SUBOPTIMAL NONLINEAR ATTITUDE CONTROL OF A 3-AXIS STABILIZED SATELLITE CONSIDERING FAILURE IN THE PROPULSION SYSTEM

Victoria de Souza Rodrigues

André Fenili

victoria.rodrigues@aluno.ufabc.edu.br

andre.fenili@ufabc.edu.br

Federal University of ABC (UFABC)

Avenida dos Estados, 5001, 09210-580, Santo André - SP, Brazil

Abstract. *This paper investigates the performance of the suboptimal nonlinear control named SDRE in the attitude of a 3-axis stabilized satellite. The control acts in two different situations related to attitude: 1) in the gas jet operation; 2) in the rebalance of liquid propellant inside the tanks and considering the connection line between them. The satellite studied here is modeled with four liquid propellant tanks, distributed symmetrically in the satellite structure. The tanks have the same dimensions and are made of the same material; it is initially assumed that they all have the same volume of propellant. The numerical simulations of the problem consider that the masses of the tanks are unbalanced because of propulsion failures. In this condition, results show that the SDRE control is able to take the satellite to any desired reference state.*

Keywords: *Flow control, Nonlinear control, SDRE, Unbalanced tanks*

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INTRODUCTION

The importance of research in the field of aerospace engineering is increasingly remarkable, since other areas of technology (such as communication and information, for example) depend on the constant upgrading and use of space vehicles (Almeida, 2016).

In the planning of a space mission, minimizing fuel consumption is always a high priority, since the higher consumption leads to the higher cost of transporting more mass into space. Poor planning of the fuel consumption of a satellite in operation can also lead to the end of a mission ahead of schedule, which may jeopardize its objectives (Yip et al, 2004).

In the design of liquid fuel tanks in satellites, one way of controlling the outflow from the propellant to the combustion chamber is by controlling the gas pressurization above this liquid column (Harland, 2005). The orifice at the bottom of each tank has an opening that can only present two states: completely open or completely closed. The present work proposes that the diameter of this orifice be variable, presenting more intermediary states and contributing directly to the flow control without depending on the change of pressurizing of the gas above the liquid column as the propellant leaves the tank.

A satellite can have different types of stabilization. One is the three-axis stabilization (Wertz, 1978). This type is mainly used in satellites whose mission is to maintain a certain attitude for a certain time, such as mapping satellites. Attitude control can be done by different actuators, like jet thrusters. These jets can adjust the attitude in manoeuvres, which requires large changes in the value of the angles, or adjust the attitude to increase precision, making only small variations at these same angles. In both cases, the more uniformly distributed the mass is on the satellite, the less energy the controllers will need to correct the attitude. This distribution, however, may be non-uniform when the mass of liquid propellant within different tanks is not the same.

In order to overcome the problem of mass balance, the present project proposes a connection between the tanks of the same liquid propellant so that it is possible for the liquid contained in one tank to pass to the other. In this way, when the tanks do not present the same mass, it is possible that the balance between them is re-established.

The model presented here is made for the rebalancing between the tanks that store the liquid propellant responsible for propulsion. Two configurations of the same problem are investigated: i) the control of propellant flow between different tanks in order to rebalance them followed by the attitude control of the satellite with the mass already evenly distributed; ii) the control of propellant flow between different tanks happening simultaneously to the attitude control of the satellite. A comparison between these two configurations is presented here, showing which is more economical in terms of energy expenditure with the control.

The control law investigated here is the nonlinear control law SDRE (Çimen, 2008), since both models (flow control and attitude control) have strongly non-linear characteristics.

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1 MATHEMATICAL MODELLING

1.1 Two tanks model

Figure 1 illustrates two tanks of the system studied. The model of the other two tanks is analogous to the shown here in this section. The tanks of the system are cylindrical and identical. They have an orifice at the bottom, which enables the flow of the liquid either to the independent line or to the connecting line between them. When the liquids are flowing independently, there may be a difference in flow rate between them, which will lead to an unbalance. The connection line enables the flow of liquid from one tank to another in order to restore the original balance.

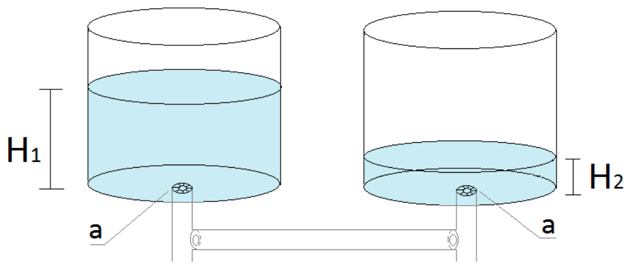

Figure 1. System composed by two tanks with a connecting line between them. H_1 and H_2 express the heights of the liquid columns in each tank; the nonlinear control acts on the variable diameter of the area "a"

The parameters of the system indicated with the index "1" are related to the first tank and the index "2" represents the second tank parameters. Both tanks are identical, so the equations demonstrated in this section shows the index 1, but is also valid for the second tank.

Equation (1) is the continuity equation (Fox et al., 2011) for tank 1 and Eq. (2) is the Bernoulli Equation (Fox et al., 2011) applied in this case study. The relation between the forces inside the tank results in Eq. (3) and its demonstration is shown in (Rodrigues and Fenili, 2013).

$$A_1 \frac{dH_1}{dt} + Q_1 = 0 \tag{1}$$

$$\frac{1}{2}\rho V_{1A}^2 + P_{1A} + \rho g H_1 = \frac{1}{2}\rho V_{1B}^2 + P_{1B} \tag{2}$$

$$\rho Q_1 (V_{1B} - V_{1A}) = \rho A_1 g H_1 + P_{1A} A_1 - P_{1B} a_1 \tag{3}$$

Boyle's Equation (Atkins and Paula, 2014) is used to obtain an equation that relates the pressure inside the tank (which is above the liquid column) and the height of this column. Denoting by P_{1A} the internal pressure of the gas in tank 1, P_{max1} being the maximum pressure internally supported by tank 1, H_{t1} is the total height of tank 1 and H_{max1} is the total height of the liquid column of tank 1, expression (4) shows the relation between these variables.

$$P_{1A} = \frac{P_{max1} \times (H_{t1} - H_{max1})}{(H_{t1} - H_1)} \tag{4}$$

Manipulating equations (1), (2), (3) and (4), and considering that the control acts on the orifice diameter at the bottom of tank 1, it is possible to obtain the expressions for the rate of change of the height of the liquid in tank 1 ($\dot{H}_1$) and the flow rate of tank 1 ($\dot{Q}_1$).

In this manipulation, the control force substitutes the force of the liquid column added to the force of the pressurized gas above this column; all these forces are acting on the orifice area at the bottom of the tank; this area is where the control acts physically. Equations (5) and (6) show the resulting expressions for the rates of change mentioned.

$$\dot{H}_1 = \frac{-Q_1}{A_1} \tag{5}$$

$$\dot{Q}_1 = \frac{1}{G_1}(H_{t1} - H_1)^2 \rho g A_1^2 \dot{a}_1 H_1 + \frac{1}{G_1}(H_{t1} - H_1) e_1 A_1^2 \dot{a}_1 - \frac{1}{G_1}(H_{t1} - H_1)^2 A_1 \dot{a}_1 F_{controle_1} - \frac{1}{G_1}(H_{t_1} - H_1)^2 A_1 a_1 \dot{F}_{controle_1} + \ldots$$
$$+\left[(H_{t_1} - H_1)^2 \rho \dot{a}_1 Q_1 - (H_{t_1} - H_1)^2 \rho g A_1 a_1 + e_1 A_1 a_1\right] \frac{1}{G_1} Q_1 \tag{6}$$

Where:

$$G_1 = (H_{t_1} - H_1)^2 \left(2\rho Q_1 A_1 - 2\rho Q_1 a_1\right) \tag{7}$$

Equations (5) and (6) for tank 1 do not depend on any variable related to the second tank. To couple the equations of both tanks, which means to connect the tanks physically, it is necessary to add mathematically an intermediate force in the control of each tank. The intermediate forces are denoted by F_1 and F_2 and are given by equations (8) and (9) respectively.

$$\alpha \dot{F}_{controle_1} + \beta F_{controle_1} = F_1 \tag{8}$$

$$\gamma \dot{F}_{controle_2} + \delta F_{controle_2} = F_2 \tag{9}$$

The values of α, β, γ and δ are given by equations (10), (11), (12) and (13), respectively.

$$\alpha = \left[(H_{t_1} - H_1)^2 \rho g A_1^2 \dot{a}_1 - e_1 A_1^2 \dot{a}_1\right] \frac{1}{G_1} \tag{10}$$

$$\beta = \left[(H_{t1} - H_1)^2 \rho \dot{a}_1 Q_1 - (H_{t_1} - H_1)^2 \rho g A_1 a_1 + e_1 A_1 a_1\right] \frac{1}{G_1} \tag{11}$$

$$\gamma = \left[(H_{t_2} - H_2)^2 \rho g A_2^2 \dot{a}_2 - e_2 A_2^2 \dot{a}_2\right] \frac{1}{G_2} \tag{12}$$

$$\delta = \left[(H_{t_2} - H_2)^2 \rho \dot{a}_2 Q_2 - (H_{t_2} - H_2)^2 \rho g A_2 a_2 + e_2 A_2 a_2\right] \frac{1}{G_2} \tag{13}$$

The equations, now coupled, are rewritten for each tank, as shown by equations (14), (15), (16) and (17).

$$\dot{H}_1 = \frac{-Q_1}{A_1} \tag{14}$$

$$\dot{Q}_1=\frac{1}{G_1}e_1A_1^2\dot{a}_1H_{t_1}+\left[(H_{t_1}-H_1)^2\rho gA_1^2\dot{a}_1-e_1A_1^2\dot{a}_1\right]\frac{1}{G_1}H_1+F_1+\left[(H_{t_1}-H_1)^2\rho\dot{a}_1Q_1-(H_{t_1}-H_1)^2\rho gA_1a_1+e_1A_1a_1\right]\frac{1}{G_1}Q_1 \tag{15}$$

$$\dot{H}_2=\frac{-Q_2}{A_2} \tag{16}$$

$$\dot{Q}_2=\frac{1}{G_2}e_2A_2^2\dot{a}_2H_{t_2}+\left[(H_{t_2}-H_2)^2\rho gA_2^2\dot{a}_2-e_2A_2^2\dot{a}_2\right]\frac{1}{G_2}H_2+F_2+\left[(H_{t_2}-H_2)^2\rho\dot{a}_2Q_2-(H_{t_2}-H_2)^2\rho gA_2a_2+e_2A_2a_2\right]\frac{1}{G_2}Q_2 \tag{17}$$

1.2 The satellite model

The satellite is modeled as a cube, where the four cylindrical tanks occupy its edges, as shown in Fig. 2.

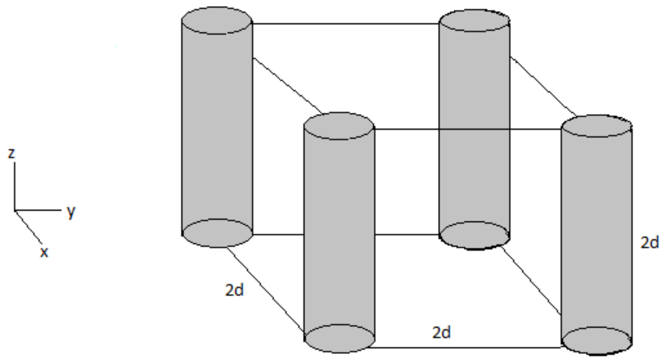

Figure 2. The satellite model

The Euler's equations for a rigid body (Beer et al., 2006) of this satellite are written from the sequence of Euler 3-1-3 (Equations 18,19 and 20), where ω is the angular velocity related to each axis in the fixed system of the body.

$$\omega_x=\dot{\phi}\sin\theta\sin\psi+\dot{\theta}\cos\psi \tag{18}$$

$$\omega_y=\dot{\phi}\sin\theta\cos\psi-\dot{\theta}\sin\psi \tag{19}$$

$$\omega_z=\dot{\phi}\cos\theta+\dot{\psi} \tag{20}$$

Moments and products of inertia are calculated considering the simplification that the propellant of each cylinder totally composes the mass of each one of them; in addition, it is considered that the center of mass of each cylinder is always positioned at its the geometric center. The inertia moments and products of the cube with four cylinders are calculated as:

$$I_{xy}=I_{yx}=d^2(m_1-m_2+m_3-m_4) \tag{21}$$

$$I_{xz}=I_{zx}=0 \tag{22}$$

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$$I_{yy} = \left[\frac{1}{4} R^2 + \frac{1}{12} \times (2d)^2 + d^2 \right] \times (m_1 + m_2 + m_3 + m_4) + \frac{m_{cube}}{12} (2d)^2 \quad (23)$$

$$I_{yz} = I_{zy} = 0 \quad (24)$$

$$I_{zz} = \left[\frac{1}{2} R^2 + (2d)^2 \right] \times (m_1 + m_2 + m_3 + m_4) + \frac{m_{cube}}{12} \times (2d)^2 \quad (25)$$

The satellite thrusters, responsible for attitude control, are represented by forces centered on the faces of the cube (omitted in figure 2). The moments that each jet produces for each angle of Euler are then defined as:

$$\sum M_{\phi} = d(F_{1B} + F_{2B} + F_{3B} + F_{4B}) \quad (26)$$

$$\sum M_{\theta} = d(F_{2C} + F_{4C} + F_{5C} + F_{6C}) \quad (27)$$

$$\sum M_{\psi} = d(F_{1B} + F_{2B} + F_{3B} + F_{4B}) \quad (28)$$

The thrusters that generate forces around the Z axis are called F_{1B}, F_{2B}, F_{3B} and F_{4B}. The thrusters around the X axis are called F_{2C}, F_{4C}, F_{5C} and F_{6C}. Due to the 3-1-3 rotation, the moments around ϕ and ψ are the same.

For the case where the satellite has mass not evenly distributed, when the coordinate axes of the body is placed in the center of the cube, these same axes do not coincide with the main directions of the satellite. Therefore their moments are described as (Beer et al., 2006):

$$M_{\phi} = \omega_y(-I_{zx}\omega_x - I_{zy}\omega_y + I_{zz}\omega_z) - \omega_z(-I_{yx}\omega_x + I_{yy}\omega_y - I_{yy}\omega_z) + I_{xx}\dot{\omega}_x - I_{xy}\dot{\omega}_y - I_{xz}\dot{\omega}_z \quad (29)$$

$$M_{\theta} = \omega_z(I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z) - \omega_x(-I_{zx}\omega_x - I_{zy}\omega_y + I_{zz}\omega_z) - I_{yx}\dot{\omega}_x + I_{yy}\dot{\omega}_z - I_{yz}\dot{\omega}_z \quad (30)$$

$$M_{\psi} = \omega_x(-I_{yx}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z) - \omega_y(-I_{zx}\omega_x - I_{zy}\omega_y + I_{zz}\omega_z) - I_{zx}\dot{\omega}_x - I_{zy}\dot{\omega}_y + I_{zz}\dot{\omega}_z \quad (31)$$

And considering as state variables the following:

$$x_1 = \phi$$
$$x_2 = \dot{\phi}$$
$$x_3 = \theta$$
$$x_4 = \dot{\theta}$$
$$x_5 = \psi$$
$$x_6 = \dot{\psi}$$

The moments can be written as a function of x_1, x_2, x_3, x_4, x_5 and x_6 in the matrix form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a & 0 & b & 0 & c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d & 0 & e & 0 & f \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & 0 & hh & 0 & i \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j & 0 & k & 0 & l \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & nn & 0 & o \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p & 0 & q & 0 & r \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} = \begin{bmatrix} 0 \\ \sum M_x \\ 0 \\ \sum M_y \\ 0 \\ \sum M_z \end{bmatrix} \qquad (32)$$

Where a, b, c, d, e, f, g, hh, i, j, k, l, m, nn, p, q and r are functions of state variables and inertia moments and products. From this matrix form, the vector is isolated and its result is calculated from a numerical integration made in *Matlab*. Thus, the positions and velocities of each angle can be calculated over a period of time.

2 SUBOPTIMAL NONLINEAR CONTROL

The nonlinear control technique used for flow control and attitude control in this paper is called SDRE – State Dependent Riccati Equation (Mracek and Cloutier, 1998). The SDRE can be understood as an extension of LQR (Linear Quadratic Regulator) for control of nonlinear systems (Shamma and Cloutier, 2003).

The control SDRE solves optimal control problems, like the one shown in Eq. (33). This problem is of optimal control with infinite horizon for a nonlinear autonomous system (Mracek and Cloutier, 1998).

$$\dot{x}=A(x)x+B(x)u \qquad (33)$$

The equation of the two tanks system can be written on the same format of Eq. (33), as presented in Eq. (34).

$$\underbrace{\begin{Bmatrix} \dot{H}_1 \\ \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \end{Bmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} 0 & -\frac{1}{A_1} & 0 & 0 \\ \alpha & \beta & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{A_2} \\ 0 & 0 & \gamma & \delta \end{pmatrix}}_{A(x)} \underbrace{\begin{Bmatrix} H_1 \\ Q_1 \\ H_2 \\ Q_2 \end{Bmatrix}}_{x} + \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}}_{B} \underbrace{\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}}_{u} \qquad (34)$$

The vector $\dot{x}$ contains the rates of the height of the liquid propellant and the flow rates in tanks 1 and 2 (analog for tanks 3 and 4). The matrix A contains the coefficients of the vector ‘x’ and it is a function of time. The matrix B is the matrix of the coefficients that multiplies the vector ‘u’, which contains the intermediate control forces (F_1 and F_2). The matrix B can be either constant or variable in time. For this case, it is assumed that B is constant.

The nonlinear regulator problem with an infinity horizon is defined through the minimization of the non-quadratic functional related to x, but quadratic related to u, expressed in Eq. (35).

$$J=\min_{u(t)}\int_0^{\inf}[x^TQ(x)x+u^TR(x)u]dt \tag{35}$$

The matrices of the weight coefficients for the state variables and the control, in the functional shown in Eq. (35), depend on the state variables as well as the $Q:\Re^n \to \Re^{n\times n}$ is positive and semi-defined and the matrix $R:\Re^n \to \Re^{n\times m}$ is positive defined for all x. It is assumed that the functions f(x), B(x), Q(x) and R(x) are smooth enough.

With these matrices defined above, the 'u' vector must be written in order to allow the implementation of the SDRE control, i. e., 'u' must be written as shown in Eq. (36).

$$u=-R^{-1}B^TPx=\begin{Bmatrix} F_1' \\ F_2' \end{Bmatrix} \tag{36}$$

The matrix R is originated from the integral in Eq. (37) (Mracek and Cloutier, 1998).

$$\int x^T(t)Q(x)x(t)+u^T(t)R(x)u(t) \tag{37}$$

The matrices Q(x) and R(x) are the weighting matrices, which are responsible for giving different weights for different terms that compose the system. The matrices used in the numerical simulation for the flow control are presented in Eq. (38) and Eq. (39). Equations (40) and (41) present the matrices used in the attitude control in both situation investigated here.

$$R(x)=\begin{pmatrix} 0.1 & 0.04 \\ 0.04 & 1 \end{pmatrix} \tag{38}$$

$$Q(x)=\begin{pmatrix} 60 & 0 & 0 & 0 \\ 0 & 4.5\times10^5 & 0 & 0 \\ 0 & 0 & 150 & 0 \\ 0 & 0 & 0 & 1.5\times10^6 \end{pmatrix} \tag{39}$$

$$R(x)=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{40}$$

$$Q(x)=\begin{pmatrix} 10^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10^4 \end{pmatrix} \tag{41}$$

3 NUMERICAL SIMULATION

This section shows the results of numerical simulations made in *Matlab* software. These simulations present the performance of the flow control and attitude control in the satellite model studied here.

The dynamics of the satellite and the control force of the actuators are compared in two situations of interest: the first (situation 1) shows the flow control acting first and exclusively on the satellite, followed by the attitude control; that is, the attitude control acts only after the mass balance between the satellite tanks. The second situation (situation 2) shows the flow control acting simultaneously to the attitude control, so part of the attitude control occurs with the unbalanced satellite mass.

One of the considerations made for these numerical simulations is that the flow control influences the attitude control, since the inertial tensor of the satellite changes, but it is not considered that the attitude control influences the flow control. Therefore, the behavior of the fluid inside the tanks in both situations studied here is the same.

Table 1 shows the initial conditions for flow control. These conditions are used in both situations investigated.

Table 1. System constant parameters for numerical simulation of the flow control and initial conditions

Parameter	Tank 1	Tank 2
Cross Sectional Area of the Tank (m^2)	0.01	0.01
Density of the Liquid (kg/m^3)	1032	1032
Total Height of the Tank (m)	5	5
Maximum Pressure (Pa)	10^5	10^5
Maximum Height of the Liquid Column in the Tank (m)	4	4
Orifice Area (m^2)	0.009	0.009
Height of the Liquid Column (m)	3	1
Flow Rate (m^3/s)	0.1	0.1

The following flow control diagrams show two curves: one representing tanks 1 and 3 and the other representing tanks 2 and 4. Figure 3 shows how the flow of liquid in the tanks occurs over time. It is possible to notice that in approximately 4 seconds the flows have zero values, which means that the tanks have already reestablished the balance.

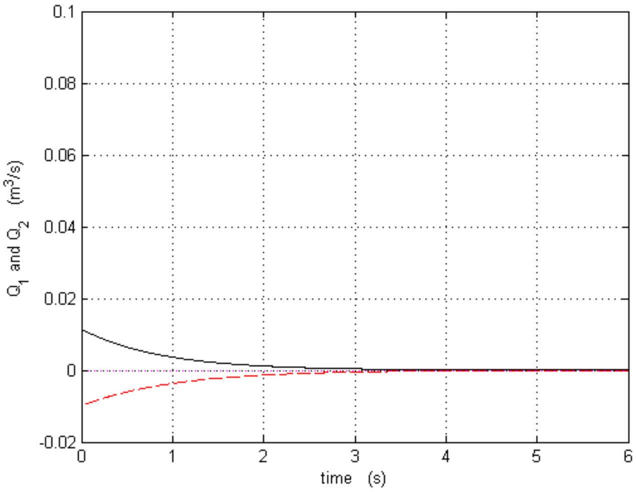

Figure 3. Flow rate curves converging for both tanks. Solid line for tank 1 and dashed line for tank 2. Dotted line represents the average of both flow rates

Figure 4 shows the heights of the liquid column in the tanks. In addition to the analysis of flow behavior, the liquid heights equalize before 5 seconds.

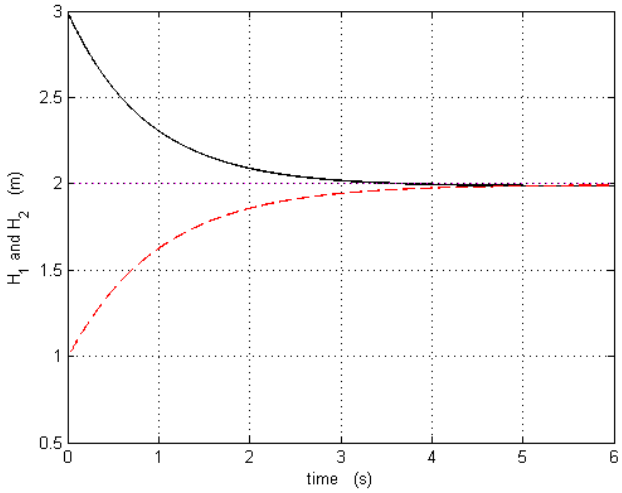

Figure 4. Heights of the liquid column of each tank. Solid line for tank 1 and dashed line for tank 2. Dotted line represents the average of both heights

The areas of the variable diameter orifice at the bottom of each tank are responsible for flow control. Figure 5 shows how these areas vary over time. Balance of heights occurs when the area of the orifice of the tank 1 (and of the tank 3) reaches about half the value of the area of the orifice of the tank 2 (and of the tank 4).

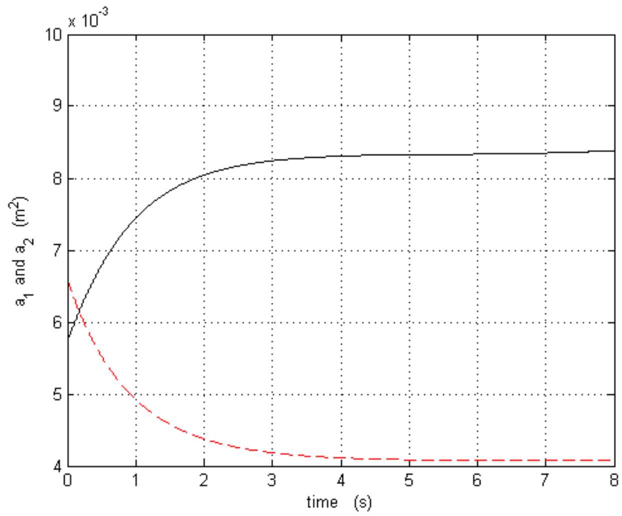

Figure 5. Variation in the area of the orifices at the bottom of each tank. Solid line for tank 1 and dashed line for tank 2

Figure 6 shows the control forces acting on the orifices of each tank. They have little variation over time.

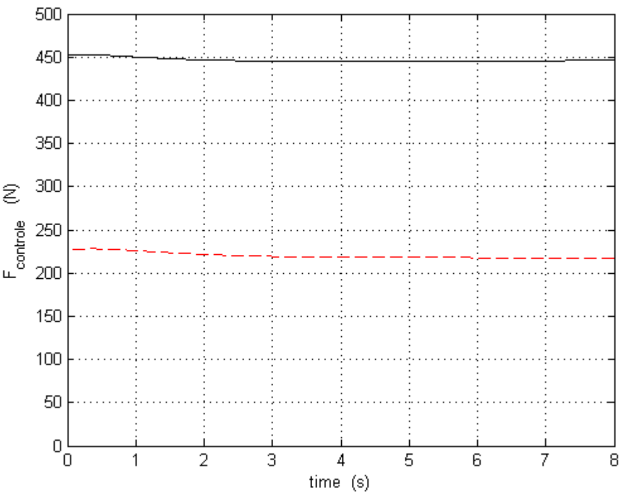

Figure 6. Control force for each tank. Solid line for tank 1 and dashed line for tank 2

Attitude control is different for each situation investigated here. Table 2 shows the initial and reference conditions for this control. The control implemented here aims to bring the satellite from the initial position of 5 degrees at each angle of rotation to the 0 degree position in all of them.

Table 2. Initial conditions for attitude control

Parameter	
Cube mass (kg)	10
Mass of tanks 1 and 3 (kg)	30.96

Mass of tanks 2 and 4 (kg)	10.32
Phi, theta and psi positions (°)	5
Phi, theta and psi velocities (rad/s)	0
Phi, theta and psi position references (°)	0
Phi, theta and psi velocity references (rad/s)	0

Figures 7 and 8 show respectively the position and velocity of the rotation angle phi. The dynamics of phi is very similar in both situations, but for the situation of the controls acting simultaneously, the maximum velocity (in modulus) that phi achieves in this time interval is slightly higher.

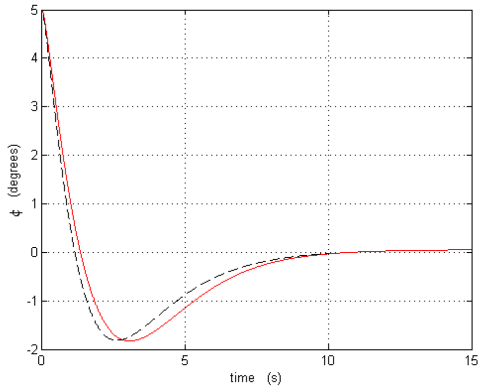

Figure 7. Position of phi. Solid line for situation 1 and dashed line for situation 2

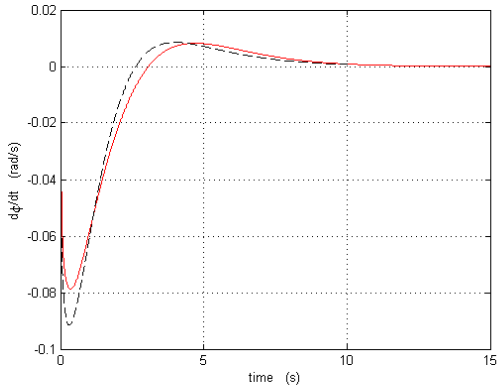

Figure 8. Velocity of phi. Solid line for situation 1 and dashed line for situation 2

Figures 9 and 10 show respectively the position and velocity of the rotation angle theta. Again, the dynamics are very similar in both situations, with the theta velocity for the condition of simultaneous controls being slightly higher (in modulus).

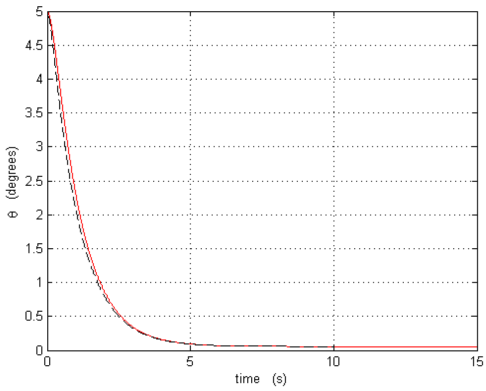

Figure 9. Position of theta. Solid line for situation 1 and dashed line for situation 2

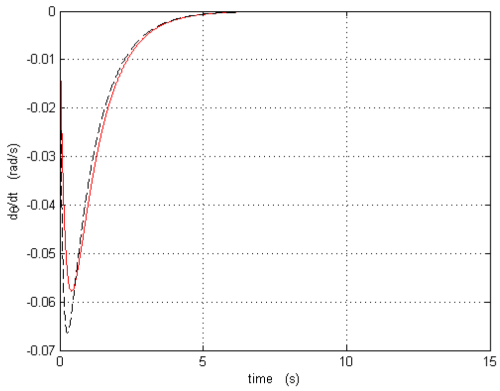

Figure 10. Velocity of theta. Solid line for situation 1 and dashed line for situation 2

Figures 11 and 12 show respectively the position and velocity of the rotation angle psi. This angle reaches higher values for the condition in which the controls act separately and present similar velocities for both situations.

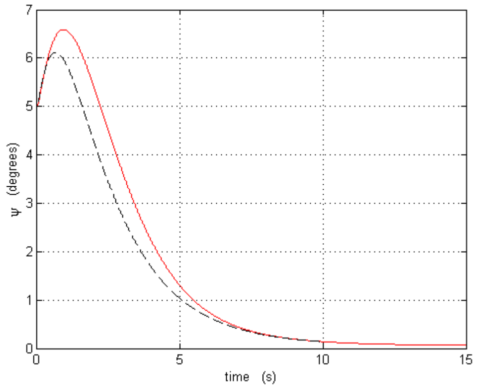

Figure 11. Position of psi. Solid line for situation 1 and dashed line for situation 2

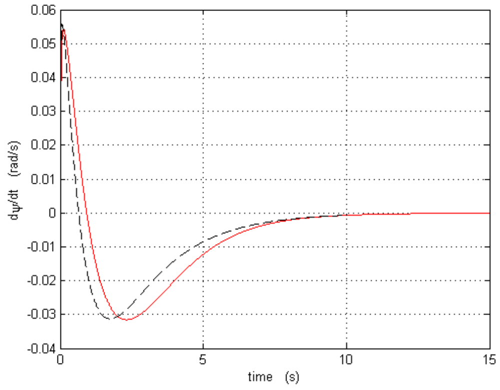

Figure 12. Velocity of psi. Solid line for situation 1 and dashed line for situation 2

The control force for the phi angle is shown in Fig. 13. The curves for both situations are similar, but for the first situation (controls acting separately), this control force is slightly higher.

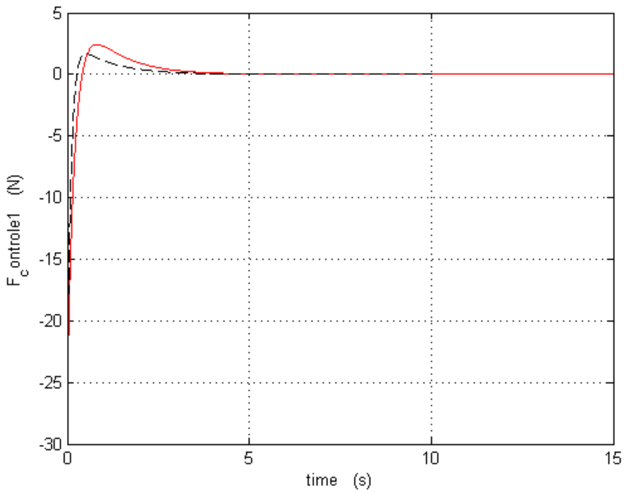

Figure 13. Force control for phi. Solid line for situation 1 and dashed line for situation 2

The control force for the theta angle is shown in Fig. 14. The curve of the first situation is also that which reaches the highest value (in modulus).

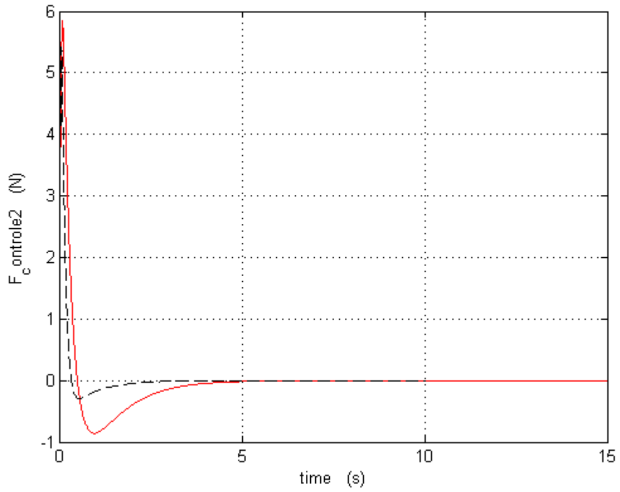

Figure 14. Force control for theta. Solid line for situation 1 and dashed line for situation 2

The control force for the psi angle is shown in Fig. 15. Again, the curves are very similar, but the curve of the first situation reaches a maximum value slightly higher than the curve of the second situation (simultaneous controls).

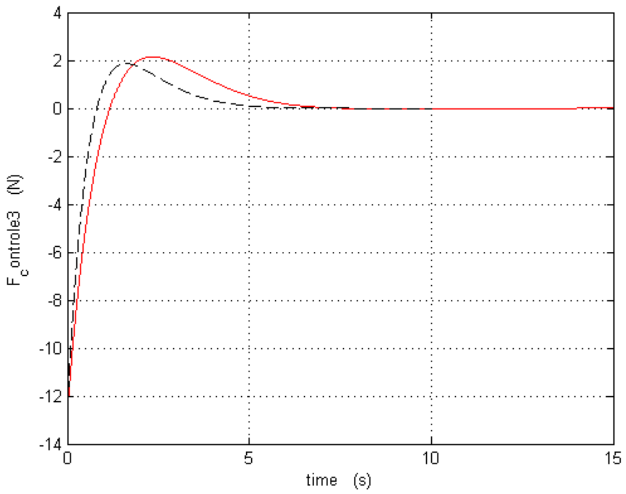

Figure 15. Force control for psi. Solid line for situation 1 and dashed line for situation 2

4 CONCLUSIONS

Satellites with more than one tank may have their mass unbalanced due to a failure in the propulsion system, which leads to a difference in the volume of liquid propellant in each of them. This imbalance can be detrimental to attitude control. Assuming all tanks have the same propellant, one solution is to interconnect them in order to provide liquid flow from one tank to the other. In this way, the mass balance can be reestablished. The flow control can be done by the suboptimal nonlinear control SDRE through an orifice of variable diameter, which is efficient in the numerical simulations for the initial condition of tanks with different heights of liquid. Attitude control was investigated in two ways in this paper: acting separately from the flow control and acting simultaneously. The results of the numerical simulations show that the satellite dynamics for the initial conditions adopted are similar in both proposed research situations. By analyzing in particular the control forces in the attitude control actuators (the gas jet forces), it is possible to observe that, when the flow and attitude controls are used simultaneously, the actuators use less energy, thus presenting the situation more economical in this sense.

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