TAGUCHI'S QUALITY MEASURES (LOSS FUNCTION AND SIGNAL-TO-NOISE RATIO) USING AN APPLICATION WITH A GRAPHICAL INTERFACE

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Abstract. *The constant technological advances these days make the business much more dynamic and aggressive. The companies that do not keep up with the constant progress and self-development can be out of the market and consequently leaded to bankruptcy. Specifically, in quality engineering, Genichi Taguchi has made great advances on methods of measuring and comparing the company's quality. Taguchi developed the quality loss function and the signal-to-noise ratio. In this context, one of the objectives of this work is to use the computationally intensive bootstrap technique to estimate confidence intervals of the quality loss function and signal-to-noise ratio, as well to develop an application with a graphical interface for easier utilization of these techniques. This article presents the results obtained by the development application "Experimento Taguchi" using data from five food products produced in three different work shifts. Thus, we obtained the quality loss function, signal-to-noise ratio and process capability for each product in each work shift. The results were compared showing which processes have better quality and which need optimization in their process.*

Keywords: *Application, Loss Function, Quality, Signal-to-noise ratio, Taguchi*

INTRODUCTION

Genichi Taguchi worked almost all of his life in quality field and he was inspired by huge minds, like W. Edwards Deming. Taguchi invented new ways of measure, compare and improve the quality. He separates the quality control processes in two areas: "on-line", that is, on production line where is applied the statistical process control techniques (SPC) and "off-line", out of production line, where there are the phases of system design and parameter optimization. Understanding that a product/process with low quality does not only damage the final consumer, but all society, brought new methods for quality field with the loss function, orthogonal arrays and signal-to-noise ratio. Taguchi developed an experimental design using orthogonal arrays to evaluate the process characteristics and find the best factor combinations to achieve the best quality index. This Taguchi's method allows resources, time and money savings because it is an experimental strategy improved.

Nowadays, the competitively for the market is each time bigger and it is necessary ally technology and knowledge to be ahead of your competitors. In this scenario, the development of an app for a specific objective turns to be advantageous. A good app makes your aims achieved in the most precise and fast way, that is, by turning your company more competitive and efficient.

In this pretext, it was developed the app "*Experimento Taguchi"* to provide a Taguchi's analysis. The app have the goal to provide a fast and easy way to apply the methodology that is aimed for students, professionals and researchers.

1 THE QUALITY LOSS FUNCTION

In the classic conception, we seek produce products that are inside the specifications. For Taguchi's sight, the search for improvement only ends when achieved the perfection, that is, a centered process with variability equals to zero. The loss function implies a philosophy of continuous quality improvement.

The Taguchi's quality loss function, Fig. 1, represents the loss generated when a quality's characteristic has a deviation from the target. This loss is occasioned for production's cost and damage suffered by the consumers (repairs, loss of business and others) (Phadke, 1989).

The loss function is given by:

$$L(y) = k(y - T)^2, \tag{1}$$

where L is the loss in monetary units when the observation is at y, T is the target and k is a constant to be defined in the future for different cases.

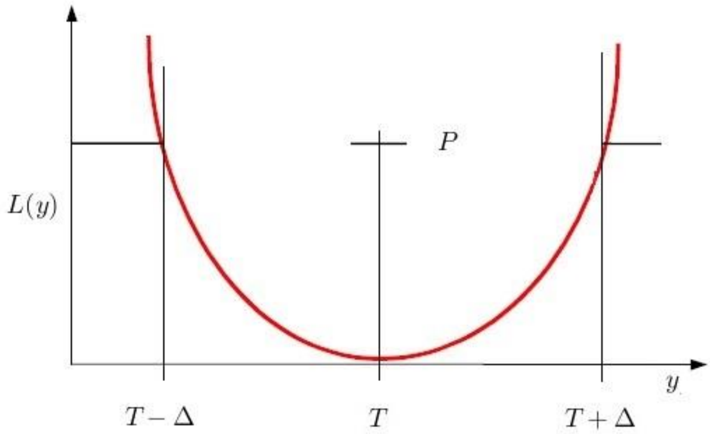

Figure 1. Taguchi's Loss Function

With this formulation, the loss function is calculated for each observation (product individually). In the most of cases, it have many observations, so the loss function goes by the average quality loss function and it is calculated by mean square deviation (MSD) from the sample.

$$\bar{L}(y) = k(MSD) \tag{2}$$

There are three types of loss functions according with the type of functional characteristic analyzed. They are:

i) Nominal-the-Better (NTB)

$$\bar{L}(\mathrm{y}) = \frac{P}{\Delta^2}[S^2 + (\bar{y} - T)^2]; \tag{3}$$

ii) Smaller-the-Better (STB)

$$\bar{L}(y) = \frac{P}{\Delta^2}[S^2 + \bar{y}^2\ ]; \tag{4}$$

iii) Larger-the-Better (LBT)

$$\bar{L}(y) = P\Delta^2\left[\frac{1}{y^2}\right]. \tag{5}$$

Where P represents the maximum loss value, Δ the tolerance of targets' deviation and T the target value.

The function have some interesting aspects as it exposes the quality and cost at the same time, which allows better comprehension of the importance of process enhancements, such as the economic aspects and engineering (Creveling and Fowlkes, 1995).

2 THE SIGNAL-TO-NOISE RATIO

According to Taguchi:

"The signal-to-noise ratio (S/N) is a measurement scale that has been used in the communication industry for nearly a century. A radio measures the signal or the wave of voice transmitted from a broadcasting station and converts the wave into sound. The larger the voice sent, the larger the voice received. […] Actually, the input is mixed with the audible noise in the space. […] Taguchi has generalized the concept of S/N ratio as used in the communication industry and applied it for the evaluation of measurement systems as well as for the function of products and processes" (Taguchi, 1924).

The S/N ratio is a proceeding utilized to enhance the robustness of a product or process. It reflex the response variability of the system made by the noises and doesn't depends of adjustments on the mean. It is utilized to compare the proposes and doesn't accounts the interactions between the analyzed factors. The signal-to-noise ratio have the functional characteristics:

i) Nominal-the-Better Type I (NTB-I)

$$S/N_{NTB-I} = 10\, log\left(\frac{\bar{y}^2}{S^2}\right); \tag{6}$$

ii) Nominal-the-Better Type II (NTB-II)

$$S/N_{NTB-II} = -10\, log(S^2\,); \tag{7}$$

iii) Smaller-the-Better (STB)

$$S/N_{STB} = -10\, log(S^2 + \bar{y}^2); \tag{8}$$

iv) Larger-the-Better (LTB)

$$S/N_{LTB} = -10\, log\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{y_i^2}\right)\right]. \tag{9}$$

3 THE BOOTSTRAP

The bootstrap is a computationally intensive non-parametrical statistical technique that permits the evaluation of statistics variabilities based on data from one single existing sample. Efron introduced this technique in 1979 and since then it has been part of theoretical and applied statistical studies. The bootstrap surged when Efron studied the estimative of sample distribution of a statistic $S(x)$ based on sample data of n size from an unknown distribution (Chaves Neto, 1991).

The bootstrap is indicated for problems that standard statistical methods doesn't exist or when is difficult its application. The purpose of this method is to obtain the standard deviation, confidence interval, estimative of probability distribution and statistic calculus through the computational use.

Operationally, the bootstrap consists of NBS resampling with replacement of the original sample $x = [x_1, x_2, \ldots, x_n]$ of size n. The resamples x_i^* are obtained with the same n size. Then, we calculate the statistics of interest $S(x_i^*)$ for each bootstrap resample. The bootstrap values set of the statistic correspond to a distribution's estimative of the statistic in question. The Figure 2 represents the bootstrap procedure to obtain NBS statistic of interest.

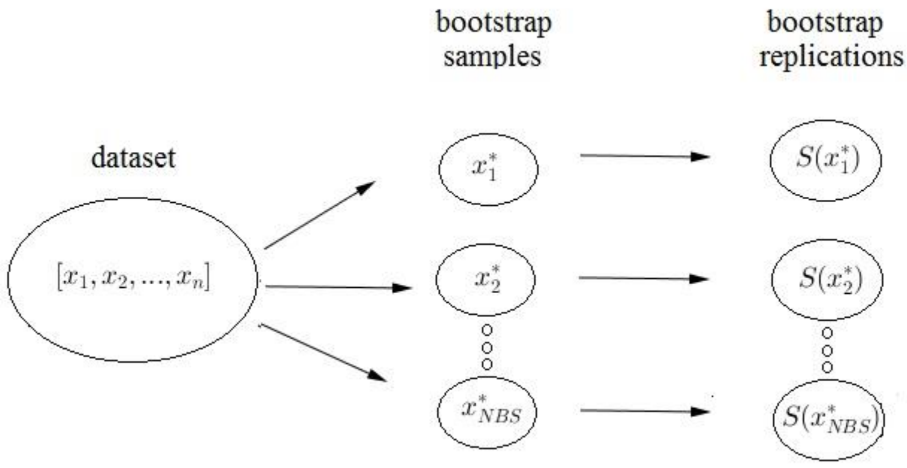

Figure 2. Methodology resample Bootstrap, adapted (Efron and Tibshirani, 1993)

The bootstrap confidence intervals present on this work are:

i) Standard Confidence Interval (Z)

$$IC_{bootZ} = (S(x) \pm Z_\alpha SE_{boot}), \tag{10}$$

where Z_α is the o $\alpha - th$ value from the score of standard normal distribution.

ii) Confidence Interval t

The efficacy of this method to calculate the confidence interval is reliable since the statistical distribution of interest being approximately normal and small addictions (Efron and Tibshirani, 1993).

$$IC_{boot\,t} = \big(S(x) \pm t_{df} SE_{boot}\big), \tag{11}$$

being n the master sample size, $df = (n-1)$ degrees of freedom and SE_{boot} the standard deviation bootstrap.

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iii) Percentile Confidence Interval (P)

$$IC_{boot\ percentil} = \left[S(x^*) - P_{97,5\%}diferenças;\ S(x^*) - P_{2,5\%}diferenças\right], \tag{12}$$

where $diferenças$ is the difference between of the average bootstrap estimative with each bootstrap estimative and $S(x^*)$ is the bootstrap estimative itself.

iv) Bias Corrected Confidence Interval (BC)

The estimative method for the BC confidence interval utilizes as interval the extremes of the percentiles of adjusted bootstrap distribution, to correct the addiction and the asymmetry of the distribution in question (Mendes, 2015).

$$IC_{BC} = \left[P_{P_i}\left(S(x^*)\right), P_{P_s}\left(S(x^*)\right)\right], \tag{13}$$

where P_i and P_s are the selected percentiles calculated by the correlation parameters from the addiction and $Z_{\alpha/2}$.

v) Bias Corrected and Accelerated Confidence Interval (BCa)

Another method to estimate the confidence interval is denominated BCa, being convenient its use when the asymmetry is present in a very strong way.

$$IC_{BCa} = \left[P_{P_i}\left(S(x^*)\right), P_{P_s}\left(S(x^*)\right)\right] \tag{14}$$

The interval is similar to the BC method, however the difference is an adjustment on the acceleration constant a on the calculus of the percentiles P_i and P_s (Andrews and Buchinsky, 2002).

4 THE PROCESS CAPABILITY INDEX

Indexes that quantify the quality are very useful when is desired to measure, compare and enhance the quality. The potential capacity ratio of the process is one of those indexes. It measures the relation between the specification dispersion of the process and the real dispersion of the process. It's an index that is highly influenced by the variability that the process presents.

When the value is smaller than 1 it's said that the process is inadequate to produce what is proposed, if the index value is between 1 and 1.33 the process is capable and above 1.33 it's satisfactory.

The first index, Eq. 15, represents only the ratio between the project specifications and the data dispersions.

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{15}$$

where USL is the upper specification limit, LSL the lower specification limit and σ the standard deviation.

As the index $C_p$does not reflect other important informations, such as the centrality of the process, it is necessary another index. The C_{pk}, Eq. 16, jointly with the C_p provides the centrality of the process, it means, when $C_p = C_{pk}$ the process is centered.

$$C_{pk} = Min\left[\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right], \tag{16}$$

where μ is the sample's mean.

Even with the two indexes above mentioned, it is not possible to represent the true efficiency of the process. Taguchi proposed the index C_{pm}, Eq. 17, in the same way as loss function, it takes in consideration the mean deviation of the target, thus penalizing it.

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \tag{17}$$

where T is the target value of the process.

It is important not assuming wrong results, therefore the data needs a distribution approximating the normal and the process under statistical control.

5 THE APP EXPERIMENTO TAGUCHI

The development of the app "*Experimento Taguchi*" was realized on the period from January to July of 2017, in Windows environment, using the programming language MATLAB, as it permits the creation of graphic interfaces and is currently one of most powerful computational systems of math and engineering.

The app utilizes the classic concepts of statistics and the new concepts presented by Taguchi. The aim was the construction of a simple interface to the user without unnecessary information noticeable. All the information is concentrated in just one place in a non-confusing way for data analysis, making all the process as simple as possible.

The main window of the app, Fig. 3, have fields to select the number of factors, the levels, the degree of interactions between the level of factors, the entry or importation of data (.xls archives), the number of bootstrap resamples, nominal value. As well as buttons for calculus of analysis of variance (ANOVA), graphic options, characteristics selection of the process ("*Opções*" button) and exportation of the results. The main window also have an individual tab to exhibit data and the result of each step made. They are: "*Matriz Experimento"* (experimental array), "*Amostra"* (sample), "ANOM" (analysis of mean), "ANOM-SN" (analysis of mean for S/N ratio), "ANOVA", "ANOVA-SN" (analysis of variance for S/N ratio) and "*Geral*" (general).

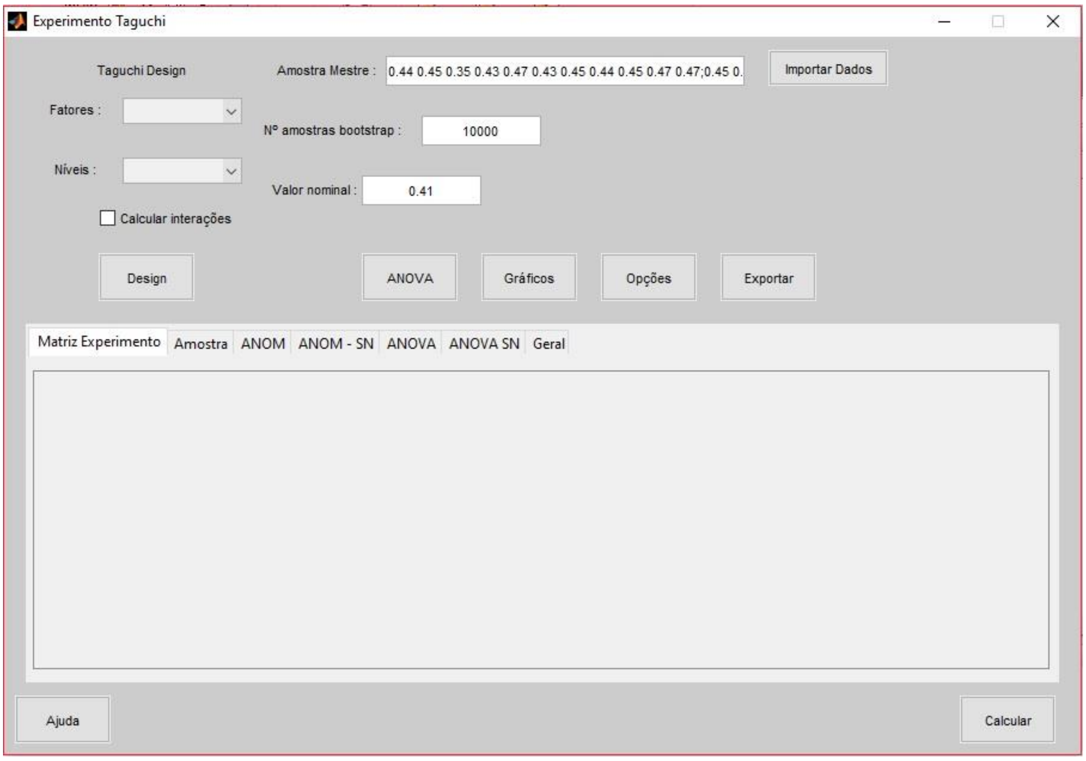

Figure 3. Main window

In the option selection window, Fig. 4, there are present the choices of process characteristics (NTB-I, NTB-II, STB, LTB), that will be applied to all analysis. As an option, there is the average loss function (which is mandatory the entry of proportionality constant k, or loss parameters P and target deviation Δ), process capability index Cp, Cpk, Cpm (that need the entry of the specification limits USL and LSL). The confidence intervals of parameters are calculated from bootstrap samples generated with confidence levels 95% and 99%, with the choices: confidence interval Z, confidence interval t, percentile confidence interval, BC confidence interval, BCa confidence interval and automatic mode (the program choose the best option for each situation). The confidence intervals will only be calculated for the parameters selected.

The result exhibition is shown in the last tab on main window, denominated "Geral", Fig. 5.

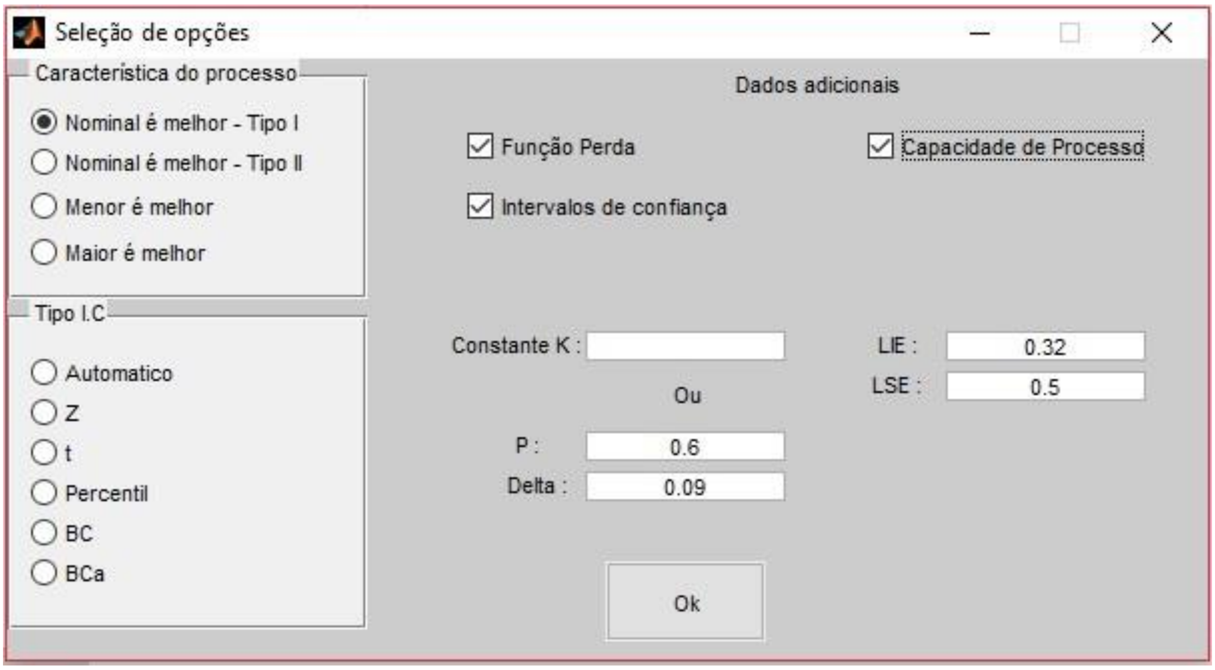

Figure 4. Options window for choosing fundamental characteristics in the analysis and additional options

The "Geral" tab presents: the orthogonal array selected together with the attribution of factors and interactions, mean of the sample, variance of the sample, S/N ratio, confidence intervals level 95% of S/N ratio lower bound (SN-LI95%) and upper bound (SN-LS95%), confidence interval level 99% of S/N ratio lower bound (SN-LI99%) and upper bound (SN-LS99%). Also exhibited average loss function, confidence intervals level 95% of average loss function lower bound (FP-LI95%) and upper bound (FP-LS95%), confidence intervals level 99% of average loss function lower bound (FP-LI99%) and upper bound (FP-LS99%). As well as Cp, confidence intervals level 95% of Cp index lower bound (Cp-LI95%) and upper bound (Cp-LS95%), confidence intervals level 99% of Cp index lower bound (Cp-LI99%) and upper bound (Cp-LS99%), Cpk, confidence intervals level 95% of Cpk index lower bound (Cpk-LI95%) and upper bound (Cpk-LS95%), confidence intervals level 99% of Cpk index lower bound (Cpk-LI99%) and upper bound (Cpk-LS99%), Cpm, confidence intervals level 95% of Cpm index lower bound (Cpm-LI95%) and upper bound (Cpm-LS95%) and confidence intervals level 99% of Cpm index lower bound (Cpm-LI99%) and upper bound (Cpm-LS99%).

	Fator 1	Fator 2	e	Média	Variância	S/N	SN-LI95%	SN-LS95%	SN-LI99%	SN-LS99%
1	1	1	1	115	3.1667	36.2079	33.3241	44.3631	31.5303	44.3631
2	1	2	2	113	1	41.0616	36.2465	44.3242	36.2465	45.6278
	Função Perda	FP-LI95%	FP-LS95%	FP-LI99%	FP-LS99%	Cp	Cp-LI95%	Cp-LS95%	Cp-LI99%	Cp-LS99%
1	0.7917	0.4808	1.3809	0.3846	1.5385	3.7463	2.9532	5.2874	2.7931	7.0293
2	1.2500	0.7885	1.7308	0.6538	1.9158	6.6667	5.2662	10.2494	5.1247	14.5896
	Cpk	Cpk-LI95%	Cpk-LS95%	Cpk-LI99%	Cpk-LS99%	Cpm	Cpm -LI95%	Cpm -LS95%	Cpm -LI99%	Cpm -LS99%
1	3.7463	2.7180	4.8050	2.7115	5.1640	3.7463	2.9814	5.5635	2.8886	7.6285
2	6	4.5233	7.5580	4.5233	8.1680	2.9814	2.8165	3.1699	2.7912	3.2374

Figure 5. Example of results display

6 CASE STUDY

For this work, it was analyzed data from food industry. There were five same type of products (product A, product B, product C, product D and product E) that differ only in their flavors. The observations refer to acidity levels, in which each flavor has its specification. Each product is produced in three different work shifts: work shift 1 (6 A.M. to 12 P.M.), work shift 2 (1 to 9 P.M.) and work shift 3 (11 P.M. to 5 A.M.). The measurements were made each 2 hours at production line.

In that particular industry, 64% of the times the product was withheld for not fit in the specifications, after reanalysis it was totally released. The other 36% was partially or completely discarded.

The cost for production doesn't evolve only the raw material and packing cost. Analyze and improve the production can reduce another expenses such as: time spend on reanalysis, consumer complaints and others. Clearly, the improvement of production brings economic gains not only for the better utilization of the raw material, but also brings a better product and competitively for the brand.

The analysis was realized with the app "*Experimento Taguchi*" described in the previous section, with the S/N ratio nominal-the-better type I, average loss function, Cp, Cpk, Cpm and 10.000 bootstrap resamples to generate the confidence intervals for the analyzed parameters.

7 RESULTS

The Table 1 shows the results of signal-to-noise ratios for each work shift and product. The product A on work shift 1 has the best performance (higher S/N ratio) in comparison to all the products at every work shift. The work shift 1 has the majority of best process executions (for products A, C and D), soon after the second work shift (products B and E). The third work shift does not exhibited a better execution for any product.

Table 1. Signal-to-noise ratio for the products

S/N ratio	Product A	Product B	Product C	Product D	Product E
1 Shift	32.11608	18.87027	26.67686	26.75233	22.35969
2 Shift	27.97406	28.21399	22.72312	24.67096	30.11286
3 Shift	28.60937	23.81632	23.72773	23.34994	26.68

Consequently, to the performance of their processes, the Table 2 shows the results of the average loss function. Work shift 1 has the higher expenses for products B and E, work shift 2 has the higher expenses for the product C and work shift 3 has the higher expenses for the products A and D.

Table 2. Average loss function for the products

Loss Function	Product A	Product B	Product C	Product D	Product E
1 Shift	0.293113	8.16446	0.657635	0.546979	1.544047
2 Shift	2.080661	0.435953	1.07274	0.864873	1.214486
3 Shift	2.958678	1.799541	0.885333	1.162616	0.781635

When analyzed the C_p (Table 3) and C_{pk} (Table 4) indexes for all the processes for every work shift, it shows that none of the process are centered.

Table 3. Index result C_p for the products

Cp	Product A	Product B	Product C	Product D	Product E
1 Shift	1.568231	0.383842	1.282733	1.1535	0.892805
2 Shift	1.024864	1.247152	0.825423	0.88752	2.145291
3 Shift	1.122683	0.72414	0.925292	0.782586	1.486547

Table 4. Index result C_{pk} for the products

Cpk	Product A	Product B	Product C	Product D	Product E
1 Shift	1.473187	0.084888	1.019608	1.052568	0.586185
2 Shift	0.521749	1.203545	0.72197	0.838214	1.239501
3 Shift	0.408248	0.509286	0.801919	0.695632	1.066109

The Table 5 shows the C_{pm} index, the true process capability. First work shift is satisfactory to produce only one of the products, capable to produce 2 products and inadequate for the other 2 products. Second work shift is inadequate to produce 4 of 5 products e capable for one product. Third work shift is inadequate to produce any of them.

It was also made the calculus of confidence intervals for all the parameters before mentioned, that are presented on Tables 6 to 10.

Table 5. Index result C_{pm} for the products

Cpm	Product A	Product B	Product C	Product D	Product E
1 Shift	1.508123	0.285753	1.006844	1.104	0.399389
2 Shift	0.566049	1.236615	0.788328	0.877967	0.71723
3 Shift	0.474685	0.608658	0.867763	0.757245	0.816653

Table 6. Confidence intervals for the S/N ratio

S/N ratio	Confidence interval Z 95%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	-	-	12.79367	24.94686	22.0643	31.28942	11.9683	41.53637	5.842239	38.87714
2 Shift	-6.30376	62.25188	18.87881	37.54917	17.16084	28.28541	14.06534	35.27658	20.66481	39.5609
3 Shift	8.867224	48.35151	16.18412	31.44852	18.64732	28.80814	13.5486	33.15128	20.76807	32.59194

Table 7. Confidence intervals for the average loss function

Loss Function	Confidence interval t 95%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	0.085633	0.500593	1.929066	14.39985	0.403037	0.912232	0.240725	0.853233	0.90717	2.180924
2 Shift	0	4.240739	0	2.458983	0.49094	1.654539	0.297328	1.432418	0.575541	1.85343
3 Shift	0	6.095227	0	5.828031	0.330905	1.439762	0.158708	2.166523	0.219585	1.343684

Table 8. Confidence intervals for the C_p

Cp	Confidence interval BCa 95%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	1.115731	2.435441	0.30671	0.489608	0.943078	1.54906	0.798339	1.766904	0.531995	2.181654
2 Shift	0.836798	1.296362	0.818713	1.906972	0.527272	1.044998	0.662652	1.377061	1.54424	2.642491
3 Shift	0.793857	1.833333	0.521133	0.942411	0.648072	1.155815	0.475131	1.223551	1.055097	1.85405

Table 9. Confidence intervals for the C_{pk}

Cpk	Confidence interval BCa 95%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	1.115731	2.797155	0	0.34767	0.855355	1.413816	0.802677	1.71649	0.402627	1.193698
2 Shift	0.33541	1.341641	0.85347	2.219334	0.523487	0.938809	0.625493	2.115351	0.944757	1.81831
3 Shift	0.166667	1.166667	0.241392	1.685303	0.59788	1.325987	0.43438	1.222222	0.791442	1.883007

Table 10. Confidence intervals for the C_{pm}

Cpm	Confidence interval BCa 95%									
	Product A		Product B		Product C		Product D		Product E	
	LB	UB	LB	UB	LB	UB	LB	UB	LB	UB
1 Shift	1.093459	2.226885	0.249295	0.322229	0.815664	1.121236	0.781527	1.602888	0.468127	0.888201
2 Shift	0.527264	0.601497	0.815141	1.869928	0.517195	0.972613	0.658648	1.342165	0.701121	0.75537
3 Shift	0.437211	0.503655	0.472748	0.735764	0.627326	1.049119	0.469282	1.132945	0.786106	0.994626

8 CONCLUSION

This paper presented a first analysis of five similar products in food industry. It was applied classical techniques for quality control (capability process index), Taguchi's concepts and advanced techniques, like bootstrap.

The process capability analysis indexes provides a procedure to evaluate the current process. The information given serve as a basis for the process improvement. Conjointly with the S/N ratio and average loss function, they might open up a new view of quality, especially in terms of monetary loss.

After analysis, it was observed that the second work shift presented a better average performance for signal-to-noise ratio and lower loss in average loss function. The work shift 1 presented the worst performance (S/N ratio) and higher average loss function. Most of the process are inadequate or capable in its production, generating a high defect rate. All the processes also were decentralized.

The utilization of the developed app "*Experimento Taguchi*" facilitates all the calculus presented in this article, thus for future analysis is recommended its use as well to apply the whole Taguchi's method. It is great methodology to improve product's quality and diagnose the cause of variation during production.

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