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RELIABILITY ANALYSIS OF REINFORCED CONCRETE BEAMS USING FINITE ELEMENT MODELS

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Abstract. *Modelling of reinforced concrete (RC) structures for reliability analysis purposes is a challenging task, for which in many cases only numerical solutions are available. This paper presents an application of reliability analysis to the specific case of RC beams subject to four-point bending tests, and considers two different finite element models developed using ANSYS. The first model employs the SOLID65 element to model the concrete, combining the Willam-Warnke and the Von Mises criteria, while the second employs the SOLID185 element and the microplane theory. In both cases, the steel reinforcement is modeled by LINK180 elements, considering a bilinear isotropic material and the Von Mises failure criteria. The models are calibrated using experimental results and applied in the reliability analysis of a RC beam under various load levels. A limit state function based on a maximum allowable displacement is adopted. The results indicate that only the Microplane model is suitable for reliability analysis of RC structures, since the Solid65 model experiences many convergence problems. Also, it is shown that, for the limit state and conditions considered, the First Order Reliability Method is accurate enough to solve the problem at hand.*

Keywords: *Structural Reliability, Reliability Analysis, Reinforced Concrete, Finite Element Method*

1 INTRODUCTION

Numerical modelling of reinforced concrete (RC) structures is a challenging task, mainly due to the highly nonlinear behavior of concrete and steel working together. It has been the subject of many papers over the years (Manfredi & Pecce, 1998; Fanning, 2001; Gomes & Awruch, 2001; Wang & Hsu, 2001). Although simplified analytical expressions may be used in some cases, a proper representation of some aspects, such as yielding of reinforcement, cracking and crushing of concrete and long-term tension stiffening effects in concrete, many times can only be achieved by means of numerical models (Biondini *et al.*, 2004; Hawileh *et al.*, 2010; Torii & Machado, 2010).

In order to deal with these aspects, finite element models have been widely used both in industry and in the field of research and education (Madency & Guven, 2007; Barbato *et al.*, 2010). However, when dealing with structural reliability analyses, the application of numerical models gives rise to much more challenging problems, since the computation of small failure probabilities requires many model evaluations. Furthermore, numerical convergence problems may appear as the reliability method changes the parameters of the deterministic model during the reliability analysis process.

The present paper presents an application of the finite element method to the specific case of reliability analysis of RC beams subject to four point bending tests. For this purpose, two different finite element models are developed using ANSYS (ANSYS, 2014), a finite element analysis software package known worldwide. The parameters of both models are calibrated by comparing the numerical results with experimental results from the literature, in a deterministic manner. After that, the load and some material properties are considered random variables and the models are applied in order to compute failure probabilities for various load levels.

In the literature, it is common to use approximated, less time-consuming, methods to solve reliability problems involving finite element models, such as the First and Second Order Reliability Methods (FORM and SORM), as can be seen in Val *et al.* (1997), Haukaas & Der Kiureghian (2006) and Gomes & Beck (2013), for example. Given the fact that these methods can lead to non-accurate results for highly nonlinear problems, in this paper the results obtained by FORM are compared with those obtained by Importance Sampling Monte Carlo.

It is also common in the literature to employ surrogate models in order to reduce the computational burden, especially when Monte Carlo methods are applied. In this case, the original limit state function is usually replaced by a less time-consuming surrogate model, which is constructed considering as few as possible evaluations of the original model. Some examples of surrogate models are: polynomial response surfaces, kriging and artificial neural networks (Soares *et al.*, 2002; Gomes & Awruch, 2004; Bucher & Most, 2008; Echard *et al.*, 2013; Gomes & Beck, 2013). This strategy seems to be very promising, particularly when the surrogate model accuracy can be increased as much as needed, which is the case, for example, of artificial neural networks (ANNs). The author intends to apply ANNs in the solution of reliability analysis of RC structures in future works.

The remainder of this paper is organized as follows. Section 2 presents, in a simplified manner, the general formulation for nonlinear finite element structural analysis. In section 3, the structural reliability problem is defined. Section 4 describes the finite element models developed and presents the calibration process considering experimental data from the literature. Reliability analyses of both models are performed in section 5. Some conclusions and discussions about the results are presented in section 6.

2 NONLINEAR FINITE ELEMENT ANALYSIS

As stated in Val *et al.* (1997), choice of the structural model depends both on the requirement of accuracy and on that of simplicity. For a reliability analysis, the structural model should be accurate enough to properly distinguish failure and survival, but also needs to present an acceptable computational cost.

For reinforced concrete structures, a proper representation of the structural behavior may demand consideration of many complex aspects. Among these aspects, the present paper deals with yielding of reinforcement and the different behaviors of concrete when subject to tensile or compressive stresses, but does not take into account, for example, debonding of reinforcement, degradation and creep. The structural models adopted herein consist of nonlinear finite element models (Bathe, 1996; Belytschko *et al.*, 2000).

In the static structural analysis by means of displacement-based finite element formulations, equilibrium is usually described by a system of linear equations which can be written as:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{f}, \tag{1}$$

where **K** is the global stiffness matrix, **u** is the vector of nodal point displacements and **f** is the vector of global nodal forces.

The global stiffness matrix is obtained by combining the individual element stiffness matrices, which depend on geometrical properties and on the constitutive laws of the materials involved. After applying the boundary conditions of the problem, the vector of displacements can be determined by solving Eq. (**1**), and this information can be used to compute other quantities of interest; for example, stresses and strains.

In many real case problems, proportionality between displacements and loads may not exist, which means that the stiffness matrix may depend considerably on the displacements. This is the case, for example, if the effects of deformations and finite displacements are included in the formulation of the equilibrium equations, which is called geometric nonlinearity, or if nonlinear constitutive laws are adopted, which leads to changes in member material properties under load and is called material or physical nonlinearity (McGuire *et al.*, 2014). For the problem considered herein, the effects of material nonlinearity are much more pronounced than those of the geometric nonlinearity, thus, only the former is considered. This occurs due to the boundary conditions, the specific configurations and characteristics of the RC beam at hand; but it may not be true for different boundary conditions or more complex structures.

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By taking into account the material nonlinearity, equilibrium is now expressed by a system of nonlinear equations, in such a way that Eq. (**1**) becomes:

$$\mathbf{K}(\mathbf{u})\cdot\mathbf{u}=\mathbf{f}\ . \tag{2}$$

Solution of Eq. (2) is usually obtained by means of iterative methods, such as the Newton-Raphson method (Bathe, 1996; McGuire *et al.*, 2014), adopted herein.

3 STRUCTURAL RELIABILITY FORMULATION

Let **X** be a vector of random variables which represents all random or uncertain parameters of a structural system. Existence of uncertainty implies possibilities of undesirable structural responses. The boundary between desirable and undesirable structural responses is defined by limit state functions $g(\mathbf{X})$, in such a way that failure and survival domains, Ω_f and Ω_s, respectively, are given by:

$$\begin{aligned}\Omega_f&=\{\mathbf{x}\mid g(\mathbf{x})\le 0\}\\ \Omega_s&=\{\mathbf{x}\mid g(\mathbf{x})> 0\}\end{aligned}\ . \tag{3}$$

Each limit state describes one possible failure mode of the structure. For each failure mode, the probability of undesirable structural responses, usually known as probability of failure, is defined as:

$$P_f=P\left[\mathbf{X}\in\Omega_f\right]=\int_{\Omega_f} f_{\mathrm{X}}(\mathbf{x})d\mathbf{x}\ . \tag{4}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function of vector **X**.

Probabilities of failure can be evaluated by means of structural reliability methods such as First and Second Order Reliability Methods (FORM and SORM) and Monte Carlo simulation (Madsen *et al.* 1986; Melchers 1999).

When simple Monte Carlo simulation (SMC) is employed, failure probabilities can be estimated via Eq. (5), by randomly generating n_{samp} samples of **X** according to its joint distribution $f_{\mathbf{X}}(\mathbf{x})$ and by considering a so-called indicator function, $I[\mathbf{x}]$, which is equal to one if **x** belongs to the failure domain and zero otherwise. It is noted that application of Eq. (**5**) requires one limit state function evaluation per sample. Besides, the smaller the failure probability to be estimated, the larger the number of samples required to achieve an acceptable accuracy. As engineering structures usually present very small probabilities of failure, the computational burden easily becomes prohibitive.

$$P_f = E\left[I\left[\mathbf{X}\right]\right] \cong \frac{1}{n_{samp}} \sum_{i=1}^{n_{samp}} I\left[\mathbf{x}_i\right]. \tag{5}$$

SMC is, in general, less efficient than FORM or SORM, but it can be easily applied to any kind of reliability problem. In order to improve its efficiency, many versions of the Monte Carlo method have been proposed in the literature, as can be seen, for example, in Engelund & Rackwitz (1993), Au & Beck (2001) and Santos & Beck (2015). In this paper, the Importance Sampling Monte Carlo method (ISMC) is adopted (Engelund & Rackwitz, 1993), since it is almost as general as, and more efficient than, the simple Monte Carlo method.

In ISMC, the samples are generated according to a sampling function, $h_{\mathbf{X}}(\mathbf{x})$, and the indicator function must be weighted so the probability with respect to the original joint distribution can be obtained. Equation (5) becomes:

$$P_f \cong \frac{1}{n_{samp}} \sum_{i=1}^{n_{samp}} I\left[\mathbf{x}_i\right] \frac{f_{\mathbf{X}}(\mathbf{x}_i)}{h_{\mathbf{X}}(\mathbf{x}_i)}. \tag{6}$$

The sampling function can be constructed by using information about design points, obtained by FORM, as described in Bourgund & Bucher (1986), which is the procedure adopted herein. By using a sampling function defined this way, the number of samples required by IMSC to compute the P_f with a given accuracy is, for most cases, many orders of magnitude less than that required by SMC. Considering that in this procedure the reliability problem is initially solved by FORM, the results obtained by this method are also presented in section 5, for comparison purposes.

4 FINITE ELEMENT MODELS

The problem at hand consists of a reinforced concrete beam subject to a four-point bending test, as illustrated in Figure 1. It was the object of study of Álvares (1993), where various experiments were carried out, for three different steel ratios, in order to validate the application of the Mazars' damage model (Mazars, 1984) to represent the concrete behavior. In the present paper, only the two experiments related to the specific amount and configuration of reinforcement bars depicted in Figure 1 are considered.

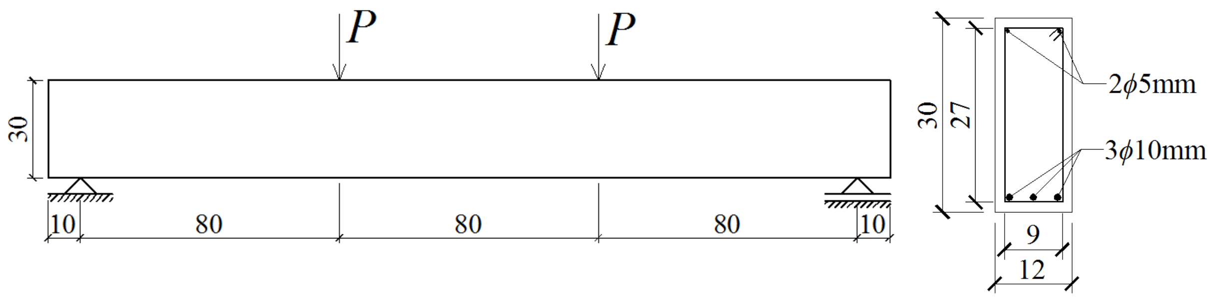

Figure 1. Reinforced concrete beam (dimensions in cm, except the diameters of the rebars).

The reinforced concrete beam is modelled by using three-dimensional finite elements. Symmetry conditions are considered to reduce the size of the model, in such a way that a quarter-symmetry model, with the discretization shown in Figure 2, is employed. The discretization consists of 2052 elements and 3151 nodes. The load is applied at the top of a plate made of linear elastic structural steel and a similar plate is used to represent the support, for which rotations are allowed. For the sake of simplicity, stirrup reinforcement bars are not included in the model, although they were present in the experiments.

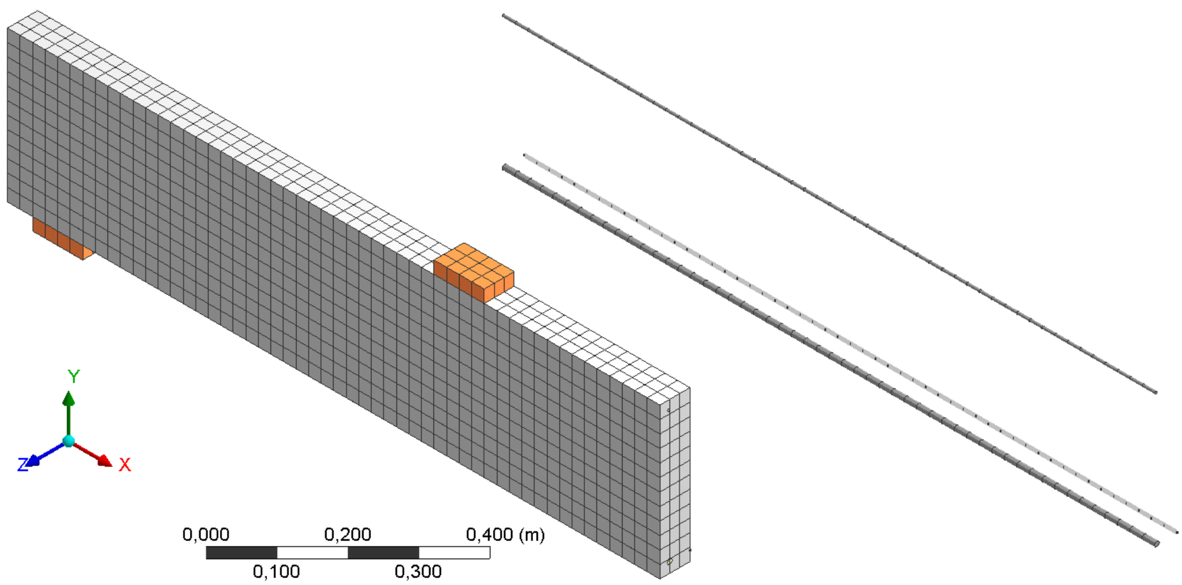

Figure 2. Discretization of concrete and rebars (quarter symmetry model).

The two models developed herein differ only by the type of element and formulation used to model the concrete. In both cases the steel reinforcement is modeled by LINK180 elements, considering bilinear isotropic materials and the Von Mises failure criteria. Thus, the stress-strain diagram for the material of the rebars is described by the Poisson's ratio, ν_{steel}, Young's modulus, E_{steel}, tangent modulus, Et_{steel}, and yield stress, f_y.

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The first model employs the SOLID65 element to represent the concrete behavior, combining the Willam-Warnke (Willam & Warnke, 1975) and the Von Mises criteria to describe the material response under tension and compression, respectively. For compression, a multilinear isotropic stress-strain curve recommended by EUROCODE 2 (*apud* Kotinda, 2006) is used. The parameters required in this case are: Poisson's ratio, ν_{concr}, Young's modulus, E_{concr}, tangent modulus, Et_{conc}, shear transfer coefficients for an open crack, s_1, shear transfer coefficients for a closed crack, s_2, , uniaxial compressive strength, f_c, uniaxial tensile cracking stress, f_t, and uniaxial crushing stress. The latter is adopted equal to −1, so the crushing capability is removed and the Von Mises criterion is adopted for compression. According to the literature (*e.g.* Kotinda, 2006), adoption of the Von Mises criterion helps to avoid convergence issues.

The second model employs the SOLID185 element and the so-called Microplane model to represent the concrete material response. The Microplane model implemented in ANSYS is based on Bazant & Gambarova (1984) and Bazant & Oh (1985). According to ANSYS (2014), in this formulation "the material behavior is modeled through uniaxial stress-strain laws on various planes and directional-dependent stiffness degradation is modeled through uniaxial damage laws on individual potential failure planes, leading to a macroscopic anisotropic damage formulation". The following parameters are required by this model: Poisson's ratio, ν_{concr}, Young's modulus, E_{concr}, , uniaxial compressive strength, f_c, uniaxial tensile cracking stress, f_t, and three damage function constants, k_0, k_1 and k_2.

For a given set of values for the parameters, the models are solved by applying the Newton-Raphson method, as stated before. However, in order to capture the nonlinear behavior of the structural response, the load intensity is multiplied by a non-dimensional load factor, λ, which varies from 0 to 1. The number of load steps is controlled by ANSYS, considering an initial number of substeps equal to 60 and the interval defined by a minimum of 60 and a maximum of 500 substeps. Also, convergence in many cases is only possible if just one equilibrium iteration is allowed per substep, so the number of equilibrium iterations per substep is set to one.

The parameters of the models are determined as explained in the next section.

4.1 Calibration of the models

Most of the parameters of the models are calibrated first by trial and error, and after that by an optimization procedure, using the *fmincon* function of MATLAB. The values of these parameters are determined by minimizing the differences between experimental results, obtained from the literature, and numerical responses, in a least squares way. Upper and lower bounds applied for each parameter during the optimization process are defined by their initial value ±25%. Table 1 summarizes the parameters and respective values. Those for which just one value is given are not calibrated; instead, values are taken from Álvares (1993) or from other references from the literature.

Table 1. Parameters of the models

Parameters of	Solid65		Microplane	
	Initial value	After calibration	Initial value	After calibration
Steel	$\nu_{steel} = 0.3$		$\nu_{steel} = 0.3$	
	$E_{steel} = 196\text{GPa}$		$E_{steel} = 196\text{GPa}$	
	$Et_{steel} = 39.2\text{GPa}$	$Et_{steel} = 39.2\text{GPa}$	$Et_{steel} = 39.2\text{GPa}$	$Et_{steel} = 51.63\text{GPa}$
	$f_y = 500\text{MPa}$	$f_y = 500\text{MPa}$	$f_y = 350\text{MPa}$	$f_y = 350\text{MPa}$
Concrete	$\nu_{concr} = 0.2$		$\nu_{concr} = 0.2$	
	$E_{concr} = 29.1\text{GPa}$		$E_{concr} = 29.1\text{GPa}$	
	$f_c = 27\text{MPa}$		$f_c = 27\text{MPa}$	
	$f_t = 3.6\text{MPa}$	$f_t = 3.6\text{MPa}$	$f_t = 2.7\text{MPa}$	$f_t = 2.162\text{MPa}$
	$s_1 = 0.7$	$s_1 = 0.787$	$k_0 = 6\times10^{-5}$	$k_0 = 6.11\times10^{-5}$
	$s_2 = 0.9$	$s_2 = 0.9$	$k_1 = 0.50$	$k_1 = 0.513$
			$k_2 = 100$	$k_2 = 98.547$

A comparison of numerical and experimental results is illustrated in Figure 3, in terms of displacement at midspan *versus* applied load. During the calibration of the Solid65 model, many convergence problems occurred, making it difficult the calibration and indicating that this model may be not suitable for reliability analysis purposes, at least for the case at hand. Although a good representation of the structural behavior is achieved by both models, the final root-mean-square error for the Solid65 model is about twice the Microplane model error.

The experimental structural behavior presented in Figure 3 is widely discussed in the literature (Álvares, 1993; Leonel *et al.*, 2003; Arivalagan, 2012). The first part of the load-displacement curve corresponds to an almost linear response of both materials. Once flexural cracks appear, a change of slope is observed and this slope remains almost linear until yielding of steel reinforcements and crushing of concrete take place. Most of the convergence problems experienced by the Solid65 model seem to occur during the cracking initiation in concrete, due to the changes on the stress-strain relations for the material in the direction perpendicular to the crack face. On the other hand, the microplane theory leads to a more gradual change of slope during the cracking process. Even though, the results indicate that for both models the behaviors of concrete and steel are properly described.

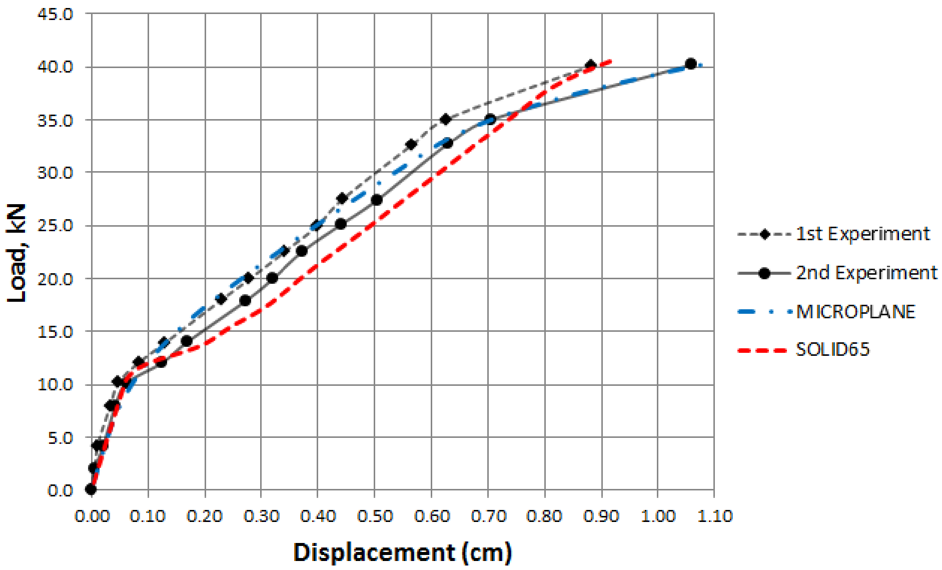

Figure 3. Experimental and numerical force-displacement diagrams

5 RELIABILITY ANALYSES

Information about failure of the beams tested in Álvares (1993) was not found. Thus a simplified limit state function is constructed, assuming that failure occurs whenever the displacement at midspan exceeds a critical value, d_{crit} = 0.9712cm, given by the average of the experimental maximum displacements. Although very simple, this kind of limit state function was already employed in similar analyses in the literature (*e.g.*, Torii & Machado, 2010; Farag & Haldar, 2016). The limit state function can be written as:

$$g\left(\mathbf{X}\right)=d_{crit}-d_{\max}\left(\mathbf{X}\right). \tag{1}$$

where d_{max} is the deflection of the beam at midspan, which depends on the random variables, **X**, of the problem.

Table **2** summarizes the random variables adopted for the reliability analyses, which are: the load, P; the uniaxial compressive strength of concrete, f_c; and the yield stress of steel, f_y. Also, variation of the load level is performed by multiplying it by a load factor, α, different from that applied in the solution of the nonlinear analysis. Thus, the vector of random variables for each load level is given by $\mathbf{X}=\{\ \alpha{\cdot}P, f_c, f_y\}$, where α varies from 0.4 to 1.0 in increments of 0.1. Furthermore, it is assumed that all random variables follow normal distributions and the parameters of the distributions are based on Santos *et al.* (2014).

Table 2. Probability distributions and theirs parameters (mean μ and coefficient of variation *c.o.v.*)

Random Variables	Distribution type	μ	*c.o.v.*
Load, P	Normal	30 kN	0.2
Uniaxial compressive strength of concrete, f_c	Normal	31.59 MPa	0.15
Yield stress, f_y	Normal	540 MPa$^{(1)}$ 378 MPa$^{(2)}$	0.05

$^{(1)}$Solid65 model; $^{(2)}$Microplane model.

Figure 4 illustrates the failure probabilities obtained for each load level. The Solid65 model presented convergence problems for all load levels, in such a way that it was not possible to find the design point during the application of FORM; thus a proper sampling function for ISMC was not constructed. As a result, all failures probabilities computed using the Solid65 model are all equal to 1.

On the other hand, results obtained for the Microplane model seem to be coherent: the failure probability increases as the load increases, as expected. Also, for all load levels it was possible to solve the reliability problem by means of FORM and ISMC, and the Microplane model did not experience any convergence issue.

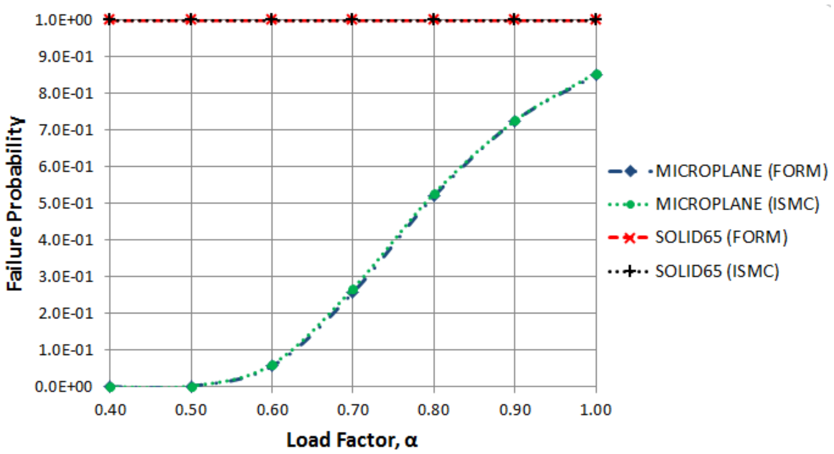

Figure 4. Failure probabilities for different load levels.

Finally, although the problem at hand presents a high degree of nonlinearity in terms of load *versus* displacement, the limit state function adopted results almost linear, at least in the vicinity of the most probable point, in such a way that very similar results are obtained by using FORM or ISMC.

6 CONCLUSIONS

In this paper, two different finite element models, called Solid65 and Microplane models, were developed in ANSYS, in order to assess the reliability of a reinforced concrete beam subject to a four-point bending test.

The parameters of both models were calibrated by means of a least squares scheme, taking into account experimental results from the literature. During the calibration, the Solid65 model experienced some convergence issues and, although both models were able to adequately represent the experimental load-displacement curves, it presented a much larger error than that obtained by the Microplane model.

After calibration, both models were employed in a reliability analysis, considering a limit state function defined by a maximum allowable displacement. Due to convergence problems of the Solid65 model, it was not possible to determine failure probabilities for this case; on the other hand, the results obtained by using the Microplane model were as expected and this model did not present convergence issues. This indicates that the Microplane model is suitable for reliability analysis of RC structures, while the Solid65 model is not.

Finally, despite the highly nonlinear structural response of the RC beam, the limit state function adopted herein results almost linear in the regions that most affect the failure probabilities. Thus, less time-consuming methods such as FORM can be employed in the solution of this kind of reliability analysis problem; there is no need to use "exact" methods such as the Importance Sampling Monte Carlo method.

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