APPLICATION OF AN INVERSE METHOD FOR THE ESTIMATION OF HEAT FLUX IN LOW-THRUST HYBRID PROPELLANT ROCKET MOTOR AND ITS ANALYTICAL VALIDATION

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Abstract. *The present study is focused on the estimation of the heat flux traversing through the walls of heat insulator from a hybrid propellant rocket motor. Experimental temperature data were used to determine the heat fluxes on internal surface of the insulator using an inverse method of heat transfer along with finite element analysis. Estimated values of heat flux were validated using the analytical solution of a heat equation problem in the cylindrical coordinates. The temperatures on internal and external surfaces of the heat insulator were calculated for long burning times of the hybrid propellant test-motor.*

Keywords: *Heat insulation, Hybrid propulsion, Heat flux, Inverse problem, Temperature distribution*

INTRODUCTION

The presented study was motivated by the necessity to increase the burning time of a low-thrust hybrid propellant rocket motor that had been initially designed in the University of Brasilia (Brazil) to operate with paraffin-and-nitrous-oxide propellant pair for 12 seconds. The replacement of paraffin grain with the polyethylene one, both with the same cross-section, have increased the motor's operation time to 40-50 seconds and required application of reliable thermal protection. Determining the heat flux through internal walls of motor is among various problems that have to be solved to establish geometry of heat insulation.

Determination of heat flux through wall of insulator is difficult due to complicated heat transfer inside combustion chamber that according to reference (Shishkov et al., 1989) is accompanied by the following processes: high gradients of velocity and pressure along motor, interaction of shock waves with boundary layer, increased level of turbulence flow, presence of areas with separated flow, phase change in materials and their ablation, influence of temperature on thermal properties of insulators etc. Numerical simulation of such heat transfer requires deep knowledge of physical and chemical processes in combustion chamber, elaboration of complex models, and finally, possession of powerful computational and time resources. Estimation of heat flux for preliminary determination of insulator’s design in hybrid propellant test-motor should be fulfilled with reduced resource consumption.

Solution of inverse heat transfer problem, whose idea is described in works of Beck (1970), Tikhonov and Arsenin (1977), Alifanov (1994), etc., could form the basis of methodology for heat flux estimation, where the unknown boundary heat flux is determined by temperature measurements taken below the boundary surface. Numerical (Stolz, 1960) or analytical methods (Monde, 2000) can be adapted for solving an inverse problem of heat conduction. In the later work, numerical method is used for analytical data validation. However, at presented work it is suggested to use analytical method for validation of heat flux values obtained by finite element analysis (FEA). Together with FEA, the trial-and-error procedure is one of the simplest methods for solving inverse problems.

The solution of inverse problem was successfully applied for determination of heat flux to solid-propellant rocket motor nozzles (Ungar, 1961 or Mehta, 1981). This approach is difficult to realize for prediction of heat fluxes to walls of combustion chamber of a solid propellant rocket motor, since insulator is protected temporarily by solid grain characterized by some value of regression rate. However, the methodology is applicable for a hybrid propellant rocket motor, whose classical design includes pre- and post-chambers that do not contain solid grain. Here, the thermal protection of structural casing is realized by insulation only.

Thus, the main objective of the study was to estimate the heat flux on internal surface of insulator installed in post chamber of low-thrust hybrid propellant rocket motor by solution of inverse problem of heat conduction with use of FEA. Procedure included test measurements of temperature on external surface of heat insulator for relatively short burning-time of hybrid propellant test-motor. Experimental data, presented as a temperature-time plot, were used to determine heat flux boundary condition at internal surface of insulator’s wall with known thermal properties by finite-element method.

Another objective of the study was to validate the values of heat flux by analytical solution of a heat equation problem in cylindrical domain. Solution was obtained using an integral transform approach, where the transform kernel was based on the solution of an associated Sturm-Liouville boundary value problem.

With obtained values of heat flux, preliminary design of heat insulator made from materials with known thermal properties becomes possible. Meanwhile, the application of results of the study is limited to preliminary evaluation of heat insulator efficiency by determination of temperature on its external surface at given conditions of motor operation with increased burning time. It is necessary to note that some considerations and assumptions suggested in the study and described in methodology limit practical application of suggested method for long-time operation of motor.

1 METHODOLOGY

1.1 Insulation material

A composite material on the basis of phenolic resin was used as a heat insulator. This material belongs to currently used thermoset-based insulations reinforced with various non-organic fillers, and whose characteristics are given in reference (Ahmed, 2009). According to the same reference, phenolic materials have good temperature and erosion resistance, but do not exhibit high strain capability. This is not a problem for small-size motors that do not experience high deformations.

Ahmed (2009) states that phenolic starts to ablate when heated to $683\ K$ $(410\ °C)$ and completely vaporize from solid to gaseous state at $1585\ K$ $(1312\ °C)$. According to literature review mentioned by Duffa (2013) there are three sets of reactions and the first one, leading to depolymerization, occurs in temperature range $600 - 800\ K$, but the production of pyrolysis gases starts at $400\ K$. These properties make possible to avoid overheating of internal surfaces of the insulator and its erosion, leaving a constant thickness for short burning times.

Two geometries of heat insulator were under consideration in the study: cylindrical with thickness $3\ mm$ and complex with thickness $5\ mm$. Both were made from Celeron (Fig. 1, a) that is a phenolic resin reinforced by cotton fabric, manufactured by pressing under high temperature and whose thermal properties are given in Table 1.

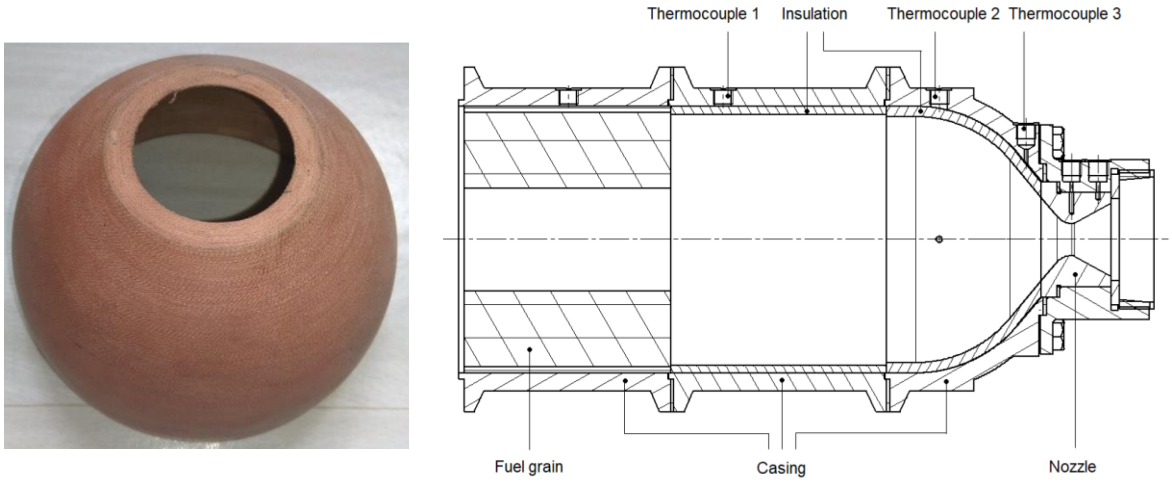
Table 1. Thermal properties of Celeron

Density, kg/m^3	Thermal conductivity, W/(m·K)	Thermal capacity, J/(kg·K)
1350	0.35	1465

1.2 Experimental measurements of temperature

Modular hybrid propellant test-motor described in reference (Andrianov, et al., 2015) was adapted for temperature measurements on external surface of insulator. A hot-fire test was arranged for low oxidizer-to-fuel ratio 1.5 in order to maintain relatively low pressure and velocities of hot-gases in combustion chamber. Together with short duration of test, this strategy was aimed to avoid erosion of insulators.

The temperature on selected points of external surface of insulator was measured by N-type thermocouples, whose arrangement for the hot-fire test is shown in Figure 1, b. Liquid nitrous oxide was used as an oxidizer and solid paraffin in form of one-port cylindrical grain as a fuel. The signals from pressure transducer, which was installed in post-chamber of motor, and thermocouples were recorded and processed by 16-channel data acquisition system.



a) b)

Figure 1. Phenolic heat insulation and its arrangement in test-motor: a) aft end bell insulator made from Celeron, b) installation of thermocouples along external surface of insulator

1.3 Numerical model

Since the heat insulators have axisymmetric geometry, the two-dimensional thermal transient analysis was arranged in Ansys Workbench. The simple square plane model was used for cylindrical insulator, where measurements were fulfilled by thermocouple 1. Since thermocouples 2 and 3 were installed in spherical and conical parts of the aft end bell insulator, the plane models with a shape of sector and rectangle were used correspondingly (Figure 2). The mesh with predominantly excellent quality was used for all models according to skewness criterion (Table 2), which is one of the primary quality measures for a mesh according to reference (ANSYS, 2012).

The heat flux applied in four steps on one of the edges of model, which corresponded to internal surface of insulator, was time synchronized with pressure data measured in combustion chamber (Figure 3). Here, the total time of burning with transition zones is 10.9 s and the heat flux is constant for 9.9 s. The data on temperature were recorded on opposite edge of model and appropriate curve of temperature vs time was plotted in numerical simulation. This curve was compared with experimental curve to determine the appropriate value of heat flux that was defined by criterion discussed below.

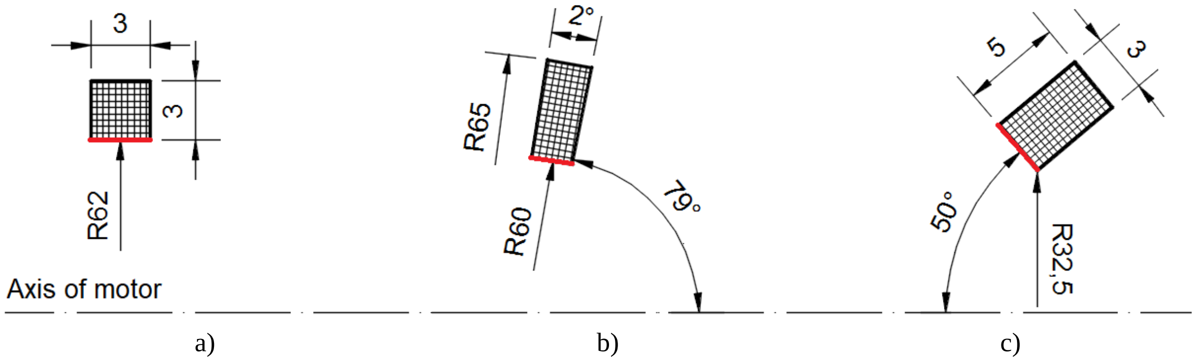


Fig. 2. The shape of plane models accepted for numerical analysis (red edge is the place of heat flux application, the opposite edge serves for temperature data collection): a) square model for thermocouple 1; b) sector model for thermocouple 2; c) rectangular model for thermocouple 3

Table 2. Mesh details for three axisymmetric plane models

Model's shape	Thermo-couple No.	Number of nodes	Number of elements	Element type	Skewness, average	Skewness, maximal
square	1	11041	3600	PLANE77	$1.3\cdot10^{-5}$	$8.3\cdot10^{-3}$
sector	2	13491	4402	PLANE77	$7.3\cdot10^{-3}$	$4.6\cdot10^{-1}$
rectangle	3	18321	6000	PLANE77	$1.3\cdot10^{-5}$	$1.3\cdot10^{-2}$

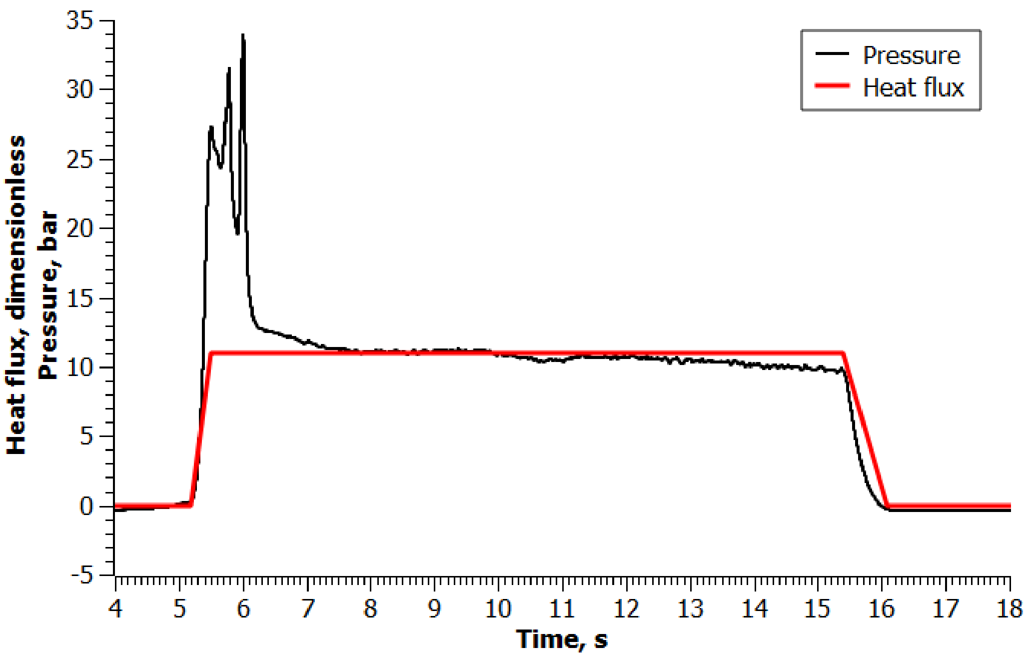


Figure 3. Synchronization of heat flux of numerical model with pressure profile in combustion chamber

1.4 Analytical validation of numerical results

Consider the heat conduction in cylindrical domain, where temperature varies only along radial direction and there are no internal sources of heat generation, then the following form of the heat equation is obtained:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{1}$$

where T is the body temperature, α is thermal diffusivity of material, r is radial coordinate and t is time variable.

The given differential equation does not have trivial solution and solution procedures are out of scope of the study. The insulator in analytical model was treated as a hollow cylinder, where the radial variable varies between the internal and external radii of the insulator. The boundary conditions for the equation are represented by the values of heat flux estimated by FEA and by initial temperature of the model. Solution presented in the study was obtained by using discrete integral Hankel–Webber transforms and has the following form

$$\begin{aligned} T(r,t) \\ = f_0 - \frac{2\alpha a q}{b^2 - a^2}[u(t-t_0)(t-t_0) - u(t-t_1)(t-t_1)] \\ + q\pi\sum_{m=1}^{\infty}\frac{[u(t-t_0)(t-t_0) - u(t-t_1)(t-t_1)]J_1(\tau_m a)J_1(\tau_m b)R_m(r)}{\tau_m[J_1^2(\tau_m a) - J_1^2(\tau_m b)]} \\ - q\pi\sum_{m=1}^{\infty}\frac{[u(t-t_0)e^{-\alpha\tau_m^2(t-t_0)} - u(t-t_1)e^{-\alpha\tau_m^2(t-t_1)}]J_1(\tau_m a)J_1(\tau_m b)R_m(r)}{\tau_m[J_1^2(\tau_m a) - J_1^2(\tau_m b)]}, \end{aligned} \tag{2}$$

where $u(t)$ is the Heaviside step function.

In equation (2), a and b are internal and external radii of heat insulator respectively, f_0 is initial constant temperature, t_o is initial moment of burning and q is a heat flux on internal surface of insulator, $R_m(r)$ are the problem eigenfunctions and also the kernel of the integral transformations associated to the corresponding τ_m eigenvalues. The function $J_1(z)$ is the first order Bessel function of the first kind. This solution provides values of temperature across the cross-section of the insulator at any moment of time t.

The validation of values of heat flux was made by comparison of temperature vs time curves obtained numerically and by analytical solution (equation (2)) for burning time $45\ s$. For those plots, numerical and analytical models were adjusted for burning interval starting at $5\ s$ and finishing at $50\ s$ and heat flux in numerical model was changed from 0 to predefined constant value abruptly in transition interval $0.01\ s$.

According to presented procedure, the following assumptions and considerations were made in the study:

– Heat transfer is one-dimensional in direction of coordinate radius. The assumption is justified by axisymmetric geometry of heat insulators. In addition, it gives possibility to

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treat conical and spherical parts of aft end bell insulator as a short cylinder in analytical validation.

- Only a portion of experimental curve of temperature that corresponded to burning time of the motor was taken into consideration in the study. It helped to simplify both numerical and analytical models, considering only conduction through the wall of insulator and avoiding complex analysis of heat transfer on internal surface of insulator due to radiation and convection, contact resistance between insulator and structural casing of motor and, finally, cooling of motor due to radiation and convection after hot-fire test.
- It is considered that the heat flux is constant during burning in numerical simulations, as the pressure in combustion chamber does not have significant changes.
- Thermal properties of heat insulation material are independent of temperature, and material does not undergo phase changes.
- There are no changes in shape and geometry of heat insulator during burning. This assumption is acceptable, since burning time of motor was short and change of insulator's thickness was not detected after tests.

2 RESULTS AND DISCUSSION

According to graphs shown in Figures 5 – 7, the measured temperature is characterized by fluctuations, whose amplitude reaches $0.5\ °C$. In addition, the shape of experimental curve has a peak at initial moment of burning that corresponds to initial growth of pressure (Figure 4). There is no clear vision of the reasons responsible for the temperature peak, which probably is a result of reaction of data acquisition system on dynamic pressure increase caused by ignition.

Couple of criterions were under consideration to determine appropriate value of heat flux by comparing experimental and numerical curves. The criterion based on equality of areas below the curves were rejected because of temperature peak at the beginning of burning. Equality of temperature derivative also was not accepted as a criterion, since it changes along the interval of burning for experimental curve. Thus, the coincidence of numerical curve with peak of temperature at the end of burning was accepted as a criterion for determination of heat flux (red curve in figures 5 – 7). Quite clear that such criterion overestimates the value of heat flux in comparison with the case when numerical curve corresponds to interpolated experimental data. However, since preliminary evaluation of heat insulator efficiency was a purpose of the study, increment of heat flux can either serve as a safety factor or compensate insulator erosion in further analysis.

The variation of heat flux between neighboring numerical curves is near 5 %for the plot of thermocouple 1 (Figure 5) and near 25 % for the plots of thermocouples 2 and 3 (Fig. 6 – 7). High variation of heat flux corresponding to amplitude of fluctuations of experimental data can be explained by very low intensity of experimental curves. The intensity can be increased by changing the conditions of experiment: by decreasing the thickness of insulator or by increasing burning time. In addition, the substitution of phenolic composite with material that has high erosion resistance (graphite) may be beneficious.

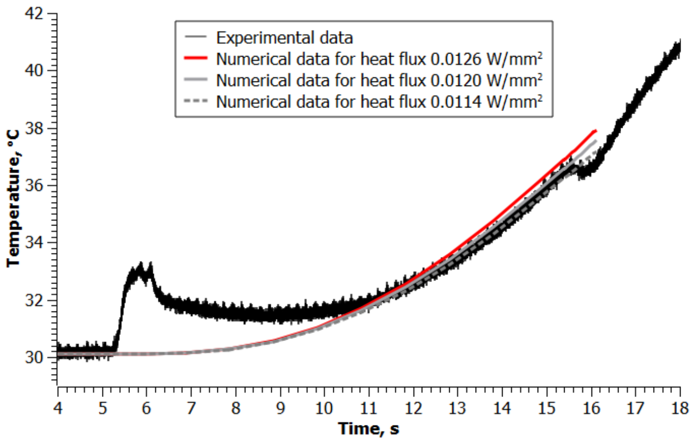


Fig. 5. Variation of temperature with time for thermocouple 1

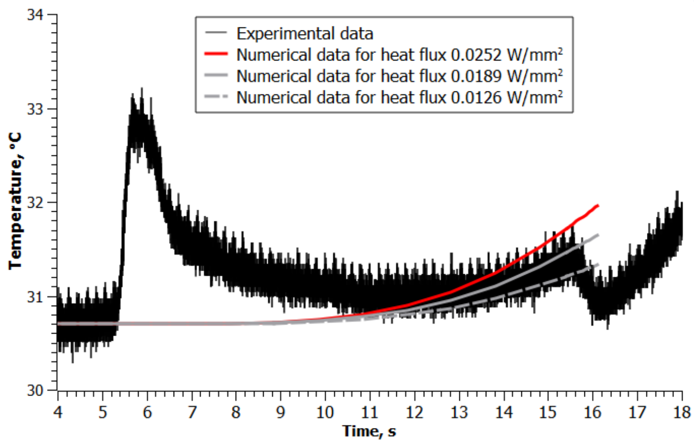


Fig. 6. Variation of temperature with time for thermocouple 2

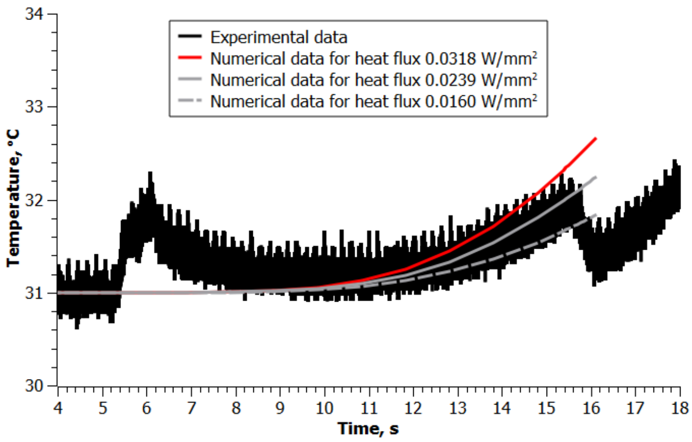
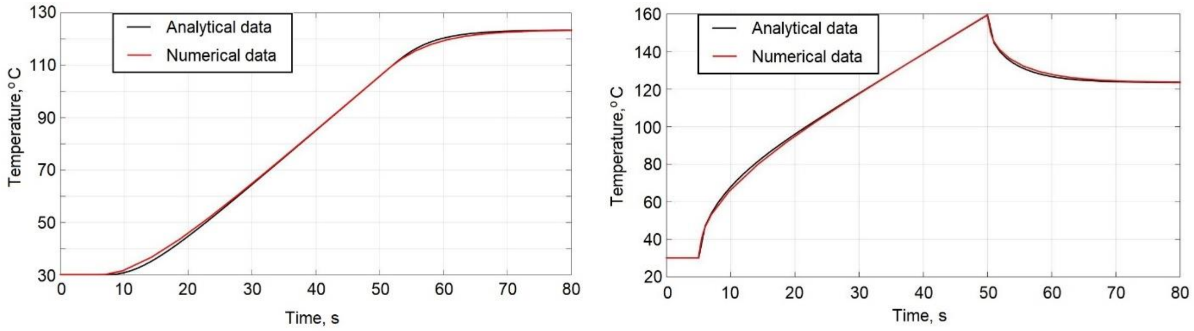


Fig. 7. Variation of temperature with time for thermocouple 3

The following values of heat flux were accepted in the study: $0.0125\,W/mm^2$ for thermocouple 1, $0.0252\,W/mm^2$ for thermocouple 2 and $0.0318\,W/mm^2$ for thermocouple 3.

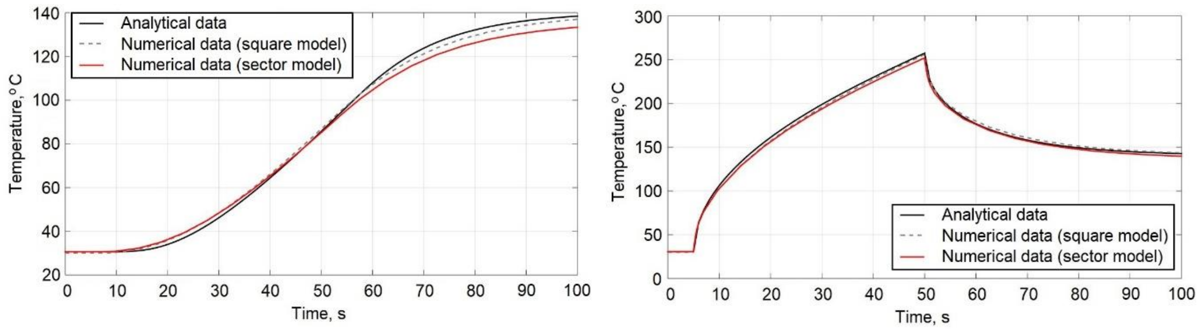
These data are in accordance with typical axial heat flux distribution for solid propellant rocket motors, where flux starts to increase in the beginning of convergent part of motor and the peak is always at the nozzle throat (Sutton, 2010).

The temperature curves based on numerical data and plotted for prolonged burning $45\ s$ are in accordance with those temperature curves calculated by analytical solution for the same value of heat flux (Fig. 8 – 10). For the case of thermocouple 2 the numerical results based on sector model provide quite high error 4 % for temperature on external surface of insulator. Thus, the numerical data was corrected by application of square plane model in cylindrical domain (grey dash curve) instead of sectoral one (Fig. 9, a). Maximal error was diminished to 1.5 %, since geometry of the square model is the same as in analytical solution. The maximal error for thermocouple 1 is not high: 3.5 % for internal surface and 1.3 % for external one.



a) b)

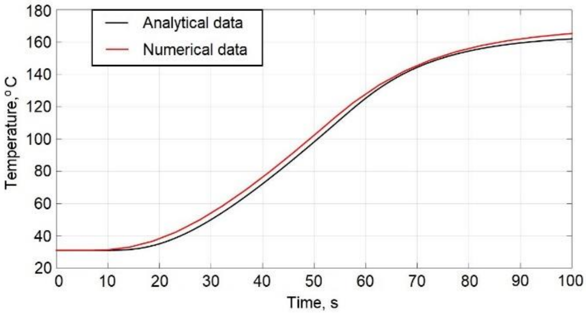
Fig. 8. Validation of numerical results by analytical solution for thermocouple 1 and heat flux 0.0126 W/mm^2: a) distribution of temperature on external surface of heat insulation; b) distribution of temperature on internal surface of heat insulation



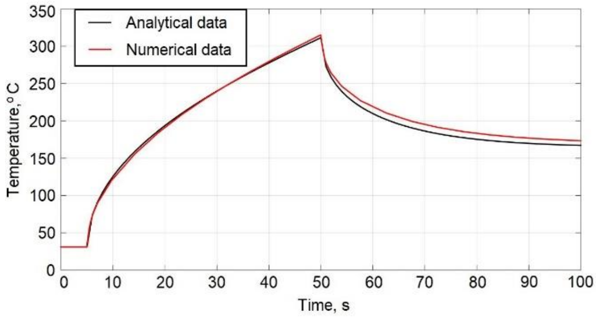
a) b)

Fig. 9. Validation of numerical results by analytical solution for thermocouple 2 and heat flux 0.0252 W/mm^2: a) distribution of temperature on external surface of heat insulation; b) distribution of temperature on internal surface of heat insulation

According to literature (Kalinichev and Yagodnikov, 2011) the permissible temperatures on internal surface of structural casing of solid rocket motors are: $150\ °C$ for aluminum alloys, $300\ °C$ for structural steel of normal quality, $450\ °C$ for heat-resistant steel. Thus, at the absence of ablation/erosion effects, application of the Celeron insulators with geometry shown in Fig. 1, b are justified for the given conditions of burning, since the maximal temperatures on the external surface of insulator are in the range $120 - 165\ °C$ and casing of test-motor is made from steel AISI 304.



a)



b)

Fig. 10. Validation of numerical results by analytical solution for thermocouple 3 and heat flux 0.0318 W/mm^2: a) distribution of temperature on external surface of heat insulation; b) distribution of temperature on internal surface of heat insulation

CONCLUSIONS

The approach based on solution of inverse problem with application of analytical and numerical methods can be extremely useful for determination of heat fluxes in pre- and post-chamber of hybrid rocket motors. In the presented study, the values of heat flux along post-chamber of hybrid propellant test-motor are in the range $0.012 - 0.032\ W/mm^2$ for the moderate conditions of combustion. These data are indispensable for the prediction of thermal state of motor's structural components. Thus, the experiments should be repeated for other burning conditions with different values of O/F ratio taking into consideration suggestions for increasing of experimental data intensity.

It is necessary to note that analytical or numerical solutions accepted in the study will provide underestimated values of temperature on external surface of insulator in case of prolonged burning times due to erosion of insulators. Nevertheless, the temperature distributions predicted by suggested method are useful for preliminary estimation of insulator geometry that can be corrected after determination of erosion rates of insulator's materials.

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