XFEM AND INTERFACE ELEMENT MODELS APPLIED TO FRACTURE OF FIBER REINFORCED CONCRETE BEAMS

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Abstract. *Since the second half of the 20th Century, Fracture Mechanics is gaining strong relevance in Structural Engineering, more recently in numerical and probabilistic analyses regarding the opening of cracks in non-conventional reinforced concrete structures. Some applications include polymeric or natural fiber reinforcement. However, the study of these problems involves nonlinear effects with complex modeling, once the fracture process introduces discontinuities in the displacement field .Different approaches have been employed in order to analyze fracture paths, peak loads and deflections in a composite fiber reinforced concrete beam. In this study, numerical fracture analysis was carried out using two distinct finite elements (FEM) formulations: Interface Elements and Extended Finite Element Method (XFEM). Mixed-mode fracture behavior is considered. These numerical methodologies are based on a constitutive damage model, adjusted to experimental results of failure mechanism in a fiber reinforced concrete. The simulations reproduce experimental procedures reported in the literature by Marangon (2011), as the four-point bending test and the direct tensile test. In this respect, the joint study of which approaches has contributed to achieve results with good accuracy, validated against experimental procedures, and predicting the beam's fracture behavior under mixed-mode.*

Keywords: *Fracture Mechanics, Extended Finite Element Method, Interface Elements, numerical analysis, concrete fracture, composite materials.*

1 INTRODUCTION

In the last decades, with the increasing advance of computational technologies, numerical analyses applied to Structural Engineering have become essential tools for a civil engineer during the execution of a structural design project. These computational techniques contribute to reaching answers and making robust analyses, accurately portraying the real-world phenomena.

One of the areas of Classic Mechanics that is gaining strong relevance in the structural area since the second half of the 20th century is Fracture Mechanics. This field studies the behavior of materials and mechanical processes that can result in the propagation of cracks and crevices, leading finally to the failure of the structural element. In this way, the interest in simulating the behavior of structures in distinct scenarios becomes a potentially useful tool to engineer's activities. This situation is reflected when it is relevant to predict the load capacity of structural components, ensuring their stability and preventing the occurrence of accidents.

Many formulations are employed to simulate and predict the opening of cracks in structural elements, especially the Finite Element Method (FEM). Therefore, these studies supply new project conceptions, where the structure is not assumed as a continuous environment, but presenting concentrated failure stresses. However, the classic formulation of FEM can be unfeasible in failure studies, once the fracture process introduces incremental changes in the finite element mesh used to obtain the approximate solution of the problem.

The Extended Finite Element Method (XFEM) emerges as a technique for fracture simulation. Its main feature is the addition of enrichment functions to the field of continuous displacements, representing discontinuities in the model. This methodology leads to good results with a low computation cost compared to the classical formulation of FEM (Moes et al, 1999).

With the recent adoption of polymeric fibers from different materials mixed to the concrete, encompassing all interactions between the specific matrix and fibers, simulating material behavior becomes even more complicated. This paper presents a numerical model for the simulation of a reinforced concrete beam with steel fibers using XFEM and Interface Elements. Mechanical behavior, peak loads, deflections and the opening of cracks are assessed. The numerical results reproduce the behavior described by experiment such as the four-point bending test and the direct tensile test, reported in the literature by Marangon (2011).

2 FIBER REINFORCED CONCRETE

Concrete is one of the most widely employed materials in the Brazilian construction industry in the last decades. This material presents many advantages that justify its extended use, such as high-speed construction, adaptability, relatively low cost, and excellent resistance against shocks and vibrations. Nevertheless, its application produces some disadvantages once the material has low ductility, providing a shorter service life of the structure. In addition, concrete has a reduced tensile strength and a faster deterioration after the appearance of cracks (Mehta & Monteiro, 2008).

Nowadays, with the increasing search for new materials in Structural Engineering, fiber reinforced concrete is becoming widespread in the construction industry. The primary purpose of the introduction of these fibers in concrete is to increase material tensile strength, as well as increasing its ductility and hindering the appearance of cracks.

Another relevant and advantageous mechanism in the use of fibers involves the phenomenon of stress transference within the concrete (Mehta & Monteiro, 2008). Fibers act moving the concentrated tensions on the cracks to the matrix, avoiding their propagation and providing a tracking control of the material. This phenomenon is known as the fiber shear transfer, as illustrated in Fig. 1.

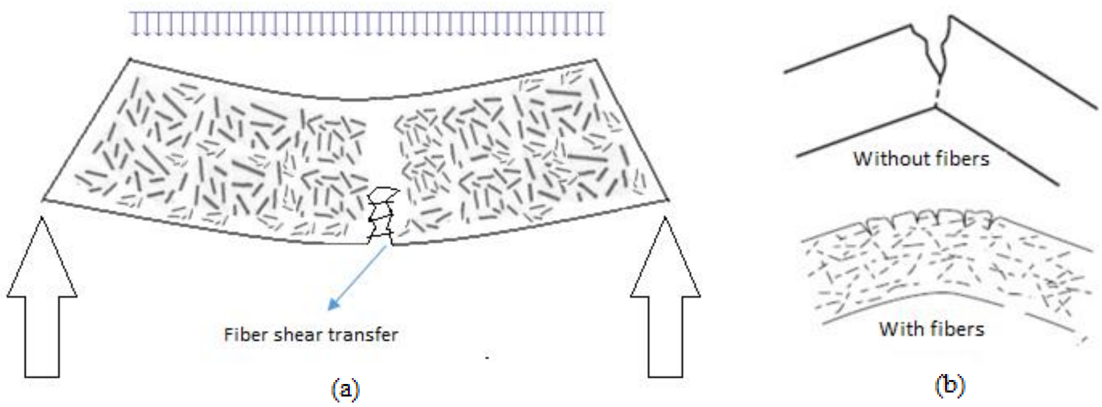

Figure 1. (a) Fiber shear transfer in concrete and (b) crack opening in concrete with and without fibers (Adapted from Johnson, 1980 apud Mehta & Monteiro, 2008).

3 NUMERICAL MODELLING OF THE FRACTURE PROCESS

In fracture modeling using the conventional FEM approach, the fracture is inserted into the finite element model through mesh refinement. An alternative methodology is the adoption of interface elements between solid elements in order to represent fractures explicitly. These features in association with the concept of cohesive fracture mechanics (CFM), define the so called "cohesive elements".

The theory of CFM affirms the existence of cohesive forces located near the crack tip, as shown in Fig. 2. Needleman (1987) was responsible for establishing the relation between cohesive stresses and their respective relative displacements. This technique is associated to a damage constitutive model.

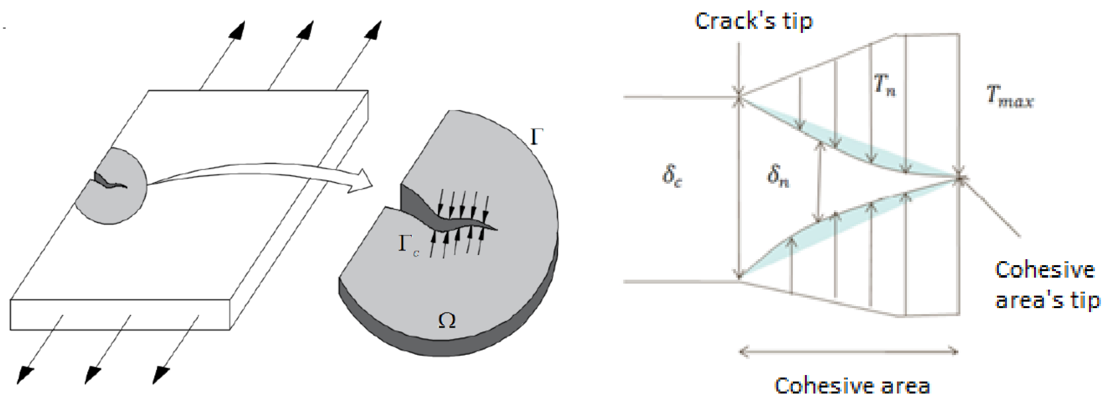

Figure 2. Cohesive forces acting on the crack tip (Adapted from Aragão, 2011).

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Softening behavior of the material can be modelled using a damage constitutive model, which is divided into three distinct phases as shown in Fig. 3. Step (1) refers to the loading phase, characterized by the linear-elastic material behavior; (2) the softening stage, where the material begins to suffer damage, and evolves until the cohesive interfaces are entirely separated. Finally, the unloading phase (3), where the damage is irreversible and the traction-separation curve follows a linear unloading until the origin.

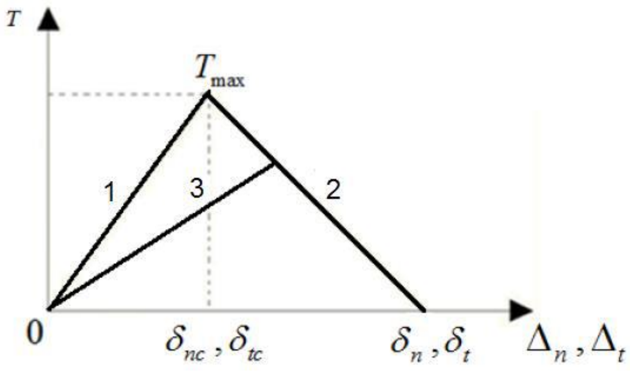

Figure 3. Damage constitutive model for interface elements (Silva & Roehl, 2015).

Several cohesive constitute models have been proposed based on traction-separation law as shown in Fig. 4. The softening phenomenon of materials is related to the loss of relative resistance, and it can be represented by different types of functions, as detailed below.

a) Linear softening model**:** disregards the cohesive tangential effects, simplifying the resistance curve and considering that the material starts losing strength from the beginning of load.

b) Exponential softening model: disregards the cohesive tangential resistance. The resistance curve decreases exponentially during crack opening. It is considered that the material starts losing strength from the beginning of loading.

c) Bi-linear softening model: disregards the contribution to resistance given by the tangential forces. Two segments represent the resistance curve.

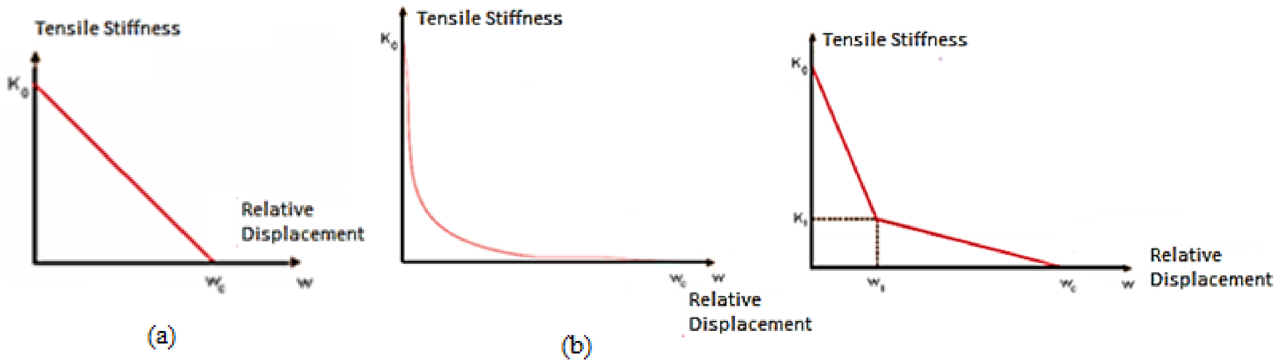

Figure 4. Damage constitutive model with (a) linear, (b) exponential and (c) bi-linear softening. (Adapted from Paliga et. al., 2007).

The Extended Finite Element Method (XFEM) emerged in 1999 from the studies of Belytschko & Black with the primary purpose of dealing with strong discontinuities. This method is an extension of the standard Finite Element Method (FEM), based on a technique which does not represent the fracture explicitly. The fracture is introduced by using nodal enrichment functions to model the discontinuous displacement field. The functions depict the asymptotic fields located in a region next to the crack tip, as shown in Fig. 5 (Melenk & Babuska, 1996; Silva & Roehl, 2015).

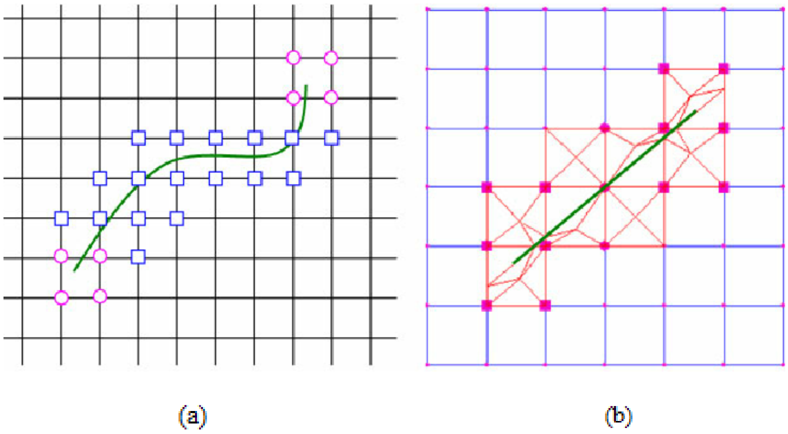

Figure 5. Crack representation on a uniform XFEM mesh, (a) enriched nodes and (b) triangular sub-division of the enriched elements. (Xiao & Karihaloo, 2005).

Some advantages of XFEM are related to the fact that the fracture can cross the elements without the need of upgrading the mesh during its propagation. Other relevant characteristics that justify the extensive usage of the method are high robustness, high precision in problems with discontinuities, ease of application to nonlinear cases and maintenance of symmetry from conventional FEM, among others (Sukumar & Prévost, 2003).

The XFEM formulation uses the same principles as the standard FEM, in addition to the representation of crack, which has its geometry regardless of the adopted mesh. In this sense, the mathematical approach in finite element displacement field is given by equation (1), which is composed by two distinct parts. The first part is associated to the standard formulation of finite elements, where u_i refers to the displacement degree-of-freedom at node i, I_n is the set of nodes in the model, and N_i is the shape function associated to node i.

The second one is related to the displacements discontinuities, with the insertion of the Heaviside function $H(x)$ ("step function"). Notice that, in this part, the additional degree-of-freedom is represented by the variable b_i, while J_n is the set of nodes of the elements crossed by the fracture.

$$u(x) = \sum_{i \in I_n} u_i \, N_i(x) + \sum_{i \in J_n} b_i \, N_i(x) \, H(x) \tag{1}$$

The Heaviside function $H(x)$, defined here as a composite function involving the distance with signal from point *x* to the fracture surface, is expressed by Equation (2):

$$H(x) = H\big(d(x)\big) = \begin{cases} 1, & \text{if } d(x) \geq 0 \\ -1, & \text{if } d(x) < 0 \end{cases} \tag{2}$$

In some of the more complex cases, the Heaviside function cannot efficiently represent the existent discontinuities. In this way, it is necessary to perform node enrichment through asymptotic functions at the crack tip. Consequently, Equation (1) is changed to Equation (3).

$$u(x) = \sum_{i \in I_n} u_i \, N_i(x) + \sum_{i \in J_n} b_i \, N_i(x) \, H(x) + \sum_{i \in K_n} N_i(x) \sum_{l=1}^{4} c_i^l \, F_l(x), \tag{3}$$

where c_i^l are the additional degrees of freedom related to the displacements of node i, $F_l(x)$ indicates the asymptotic functions used to model the crack tip, and K_n is the set of elements that contain the crack tip.

The commercial software *Abaqus* ® includes both strategies for the representation of cracks in a finite element discretization. An alternative formulation of the XFEM is implemented using the concept of “phantom nodes”. For more details to the “phantom nodes” method, refer to Song et. al, 2006.

4 NUMERICAL ANALYSES OF FRACTURE IN STEEL-FIBER REINFORCED CONCRETE

The main purpose of this section is to simulate the behavior of a steel-fiber reinforced concrete beam, assessing deflections, displacements, peak loads and crack opening in the structure. The beam is composed by a mixture containing clusters with a maximum diameter of 19 mm, enhanced with a steel fiber aspect ratio 80 (length of 60 mm; diameter of 0,75 mm) in a volumetric content of 1 %.

The simulations reproduce the experimental procedures reported in the literature by Marangon (2011): four-point bending test and direct tensile test. In order to reproduce these experimental tests using the Extended Finite Element and Interface Elements, approaches were used to model the fracture process. The computational analyses were carried out using the commercial software *Abaqus* ®.

4.1 Direct Tensile Test

The direct tensile test reported by Marangon (2011) uses a prismatic concrete specimen reinforced with steel fibers. The specimen test is restricted at one of its edges and submitted to axial tensile forces at the opposite side. The following figure exhibits the specimen geometry, the finite element mesh, constraints and loads used in the simulation.

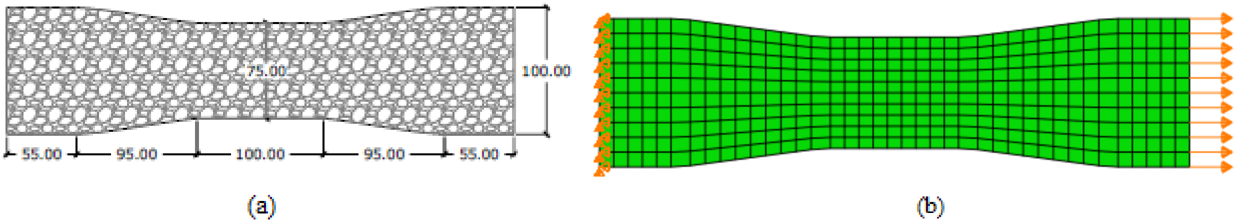

Figure 6. Specimen for the direct tensile test (Marangon, 2011): (a) geometry and (b) finite element mesh, constraints and loads.

Marangon (2011) does not establish the numerical values for the fracture energy G_C of the steel-fiber reinforced material. However, this parameter is essential to match the load-displacement curve at post-peak with the experimental results.

The mathematical procedure used to define this parameter integrates the area under the traction-separation curve from the direct tensile test. This area can be calculated by using many numerical methods, such as the Trapezoid Method, for example. In this paper, the experimental curve was fitted to an exponential function *f(x)* using the Minimum Square Method. Hence, the area under the curve was immediately calculated by the definite integral of this function in the range of $x \in [0,11]$.

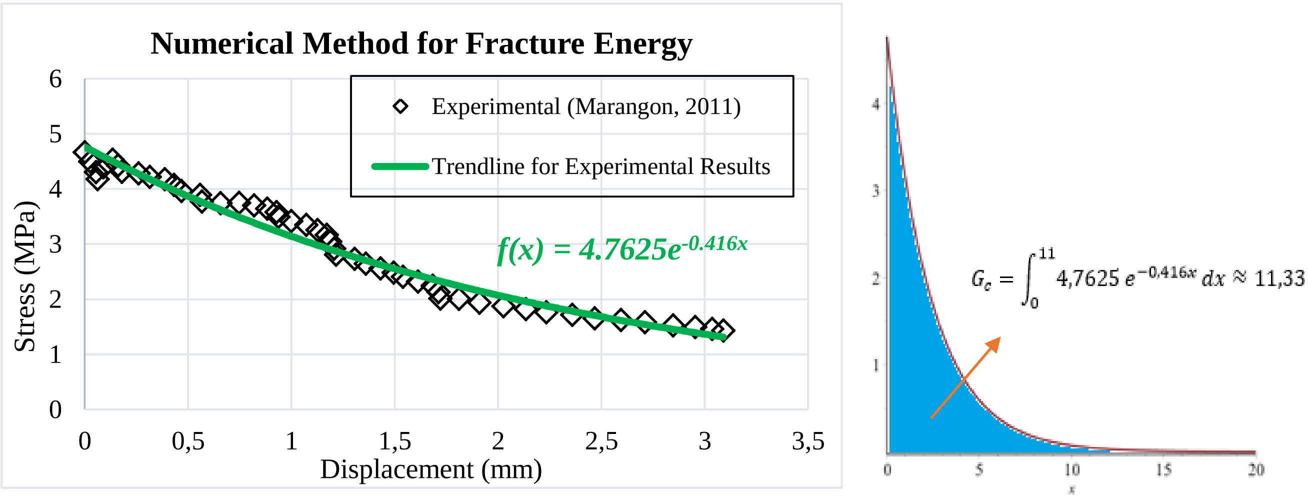

Figure 7. Fracture energy G_C estimation from experimental data.

An exponential softening model was employed to establish the relationship between traction and separation on the fracture surface, which is given by Eq. (4) and Eq. (5). Notice that, in the exponential case, α is the parameter that controls the decay rate of the curve, D is the parameter related to damage, K_0 is the stiffness coefficient, and δ is the associated displacement.

$$T = \begin{cases} (1-D)K_0 * \Delta\delta, & \delta_0 < \Delta\delta < \delta_f \\ 0, & \delta_f < \Delta\delta \end{cases} \tag{4}$$

$$D = 1 - \left(\frac{\delta_0}{\Delta\delta}\right) * \left(1 - \frac{1 - \exp\left(-\alpha * \left(\frac{\Delta\delta - \delta_0}{\delta_f - \delta_0}\right)\right)}{1 - \exp(-\alpha)}\right) \tag{5}$$

Therefore, due to the nonlinearity of Eq. (5), it is necessary to apply numerical methods to obtain iteratively the softening parameters α and the total displacement at failure $\Delta\delta$. This iteration was calculated by using the Generalized Minimum Square Method, with the assistance of the computational software MATLAB ®. The experimental points from Marangon's test were fitted to the exponential softening model, obtaining $\alpha = 1{,}3$ and $\Delta\delta = 4{,}62\ mm$.

In addition, Marangon (2011) does not specify the Poisson coefficient for the concrete beam, so a typical value for concrete ($v = 0{,}2$) was used. The mechanical properties of the reinforced concrete and of the fracture are summarized in Table 1.

Table 1. Properties of the reinforced concrete and fracture.

Reinforced Concrete	
E (GPa)	35,77
$\boldsymbol{v}$	0,2

Fracture	
$\boldsymbol{\sigma_r}$ (MPa)	4,65
$\boldsymbol{\alpha}$	1,3
$\boldsymbol{\Delta\delta}$ (mm)	4,62

The numerical model was discretized with quadrilateral elements (9,5mm x 7,5mm) as shown in Figure 8. The numerical test was performed using arc-length control (so-called Riks), with the adoption of an exponential softening model.

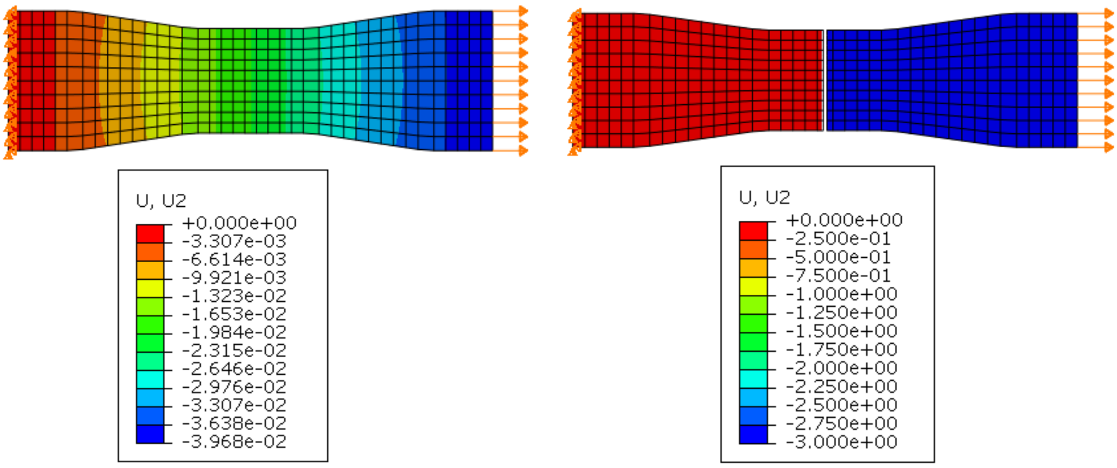

Figure 8. Direct tensile test reproduction (a) specimen before fracture initiation and (b) immediately after the fail.

Figure 9 exhibits the experimental and numerical traction-separation curves for the direct tensile test.

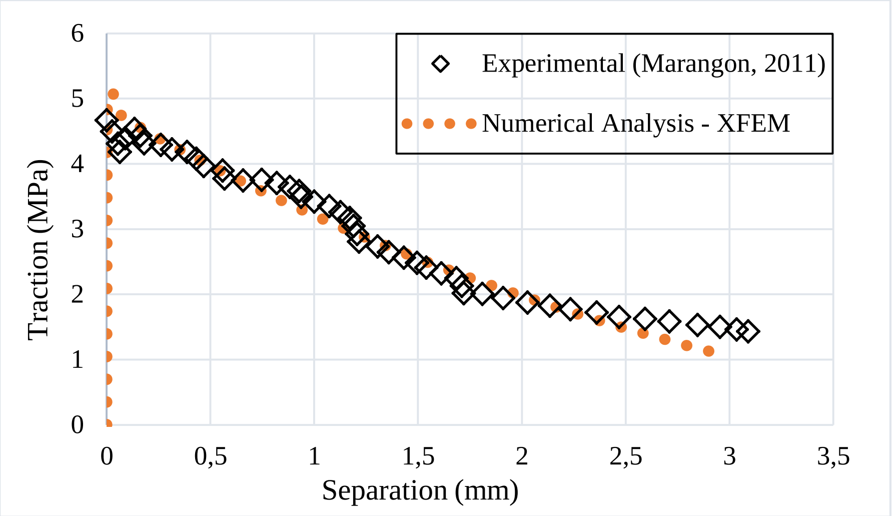

Figure 9. Traction-Separation curve obtained after the numerical simulation with ABAQUS ®.

The obtained traction-separation curve exhibits a very similar behavior to the experimental curve, presenting relative errors smaller than 5%.

4.2 Four-Point Bending Test

The properties related to the fiber-reinforced concrete bending are critical since the elements are submitted to bending forces in most structural applications. Figure 10 shows the geometry, the boundary conditions and the applied loads in the four-point bending test (Marangon, 2011).

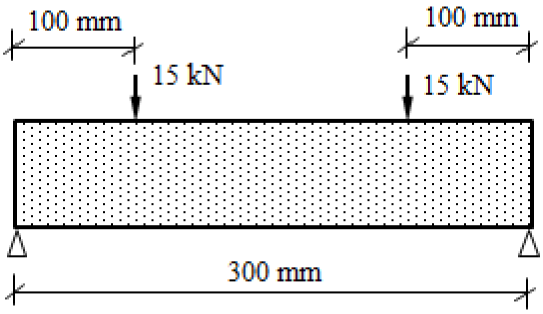

Figure 10. Schematic representation of the four-point bending test.

Firstly, in order to simulate the beam behavior, the numerical modelling was carried out by using the same mechanical properties adopted in the direct tensile test. However, the obtained results did not match the reference experimental curve, presenting incoherent peak load and an inadequate curve fitting.

According to Marangon (2011), the fiber orientation in the samples can also be considered as a significant factor in the difference in stresses in traction and bending tests. The behavior of fibrous composite materials under tension depends on fiber distribution and orientation. If this orientation is not adequate, dispersion of test results is observed. During the mixing process, fibers are scattered randomly in the concrete mixture. At the launching phase, only part of the fibers will be aligned in the direction of the tension stresses. The use of short fibers leads to a random distribution in the concrete matrix, without a preferred direction (Marangon, 2011).

Therefore, the tensile strength σ_r was obtained from the analysis of the experimental load- displacement curve (Figure 6.44 from Marangon, 2011) for a load value of 22 kN. This load was used once it indicates approximately the beginning of the damage process, which represents the end of the elastic region (linear stretch of the load-displacement curve). Adopting the same fracture energy G_C obtained in the direct tensile test, the parameters of the exponential softening model were calculated. Table 2 summarizes the mechanical properties used in the simulation.

Table 2. Mechanical properties for the bending test.

Reinforced Concrete	
E (GPa)	35,77
$\boldsymbol{\nu}$	0,2

Fracture	
$\boldsymbol{\sigma_r}$ (MPa)	6,5
α	2,2
$\Delta\boldsymbol{\delta}$ (mm)	4,62

The model was discretized with quadrilateral elements as shown in Fig. 11. In order to avoid mesh-dependent results and taking advantage of the symmetry of the model, the mesh was refined in the central region of the structure (1,5 mm x 1,5 mm), given the predictability of the crack opening in this area.

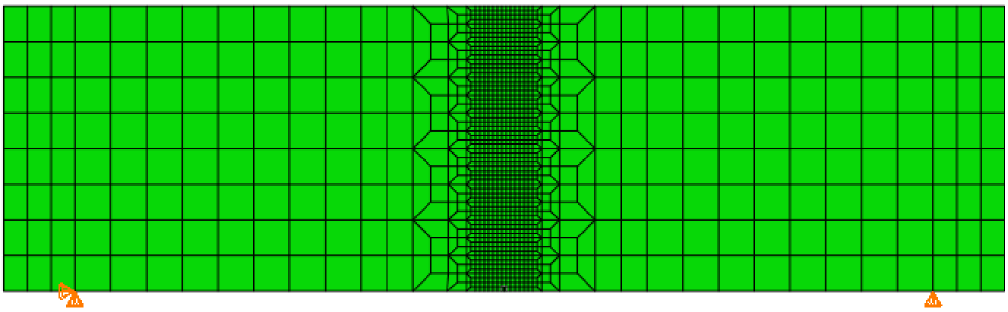

Figure 11. Finite element mesh adopted (ABAQUS ®).

The fracture process was modeled in *Abaqus®* using XFEM and, alternatively, using interface elements, in conjunction with an exponential softening model for fracture behavior. The analysis control followed with Riks' method under plane strain conditions. Figure 12 shows the final stage of loading at load values of 13.6 kN for the XFEM model. At this stage, the fracture has propagated up to 0.64 mm from the top and the crack mouth opening is 3.83 mm. Figure 13 presents the vertical displacements in the final deformed configuration with XFEM. The maximum vertical displacement is 0.65 mm.

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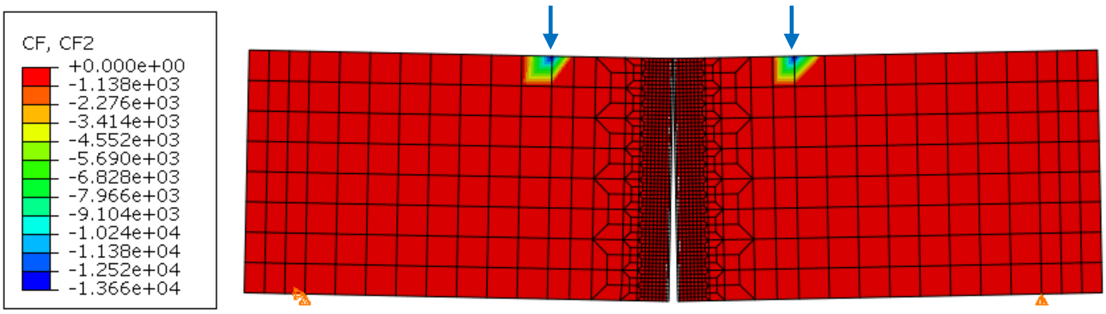

Figure 12. Final loading acting on the beam: point load 13,6kN.

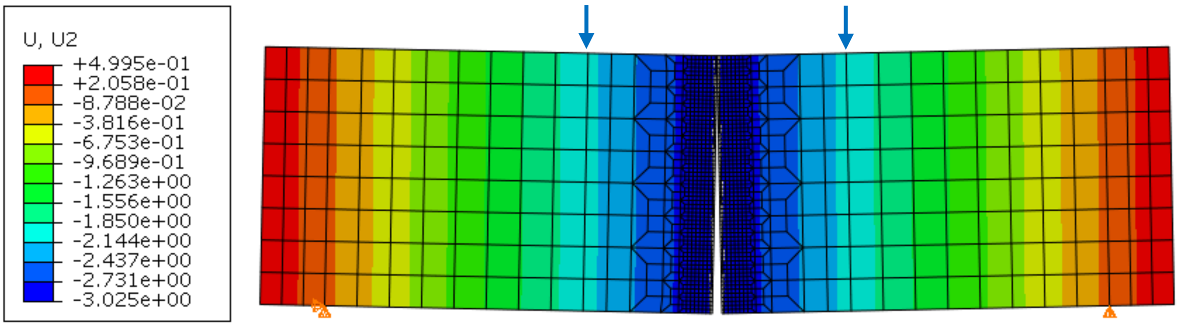

Figure 13. Vertical displacements in the beam in the final configuration. Displacements in mm.

Figure 14 compares the load-displacement curves obtained from the numerical analyses with XFEM and interface models, and the experimental results obtained by Marangon.

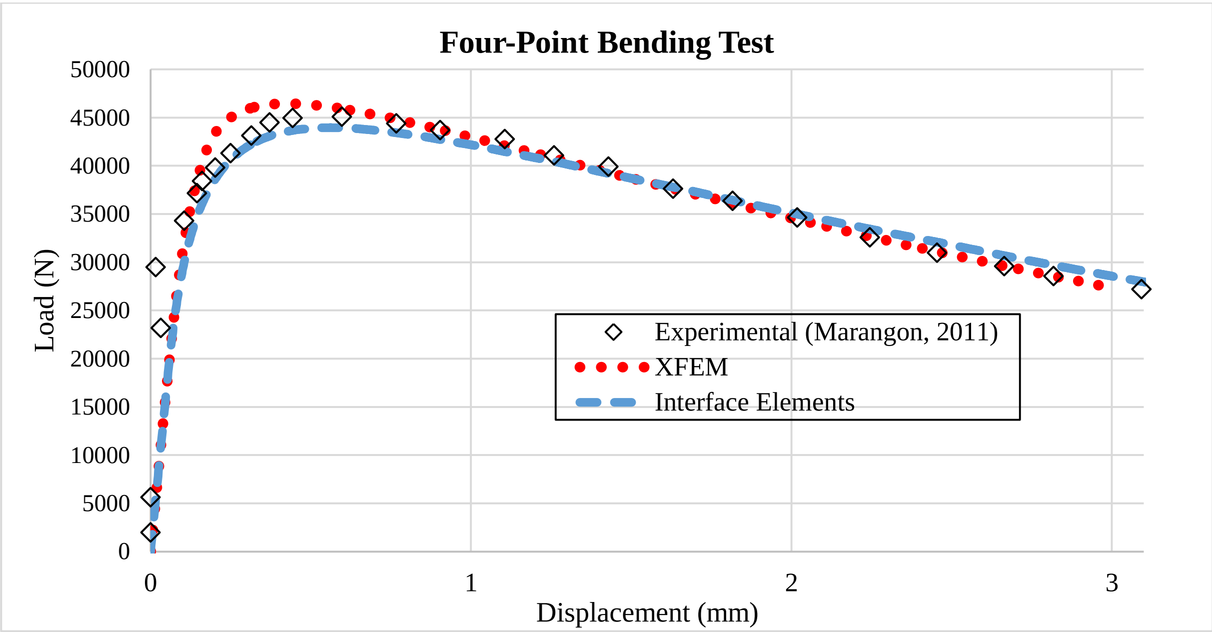

Figure 14. Comparative plots of load-displacement of the four-point bending test for Marangon's beam reinforced with steel fibers using XFEM and Interface Elements.

The comparison of the numerical and experimental load-displacement curves exhibits very good agreement. The peak load obtained with the XFEM model was 46.5 kN, with an error lower than 5% with respect to the experimental value (45.1 kN). On the other hand, the Interface Element model provided a lower peak load (44 kN), with an error of 2% referred to the experimental value. Both analyses produce post-peak responses close to the experimental curve. Finally, the numerical results report that the Extended Finite Element method allied to an exponential softening model can simulate steel-fiber reinforced concrete efficiently. The numerical crack path obtained using XFEM follows a very similar path to the one obtained experimentally. This feature is imposed in the interface model.

5 CONCLUSIONS

This study carried out a numerical analysis with the primary goal of simulating a fiber-reinforced beam behavior, reproducing the experimental procedures reported in the literature by Marangon (2011). Following the concept of Cohesive Fracture Mechanics, the Extended Finite Element method and Interface Elements were employed to model fracture propagation and predicting peak load, crack opening, deflections, among other results. In this regard, the direct tensile and the four-point bending tests were reproduced through the commercial software *Abaqus* ®, confirming the experimental procedures and achieving errors smaller than 5%.

The numerical model reached good levels of accuracy, due to the adoption of an exponential softening model able to reproduce the behavior of the reinforced concrete material when submitted to loading. The XFEM predicts an adequate crack path, which is very similar to what has been reported experimentally.

Finally, it is concluded that, for concrete structures with nonlinear behavior, an excellent approach to simulate crack propagation and structure analysis is achieved with the use of these numerical models. In this sense, numerical simulations strengthen the cohesive fracture mechanics concepts in the context of Structural Engineering nowadays, becoming a valuable tool to predict the behavior of a fiber reinforced concrete.

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