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# CONSTRUCTAL DESIGN APPLIED TO IMPROVE THE HEAT REMOVAL BY CONVECTION OF FINNED ASSEMBLIES WITH TRIANGULAR SHAPE

**E. X. Barreto**

**C. E. G. Falcão**

**I. B. Mallmann**

eduardo.barreto@ufsm.br

cegfalcao@gmail.com

igor.mallmann@gmail.com

Department of Mechanical Engineering, Federal University of Santa Maria

Avenida Roraima, 1000, Centro de Tecnologia – CT, 97105-900, Santa Maria, RS, Brazil.

**E. S. D. Estrada**

**E. D. Santos**

emanuelestrada@gmail.com

elizaldosantos@furg.br

School of Engineering, Federal University of Rio Grande

Av. Itália km 8, 96201-900, Rio Grande, RS, Brazil

**L. A. O. Rocha**

laorocha@gmail.com

Department of Mechanical Engineering, Federal University of Rio Grande do Sul

Rua Sarmento Leite, 425, 90050-170, Porto Alegre, RS, Brazil

***Abstract.*** *In the present work, a numeric study is conducted to find the best configuration of a finned assembly, aiming the maximum heat removal by means of convection. Here the cooling of a heat generating medium is investigated by sets of triangular fins with number of fins N = 3. It is assumed for both, the finned assembly and the heat generating medium, the conductivity of copper and the numerical solution considers perfect thermal contact in its interface. In order to find the setting that reduces most the overall thermal resistance of the whole system, a numerical code based on finite elements and developed in MATLAB environment was elaborated to determine the temperature field in all mathematical domain. In this context, the area of heat generating medium as well as the total area of the system and the dimensionless coefficient of heat transfer are considered constants.*

***Keywords:*** *Constructal Design, Convection, Finned assemblies, Heat removal, Overall thermal resistance.*

# 1 INTRODUCTION

Constructal Design, the method used to optimize flow systems, animated or inanimated, is based on a physics principle, the Constructal Law: "For a finite size flow system to persist in time (to survive) it must evolve in such a way that it provides easier access to the imposed currents that flow through it". Thus, based on Constructal Law, Constructal Design has been widely used to optimize flow systems in the engineering field and its aplication on extended surfaces is recent on the literature. This method is based on the exhaustive search for the optimal geometry, varying freely the degrees of freedom considering the constraints of the problem. In Bejan and Almogbel (2000), Constructal Design was applied in the optimization process of fins with a "*T*" shape in order to obtain the maximization of the overall thermal conductance of the whole system considering the imposed volume and materials constraints of the fins. Bonjour et al. (2004) performed an analytical and numerical study concerning the geometric optimization of fins with a radial and "ramified" shape inserted into heat exchangers with co-axial flows. It was analysed the relation between performance and architecture of fins. Matos et al. (2004) conducted a tridimensional study to optimize staggered arrangements of heat exchangers with circular and eliptical tubes. Lorenzini et al., (2009) proposed the geometric optimization of fins with "*T-Y*" shape, i.e., a configuration where exist a cavity between the two branches of the fins. Again, the objective was the minimization of the global thermal resistance of the system concerning the total volume and the constraints. The results showed that the cavity with a lower volume and fins with a higher volume increase the performance of the finned assembly. Lorenzini and Rocha, (2006) minimized the thermal global resistance considering the total volume and materials constraints of the fins aiming the optimization of finned assemblies with an "*Y*" shape. This was a complete optimization study performed with the exhaustive search, i.e., all degrees of freedom were optimized and the global thermal resistance as well as the optimal geometry were correlated by power laws.

In this work, Constructal Design is applied to find the best configuration of finned assembly aiming the highest heat removal by convection. Here, the cooling of a heat generating medium with rectangular shape is investigated by finned assemblies with $N = 3$ fins with triangular shape. It's assumed for both, the finned assembly and the heat generating medium, the thermal conductivity of cooper and the numerical solution considers a perfect thermal coupling between them. In order to find the configuration that reduces most the global thermal resistance of the whole system, a numerical code based on finite elements is used to determine the temperature field of the whole mathematical domain. In this context, the area of heat generating medium and the total area of the system, as well as the thermal conductivities and the dimensionless heat transfer coefficient, $\lambda$, are considered constants.

# 2 MATHEMATICAL MODEL

Consider the finned assembly shown in Fig. 1 which represents the domain to be analyzed and its dimensions. The configuration is two-dimensional, with the third dimension, $\tilde{W}$, sufficiently longer than the heights $\tilde{H}_0$, $\tilde{H}_1$ and $\tilde{H}_2$ and the widths $\tilde{L}_0$ and $\tilde{L}_1$. The set consists of a finned assembly consisting of 3 triangular fins with thermal conductivity $\tilde{k}_p$, in perfect thermal contact with a heat generating medium and thermal conductivity $\tilde{k}$.



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The rectangular element generates heat uniformly in a volumetric rate $q'''$(W/m$^3$). The lower and side surfaces are insulated and the heat generated current ($q'''$AW) is first removed by conduction and then convection through the fins.

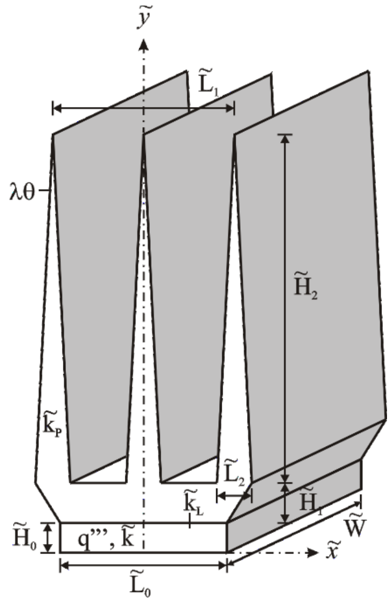


**Figure 1. Finned assembly consisting of three triangular fins in perfect thermal contact with a heat generating medium located in its lower base.**

This work consists in calculating the maximum excess dimensionless temperature ($T_{max}$-$T_{min}$)/($q'''_{A/k}$) and see which geometry ($L_1/H_1$, $L_2/H_2$ and $H_1$) best removes the generated heat. It is assumed a steady state problem with uniform heat generation. The equation for heat diffusion to the heat generating medium is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{q'''}{k}=0 \tag{1}$$

Equation (1) can be put in dimensionless form using the following dimensionless groups.

$$\left.\begin{aligned}
&\theta=\frac{T-T_\infty}{q'''A/k}\\
&\tilde{k},\tilde{k}_p=\frac{k,k_p}{k}\\
&\tilde{x},\tilde{y},\tilde{L}_0,\tilde{H}_0,\tilde{L}_1,\tilde{H}_1,\tilde{L}_2,\tilde{H}_2=\frac{x,y,L_0,H_0,L_1,H_1,L_2,H_2}{A^{1/2}}
\end{aligned}\right\} \tag{2}$$

where $A$ in Eq. (2) represents the total area of the mathematical domain. This analysis provides the excess dimensionless temperature as a function of the geometry of the finned assembly solving the heat diffusion equation with uniform heat generation for the rectangular element placed at the base of the considered domain. The heat diffusion equation in dimensionless form is represented by

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} + 1 = 0 \tag{3}$$

Assuming no heat generation in the finned assembly, the heat diffusion for this region is

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} = 0 \tag{4}$$

The maximum excess of dimensionless temperature, $\theta_{max}$, is defined as

$$\theta = \frac{T_{\max} - T_{\infty}}{q''' A / k_p} \tag{5}$$

where $k_p$ is fins thermal conductivity of the finned assembly. Evaluating the thermal resistance, Eqs. (3) and (4) are solved using the boundary conditions given considering the finned assembly subjected to convection, i.e.:

$$\frac{\partial \theta}{\partial n} = -\lambda \theta \tag{6}$$

where $\lambda$ is given by Eq. (6) as

$$\lambda = \frac{h A^{1/2}}{k} \tag{7}$$

The sides as the bottom of the heat generating medium are considered adiabatic. These boundary conditions in dimensionless terms are given by

$$\frac{\partial \theta}{\partial n} = 0 \tag{8}$$

The next step is to achieve optimization, i.e., minimizing the thermal resistance given by Eq. (5). In this context it is considered two constraints of area. The first constraint is the total area of the mathematical domain

$$A = \frac{3}{2}(L_2 H_2) + \frac{1}{2}(L_1 + L_0) H_1 + L_0 H_0 \tag{9}$$

and the second constraint is the area occupied by the rectangular heat generating element.

$$A_p = L_0 H_0 \tag{10}$$

Dividing Eq. (10) by Eq.(9) yields the fraction of area, $\phi_p$, of the heat generating element

$$\phi_o = \frac{A_p}{A} = \tilde{L}_0 \tilde{H}_0 \tag{11}$$

From Eq. (11) it is possible to express Eq. (9) in dimensionless form:

$$1 - \phi_0 = \frac{3}{2}(\tilde{L}_2 \tilde{H}_2) + \frac{1}{2}(\tilde{L}_1 + \tilde{L}_0)\tilde{H}_1 + \tilde{L}_0 \tilde{H}_0 \tag{12}$$

In the search for the optimal solution, the problem considers three degrees of freedom: $\tilde{H}_1$, $L_1/H_1$ and $L_2/H_2$, and five constants: $\phi_p$ = 0.15, $A$ = 1, $L_0/H_0$ = 15, $\tilde{k}_p = \tilde{k} = 300$. The degrees of freedom of this problem were varied by the exhaustive search method until reaching the three times optimal geometry, which provides the minimal thermal resistance, i.e., $(\theta_{max})_{mmm}$.

## 3 NUMERICAL MODEL

The numerical method was developed considering the finned assembly compound of three triangular fins. The maximum excess of dimensionless temperature expressed in Eq. (5) is determined by numerical solution of the heat conduction equation given by Eqs. (3) and (4) for the temperature field of the fins, depending on the chosen degrees of freedom. The appropriated mesh used in this work was obtained through successive refinements until the criterion $\left|\left(\theta_{\max}^{j} - \theta_{\max}^{j+1}\right)/\theta_{\max}^{j}\right| \le 5\times10^{-4}$ is satisfied, where $\theta_{\max}^{j}$ is the maximum excess of dimensionless temperature calculated using the actual mesh, $\theta_{\max}^{j+1}$ represents the maximum excess of dimensionless temperature calculated from the next mesh refinement, when the number of elements increases four times. The numerical solution of the heat conduction equation was obtained with the implementation of a finite element code based on triangular elements and developed using MATLAB. Table 1 shows tests performed to obtain mesh independence from the number of elements.

Table 1. Mesh refinements to obtain results independent of elements number, considering: $\phi_p$ = 0,15, $L_1/H_1$ = 12, $L_2/H_2$ = 0.02 and $\tilde{H}_1 = 0.09$.

| Number of nodes | Number of triangular elements | $\theta_{\max}^{j}$ | $\theta_{\max}^{j+1}$ | $\left\|\left(\theta_{\max}^{j} - \theta_{\max}^{j+1}\right)/\theta_{\max}^{j}\right\| \le 5\times10^{-4}$ |
|---|---|---|---|---|
| 150 | 166 | 0.00775 | 0.00776 | $1.46 \times 10^{-3}$ |
| 465 | 664 | 0.00776 | 0.00777 | $6.81 \times 10^{-4}$ |
| 1593 | 2656 | 0.00777 | 0.00777 | $2.83 \times 10^{-4}$ |
| 5841 | 10624 | 0.00777 | - | - |

Thus the mesh, considering the proposed independence criterion, consists of 1593 nodes and 2656 triangular elements. In the next section are performed a series of simulations, varying the degrees of freedom of the problem, in order to obtain the optimal geometry.

# 4 RESULTS

Here is presented the numerical results obtained through the simulations performed for the finned assembly considering the respective degrees of freedom as well as their constraints. During the optimization process the root mean square (RMS), shown in Eq. (13), was used as a comparison parameter considering the numerical results of the present work and those obtained by Barreto et al. (2016) when simulations involving a finned assembly with rectangular shape were performed.

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - x_j}{x_i}\right)} \tag{13}$$

The results initially considered the optimization of a single degree of freedom, followed by the simultaneous optimization of two degrees of freedom and ending with the optimization of the three degrees of freedom.

Initially, it was verified the influence of the variation of the ratio $L_2/H_2$ concerning the overall thermal resistance of the system. It was performed a set of simulations considering a range of $L_2/H_2$ ratio in order to find the value that minimizes the $\theta_{max}$, i.e., the value $(L_2/H_2)_o$ which yields $(\theta_{max})_m$ considering fixed all the other degrees of freedom. The graph of Fig. 2 presents the curve of $\theta_{max}$ generated for each value assumed by $L_2/H_2$. The curve is compared with the one obtained from Barreto et al. (2016) which considered a finned assembly with rectangular fins.

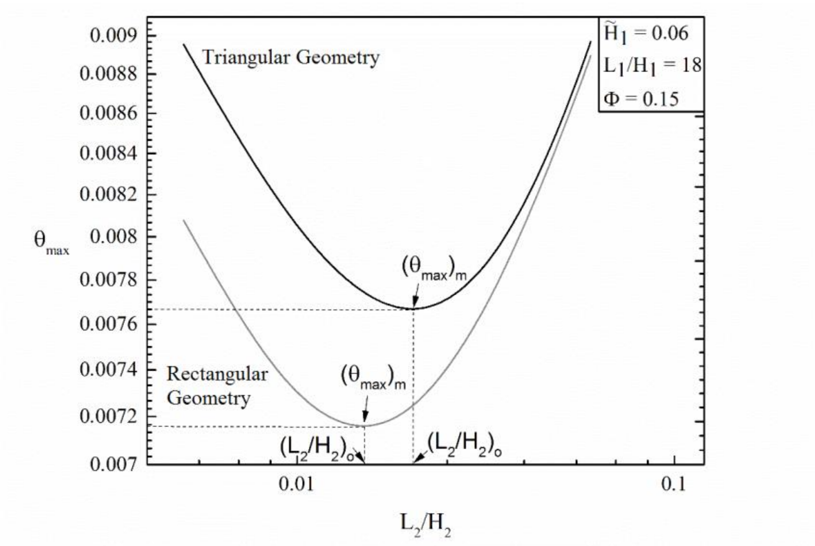


**Figure 2. Maximum excess of dimensionless temperature, $\theta_{max}$, for a range of values of $L_2/H_2$.**

Values of $\theta_{max}$ were obtained for a range of $L_2/H_2$ from 0.005 to 0.06, keeping fixed the other degrees of freedom at $L_1/H_1$ = 18 and $\tilde{H}_1 = 0.06$. This first analysis yield RMS values of the dimensionless temperature between the triangular and rectangular geometries equal to



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16.74%. The values of the maximum excess dimensionless temperature for the finned assembly with rectangular shape was $(\theta_{max})_m$ = 0.0084 while the value found for the assembly with triangular shape is $(\theta_{max})_m$ = 0.0076 which means a better performance of 10.58% for the finned assembly with triangular shape.

Figure 3 shows curves of dimensionless temperature from the finned assemblies with rectangular and triangular geometries concerning the variation of $L_1/H_1$ ratio. In this case, values of $L_2/H_2$ = 0.01 and $\tilde{H}_1$ = 0.04 remained constant.

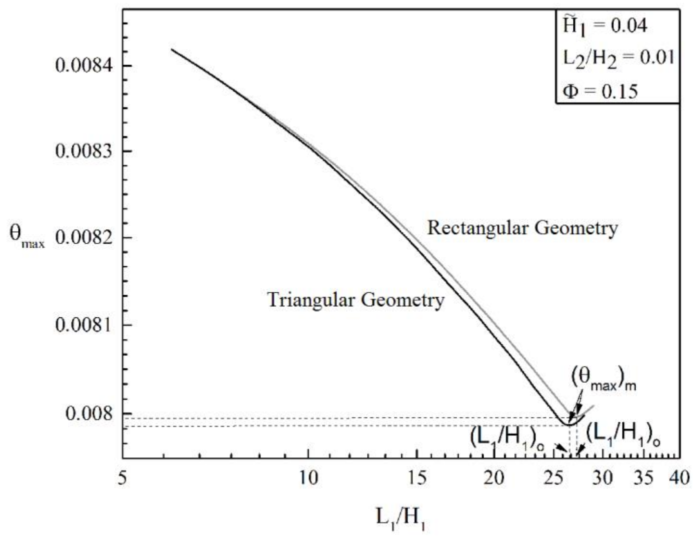


**Figure 3. Maximum dimensionless temperature considering results of the present study and results obtained by Barreto et. al, (2016), for a range of $L_1/H_1$ values.**

In the graph of Fig. 3, for the triangular geometry the optimized parameter $(L_1/H_1)_o$ = 27 presented a maximum dimensionless temperature $(\theta_{max})_m$ = 0.0079, while the rectangular geometry with $(L_1/H_1)_o$ = 26 has presented the value of $(\theta_{max})_m$ = 0.0086. The RMS values between two curves was 7,38% considering the degree of freedom $L_1/H_1$ and the fluctuations between the maximum dimensionless temperature in both cases 7.79%.

In a second moment, the geometry was doubly optimized regarding to the aspect ratios $L_1/H_1$ and $L_2/H_2$.

The doubly optimized geometry in this case is the one at which the minimized value of the dimensionless temperature with respect to a degree of freedom is minimized again by considering the parallel optimization of another degree of freedom. That is, the doubly optimized $(L_1/H_1)_{oo}$ and $(L_2/H_2)_{oo}$ ratios present a maximum excess dimensionless temperature doubly minimized with respect of $L_1/H_1$ and $L_2/H_2$ ratios.

The results obtained for the triangular geometry are again compared with those obtained by Barreto et. al, (2016), i.e., with respect to the rectangular geometry, Fig. 4. Here, simulations were developed where the dimensionless temperature variation was computed considering $L_1/H_1$ intervals between 2 and 15.

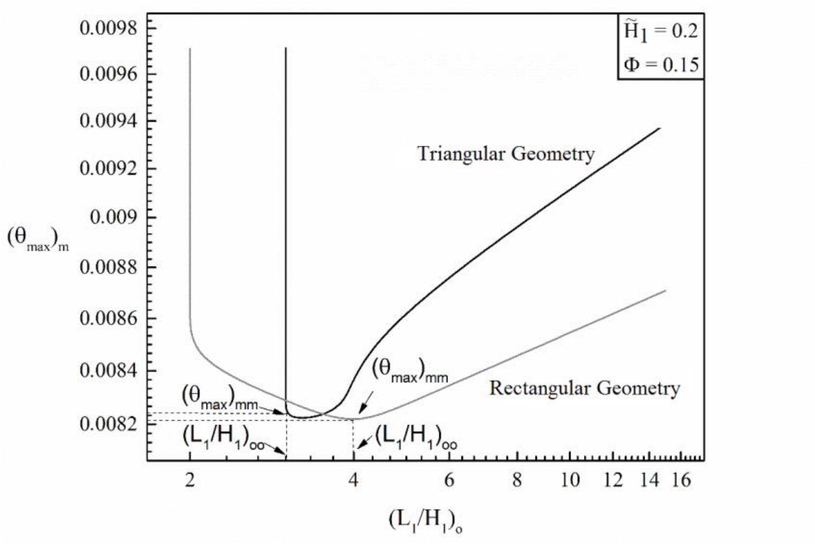


**Figure 4. Maximum dimensionless temperature twice minimized considering results of the present study and results obtained by Barreto et. al, (2016), for a range of ($L_1/H_1$)$_o$ values.**

The RMS values between the simulations involving the triangular and rectangular geometry curves were equivalent to 16.35%, considering the degree of freedom $L_1/H_1$, and the variation of $(\theta_{max})_{mm}$ between both geometries was 10.22%.

An analysis of the performance of the triangular and rectangular geometries with respect to the degree of freedom $L_2/H_2$ was also performed. Again the degree of freedom $\tilde{H}_1$ = 0.2 has been fixed, and the behavior of the geometries for a given range of $(L_2/H_2)_o$ is illustrated in the graph of Fig. 5 below.

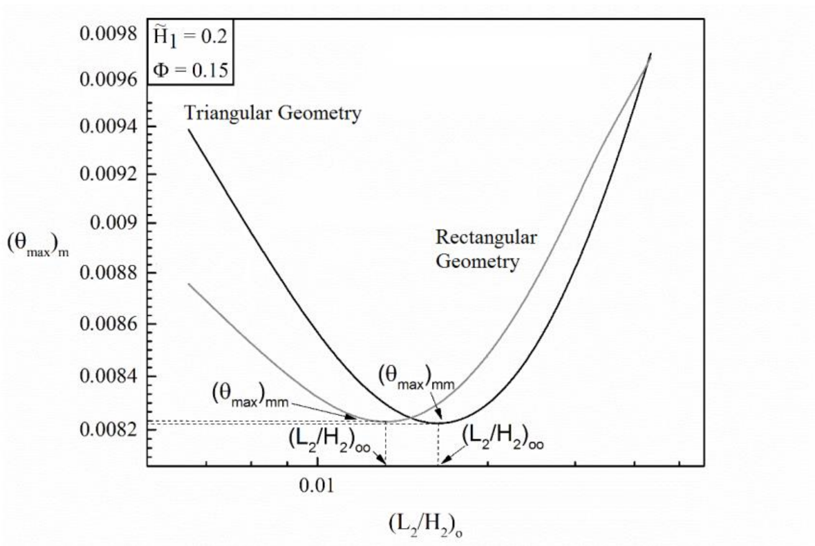


**Figure 5. Maximum dimensionless temperature twice minimized considering results of the present study and results obtained by Barreto et. al, (2016), for a range of ($L_2/H_2$)$_o$ values.**

The triangular geometry has an optimal value of $(L_2/H_2)_{oo}$ = 0.02 and $(\theta_{max})_{mm}$ = 0.0082, while the geometry obtained by Barreto et. (2016), has optimal values $(L_2/H_2)_{oo}$ = 0.015 and $(\theta_{max})_{mm}$ = 0.0090. For other geometry configurations, the optimum $(L_2/H_2)$ values found here were recurrent.

For comparison, the RMS values between the simulations involving the triangular and rectangular geometries were 15.82% between the curves, and a diminishing in the value of $(\theta_{max})_{mm}$ equal to 10.22% was favorable to the triangular geometry.

In a last analysis, the simultaneous optimization of the parameters $\tilde{H}_1$, $L_1/H_1$ and $L_2/H_2$ for the finned body was considered, in order to obtain a total optimization of the finned assembly. Just like before, the optimization was first analyzed for the case of triangular fins, and after a comparison was made with the results obtained by Barreto et. al (2016).

From the graphs below, it is evident that low values of $H_1$ provide an increase of the system performance, i.e., collaborating to minimize the maximum dimensionless temperature. It is also possible to visualize that low $\tilde{H}_1$ values are associated with high values of $L_1/H_1$ aspect ratio. The graph of Fig. 6 compares the behavior of the optimized thermal resistance concerning a range of $\tilde{H}_1$ values from 0.01 to 0.3 between the triangular finned geometry and the rectangular geometry obtained by Barreto et. al (2016).

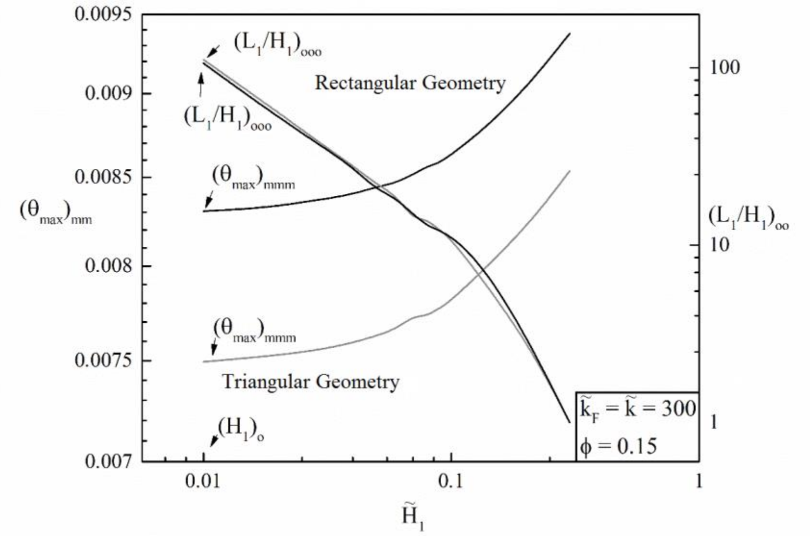


**Figure 6. Maximum dimensionless temperature three times minimized and three times optimized geometry $(L_1/H_1)_{ooo}$ considering results of the present work with those obtained in Barreto et. al, (2016), for a range of $\tilde{H}_1$ values.**

It is observed that the $L_1/H_1$ aspect ratio presents similar behavior for both geometries, and these are optimized for low values of $\tilde{H}_1$ and high values of $L_1/H_1$. A significant reduction of the dimensionless excess temperature of the system was observed when comparing the results obtained by the triangular fins with respect to the rectangular fins in the optimized situation, as can be seen in the maximum dimensionless temperature curves of Fig. 6. For this comparison, the values RMS between the simulations involving the triangular and rectangular geometries were 10.53% between the dimensionless temperature curves, and the values of $(\theta_{max})_{mmm}$ for each geometry reveals a difference of 10.83% in favor of the triangular geometry. The variation of the curve of $(L_2/H_2)_{oo}$ as the degree of freedom $\tilde{H}_1$ varies is shown in Fig.7 below.

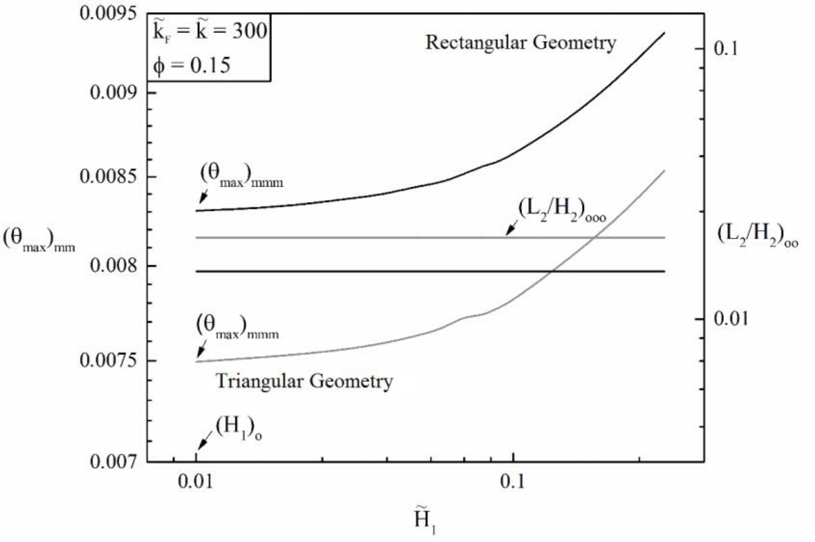


**Figure 7. Maximum dimensionless temperature three times minimized and three times optimized geometry $(L_2/H_2)_{ooo}$ considering results of the present work with those obtained in Barreto et. al, (2016), for a range of $\tilde{H}_1$ values.**

In Fig. 7 it is possible to observe that the behavior of the dimensionless temperature and degree of freedom $L_2/H_2$ for a range of $\tilde{H}_1$ values ranging from 0.01 to 0.2. The value $(L_2/H_2)_{ooo}$ = 0.02 was found at the point where we have $(\theta_{max})_{mmm}$, and no variations of this ratio were observed for different $\tilde{H}_1$ values.

For both geometries evaluated, the value of $L_2/H_2$ remains constant as different values of $\tilde{H}_1$ are experienced. For the rectangular geometry, the value of $(L_2/H_2)_{ooo}$ = 0.015 was found in the vast majority of simulations, whereas in triangular geometry this value was $(L_2/H_2)_{ooo}$ = 0.02, and the reason of this is subject to be discussed at the conclusion of this study.

Next, the relation that shows the behavior of the maximum dimensionless temperature along different values of $(L_1/H_1)_{oo}$ is presented in Fig. 8. It is possible to verify that the increase of the aspect ratio $L_1/H_1$ collaborates significantly to optimize the performance of the set.

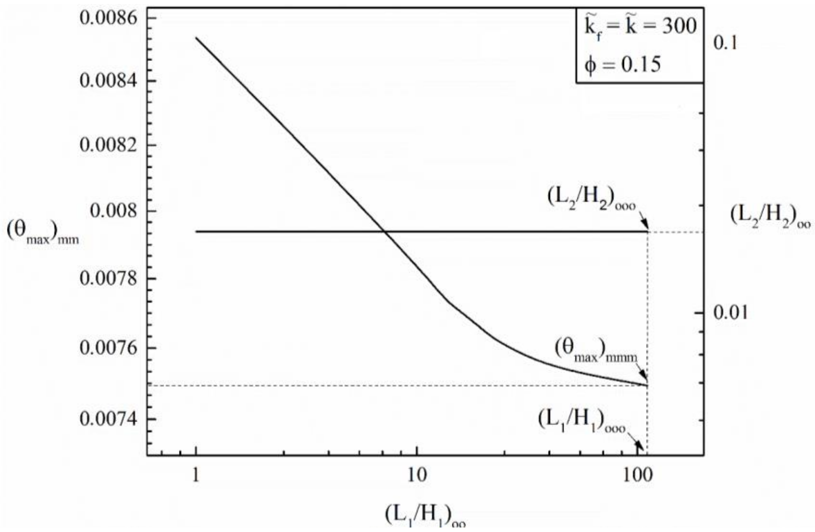


**Figure 8. Maximum dimensionless temperature three times minimized and three times optimized geometry ($L_2/H_2$)$_{ooo}$ considering results of the present work with those obtained in Barreto et. al, (2016), for a range of ($L_1/H_1$)$_{oo}$.**

In the end of the optimization process, which was based on the results obtained from exhaustive simulations, and with the analysis of the graphs presented in this session, it was possible to obtain the three-optimized geometry of the triangular finned body, i.e., the geometry which maximizes the overall performance of the system, by the variation of its degrees of freedom and respecting its constraints. This geometry, for the set of triangular fins, presented the following optimized values for its degrees of freedom: $\left(\tilde{H}_1\right)_o = 0.01$, $(L_1/H_1)_{ooo} = 111$ e $(L_2/H_2)_{ooo} = 0.02$. A visual analysis of the optimized geometries is shown in Fig. 9 below, considering different values of $\tilde{H}_1$. For each situation, the optimized ratios of $L_1/H_1$ and $L_2/H_2$ are represented.

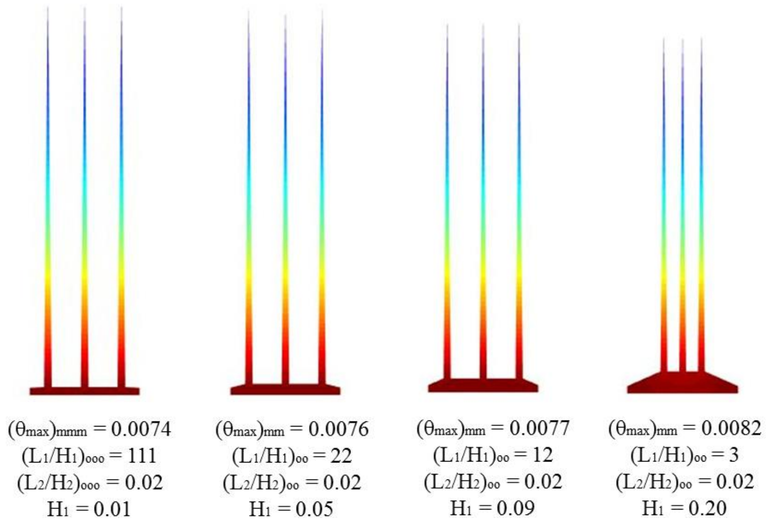


**Figure 9. Visual analysis of optimized triangular geometries considering different values of $\tilde{H}_1$.**

# 5 CONCLUSIONS

A study was carried out aiming at the thermal optimization of a finite set destined to cool a heat generating medium in its base through the exhaustive search of the optimal geometry obtained from the variation of its degrees of freedom and respect to the imposed constraints. The work had the geometric optimization of 3 degrees of freedom: $L_1/H_1$, $L_2/H_2$ and $\tilde{H}_1$ respecting in turn the arbitrated restrictions: $\phi_p = 0.15$, $\lambda = 1$, $L_0/H_0 = 15$.

In this process, the geometry counted on optimizations of only one degree of freedom, as well as the simultaneous optimization of two and finally three degrees of freedom simultaneously. Some important conclusions about the obtained results were compared with those obtained by Barreto et. al (2016), such as:

• In all the optimization processes performed, the optimum value of the $L_2/H_2 = 0.02$ ratio was obtained for the triangular geometry models, differing from those with rectangular geometry obtained by Barreto et. al, (2016) where the optimum geometry remained close to 0.015. The performance of the system increases as $L_2/H_2$ approaches 0.02. As the length of the fin increases, the cross section of the fin is reduced in the case of the triangular geometry, as well the thermal resistance due to conduction increases it also increases the area for convection which provides further convection heat removal. In this context, the balance between the increase in thermal resistance by conduction and the improvement in convection, favors the performance obtained in the triangular geometry, i.e., the decreasing of overall resistance of the system.

• Optimized values have present fluctuations with respect to the maximum dimensionless temperature of 7.79, 10.22 and 10.83% always smaller in the case of triangular geometry which proves a greater efficiency of this in regard to the removal of heat. The RMS values considering all intervals of degrees of freedom involved in each optimization process presented values of 7.38, 16.35 and 10.53% again in favor of the triangular geometry.

• In the same way as in the results obtained by Barreto et. al, (2016), lower values for the maximum dimensionless temperature were obtained for low values of $\tilde{H}_1$, since this height plays the role of a thermal resistance between the heat generating medium and the fins of the finned assembly. In the same way that $\tilde{H}_1$ decreases, optimal values of the geometry are obtained for high values of the ratio $L_1/H_1$.

The thermal performance of the finned assembly with triangular geometry was superior to the thermal performance of the finned assembly with rectangular geometry according to the results obtained here. For future work, fins with variable geometries, e.g., those whose shape can be obtained through a polynomial, may be investigated as a better alternative in the solution of problems involving the removal of heat by means of convection.

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