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REVERSE TIME MIGRATION IN VTI MEDIA USING PSEUDO-ACOUSTIC APPROXIMATIONS IN STAGGERED GRID

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Abstract. *Algorithms based on finite-difference method (FDM) using staggered grids are the most successfully applied for seismic modeling. Although the elastic theory can better represent wave propagation than acoustics, a central issue is that it is difficult to estimate all the physical parameters involved in geophysical applications, especially in anisotropic media. Additionally, this schemes demand much more CPU time and memory than acoustic schemes and, in many applications, they presuppose the use of algorithms for decoupling quasi-P and quasi-S modes, e.g., in reverse time migration (RTM). Hence, an acoustic algorithm is preferred to be used in RTM. Recently, it has been proposed equivalent staggered-grid schemes (ESG) to solve the second-order wave equations in terms of only one wavefield, resulting in schemes with lower memory requirement than the ordinary staggered-grid schemes. This paper develops a prestack RTM algorithm using ESG scheme applied to acoustic VTI media. First, the ESG numerical scheme is obtained for forward problem using coupled second-order equations in terms of normal stress components. Second, this algorithm is used to implement reverse time migration algorithms applying the excitation-time imaging condition. The developed algorithm is applied to a synthetic example, a complex model known as Hess VTI model.*

Keywords: *Reverse time migration, Seismic migration, Finite difference method, staggered grid, VTI pseudo-acoustic equations*

1 INTRODUCTION

Algorithms based on the finite-difference method (FDM) using staggered grids (Virieux, 1986) are recognized as ones of the most successfully applied for seismic modelling because of its superior numerical responses. The same grid proposed by Virieux (the standard staggered grid, SSG) is suitable to model 3D anisotropic elastic media up to the orthotropic symmetry. Although the elastic theory can better represent wave propagation than acoustics, a central issue is that it is difficult to estimate all the physical parameters involved in elastic propagation, especially in anisotropic media, and this schemes demand much more CPU time and memory than the acoustic schemes. In many applications, elastic algorithms presuppose the use of some algorithm for decoupling quasi-P and quasi-S modes, e.g., in reverse time migration (RTM) algorithms (Baysal et al., 1983). The decoupling problem (Yan and Sava, 2009) by itself is an open problem and the available algorithms are very CPU demanding. Hence, an acoustic (or pseudo-acoustic) algorithm is preferred to use in RTM algorithms, as made in the present paper.

Many authors have proposed acoustic formulations in VTI media and many of them have applied it to RTM (Alkhalifah, 2000; Duveneck et al., 2008). These algorithms intend to model only the quasi-P wave elastic mode, reducing the number of parameters involved and increasing the computational efficiency. The first algorithm was proposed by Alkhalifah (2000): it is based on a fourth-order partial differential equation with mixed space and time derivatives and, actually, it is a pseudo-acoustic algorithm (because of the existence of p-S wave artifact) and the FDM with simple grid (non-staggered grid schemes, NSG) is used to solve the equation. Other authors have proposed advantageous second-order coupled equations without mixed derivatives. These equations can be implemented easily in higher-orders. Di Bartolo et al. (2012) proposed an equivalent staggered-grid scheme (ESG) to solve the second-order acoustic wave equation for general heterogeneous media. Recently, Di Bartolo et al. (2015) developed the elastic ESG for application directly to second-order full-elastic equations for general anisotropic media and Di Bartolo et al. (2017) developed a hight-order ESG scheme for second-order equations. The proposed ESG results in schemes with lower memory requirement than the ordinary staggered-grid schemes (OSG) (Duveneck et al., 2008), providing equivalent responses.

This paper develops a prestack RTM algorithm using ESG scheme applied to acoustic VTI media. The numerical scheme for forward modeling of wave propagation is obtained applying fourth-order ESG approximations to a coupled second-order VTI equations in terms of normal stress components and this algorithm is the base of the RTM algorithms. The next section develops the mathematical descriptions for pseudo-acoustic VTI wave equations obtained using physical principles. The subsequent section develops the ESG staggered-grid scheme applied to obtain numerical solutions of the wave equation and the same formulation is applied to RTM algorithm. Then, the RTM is discussed and a numerical example is presented — the migration of the complex synthetic model known as Hess VTI model. Finally, conclusions are addressed.

2 MATHEMATICAL MODELING: WAVE EQUATIONS

From physical principles, we can obtain pseudo-acoustic VTI equations as the first-order system (?Duveneck et al., 2008)

$$\frac{\partial}{\partial t}\begin{pmatrix} \tau_H \\ \tau_V \end{pmatrix} = \begin{pmatrix} C_{11} & C_{13} \\ C_{13} & C_{33} \end{pmatrix} \begin{pmatrix} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ \frac{\partial v_z}{\partial z} \end{pmatrix}, \tag{1}$$

$$\rho\frac{\partial}{\partial t}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} \frac{\partial \tau_H}{\partial x} \\ \frac{\partial \tau_H}{\partial y} \\ \frac{\partial \tau_V}{\partial z} \end{pmatrix}, \tag{2}$$

where (τ_H, τ_V), (v_x, v_y, v_z) and C_{ij} are the components, respectively, of stress tensor, particle oscillation velocity and elasticity tensor; ρ is the density of matter of the propagation medium. Equations (1) and (2) are the double-field equations (without source term) in terms of velocity and normal stress components and describe the pseudo-acoustic wave propagation in VTI three-dimensional media.

Equations in terms of only stress components can be obtained by eliminating the velocity components in Eq. (1)-(2), resulting in

$$\frac{\partial^2 \tau_H}{\partial t^2} = C_{11}\left[\frac{\partial}{\partial x}\left(b\frac{\partial \tau_H}{\partial x}\right) + \frac{\partial}{\partial y}\left(b\frac{\partial \tau_H}{\partial y}\right)\right] + C_{13}\frac{\partial}{\partial z}\left(b\frac{\partial \tau_V}{\partial z}\right), \tag{3}$$

$$\frac{\partial^2 \tau_V}{\partial t^2} = C_{13}\left[\frac{\partial}{\partial x}\left(b\frac{\partial \tau_H}{\partial x}\right) + \frac{\partial}{\partial y}\left(b\frac{\partial \tau_H}{\partial y}\right)\right] + C_{33}\frac{\partial}{\partial z}\left(b\frac{\partial \tau_V}{\partial z}\right), \tag{4}$$

where b (“buoyancy”) is the inverse of the density ($b = 1/\rho$). It is important to note that, as long as one is not interested in using density gradient for modeling, migration or another application, the equations to be considered are simplified to

$$\frac{\partial^2 \tau_H}{\partial t^2} = c_{Px}^2\left[\frac{\partial^2 \tau_H}{\partial x^2} + \frac{\partial^2 \tau_H}{\partial y^2}\right] + c_{Pz}c_{Pn}\frac{\partial^2 \tau_V}{\partial z^2}, \tag{5}$$

$$\frac{\partial^2 \tau_V}{\partial t^2} = c_{Pz}c_{Pn}\left[\frac{\partial^2 \tau_H}{\partial x^2} + \frac{\partial^2 \tau_H}{\partial y^2}\right] + c_{Pz}^2\frac{\partial^2 \tau_V}{\partial z^2}. \tag{6}$$

These equations are very similar to the coupled equations equivalent to classical pseudo-acoustic equation (Fowler et al., 2010).

3 NUMERICAL MODELING

Figure 1 presents a 2D version of the staggered grid used here to solve wave propagation in VTI for the pseudo-acoustic case: a staggered grid very similar to that proposed by Virieux but with appropriate changes. Previously, Duveneck et al. (2008) reported the use of staggered-grid scheme for RTM. This scheme uses the staggered-grid presented in Figure 1 and applies traditional staggered-approximations to the first-order equations (named here ordinary staggered-grid scheme, OSG). In this paper, we use the equivalent staggered-grid scheme (ESG), a scheme applied directly to second-order pseudo-acoustic equation. This scheme has lower memory requirement than OSG (in three-dimensional case), providing equivalent numerical responses. It is noteworthy that staggered-grid schemes present enhanced numerical properties in relation to non-staggered FD schemes.

The properties and wavefields are considered in the same positions as the OSG, except the velocity wavefield (not considered), as depicted in Fig. 1. The ESG approximations in time is:

$$\left(\frac{\partial^2 q}{\partial t^2}\right)_{i,j,k}^{n} \cong \frac{Q_{i,j,k}^{n+1} - 2Q_{i,j,k}^{n} + Q_{i,j,k}^{n-1}}{\Delta t^2}. \tag{7}$$

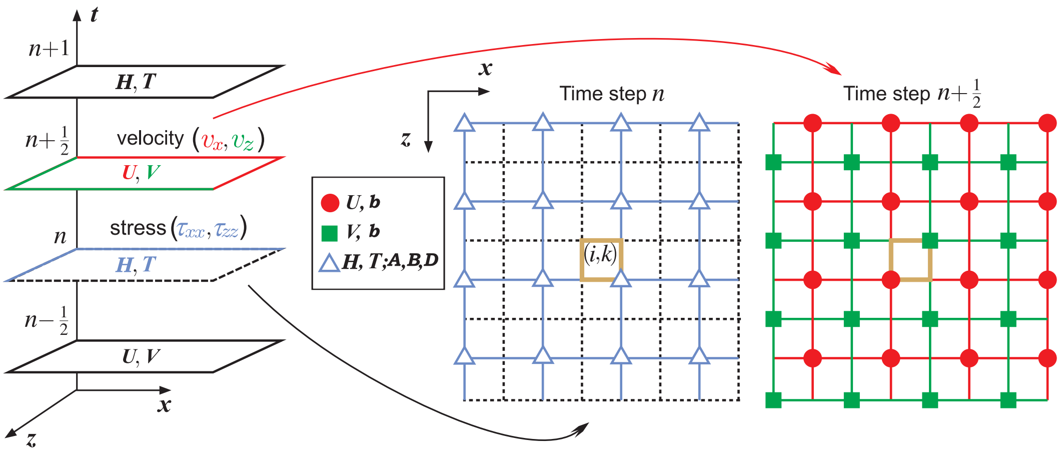


Figure 1: VTI pseudo-acoustic staggered grid: $(U, V) = (v_x, v_z)$**,** $(H, T) = (\tau_H, \tau_V)$ **and** $(C_{11}, C_{13}, C_{33}) = (A, B, D)$ **are the discrete components, respectively, of velocity, normal stress and elasticity tensor.**

All the approximations are made in the grid point $(t, x, y, z) = (n\Delta, ih, jh, kh)$. The spatial approximations adopted are fourth-order. The general approximation for second-order derivatives is very complex (**?**). However, instead using Eq. (3)-(4), we are interested in using the Eq. (5)-(6), that generates a faster scheme. In this case, for example, in the x-direction, the approximation is (Di Bartolo et al., 2012)

$$\left(\frac{\partial^2 q}{\partial x^2}\right)_i \cong \frac{1}{576h^2}\left[(Q_{i+3} + Q_{i-3}) - 54(Q_{i+2} + Q_{i-2}) + 783(Q_{i+1} + Q_{i-1}) - 1460Q_i\right]. \tag{8}$$

The ESG scheme obtained applying the approximations above is completely non-staggered and it has greater stencil than the schemes of the same order using single grid. Additionally, absorbing boundary conditions proposed by Cerjan et al. (1985) were applied to avoid reflection from the numerical boundaries.

4 REVERSE TIME MIGRATION

In this section — within the scope of the geophysical exploration method known as seismic method — the fundamental ideas about the imaging using the wave equation will be presented. In this method, the recorded energy acquired in seismic exploration survey is propagated backward in time and a imaging condition is applied. This *migration method* is called reverse time migration (RTM). Here it will be discussed prestack RTM, i.e., the imaging applied to each shot gather individually; the final image is obtained stacking all individual images. In the sequence, the changes in the algorithm of direct modeling will be discussed and the use of a imaging condition to generate the images associated with reflectors where impedance contrast is found.

In this work, unlike the standard procedure performed routinely in oil industry, the algorithms will be applied to synthetic data, generated with direct numerical modeling algorithms applied to synthetic models. This type of migration (synthetic) is widely used to validate new algorithms and is important in other many applications.

Mathematically, the problem of imaging is an inverse problem, as opposed to the direct problem of wave propagation. In the direct problem, the properties of the medium are known and the responses related to wave propagation in this medium can be obtained. In the inverse problem, the response of propagation of the wave in the model is known and we want to use it, inversely, to obtain information about the properties of medium.

Actually, the problem of imaging within the scope of the seismic method discussed in this work is an approximate inverse problem, since it uses what is called a velocity macromodel as a prior information. The main objective of the different methods applied for the imaging in seismic method, known generically as seismic migration methods, is repositioning seismic energy measured at receivers to its correct subsurface positions, removing the distortions introduced by wave propagation. That is, it is aimed to obtain an image of the reflectors and diffractors in subsurface, which in last instance is related to the local geology.

In a typical off-shore seismic acquisition campaign, typically hundreds of shots covering extensive regions are collected (e.g., see Fig. 3). The receivers record reflections, diffractions, etc. from the subsurface. The receivers and source are moved together the seismic ship. After seismic data has been recorded at the field, a seismic processing is applied in order to attenuate noise and amplify reflection signal. One of the processes carried out aims to estimate the velocities of the rock layers. This velocity model can be used for migration or some more advanced procedure for generating the final model can be applied, e.g., tomography. It is important to mention that image generated using the obtained velocity model provides clues regarding the quality of the used model, for example by checking the "focus" of the images or the existence of reflectors intersecting. When applied in the field data, it is common that this process of imaging is made more than once. A geophysical professional follows this process and at the same time is responsible for the geological interpretation of the final image generated by migration. This professional makes the decision about the location of the exploratory well.

There are many different migration methods, one of the most popular being the so-called Kirchhoff migration, a technique that uses an integral solution of the waves propagation problem. It is widely used because it is an extremely fast technique where integral solutions are implemented very efficiently through summation, without time marching. However, this technique has problems with models with hight degree of complexity, such as those associated to regions involving salt bodies. In these cases, the seismic migration methods based on the wave equation have becoming increasingly more common. Nowadays, the development of computation power (clusters) and the increasing exploratory challenges, made this techniques important in practice. These methods are known generically by wave equation migration methods (WEM). In these methods, the modeling and migration algorithms are used together, presenting exactly the same characteristics.

In this work, we apply the complete acoustic wave equation, solved in the time domain, which characterizes the so-called reverse time migration (RTM). Specifically, we use the technique that migrates separate shots and stacks the images resulting from each shot at the end to obtain the final image, i.e., the pre-stacking technique. Pre-stacking migration presents much better results than the post-stacking technique, where the data is stacked before applying the migration algorithm. The stacking process applied before the migration suffers from serious problems in focusing energy in complex regions, which ultimately degrade the final image. However, since the post-stacking migration technique is applied only once (to the entire stacked section), it demands much less computing time than the pre-stacking migration, being chosen in some practical cases for this advantage.

In the sequence, the steps of the pre-stack reverse migration are detailed. The first stage of the RTM consists of the propagation of the wave forward in time, where the transit times of the direct wave at each point in the domain are computed and recorded. In the second stage, the shot gather is reintroduced into the domain, by applying the values measured by the receivers as the source (in the position where they were measured) and propagated in the reverse direction of the time. An image for this second step is the following: considering the wave propagation from the source to the receivers as a movie, the second step is nothing more than the movie backward in time, finishing exactly with the energy at the place of source. In this step, an image condition is applied, making the propagation energy stops at the place where it was generated by reflection or diffraction. In other words, the image condition is responsible for position the energy in its correct subsurface position.

The image condition establishes that an image point P is any one in which the propagation wave time is equal to the time of the direct wave (calculated and recorded at the first step). It is called excitation-time imaging condition, represented schematically in Fig. 2.

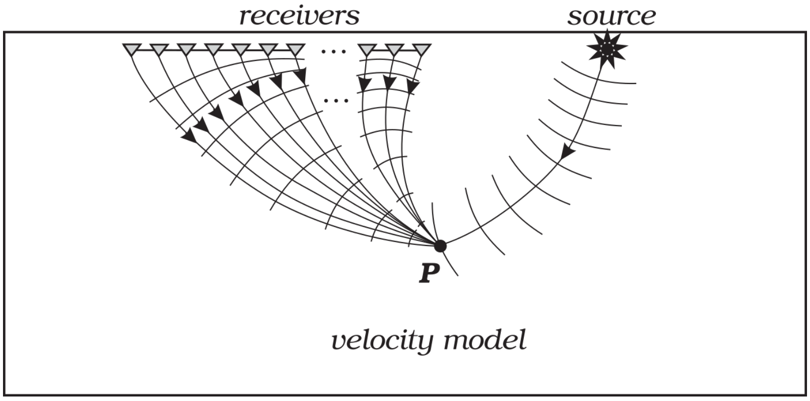


Figure 2: Excitation-time imaging condition.

5 NUMERICAL EXAMPLE

Now we present the application of the ESG algorithm to prestack RTM: we have made a synthetic acquisition of 160 shots and applied the RTM algorithm to each shot, before stacking. We used the synthetic VTI Hess model (Fig. 3) and applied the RTM algorithm discussed above. The synthetic acquisition was made using pseudo-acoustic algorithm including surface multiples and no pre-processing was carried out before migration. The geometry used is depicted in Fig. 3: doted line R represent the position of the streamer of receivers (for the first shot) and Tn are the shot positions (for shot n). All acquisition spaces are $\delta = 70$ (spacing between receivers, between shots and the minimum offset) and the size of the streamer is 2100 m (30 channels). The first shot was located at 2100 m (the last at 13300 m). The total time of acquisition was $t = 3.42$ s. The numerical parameters used were $h = 7$ m, $\Delta t = 3.8 \times 10^{-4}$ s, and hence, $N_t = 9000$ and $(N_x, N_z) = (2001, 601)$. A Ricker pulse (second derivative of Gaussian function) with cutoff frequency of 40 Hz was applied as source.

The synthetic acquisition generated 160 shot gathers. Figure 4 shows the snapshots for the a central shot (number 60) at different times and Fig. 5 shows some seismograms (shot gathers) for shots indicated in Fig. 3. This shot gathers were migrated using the RTM algorithm discussed above, as will be detailed below.

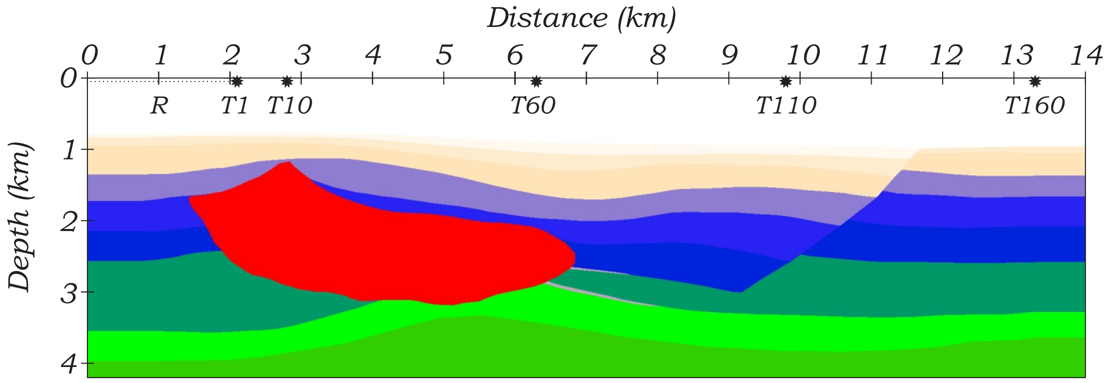


Figure 3: VTI Hess model and geometry: R **and** Tn **represent, receptively, the receivers and the shots.**

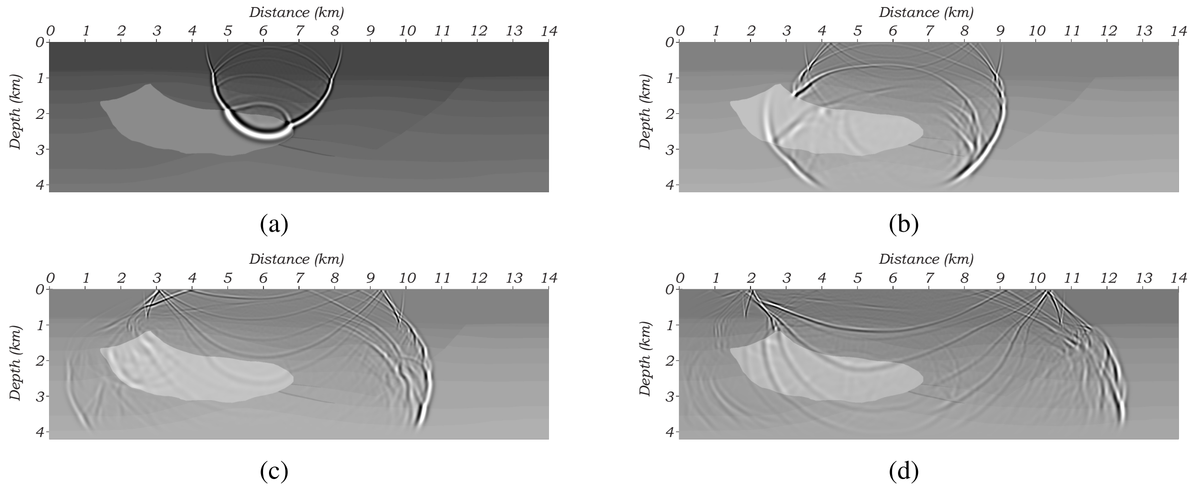
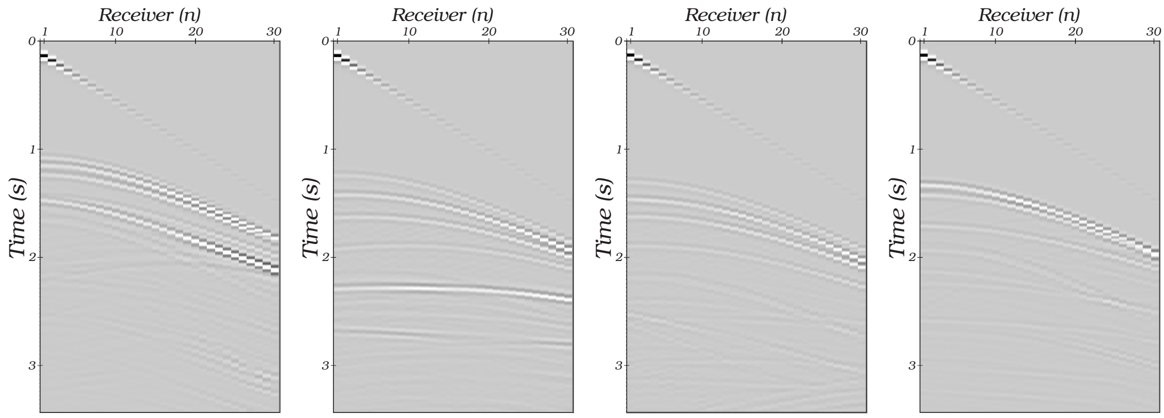


Figure 4: Snapshots of the shot 60 at the times: (a) $t = 1.33\,\mathrm{s}$**, (b)** $t = 1.90\,\mathrm{s}$**, (c)** $t = 2.47\,\mathrm{s}$ **e (d)** $t = 3.04\,\mathrm{s}$**.**

The migration algorithm applied here is based on the ESG formulation of fourth order. The first step of RTM, the forward modeling, applied a smoothed velocity model to obtain the transit times. It is noteworthy that the smoothing was applied to the slowness model (the inverse of the velocity model) to avoid the same time the distortion of the transit times and the interference between reflected waves, that are attenuated in smoothed model. The condition used to obtain the transit times was to consider the time of the wave with great amplitude in the proximities of the first break. Figure 6 shows the transit times obtained for the shot 60, where can be seen the better continuity in the smoothed model.

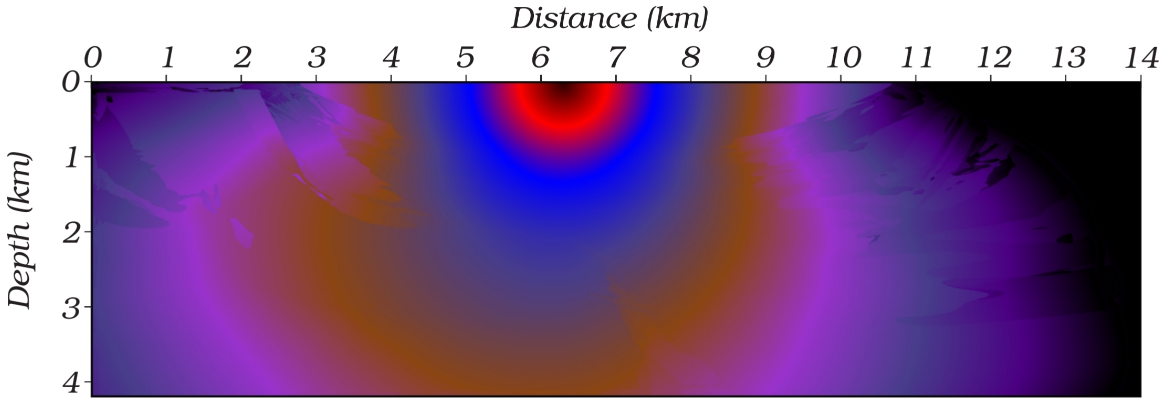
The second step of the RTM also applied a smoothed model. It uses the obtained transit times to apply the image condition and obtain the image of each shot. Some images of selected shots are shown in Fig. 7, where can be seen the images of the reflectors near the shot.

The final image, obtained after stacking 160 shots, is shown at Fig. 8. Al the reflectors were clearly imaged by the algorithm, including reflectors below the salt dome. There is only a minor shadow zone left bottom the salt. We stress the fact that it is a raw image and no filtering was applied to that to attenuate the known noise generated by RTM image.

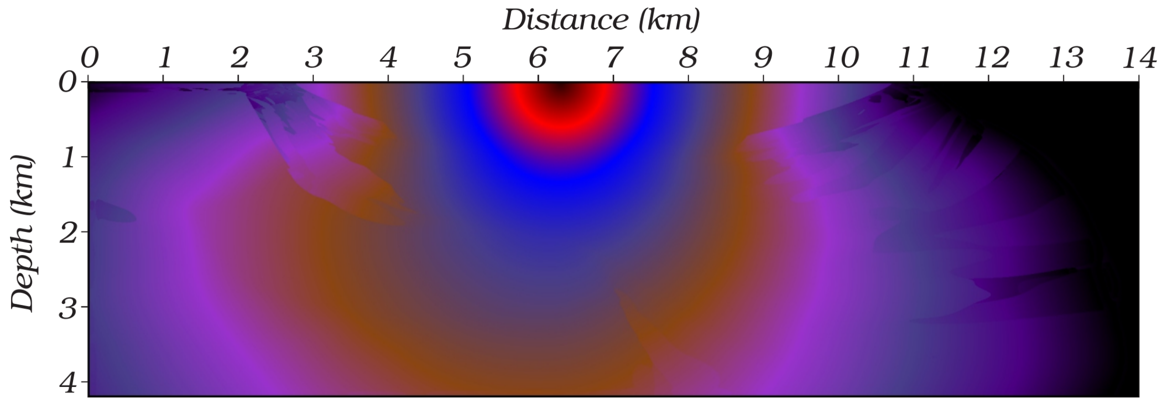


(a) (b) (c) (d)

Figure 5: Seismograms of the shots: (a) 10, (b) 60, (c) 110, (d) 160.



(a)



(b)

Figure 6: Transit times for shot 60 using velocity model (a) no smoothed and (b) smoothed.

6 CONCLUSION

We applied a finite-difference staggered-grid scheme to reverse time migration in transverse isotropic media using pseudo-acoustic wave equations. The numerical scheme used is the ESG, that provide numerically equivalent responses that most common staggered-grid scheme. A clear advantage of ESG applied here is its lower memory requirement (20% less) compared to

(a) (b) (c) (d)

Figure 7: Images of individual shots shown at the Fig. 3, i.e., shots (a) 10**, (b)** 60**, (c)** 110 **e (d)** 160**.**

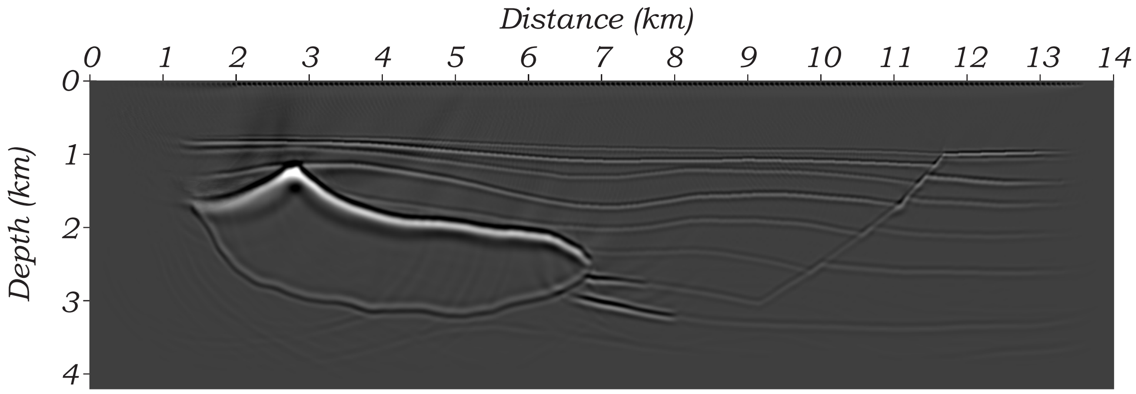


Figure 8: Final image of the Hess VTI after stacking 160 shots.

OSG, for 3D media. Although we have implemented only 2D RTM algorithm, the application to 3D case can be made using the 3D ESG algorithm in a similar way. However, it is imperative that the computational resources to be more than the a simple PC (used for the example presented here). We stress the fact that, in real case data, powerful clusters must be applied.

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