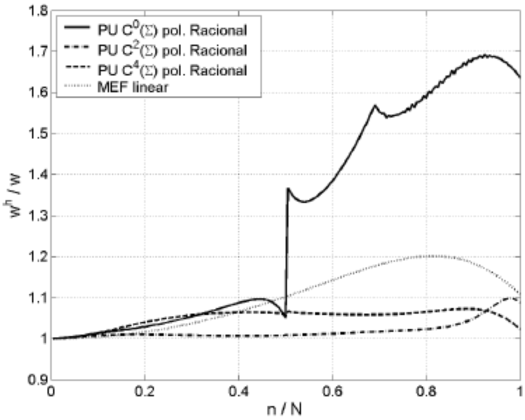

Figure 3: Frequency spectrum of Garcia, Rossi and Linzmaier (2010).

From Figures 2 and 3, it can be concluded that through the GFEM, the range of modes with good approximation is about 50%. Thus, it is expected that with the presence of a few modes, the transient response obtained through the GFEM may be as accurate as the analyzes made with the full modal matrix. Looking still at Figure 2, of which both authors use trigonometric enrichment, the range of acceptable modes is about 60%. This percentage is used as a parameter to study the efficiency of GFEM in the analysis of this work.

5 APLICATIONS

To analyze the behavior of GFEM in transient dynamic analysis, three examples of Euler-Bernoulli beam are discussed. The applications presented below are simple structures due to the initial state of the research, but are useful to compare the GFEM performance and, once the proposal of this work is verified, it can be applied in more complex structures.

For the examples, different percentages of vibration modes are left in the modal matrix, from the previously analyzed value of 60%. In this way it is possible to analyze the accuracy of the transient response, since the displacement values are compared with a reference response.

In all examples the beam has length $L = 5{,}0$ meters, elasticity modulus $E = 210$ GPa, mass density $\rho = 8000$ kg/m³, moment of inertia $I = 4{,}1667 \times 10^{-6}$ m^4 and uniform cross section area $A = 0{,}05$ m². The number of degrees of freedom considered in each analysis is the total number of effective degrees of freedom after introduction of boundary conditions. Also, the time interval considered was 10 seconds, with $\Delta t = 2{,}5 \times 10^{-3}$. To facilitate the understanding and visibility of the results, the first example is shown in a range of *9,0 to 10* seconds, and other examples are shown in a range *9,5 to 10* seconds.

Also, in all analyzes the GFEM with just one enrichment level was used.

5.1 Simply supported beam

As a first example, there is a simply supported Euler-Bernoulli beam with a step force applied at this center, as show in the Fig. 4.

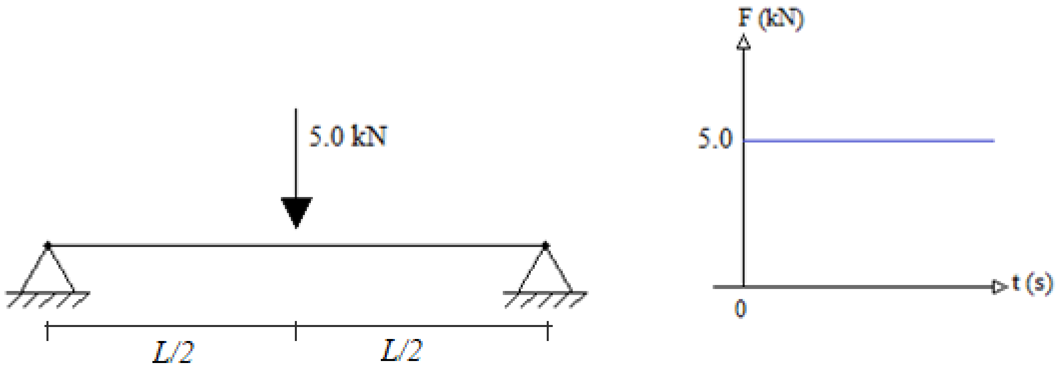

Figure 4. Simply supported Euler Bernoulli beam with a step force

For this example, the reference solution is the analytical solution, presented in Chopra (1995), and given by:

$$u_{L/2}(t) = \frac{2F_0L^3}{\pi^4 EI}\left[\frac{1-\cos(\omega_1 t)}{1^4} + \frac{1-\cos(\omega_3 t)}{3^4} + \frac{1-\cos(\omega_5 t)}{5^4} + \frac{1-\cos(\omega_7 t)}{7^4} + \ldots\right] \quad (16)$$

where $\omega_1, \omega_2, \ldots \omega_n$ are the beam vibration natural frequencies, F_0 is the step force applied, L is the beam length.

The results of displacement as a function of time are shown in Fig. 5-8. The percentages of modes contained in the modal matrix are 100%, 60%, 30%, 20%, 12%, 8%, 4% and one mode only, respectively. In these figures, GFEM24 stands for the number of degrees of freedom used in the analysis, and the percentage value means the number of modes in the modal matrix. The results of this example refer to the displacements in the middle of the beam ($x = 2,5$ m.) .

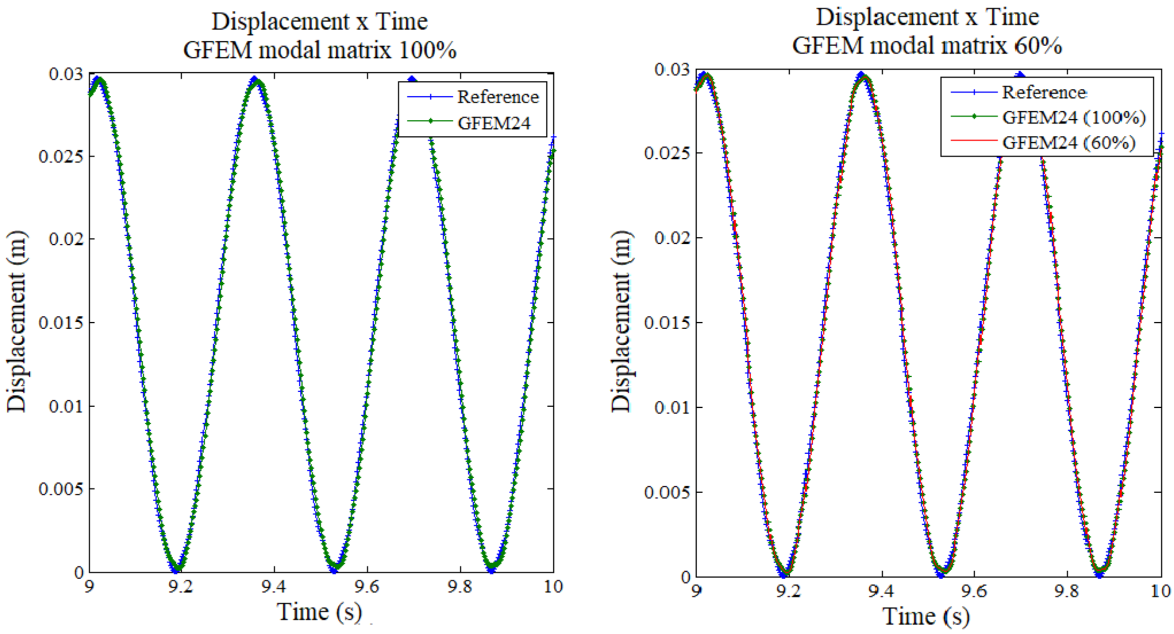

Figure 5. Displacements as a function of time for the complete modal matrix, and with 60% of the modes

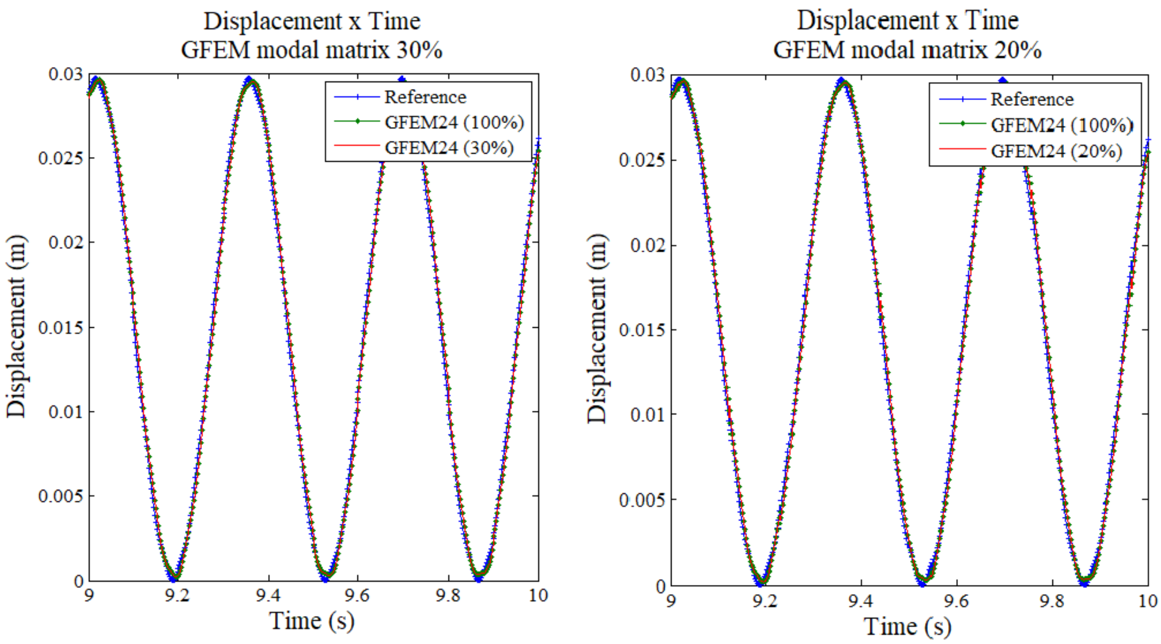

Figure 6. Displacements as a function of time for the modal matrix with 30% and 20% of the modes

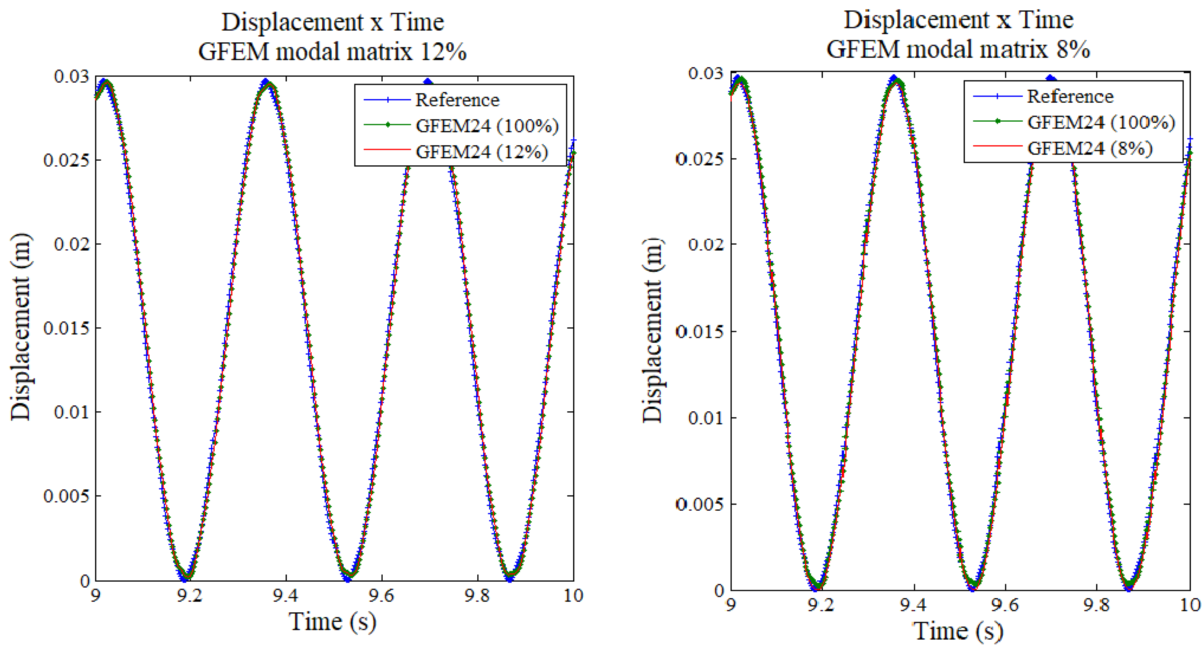

Figure 7. Displacements as a function of time for the modal matrix with 12% and 8% of the modes

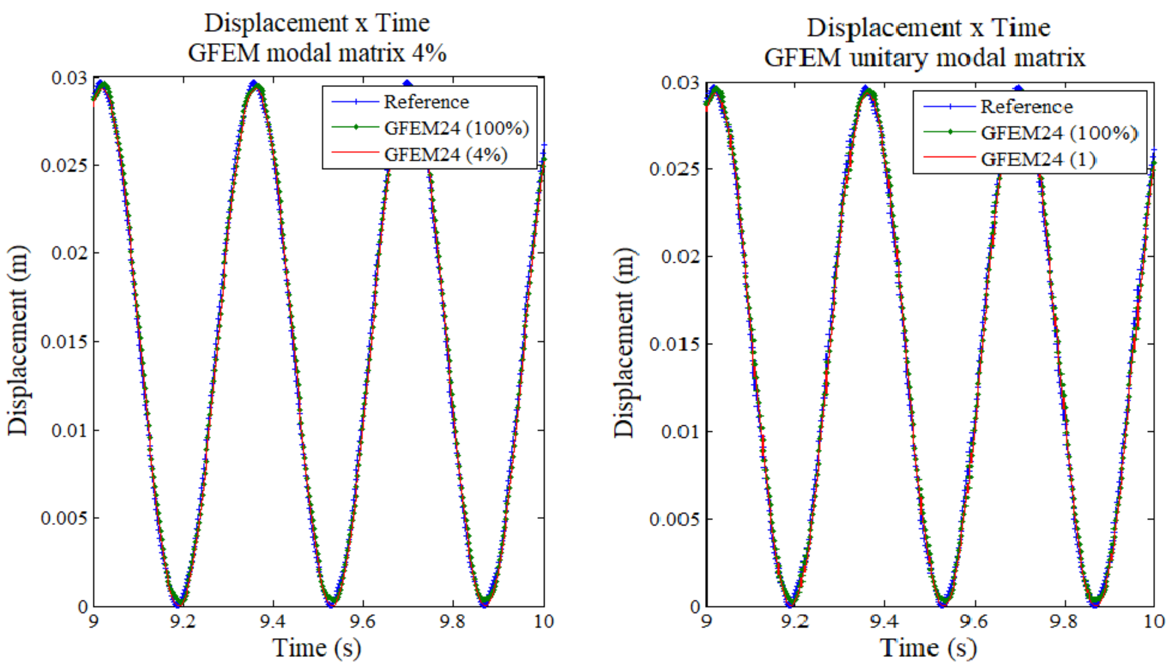

Figure 8. Displacements as a function of time for the modal matrix with 4% of the modes and only one

Observing the Fig. 5-8, it can be seen that for this beam example, the displacement results do not present significant change whereas the percentage of modal matrix modes varies. The results of the analysis using GFEM with only one mode of vibration (Fig. 8), for example, are very similar to the results obtained with the complete modal matrix. To show these results, Table 1 shows the absolute error of the displacement values at four different time instants, compared to the reference solution.

Table 1. Displacement error with reference solution (%)

Time (s)	2,5	5	7,5	10
GFEM 100%	1,37	3,38	6,45	2,89
GFEM 60%	1,38	3,38	6,56	2,90
GFEM 30%	1,40	3,36	7,04	2,91
GFEM 20%	1,43	3,33	7,79	2,92
GFEM 12%	1,62	3,17	7,81	2,94
GFEM 8%	1,83	1,90	9,47	3,66
GFEM 4%	1,83	1,90	9,59	3,66
GFEM 1	1,83	1,90	10,12	3,66

For the simply supported beam example, which is a relatively simple structure, where its deformation tendency is quite similar with its tendency to free vibration, it can be concluded that with only the first mode of vibration present in the modal matrix the results of the analysis are satisfactory. The transient response of this type of structure using only the first mode of vibration in the modal matrix results in a gain in computational efficiency in the analysis. This gain is presented in Table 2, which shows the solution time of the analysis using the complete modal matrix, as compared to the modal matrix with only one mode.

Table 2. Elapsed time

Analysis	Elapsed time difference (%)
GFEM 100% and GFEM 1	48,66

The computational efficiency presented for this example (double the efficiency, approximately) may be more significant when extended to an analysis for different types and arrangements of structure, with the more complex deformed configuration. In this context, examples 5.2 and 5.3 presents similar analyzes applied on Euler-Bernoulli beams with different types of excitation.

5.2 Cantilever and supported beam with harmonic excitation

For this example, it is considered a cantilever and supported beam with harmonic excitation applied to 1/4 of the total length, according to the Fig. 9.

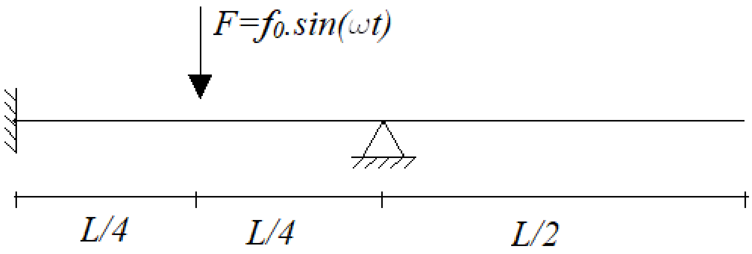

Figure 9. Cantilever and supported beam with harmonic excitation

In this example, $f_0 = 1\ kN$, $\omega = 6000$ rad/s and the transient response graphs refer to the displacement at the free end of the beam. As the analytical solution is unknown by the author, the reference solution is obtained using the FEM at 1600 degrees of freedom. For the solution to be reliable, a convergence test was performed analyzing the problem with 50, 100, 300, 600, 1000, and 1600 degrees of freedom with FEM. The method demonstrated a certain instability in the convergence of results with the lowest values of degrees of freedom. Only from 600 degrees of freedom did the displacement values stabilize. Thus, the values obtained with 1600 degrees of freedom were used as reference.

The analyzes of the modal matrix are the same as those of example 5.1, starting with the complete modal matrix and obtaining the displacement values for the percentages of modes of

60%, 30%, 20%, 12%, 8%, 4% and with the first mode only. The Figures 10-13 show the analyzes mentioned.

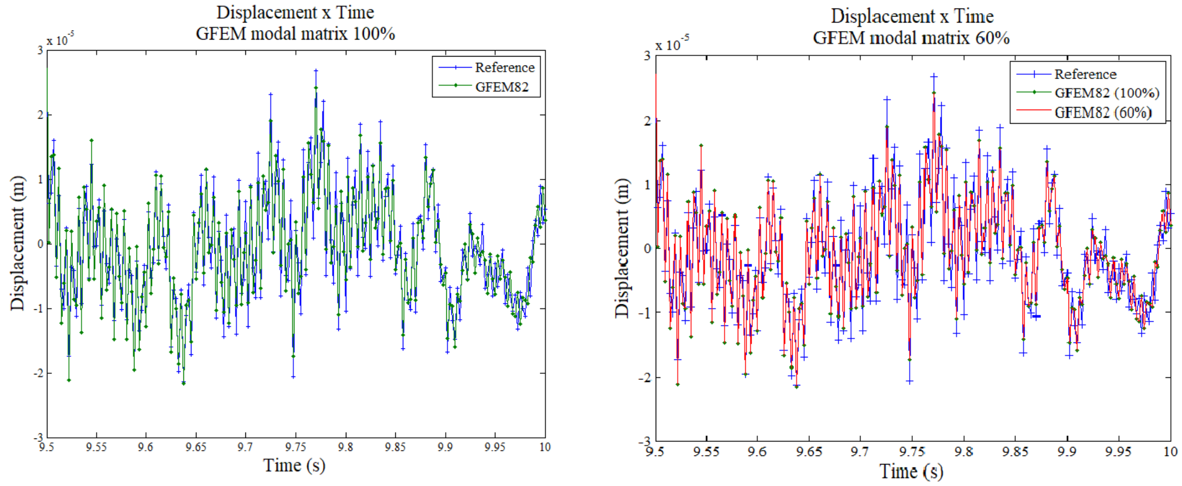

Figure 10. Displacements as a function of time for the complete modal matrix, and with 60% of the modes

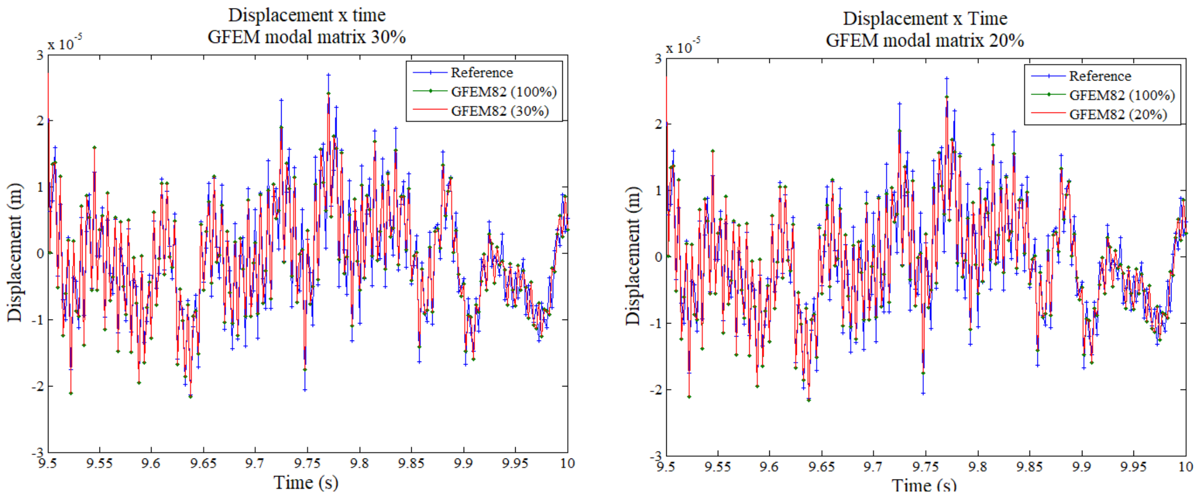

Figure 11. Displacements as a function of time for the modal matrix with 30% and 20% of the modes

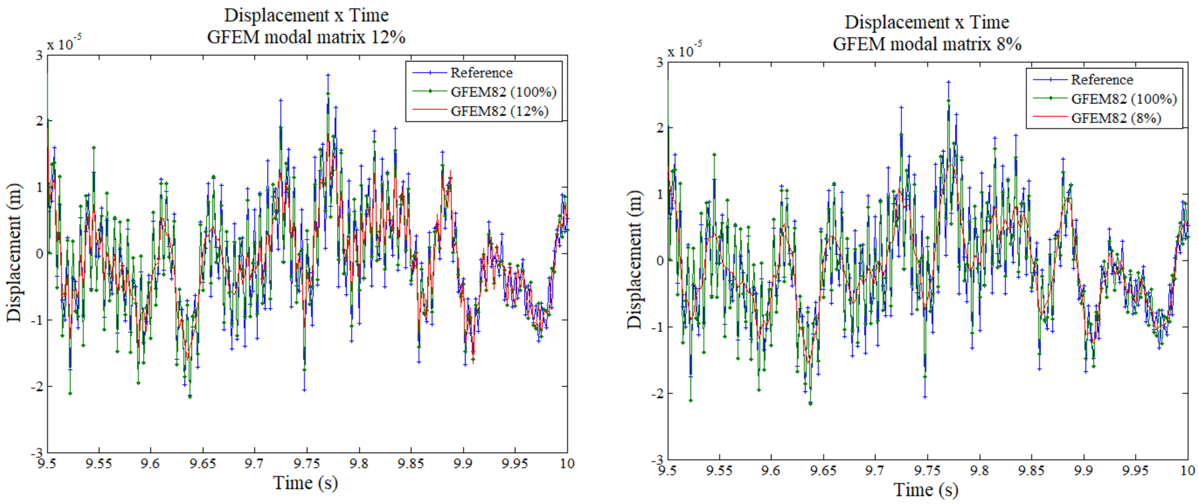

Figure 12. Displacements as a function of time for the modal matrix with 12% and 8% of the modes

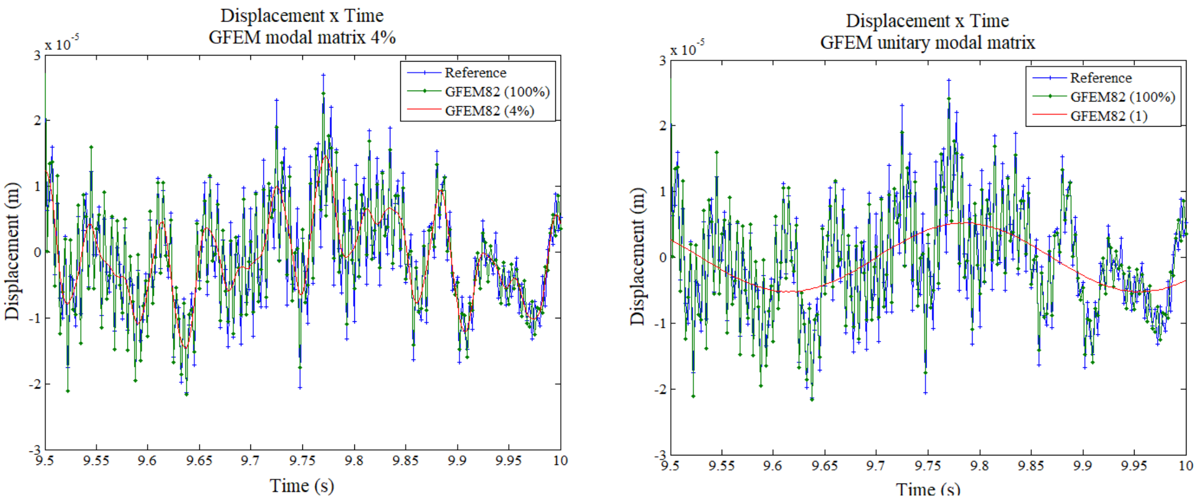

Figure 13. Displacements as a function of time for the modal matrix with 4% of the modes and only one

When observing the results of this example, it is perceived that the displacement values begin to present a bad approximation when the modal matrix has 20% of modes. Unlike example 5.1, where only one mode was sufficient to obtain good results, this configuration of structure requires the presence of more modes in the modal matrix. Table 3 shows the displacement values in four instants of time compared to the reference solution, reinforcing the conclusions about this example.

Table 3. Displacement error with reference solution (%)

Time (s)	2,5	5	7,5	10
GFEM 100%	3,18	2,32	5,77	16,22
GFEM 60%	3,18	2,31	5,76	16,22
GFEM 30%	3,14	2,52	5,85	16,21
GFEM 20%	3,68	2,73	6,10	16,24
GFEM 12%	2678,78	380,05	184,89	-80,86
GFEM 8%	3116,76	409,77	203,61	-89,71
GFEM 4%	3456,66	333,05	230,56	-72,13
GFEM 1	1691,19	308,96	120,09	213,95

The difference in the results of examples 5.1 and 5.2 is justified in part by the complexity of the applied load, which forces the beam in question to deform different from its natural tendency to vibrate. Even with the need for more modes, the computational efficiency is greater than that of example 5.1. Again, this efficiency can become more significant when expanded to a more complex structure or analysis. Table 4 shows this efficiency in a quantified way, when comparing the analyzes made with the full modal matrix, and with the modal matrix with 20% of the modes, which still showed good results.

Table 4. Elapsed time

Analysis	Elapsed time difference (%)
GFEM 100% and GFEM 20%	65,15

Also, in order to study the computational efficiency when using the reduced modal matrix in the GFEM formulation, a last example is analyzed.

5.3 Cantilever and supported beam with impulse excitation

A last example of Euler Bernoulli beam is analyzed, with an applied impulse load. The analysis regarding the modal matrix is the same as the previous examples, and therefore, it is intended to study the computational efficiency of the GFEM, when different percentages of modal matrix modes are extracted. Figure 14 shows the beam to be analyzed.

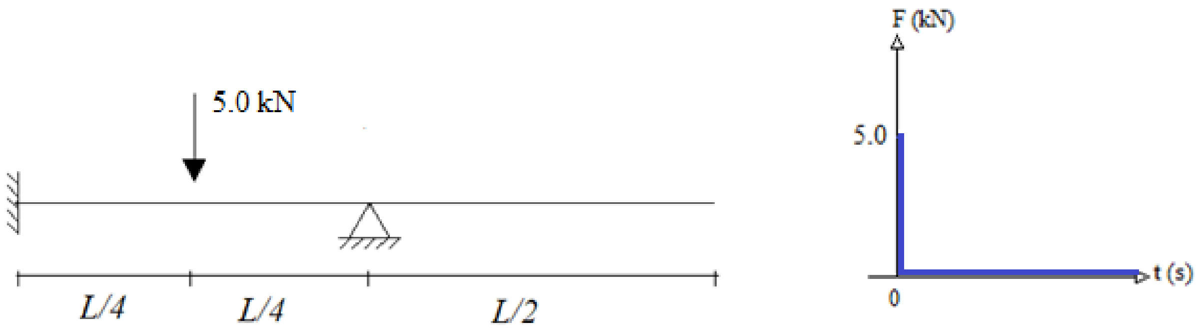

Figure 14. Cantilever and supported beam with impulse excitation

Again, the reference solution was obtained with the FEM, from a convergence analysis with 50, 100, 300, 600, 1000, and 1600 degrees of freedom. For this example, a further analysis was required, with 1800 degrees of freedom, since the FEM presented some instability in the responses. The reference response, therefore, is the one modeled from the FEM, with 1800 degrees of freedom. The Figures 15-18 show the response of the displacements as a function of time for the same percentages of the modal matrix modes of the previous examples.

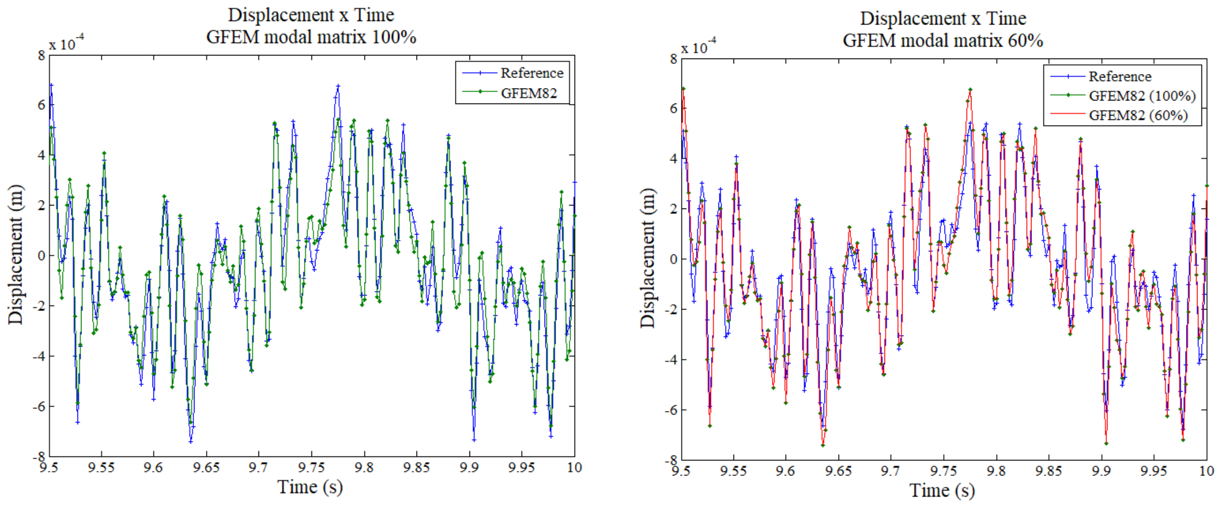

Figure 15. Displacements as a function of time for the complete modal matrix, and with 60% of the modes

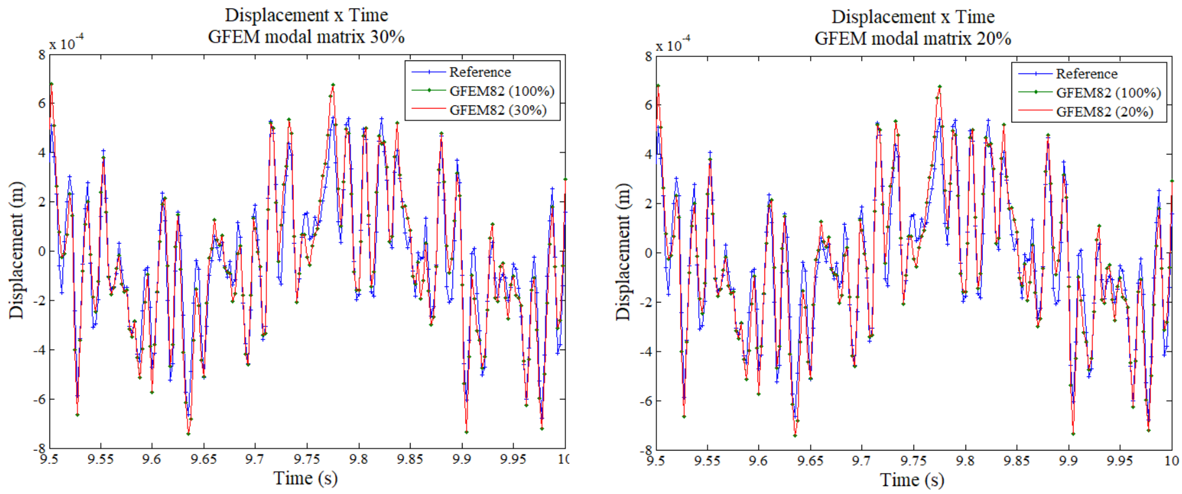

Figure 16. Displacements as a function of time for the modal matrix with 30% and 20% of the modes

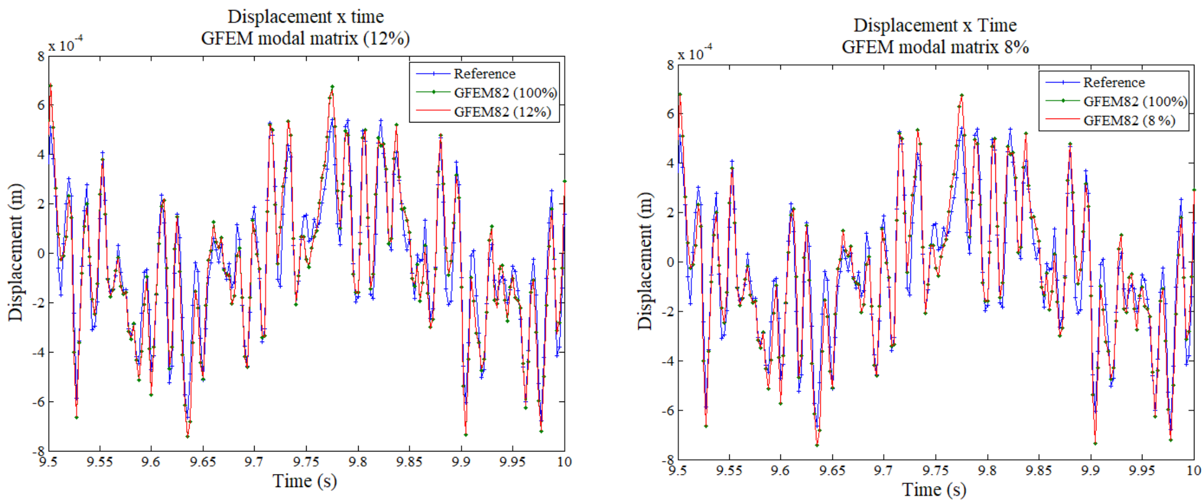

Figure 17. Displacements as a function of time for the modal matrix with 12% and 8% of the modes

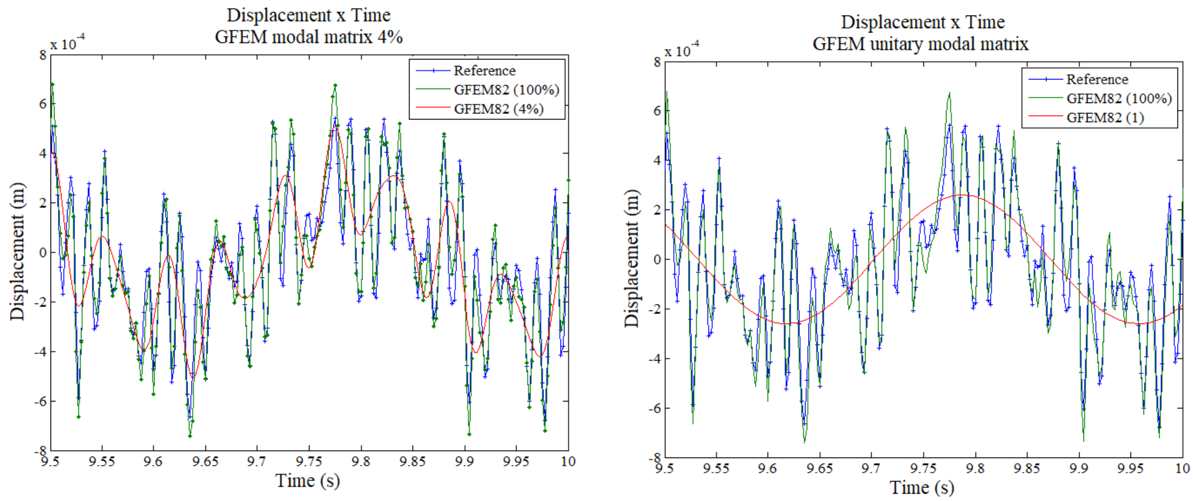

Figure 18. Displacements as a function of time for the modal matrix with 4% of the modes and only one

Analyzing Figures 15-18, it can be seen that, for this example, the displacement results as a function of time begin to deteriorate with the presence of 4% of the modes in the modal matrix. Which means, with only 8% of modes the results are very close to the reference solution. To further elucidate these differences in values, Table 5 shows the displacement values at four different time instants, as in examples 5.1 and 5.2.

Table 5. Displacement error with reference solution (%)

Time (s)	2,5	5	7,5	10
GFEM 100%	6,07	0,12	5,73	5,80
GFEM 60%	6,07	0,10	5,73	5,80
GFEM 30%	6,06	0,00	5,85	5,80
GFEM 20%	6,06	0,43	5,81	5,83
GFEM 12%	6,68	2,95	4,40	5,68
GFEM 8%	6,68	2,95	4,40	5,68
GFEM 4%	277,06	303,48	22,17	141,53
GFEM 1	255,48	244,36	88,64	36,68

Again analyzing the gain in computational efficiency, Table 6 shows the percentage of increase in lower computational cost when the problem is solved with the matrix full and with the matrix with 8% of the modes.

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Table 6. Elapsed time

Analysis	Elapsed time difference (%)
GFEM 100% and GFEM 8%	75,67

To obtain good displacement results, in the latter example a smaller number of modes were required in the modal matrix, compared to example 5.2. Thus, the computational efficiency was higher. The efficiency gain of more than 75% in this example can also be more significant when analyzing is a larger structure with more elements and loads.

6.0 CONCLUSIONS

The main contribution of the present study consists in the analysis of the behavior of the GFEM in the dynamic analysis of Euler Bernoulli beams. This study assumed, based on literature, that only a part of the modes of vibration obtained through GFEM have a good approximation. Thus, the tested that when the great majority of these modes of the modal matrix are removed, the solution of the displacements as a function of time would continue with good approximation. This assumption, proven by the present work, reflects in a gain in the agility of the numerical analysis, thus improving the computational efficiency of the method.

The modal matrix, responsible for decoupling the system from equations before applying a direct integration method, was used with several percentages of modes, and the results were compared to each other, and with a reference solution. When this analysis was done for a simple type of structure, in the case of the stepped load beam, only one mode of vibration was enough so that the displacement response had a good approximation. When the analyzes extended to different cases, such as the cantilever and simply supported beam, the presence of 12 - 8% of the modes resulted in good approximations.

The use of a few modes in the modal matrix to obtain the transient response of the structure is already a practice known in the literature. However, it was necessary to test this premise in the use of GFEM. The examples of beams tested in this work are relatively simple, however, they have been able to demonstrate that when using GFEM with a very reduced number of degrees of freedom, with only one level of enrichment and with a small percentage of modes present in the modal matrix, the results are as accurate as those obtained with reference solutions. The research will continue to study the behavior of GFEM in the transient response, extending the analysis to more complex structures such as trusses, frames, shells and plates.

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