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AUTOMATIC MEMBER GROUPING FOR TRUSS OPTIMIZATION WITH MULTIPLE FREQUENCY CONSTRAINTS

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Abstract. *This paper deals with continuous sizing structural optimization problems with respect to the minimization of the masses of truss structures considering multiple natural frequencies as constraints. It can be attractive to use a reduced number of distinct cross-sectional areas minimizing costs of fabrication, transportation, storing, checking, welding, and so on. Also, it is expected a labor-saving when the structure is welded, checked and so on. In order to obtain an automatic variable linking searching for the best member grouping of the bars of the trusses analyzed in this paper, a special encoding by using cardinality constraints is considered. A CRPSO (Craziness based Particle Swarm Optimization) is the search algorithm adopted here that uses a modified velocity expression and an operator called "craziness velocity". Some computational experiments are analyzed, presenting very interesting results providing curves of tradeoff between the optimized weights and the number of distinct cross-sectional areas used in these solutions. .*

Keywords: *Particle swarm optimization, Cardinality constraints, Automatic member grouping*

1 INTRODUCTION

The introduction of multiple natural frequencies as constraints in structural optimization problems leads to a highly nonlinear problem that has been investigated (Bellagamba and Yang (1981) and Kaveh and Zolghadr (2012)). This paper aims to address structural optimization problems of plane trusses subject to multiple natural frequencies of vibration as constraints. The problems analyzed in this paper correspond to sizing optimization of truss structures where the continuous cross-sectional areas of the bars.

In this paper, a modified PSO called CRPSO (Craziness based Particle Swarm Optimization) proposed by Kar *et al.* (2012) is used as a search engine. The CRPSO uses a modified velocity expression associated with a set of random numbers and an operator, with a predefined probability, called "craziness velocity".

Also, in this paper a special encoding, proposed originally by Barbosa and Lemonge (2005), is adopted for the candidate particles limiting the maximum number of distinct cross-sectional areas to be used in an optimized solution.

This type of constraint is important because it can lead to economies of bulk purchasing, checking, welding, and freeing the designer of the task of deciding how to group members and/or design variables. A new test-bed for the literature is provided in this paper by using cardinality constraints to discover new member groupings. The constrained optimization problem are replaced by an unconstrained problems by introducing an Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge (2002).

The remainder of the paper is organized as follows. Section 2 presents a brief review the structural optimization problems similar to those discussed in this paper. Section 3 describes the optimization problem discussed in this paper. Section 3.1 summarizes the automatic member grouping by using cardinality constraints. Section 4 describes the CRPSO algorithm and the penalty scheme. The computational experiments are presented 5. Finally, the paper ends with conclusions and future work in Section 6.

2 RELATED WORK

This section shows some recent papers from the literature on Evolutionary Algorithms (EAs) to solve structural optimization problems with multiple natural frequencies as constraints, as well as, proposed methods or strategies to find optimal member groupings of bars.

Gomes (2011) investigated the use of a particle swarm optimization (PSO) as an optimization engine for structural truss mass optimization considering size and shape design variables and with frequency constraints. The Harmony Search (HS) and Firefly Algorithm (FA), were proposed by Miguel and Miguel (2012) to solve truss shape and sizing optimization with multiple natural frequency constraints.

A Democratic Particle Swarm Optimization for structural optimization with frequency constraints was proposed by Kaveh and Zolghadr (2014).

An algorithm called Symbiotic Organism Search was proposed by Tejani *et al.* (2016a). A modified sub-population teaching-learning-based optimization (MS-TLBO) algorithm is proposed by Tejani *et al.* (2016b) to improve exploration and exploitation capacities. A multi-class teachinglearning-based optimization (MC-TLBO) technique for structural optimization with frequency constraints was proposd by Farshchin *et al.* (2016).

The automatic member grouping in structural optimization is clearly desirable but there are few papers in the literature presenting methods to solve this issue. Advantages in fabrication, checking, assembling, transportation and welding, which are usually not explicitly included in the cost function, are thus expected. Barbosa & Lemonge *et al.* presented several papers in the literature to solve structural optimization problems considering single and multiple cardinality constarints Barbosa and Lemonge (2005), Barbosa *et al.* (2008), Lemonge *et al.* (2010) and Lemonge *et al.* (2011).

Herencia and Haftka (2010) and Herencia *et al.* (2013) proposed a combined genetic algorithm-gradient strategy to find the minimum weight of the well known ten-bar truss for various numbers of cross-sectional areas. Liu *et al.* (2012a) studied the optimum conceptual design of pile foundations at the initial design stage. Singular optimum topology of skeletal structures with frequency constraints was presented by Liu *et al.* (2012b). Kripka *et al.* (2013) and Kripka *et al.* (2015) demonstrated the application of optimization strategies for the cost of beams in reinforced concrete buildings by using cardinality constraints.

Two Ant Colony Optimization algorithms are proposed by Angelo *et al.* (2015) to tackle multi-objective structural optimization problems with cardinality constraint. Optimal design of dome structures with dynamic frequency constraints was discussed by Kaveh and Ghazaan (2016) using a strategy based on cascading that reduces the objective function value over a number of optimization stages.

Carvalho *et al.* (2015b) used a PSO in to solve structural optimization problems with multiple natural frequencies of vibration without the consideration of cardinality constraints.

Recentlly, Carvalho *et al.* (2017) proposed a CRPSO to solve sizing structural optimization problems with multiple frequency constraints using cardinality constraints to find the best member groupings. Several set of numerical experiments were analyzed including large scale problems. The experiments considered discrete sizing design variables as well as, for some of them, sizing (discrete) and shape (continuous), simultaneously.

3 THE STRUCTURAL OPTIMIZATION PROBLEM

The problems analyzed in this paper refer to sizing optimization, i.e. the design variables are the cross-sectional areas (continuous) of the bars. The optimization problem can be written as: find a set of cross-sectional areas $\mathbf{a} = \{A_1, A_2, ..., A_N\}$ that minimizes structure's weights W as it follows:

$$W(\mathbf{a}) = \sum_{i=1}^{N} \rho A_i L_i \tag{1}$$

subject to natural frequency constraints

$$1 - \frac{f_j}{\bar{f}} \leq 0, \quad j = 1, ..., n_c \tag{2}$$

where ρ is the density of material, A_i is the cross-sectional area and L_i is the length of the i-th bar. The j-th natural frequency of the structure is denoted by f_j and $\bar{f}$ is the lower (or upper) limit for the j-th allowed natural frequency of the structure (in case of $\bar{f}$ being the upper limit, the equation 2 is multiplied by -1). The number of bars of the structure is denoted by N and n_c is the number of constraints of the problem.

The natural frequencies are obtained by the evaluation of the eigenvalues of the matrix $[(f_{n_f}^2 * M_a) + K]$ (Bathe, 2006), where M_a and K are the mass and stiffness matrices, respectively, and f_{n_f} are the equivalent eigenvectors with respect to the n_f natural frequencies of the structure.

3.1 Automatic member grouping and cardinality constraints

In order to explore desirable advantages during the design configurations, a designer may require that at most a given m of distinct cross-sectional areas should be used in the final structural configuration, grouping the bar in m groups.

Hence, the optimization problem must consider additional constraints requiring that no more than m ($m \leq N$) different cross-sectional areas should be used, that is,

$$A_i \in C_m = \{S_1, S_2, \ldots, S_m\}, \quad i = 1, 2, \ldots, N$$

where the cross-sectional areas $S_j, j = 1, 2, \ldots, m$ are unknown but:

- belong to a larger $(M > m)$ given set $S = \{A1, A2, \ldots, A_M\}$, for the discrete case, or
- are to be found within a prescribed range $C = \{A_{min}; A_{max}\}$, for the continuous variable case.

The standard structural optimization problem (without the cardinality constraint) is recovered when $m = N$. These additional constraints are defined as cardinality constraints.

This paper uses a PSO algorithm as an optimization algorithm and an example of encoding of a candidate particle in the PSO with 13 design variables is presented in Fig. 1. This encoding is inspired by the chromosome proposed by Barbosa and Lemonge (2005). The cardinality constraints, for instance $m = 3$, are represented by the three continuous designing variables at the left part of the encoding.

At the right part of the encoding there are, in this illustration, 10 indexes that point to the three design variables of the left part of the encoding. These indexes point to the continuos values (cross-sectional areas of the bars) able to be chosen from a lower and an upper bounds $A_{min} \leq A_i \leq A_{max}$. In this example, the design variables from 1 to 3 are equal to 3.2164 cm^2, 2.2432 cm^2 and 0.1311 cm^2, respectively. At the right part of the encoding, for example, the fourth design variable points to the first design variable of the left part and the bar referred by this design variable has the cross-sectional area equals to 3.2164 cm^2. On the other hand, the first design variable of the right part points to the third design variable of the left part receving 0.1311 cm^2 for the cross-sectional area.

4 THE CRPSO ALGORITHM AND THE PENALTY SCHEME

The optimization algorithm used to solve the constrained optimization problems presented in this paper is a Craziness Particle Swarm Optimization (CRPSO), which is a modified version of the conventional PSO algorithm proposed by Kar *et al.* (2012), in order to improve the PSO behavior, thus avoiding premature convergence. Particle Swarm Optimization (PSO) was originally introduced by Eberhart and Kennedy (1995). The algorithm was inspired by the

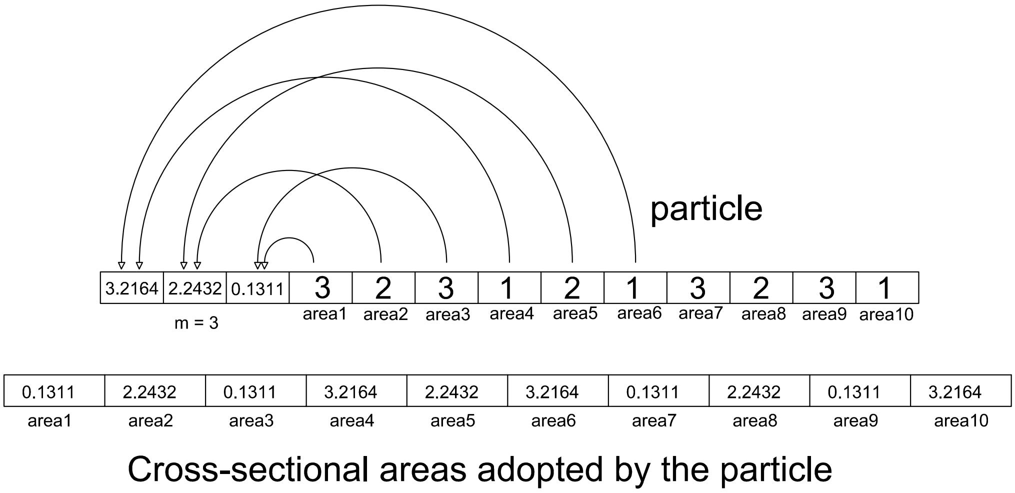

Figure 1: Example of the encoding of a particle with 13 design variables. The cardinality constraints ($m = 3$) are represented by the three variables at the left part of the vector which represents the particle. At the right part of the vector there are, for instance, 10 indexes that point to the the design variables at the left part.

social behavior of birds flocking. PSO is a population-based algorithm with fewer parameters to be set and is easier to implement than other EA's. The objective function is evaluated for each particle and the fitness value of particles is obtained in order to determine which position in the search space is the best.

In each iteration t of the CRPSO, the swarm is updated using the following equations:

$$v_j^{(i)}(t+1) = v_j^{(i)}(t) + c_1 \cdot r_1(x_{pbest}^{(i)}(t) - x_j^{(i)}(t)) + c_2 \cdot r_2(x_{gbest}(t) - x_j^{(i)}(t)) \tag{3}$$

$$x_j^{(i)}(t+1) = x_j^{(i)}(t) + v_j^{(i)}(t+1) \tag{4}$$

where $v_j^{(i)}$ and $x_j^{(i)}$ represent the current velocity and the current position of the jth design variable of the ith particle, respectively. $x_{pbest}^{(i)}$ is the best previous position of the ith particle (called *pbest*) and x_{gbest} is the best global position among all the particle in the swarm (called *gbest*); c_1 and c_2 are coefficients that control the influence of cognitive and social information, respectively, and r_1 and r_2 are two uniform random sequences generated between 0 and 1.

In CRPSO, a new velocity expression v_i, associated with a set of random numbers and an operator called "craziness velocity", is considered. The operator has a predefined probability of craziness. In this case the velocity can be expressed as follows Kar *et al.* (2012):

$$\begin{aligned} v_j^{(i)}(t+1) = \quad & r_2 \cdot sign(r_3) \cdot v_j^{(i)}(t) + (1-r_2)c_1 \cdot r_1(x_{pbest}^{(i)} - x_j^{(i)}) + (1-r_2) \cdot c_2 \cdot \\ & (1-r_1)(x_{gbest} - x_j^{(i)}) + P(r_4) \cdot sign2(r_4) \cdot v_j^{craziness} \end{aligned} \tag{5}$$

where r_1, r_2, r_3 and r_4 are the random parameters uniformly taken from the interval [0,1), $sign(r_3)$ is a function defined as

$$sign(r_3) = \begin{cases} -1, & r_3 \leq 0.5 \\ 1, & r_3 > 0.5 \end{cases} \tag{6}$$

$v_j^{craziness}$, the craziness velocity, is a user defined parameter from the interval [v^{min}, v^{max}] and

$P(r_4)$ and $sign2(r_4)$ are defined, respectively, as

$$P(r_4) = \begin{cases} 1, & r_4 \leq Pcr \\ 0, & r_4 > Pcr \end{cases} \tag{7}$$

$$sign2(r_4) = \begin{cases} -1, & r_4 \geq 0.5 \\ 1, & r_4 < 0.5 \end{cases} \tag{8}$$

and Pcr is a predefined probability of craziness. Although the parameter Pcr is fixed, $P(r_4)$ is defined every time the velocity is calculated.

To handle the constraints, a variant of the Adaptive Penalty Method (Lemonge and Barbosa (2004)), introduced by Carvalho *et al.* (2015a), is adopted here to enforce all the mechanical constraints considered in the numerical experiments. The fitness function $F(x)$ is defined as:

$$F(x) = \begin{cases} f(x), & \text{if} \quad x \quad \text{is feasible} \\ \bar{f}(x) + \sum_{j=1}^{n_c} k_j v_j(x), & \text{otherwise} \end{cases} \tag{9}$$

and

$$\bar{f}(x) = \begin{cases} f(x), & \text{if} \quad f(x) > \langle f(x) \rangle \\ \langle f(x) \rangle, & \text{if} \quad f(x) \leq \langle f(x) \rangle \end{cases} \tag{10}$$

where $\langle f(x) \rangle$ is the average value of the objective function of the current population. The penalty parameter k_j is defined as

$$k_j = |\langle f(x) \rangle| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{n_c} [\langle v_l(x) \rangle]^2} \tag{11}$$

where $\langle v_j(x) \rangle$ means the violation of the j-th constraint averaged over the current population considering only infeasible individuals.

5 COMPUTATIONAL EXPERIMENTS

The computational experiments refer to the structural optimization of two truss structures where one of them is a planar structure (10- bar truss), and the other one is a space structure (72-bar truss). The experiments analyzed in this paper are similar to those presented in Kaveh and Zolghadr (2014), Kaveh and Javadi (2014) and Kaveh *et al.* (2015a).

The CRPSO adopted here uses continuous design variables, $v_j^{craziness} = 0.001$, $c_1 = c_2 = 2.05$ and $Pcr = 0.5$.

The numerical experiments are divided in two sets.

- The first one discuss the benchmark test-problems with the same formulation presented in Kaveh *et al.* (2015a) and Kaveh and Javadi (2014) for the 10- and 72-bar trusses. The performance of the CRPSO is compared to the results found by Kaveh *et al.* (2015a) in this section.
- The second one refers to the use of CRPSO to solve the optimization problems considering cardinality constraints to find the best member groupings for the test-problems

analyzed in the first set of the numerical experiments.

The following subsections present the description of the optimization problems discussed above. For these test-problems, the number of particles is defined for each experiment (based on (Carvalho *et al.*, 2015b)): 70 for the 10-bar truss and 90 for the 72-bar truss and the number of function evaluations is displayed in the tables of results. The number of independent runs for each experiment is 50 and the best solutions presented in the comparisons are rigorously feasible. The continous search spaces are decipted in each experiment's subsection.

5.1 10-bar truss

The first experiment is the 10-bar truss depicted in Fig. 2. The material has Young's modulus E = 68.9 GPa and density ρ = 2770.00 kg/m^3. A nonstructural mass of 454.0 kg is added in each node. The sizing design variables are the cross-sectional areas of the bars. The values can be chosen from the continuous range from 0.6452 cm^2 to 50.0000 cm^2 (0.1 to 7.75 in^2). The limits for natural frequencies are $f_1 \geq 7$ Hz, $f_2 \geq 15$ Hz and $f_3 \geq$20 Hz.

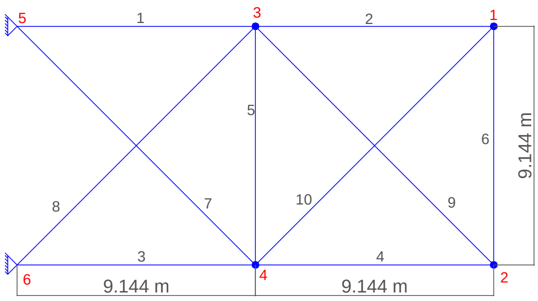

Figure 2: 10-bar truss.

5.2 72-bar truss

The last experiment corresponds to weight minimization of the 72-bar truss depicted in Figures 3 and 4 where the bars are linked as indicated in Table 5. A nonstructural mass of 2270.00 kg (5000 lbs) is added at the four nodes at the top of the truss. The material has Young's modulus E = 68.9 GPa and density ρ = 2770.00 kg/m^3. The discrete search for the sizing design variables can be chosen from the continuous range from 0.6452 cm^2 to 50.0000 cm^2 (0.1 to 7.75 in^2). The natural frequencies have the limits $f_1 \geq 4$ Hz and $f_3 \geq 6$ Hz.

5.3 The first set of numerical experiments

In order to assess the performance of the CRPSO, the results obtained by this algorithm are compared with those presented by Kaveh *et al.* (2015b).

The results are presented in Tables 1 and 2, where dv means design variable, W is the final weight and nfe means the number of objective function evaluations. The thickness of the lines representing the bars are proportional to the found areas. For the 10-bar truss, 532.1807 kg and 324.3466 kg for the 72-bar truss. The results show to be very competitive when compared with the literature, where the 10-bar truss presents the best final weight and the 72-bar truss reached the second place in the comparisons.

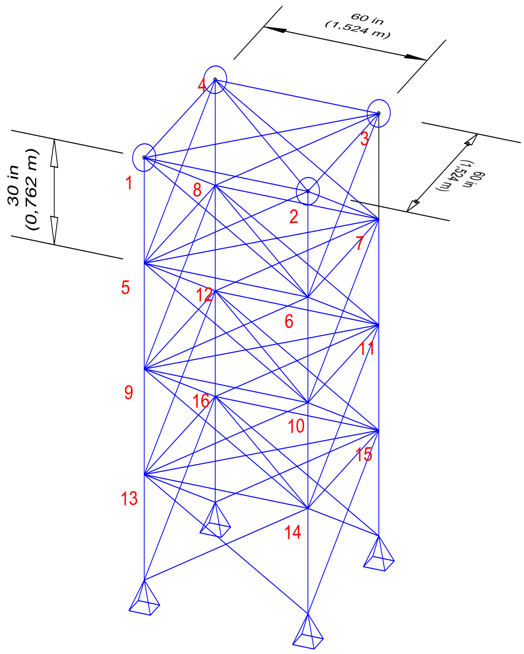

Figure 3: 72-bar truss.

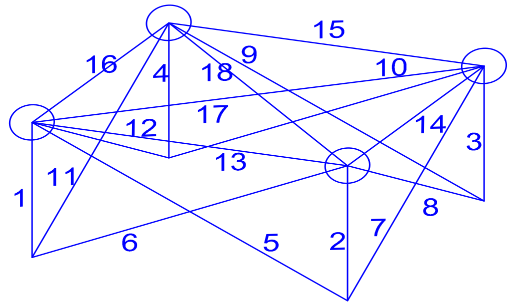

Figure 4: 72-bar truss - pattern module.

Table 3 presents the statistical results for the first set of the numerical experiments where all independent runs reached a feasible solution. Based on the performance obtained by CRPSO in the first set of numerical experiments, the same parameters of CRPSO used in this case are used in the second set of numerical experiments.

5.4 The second set of numerical experiments

The second analysis refers to the use the CRPSO to solve the optimization problems considering cardinality constraints to find the best member groupings. The results for the best solutions are presented in Tables 4 and 5 where dv means design variable, nfe is the number of objective function evaluations, m is the cardinality constraint and “no c.c.” indicates no cardinality constraints. Just after the label “no c.c.” appears the number of distinct cross-sectional areas found when no cardinality constraint was set. Each different color represents a distinct cross-sectional area group as well as the thickness of the lines representing the bars are proportional to the found areas.

Table 4 presents the results for the 10-bar truss. The number in parentheses, in the last column, just after “no.c.c” means the number of the distinct cross-sectional areas found when no cardinality constraints was set. Figure 5 shows the best configuration when no cardinality constraints are adopted. Figures 6 to 9 show the best configurations considering m = 1, 2, 3 and 4, respectively. One can observe that when m = 4, the percentual gain of final weight (i.e. the percentual decrease in final weight) is not very significant if compared to when m = 3.

The best results with and without cardinality constraints for the 72-bar truss are presented in Table 5. Figure 10 presents the best solution without cardinality constraints whereas Figs. 11 to 14 show the best results considering cardinality constraints. Again when m = 4, the percentual gain of final weight (i.e. the percentual decrease of final weight) is not very significant when compared to the final weight setting m = 3.

Table 1: Optimum results for 10-bar truss. Cross-sectional areas in (cm^2). Where A (Kaveh and Farhoudi (2013)), B (Sedaghati (2005)), C (Kaveh and Farhoudi (2011)), D (Kaveh and Javadi (2014)), E (Kaveh and Zolghadr (2014)) and F (Kaveh *et al.* (2015b)).

	References						
dv	A	B	C	D	E	F	CRPSO
A_1	36.584	38.245	32.456	35.944	35.540	35.300	35.2019
A_2	24.658	9.916	16.577	15.530	15.293	15.100	14.8800
A_3	36.584	38.619	32.456	35.285	35.784	36.500	36.4457
A_4	24.658	18.232	16.577	15.385	14.605	15.400	15.5806
A_5	4.167	4.419	2.115	0.648	0.645	0.645	0.6465
A_6	2.070	4.194	4.467	4.583	4.625	4.600	4.5787
A_7	27.032	20.097	22.810	23.610	24.778	23.700	23.0122
A_8	27.032	24.097	22.810	23.599	23.310	24.000	24.4929
A_9	10.346	13.890	17.490	13.357	12.482	11.500	12.4135
A_{10}	10.346	11.451	17.490	12.357	12.674	13.500	12.5516
	Natural frequencies						
f_1 (Hz)	7.059	6.992	7.011	7.000	6.999	7.000	7.0000
f_2 (Hz)	15.895	17.599	17.302	16.187	16.175	16.208	16.1981
f_3 (Hz)	20.425	19.973	20.001	20.000	19.999	20.005	20.0006
W (kg)	594.000	537.010	553.800	532.390	532.110	532.814	532.1807
nfe	-	-	-	21000	-	21000	26250

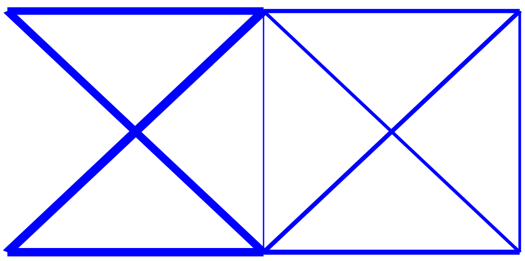

Figure 5: The best result for the 10-bar truss without cardinality constraints.

Table 2: Optimum results for 72-bar truss. Cross-sectional areas in cm^2. Where A(Konzelman (1986)), B (Sedaghati (2005)), C (Gomes (2011)), D (Kaveh and Zolghadr (2012)), E (Kaveh *et al.* (2015b)),

	References					
dv	A	B	C	D	E	CRPSO
A_1-A_4	3.499	3.499	2.987	3.949	3.60	3.3764
A_5-A_{12}	7.932	7.932	7.749	7.968	8.100	7.9036
A_{13}-A_{16}	0.645	0.645	0.645	0.645	0.645	06.460
A_{17}-A_{18}	0.645	0.645	0.645	0.647	0.645	0.6453
A_{19}-A_{22}	8.056	8.056	8.765	7.525	8.85	8.0960
A_{23}-A_{30}	8.011	8.011	8.153	7.863	8.50	8.0180
A_{31}-A_{34}	0.645	0.645	0.645	0.645	0.70	0.6455
A_{35}-A_{36}	0.645	0.645	0.645	0.652	0.645	0.6472
A_{37}-A_{40}	12.812	12.812	13.450	12.966	11.850	13.2238
A_{41}-A_{48}	8.061	8.061	8.073	8.347	8.10	8.0761
A_{49}-A_{52}	0.645	0.645	0.645	0.645	0.645	0.6458
A_{53}-A_{54}	0.645	0.645	0.645	0.645	0.645	0.6457
A_{55}-A_{58}	17.279	17.279	16.684	17.389	17.60	17.0099
A_{59}-A_{66}	8.088	8.088	8.159	8.006	7.55	8.1246
A_{67}-A_{70}	0.645	0.645	0.645	0.645	0.645	0.6452
A_{71}-A_{72}	0.645	0.645	0.645	0.645	0.645	0.6472
	Natural frequencies					
f_1 (Hz)	4.000	4.000	4.000	4.000	4.000	4.0000
f_3 (Hz)	6.000	6.000	6.000	6.000	6.0080	6.0000
W (Kg)	327.605	327.605	328.823	328.589	329.422	328.3466
nfe	-	-	-	-	27000	33750

Table 3: Statistical results and computational time, for each independent run, for the first analysis of the numerical experiments.

Truss	Best	Median	Average	Std	Worst	Time (s)
10-bar	532.1807	539.1885	541.2551	12.9433	609.2109	70
72-bar	328.3548	328.5684	330.5974	22.0378	379.2522	637

Table 4: Results for the 10-bar truss, where areas A_i are in cm^2.

dv	m = 1	m = 2	m = 3	m = 4	no c.c. (10)
A_1	28.9154	38.7420	41.7608	40.2032	35.2019
A_2	28.9154	12.9193	16.7480	19.9652	14.8800
A_3	28.9154	38.7420	41.7608	40.2032	36.4457
A_4	28.9154	12.9193	16.7480	8.9868	15.5806
A_5	28.9154	12.9193	5.1662	5.2685	0.6465
A_6	28.9154	12.9193	5.1662	5.2685	4.5787
A_7	28.9154	12.9193	16.7480	19.9652	23.0122
A_8	28.9154	38.7420	16.7480	19.9652	24.4929
A_9	28.9154	12.9193	16.7480	8.9868	12.4135
A_10	28.9154	12.9193	16.7480	19.9652	12.5516
			Natural frequencies		
f_1 (Hz)	7.0000	7.0000	7.0000	7.0000	7.0000
f_2 (Hz)	21.0369	18.8177	18.5512	17.2967	16.1981
f_3 (Hz)	22.5875	20.0000	20.0001	20.0002	20.0006
W (kg)	854.9157	605.5921	563.3052	551.1784	532.1807
nfe	7000	11200	16800	26250	26250

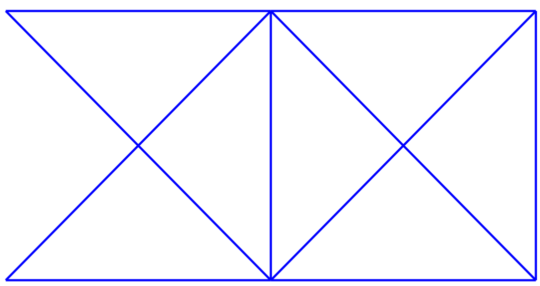

Figure 6: The best result for the 10-bar truss with cardinality constraints for m = 1.

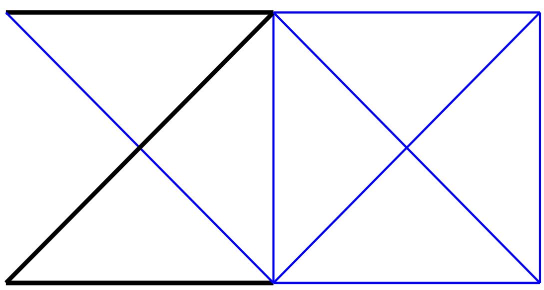

Figure 7: The best result for the 10-bar truss with cardinality constraints for m = 2.

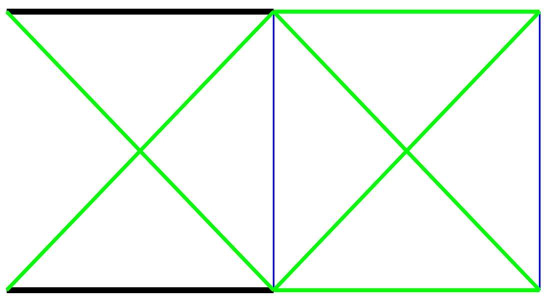

Figure 8: The best result for the 10-bar truss with cardinality constraints for m = 3.

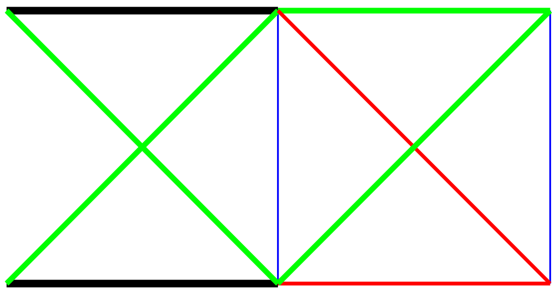

Figure 9: The best result for the 10-bar truss with cardinality constraints for m = 4.

Table 5: Best results for the 72-bar truss, where areas A_i are given in cm^2.

dv	m = 1	m = 2	m = 3	m = 4	no c.c. (16)
A_1-A_4	10.7549	10.9298	8.0362	3.4249	3.3764
A_5-A_{12}	10.7549	10.9298	8.0362	8.0312	7.9036
A_{13}-A_{16}	10.7549	0.6452	0.6452	0.6452	0.6460
A_{17}-A_{18}	10.7549	0.6452	0.6452	0.6452	0.6453
A_{19}-A_{22}	10.7549	10.9298	8.0362	8.0312	8.0960
A_{23}-A_{30}	10.7549	10.9298	8.0362	8.0312	8.0180
A_{31}-A_{34}	10.7549	0.6452	0.6452	0.6452	0.6455
A_{35}-A_{36}	10.7549	0.6452	0.6452	0.6452	0.6472
A_{37}-A_{40}	10.7549	10.9298	14.4609	15.4403	13.2238
A_{41}-A_{48}	10.7549	10.9298	8.0362	8.0312	8.0761
A_{49}-A_{52}	10.7549	0.6452	0.6452	0.6452	0.6458
A_{53}-A_{54}	10.7549	0.6452	0.6452	0.6452	0.6457
A_{55}-A_{58}	10.7549	10.9298	14.4609	15.4403	17.0099
A_{59}-A_{66}	10.7549	10.9298	8.0362	8.0312	8.1246
A_{67}-A_{70}	10.7549	0.6452	0.6452	0.6452	0.6452
A_{71}-A_{72}	10.7549	0.6452	0.6452	0.6452	0.6472
			Natural frequencies		
f_1 (Hz)	4.0000	4.0000	4.0000	4.0000	4.0000
f_3 (Hz)	6.9105	6.3896	6.0000	6.0000	6.0000
W (kg)	646.4120	419.4289	334.0600	329.4127	328.3466
nfe	9000	14400	21600	33750	33750

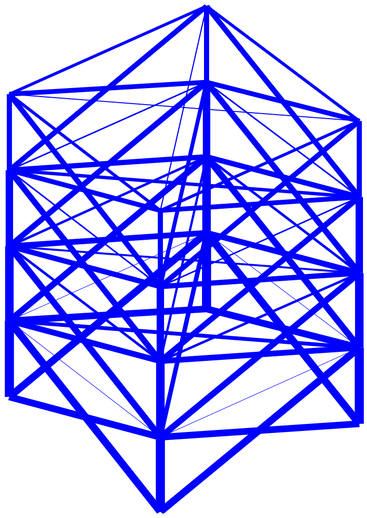

Figure 10: The best result for the 72-bar truss without cardinality constraints.

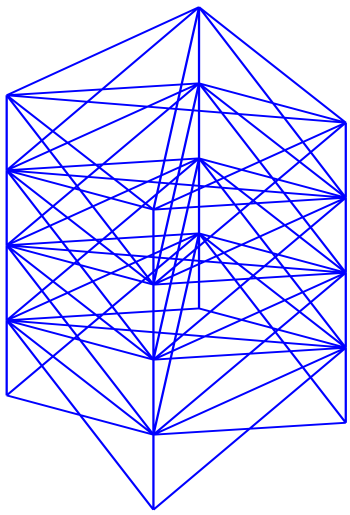

Figure 11: The best result for the 72-bar truss with cardinality constraints for m = 1.

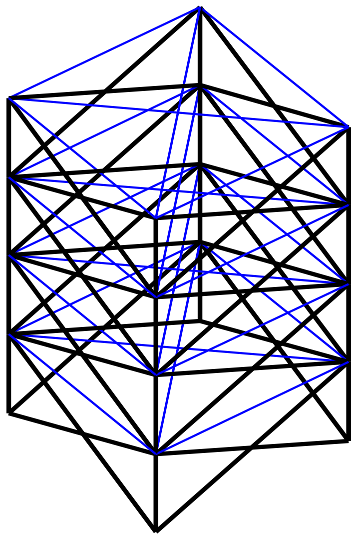

Figure 12: The best result for the 72-bar truss with cardinality constraints for m = 2.

Figures 15 and 16 shows the tradeoff for the 10- and 72-bar trusses indicating the final weights and the number of distinct cross-sectional areas. One can observe the weight reduction as the number of distinct cross-sectional areas increases.

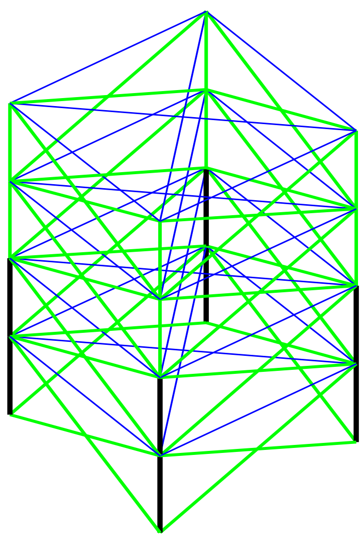

Figure 13: The best result for the 72-bar truss with cardinality constraints for m = 3.

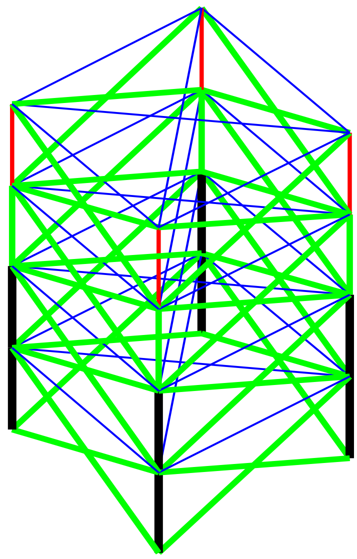

Figure 14: The best result for the 72-bar truss with cardinality constraints for m = 4.

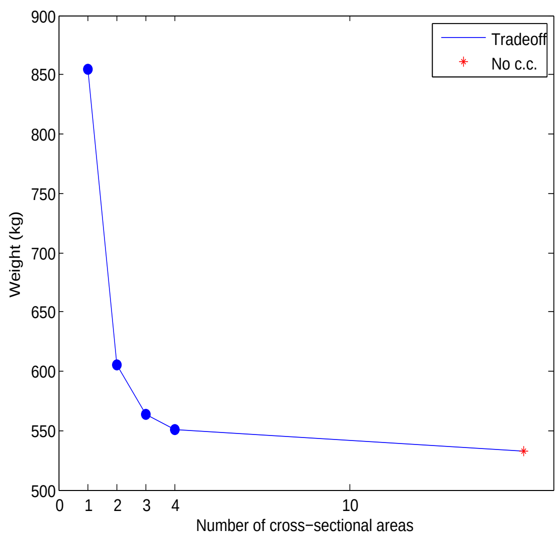

Figure 15: The tradeoff between the final weight and the number of cross-sectional areas for the 10-bar truss.

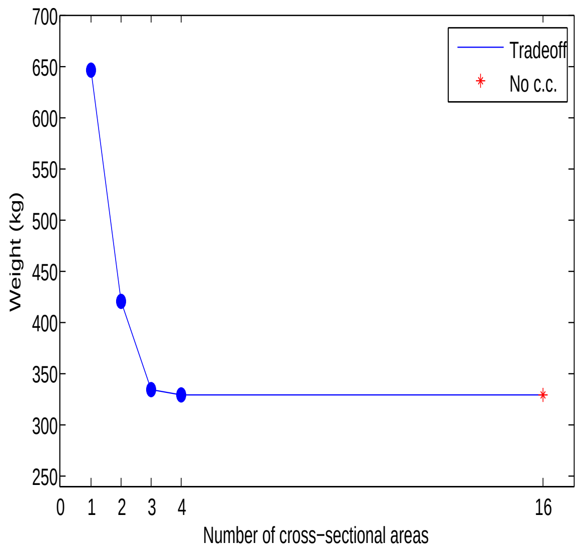

Figure 16: The tradeoff between the final weight and the number of cross-sectional areas for the 72-bar truss.

6 CONCLUSIONS AND FUTURE WORKS

Economy in fabrication, checking, transportation and so on, using a reduced number of distinct cross-sectional areas are very important aspects to be considered in the optimized structural configurations. Also, it is expected a labor-saving when the structure is welded, checked and so on.

A continuous search space for sizing design variables is used in this paper. A CRPSO is used with a special encoding for the candidate particles, considering cardinality constraints lim-

Table 6: Statistical results and computational time for the numerical experiments.

	Best	Median	Average	Std	Worst	Time (s)
			10-bar truss			
$m = 1$	854.9157	854.9176	854.9269	0.0305	855.0904	19
$m = 2$	605.5921	625.2052	626.9118	25.4992	690.8852	30
$m = 3$	563.3052	594.6670	593.6617	13.8510	640.7576	44
$m = 4$	551.1784	580.8409	584.1444	26.0484	684.9669	72
			72-bar truss			
$m = 1$	646.4120	646.4121	646.4123	0.0004	646.4134	149
$m = 2$	419.4289	505.9693	492.2180	35.5780	563.2691	232
$m = 3$	334.0600	407.1472	417.8215	56.8582	511.6995	369
$m = 4$	329.4127	366.9505	376.3824	34.7613	459.2720	550

iting the maximum number of distinct cross-sectional areas to be used in an optimized solution.

Future work will consider more complex structures as those analyzed by Carvalho *et al.* (2017) and, as in that paper, new member grouping for the benchmark problems will be proposed so as to reformulate the original description of these problems in terms of the number of sizing design variables, making the search easier, as can be observed in Carvalho *et al.* (2017).

Finally, it is important to observe that it is possible to formulate a new optimization problem by using a more sophisticated fitness function that reflects the total costs associated with aspects related to fabrication, transportation, storing, checking, etc.

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