A STRUCTURAL RELIABILITY ANALYSIS FOR THE DIFFERENTIAL SETTLEMENT OF A MULTI-STOREY BUILDING WITH SHALLOW FOUNDATION

Mauricio Rogerio Ramos Ribeiro

mauricio3r@usp.br

School of Engineering of the University of São Paulo (EPUSP)

Av. Prof. Luciano Gualberto, 3, São Paulo, SP – Brazil, 05508-010

School of Engineering of the University of São Paulo (EPUSP)

Valério Silva Almeida

valerio.almeida@usp.br

School of Engineering of the University of São Paulo (EPUSP)

Av. Prof. Luciano Gualberto, 3, São Paulo, SP – Brazil, 05508-010

School of Engineering of the University of São Paulo (EPUSP)

Hélio Luiz Simonetti

helio.simonetti@ifmg.edu.br

Instituto Federal de Minas Gerais – IFMG

Campus Betim, Rua Itamarati, 140, São Caetano Betim, MG-Brasil, 32677-564

Abstract. *This work is based on the reliability analysis for the differential settlements from a multi-storey building, with the FOSM (First Order Second Moment) method applied in conjunction with FEM (Finite Element Method) that evaluates the efforts and settlements of the building in conjunction with the soil-foundation. The FOSM method contains gradient vectors and is based on optimization iterative procedure, and the ultimate state function depends on the FEM results, what makes a direct differentiation not possible for the function analysis. To do it so, the FDM (Finite Difference Method) is formulated in the FOSM algorithm to obtain the gradient vector and the optimization as well. Once it has the reliability results, it is possible to discuss about the probability of failure, when applied the variability of the parameters of the soil described in the literature commonly used in foundation designs.*

Keywords: *Reliability, First Order Second Moment (FOSM), Soil-Structure Interaction, Finite Difference Method*

INTRODUCTION

With the persuade of making more realistic analysis from numerical simulations, the Reliability methods introduce the statistical parameters of the variables that compound the problem's formulations. In this context, the Structural Reliability method is chosen herein to analyze the differential settlement that occur in a multi-storey building, contemplating the variations of the more influent coefficients of the problem. As said in Song (2004), probability and statistics are important tools to help us predicting scientific phenomenon behavior and solving the increasing complexity mathematical problems. Although it is needed to have the right analysis of the variables for the problem, according to JCSS (2001), their material is intended to help engineers to design structures with the probabilistic point of view, and to be used by those who want to apply new approaches in future works. Having the necessary information about the general variables probabilistic behavior, an attempt to bring them to Brazilian environment and standards is made in this work, as recent works (Ramirez, 2016), (Santos *et al*., 2014) and a classical work (Hachich, *et al.,* 1998) allied with Brazilian technical standards NBR8681:2003, NBR6118:2014, NBR6122:2010 and NBR6123:1988. To make a statistic study it is necessary a probabilistic method, in this work the FOSM (First Order Second Moment) method will be used. According to Beck (2011), FOSM is a simpler method which is the basis for more complex ones. Still, a simulation routine is necessary to run the model, so in Almeida (2003) is possible to simulate the shallow foundation interacting with the multi-storey building and soil influence by means of Finite Element, with the last using the winkler's model. Finally, a macro routine is built to integrate those procedures, thus an algorithm is made in MATLAB to run FOSM method with the limit-state equation that depends on an implicit with the model's result, what makes a Finite Difference Method (FDM) necessary to deal with the gradient vector within FOSM method as assumed in Beck (1999). A numerical example is done with hypothetical case of a multi-storey building with shallow foundation to designed in Brazil in order to analyze and discuss the results afterwards. With those results obtained, an appropriate interpretation is done and compared with the references making a suitable conclusion with new works proposal.

1 STRUCTURAL RELIABILITY

Considering a real structural problem, we have various parameters, among them the section geometries, material physical characteristics, loads etc.; in which can be represented by aleatory variables according to Fig.1, that each variable can be represented by a density probability function $f_{x_i}(x_i)$ and the respective parameters.

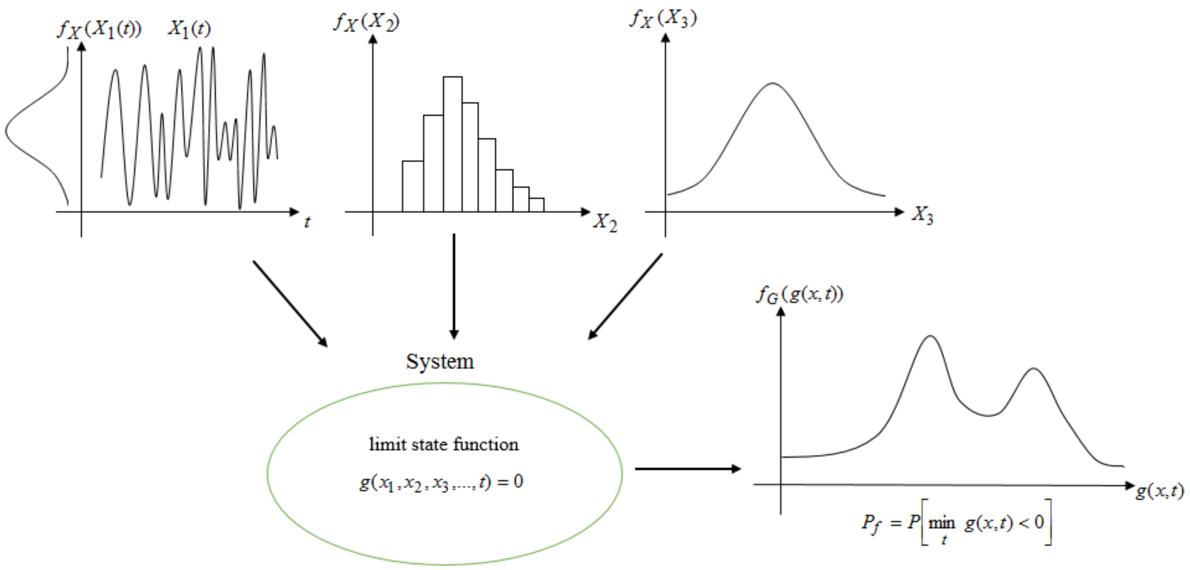

Figure 1. Scheme of a structural reliability problem (Beck, 2015, adapted)

The variable vector of the problem is named by $\boldsymbol{X}$ and a possible correlation between a pair of variables is indicated by the correlation matrix $\boldsymbol{C}$. The usage of the model simulation (in this case, a FEM) will produce a range of response $s(\boldsymbol{X})$, whose can be deformations, stresses, displacements, internal forces or any phenomenon which can be analyzed, according to the failure mode settled (excessive displacement, limit stress, etc.). The limit state function is formulated in terms of the effects of the loads in the model and the critical values (or limits) $x_{\lim}$:

$$g(s(\mathbf{x}), x_{\lim}) = 0 \tag{1}$$

The limit value of x_{lim} can be aleatory or deterministic, like a stress value or standard limit values. It is important that the function $s(\boldsymbol{x})$, that depends on the model response, is explicit for the results and analysis, otherwise contains implicitly the variables vector $\boldsymbol{X}$, in which is applied in the model to solve the problem, in this case, the FEM.

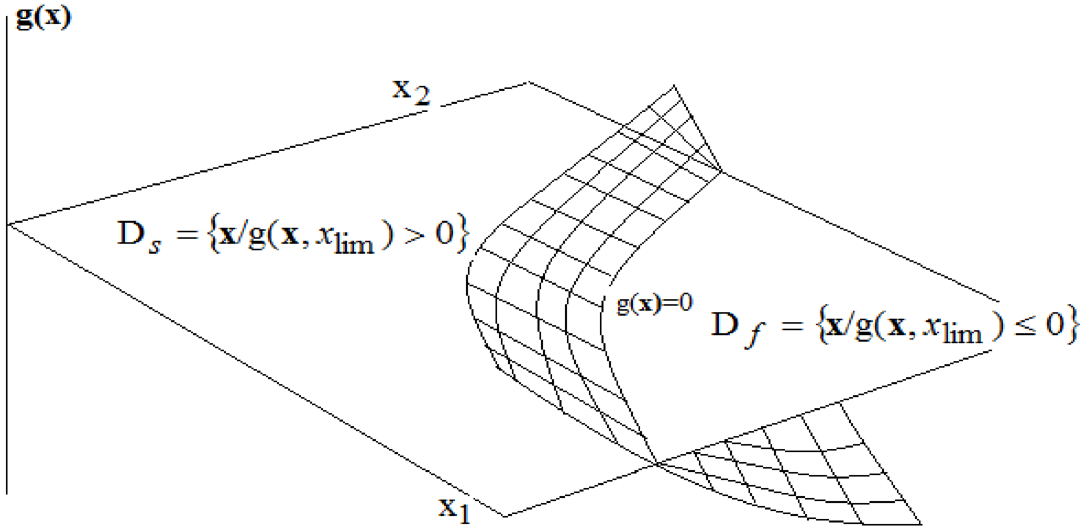

Figure 2. Domains of the limit state equation (Beck, 2015, adapted)

The state limit equation is defined so that can be divided in a Domain of Failure (D_f) and Domain of Safety (D_s), as follows in Eq. (2):

$$D_f = \{X \,/\, g(\mathbf{X}, x_{\lim}) \le 0\}$$
$$D_s = \{X \,/\, g(\mathbf{X}, x_{\lim}) > 0\} \tag{2}$$

In which-the probability of failure can be calculated:

$$P_f = \int_{g(\mathrm{x}, x_{\lim}) \le 0} f_X(\mathbf{X}) dX \tag{3}$$

Whereby $f_X(\mathbf{X})$ is the joint probability density function of the aleatory variables, of which generally is approximated for a probability density function.

The integration of the Eq.(3) is not solved exactly, however is approximated numerically by the FOSM (First Order Second Moment) method, i.e., for each value assigned for de elements of $\boldsymbol{X}$ there will be different results for $f_X(\mathbf{X})$ solved by the FEM model, in displacements, forces or deformations, for the limit state function. With this iterative process, values for $\boldsymbol{X}$ settled, thus $\boldsymbol{X} = \mathbf{x}$, that are applied in the FEM model and the results are used to solve the Eq.(3). That process, therefore, has the advantage of using an algorithm which there is no need to modify the code of the FEM program, but an external routine that runs the program externally to use only the results provided. The FOSM method will be presented below, in with solve Eq.(3) by an optimization algorithm.

2 FIRST ORDER SECOND MOMENT METHOD

According to Beck (2015), solving structural reliability problems involves the determination of the approximation of the joint probability density function $f_{Xi}(x_i)$, so the FOSM method is one transformation kind of solution, that approximate the limit sate function for a linear function, restricting the statistical values of mean and standard deviation to build $f_{x_i}(x_i)$.

2.1 Hassofer Lind transformation

The transformation methods, as FOSM, are based in the Hassofer Lind transformation, that transforms a vector of Gaussian variables $\boldsymbol{X}$, with respective mean and standard deviation, in a joint $\boldsymbol{Y}$ of normal aleatory variables with null mean and unitary standard deviation:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{4}$$

Figure 3 shows the transformation for a limit state function $g(y) = r - s$, as a general function, and assigning $r = y_1$ and $s = y_2$.

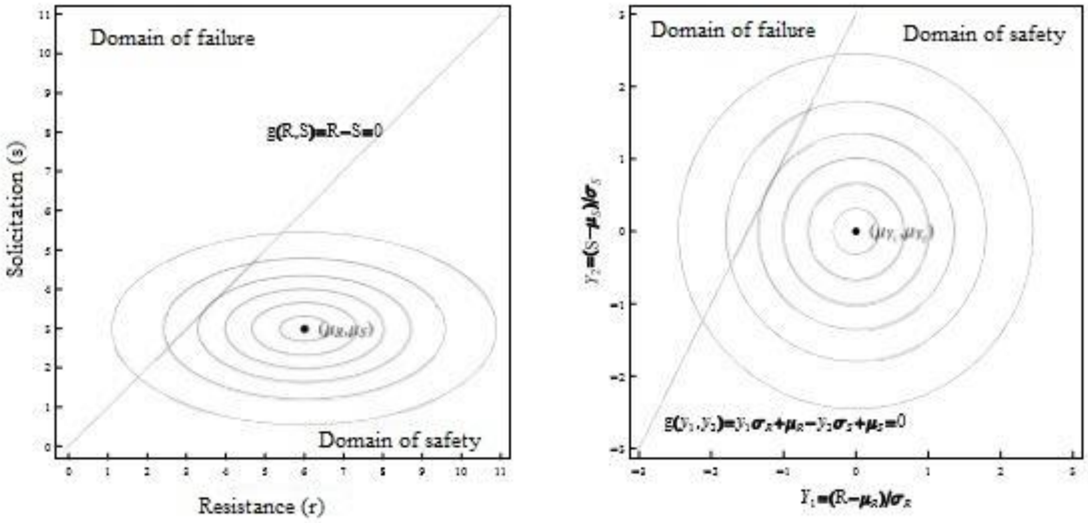

Figure 3. Hassofer Lind transformation from a $\mathbb{X}$ to a $\mathbb{Y}$ space (Beck, 2015)

2.2 Project Point and reliability index for n-dimensional problems

Reliability index β can be obtained applying the Hassofer Lind transformation (Eq. 4) in the main problem (Eq.3) and solving the optimization problem on finding the project point y^* that minimizes $d = \|y\|$ which nulls the limit state function $g(y)$. Where d is the distance between a point y to any origin, thus that is an optimization problem with restrictions, so β is the minimal distance:

$$d = \|y\| = \sqrt{y^{*T} \bullet y^*} \tag{5}$$

And the gradient vector of the limit state function is given by:

$$\nabla g(y) = \left\{ \frac{\partial g}{\partial y_1}, \frac{\partial g}{\partial y_2}, \ldots, \frac{\partial g}{\partial y_n} \right\}^T \tag{6}$$

The director cosines of any point on the limit state function can be obtained:

$$\alpha(y) = \frac{\nabla g(y)}{\|\nabla g(y)\|} \tag{7}$$

Finally, the project point y^* is the limit state function point nearest of the origin in the $\mathbb{Y}$ space, thus in that point the limit state function is projected about the origin as follows:

$$y^* = -\alpha\beta \tag{8}$$

Where $\beta = \|y^*\|$ is the distance between y^* and the origin of the $\mathbb{Y}$ space.

2.3 Lagrange method for optimization problems with restrictions

The optimization can be solved by the Lagrange method, applying the Lagrange's multiplicator λ, the Lagrangian of this problem follows:

$$\mathcal{L}(y,\lambda) = \sqrt{\mathrm{y}^{\mathrm{T}} \bullet y} + \lambda g(y) \tag{9}$$

A stationary point can be obtained for Eq.(9):

$$\frac{\partial \mathcal{L}}{\partial y} = yd^{-1} + \lambda \nabla g(y) = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = g(y) = 0 \tag{11}$$

The Eq.10 is satisfied if y is on the limit station function, and determines the coordinates of the stationary point, and assuming that this point is the project point, i.e. a minimum point, then is obtained:

$$d_{\min} = -\frac{\nabla g(y)}{\|\nabla g(y)\|} \cdot y^* = -\alpha^T \cdot y^* \tag{12}$$

2.4 Linearization of the limit state function

An expansion of the limit state function at a random point y^p, in Taylor series limited for linear terms:

$$\tilde{g}(y) = g(y^p) + \nabla g^T \cdot (y - y^p) \tag{13}$$

For the expected value:

$$E[\tilde{g}(y)] = g(y^p) - \nabla g^T \cdot y^p \tag{14}$$

And the variance:

$$Var[\tilde{g}(y)] = \nabla g^T \cdot \nabla g \tag{15}$$

By β definition, with Eq.14 and Eq.15:

$$\beta = \frac{E[\tilde{g}(y)]}{\sqrt{Var[\tilde{g}(y)]}} = -\alpha^T \cdot y^p \tag{16}$$

Therefore, if $y^p = y^*$ the Eq.(16) is equal to Eq.(12), proving that the distance between the project point and the origin of the space represents the reliability index.

2.5 The Hassofer, Lind, Rackwitz and Fiessler (HLRF) Algorithm

The HLRF algorithm was developed specifically to solve structural reliability problems. Be y_k any point, out of the failure surface but candidate to a project point, a Taylor expansion of $g(y_{k+1})$ for linear terms is given by:

$$\tilde{g}(y_{k+1}) = g(y_k) + \nabla g(y_k)^T \cdot (y_{k+1} - y_k) = 0 \tag{17}$$

A new point is obtained, that $\tilde{g}(y_{k+1}) = 0$. Assuming $\beta_k = \sqrt{y_k^T \bullet y_k}$ allied with Eq.8 applying in Eq.(17) resulting in:

$$\nabla g(y_k)^T . y_{k+1} = -\nabla g(y_k)^T \cdot (\alpha_k \beta_k) - g(y_k) \tag{18}$$

Whereas the director cosine has an unitary length, it can be inserted in Eq.(18) without modifications:

$$\alpha_k^T \cdot \alpha_k = \frac{\nabla g(y_k)^T \cdot (\alpha_k)}{\|\nabla g(y_k)\|} = 1 \tag{19}$$

Thus, applying in Eq.(18):

$$\nabla g(y_k)^T . y_{k+1} = -\nabla g(y_k)^T \cdot (\alpha_k \beta_k) - \frac{\nabla g(y_k)^T \cdot (\alpha_k)}{\|\nabla g(y_k)\|} g(y_k) \tag{20}$$

Rearranging Eq.(20):

$$y_{k+1} = -\alpha_k \left[\beta_k + \frac{\nabla g(y_k)}{\|\nabla g(y_k)\|} \right] \tag{21}$$

Figure 4 illustrates the geometric interpretation for the iterative procedure.

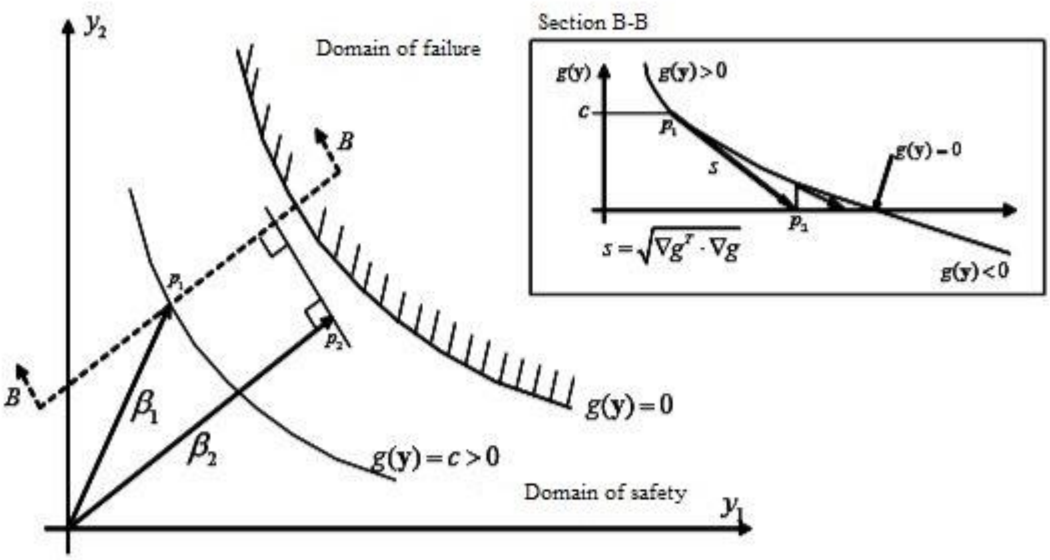

Figure 4. Geometric interpretation for HLFR iterative algorithm (Beck, 2015)

Other expression can be found by Eq.(18):

$$\nabla g(y_k)^T \cdot y_{k+1} = \left[\nabla g(y_k)^T \cdot y_k - g(y_k)\right] - \frac{\nabla g(y_k)^T \cdot \alpha_k}{\|\nabla g(y_k)\|}$$
$$= \frac{\nabla g(y_k)^T \cdot y_k - g(y_k)}{\|\nabla g(y_k)\|^2} \cdot \nabla g(y_k)^T \cdot \nabla g(y_k) \tag{22}$$

Finally, rearranging Eq. 22:

$$y_{k+1} = \frac{\nabla g(y_k)^T \cdot y_k - g(y_k)}{\|\nabla g(y_k)\|^2} \cdot \nabla g(y_k) \tag{23}$$

The convergence can be satisfied if:

$$1 + \left| \frac{\nabla g(y_{k+1}) \cdot y_{k+1}}{\|\nabla g(y_{k+1})\| \cdot \|y_{k+1}\|} \right| < \varepsilon$$
$$|g(y_{k+1})| < \delta \tag{24}$$

2.6 Matricial Hassofer-Lind transformation

In order to work with multiple values, it is necessary to make some transformation of the space $\mathbb{X} \rightarrow \mathbb{Y}$ and $\mathbb{Y} \rightarrow \mathbb{X}$, so that is convenient to introduce matrix variables. First, introducing the mean vector $\boldsymbol{M}$ and the diagonal matrix of standard deviations $\boldsymbol{D}$:

$$\mathbf{M} = \{\mu_{x_1}, \mu_{x_2}, ..., \mu_{x_n}\} \tag{25}$$

$$\mathbf{D} = \begin{bmatrix} \sigma_{X_1} & 0 & ... & 0 \\ 0 & \sigma_{X2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \sigma_{X_n} \end{bmatrix} \Rightarrow \mathbf{D}^{\mathbf{-1}} = \begin{bmatrix} \frac{1}{\sigma_{X_1}} & 0 & ... & 0 \\ 0 & \frac{1}{\sigma_{X2}} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \frac{1}{\sigma_{X_n}} \end{bmatrix} \tag{26}$$

So, the Hassofer-Lind transformation in matrix form is given by:

$$x = \mathbf{J}_{xy} \cdot y + \mathbf{M}, \qquad \mathbf{J}_{xy} = \left[\frac{\partial x_i}{\partial y_j}\right]_{i=1,2,...,n;\ j=1,2,..,n} = \mathbf{D}$$
$$y = \mathbf{J}_{yx} \cdot \{x - \mathbf{M}\}, \qquad \mathbf{J}_{yx} = \left[\frac{\partial y_i}{\partial x_j}\right]_{i=1,2,...,n;\ j=1,2,..,n} = \mathbf{D}^{\mathbf{-1}} \tag{27}$$

Making the limit state function gradient vector in the $\mathbb{Y}$ space:

$$\nabla g(y) = \left\{ \sum_{j=1}^{n} \frac{\partial g}{\partial x_i} \frac{\partial x_i}{\partial y_j} \right\}_{j=1,2,\ldots,n} = (\mathbf{J}_{xy})^{-1} \cdot \nabla g(x) \tag{28}$$

2.7 Finite Difference method applied in structural reliability problems

As mentioned briefly, since the limit state function has implicit results obtained by the FEM, the direct differentiation of $g(x)$ is not possible to calculate the gradient vector $\nabla g(x)$, according to Eq.27. To solve this issue, the FDM can be one way to make an approximate differentiation of the limit state function by finite central difference, applying a small difference Δx for each aleatory variable, with the following approximation:

$$\frac{\partial g}{\partial x_i} \approx \frac{g(x + \Delta x_i) - g(x)}{\Delta x_i} \tag{29}$$

Once that is convenient to set Δx as a percentage of the standard deviation:

$$\Delta x_i = p\sigma_{x_i} \tag{30}$$

Thus, for each value of the approximation for a gradient vector of the limit state function in the $\mathbb{X}$ space, it is necessary to calculate it using the FEM program.

3 FOSM INTREGRATION WITH THE FEM FOR SOLVING MULTI-STOREY BUILDINGS

The multi-storey building is analyzed using the program proposed by Almeida (2003), that is based on FEM with Lagrangian formulation, that uses frame and shell elements to make linear and non-linear simulations for multi-storey buildings with direct foundation, which is simulated by Winkler's model or Boundary Element Method for representing the soil that is coupled with the FEM. Then, this mechanical analysis is coupled with the FOSM method as shown at the flowchart bellow:

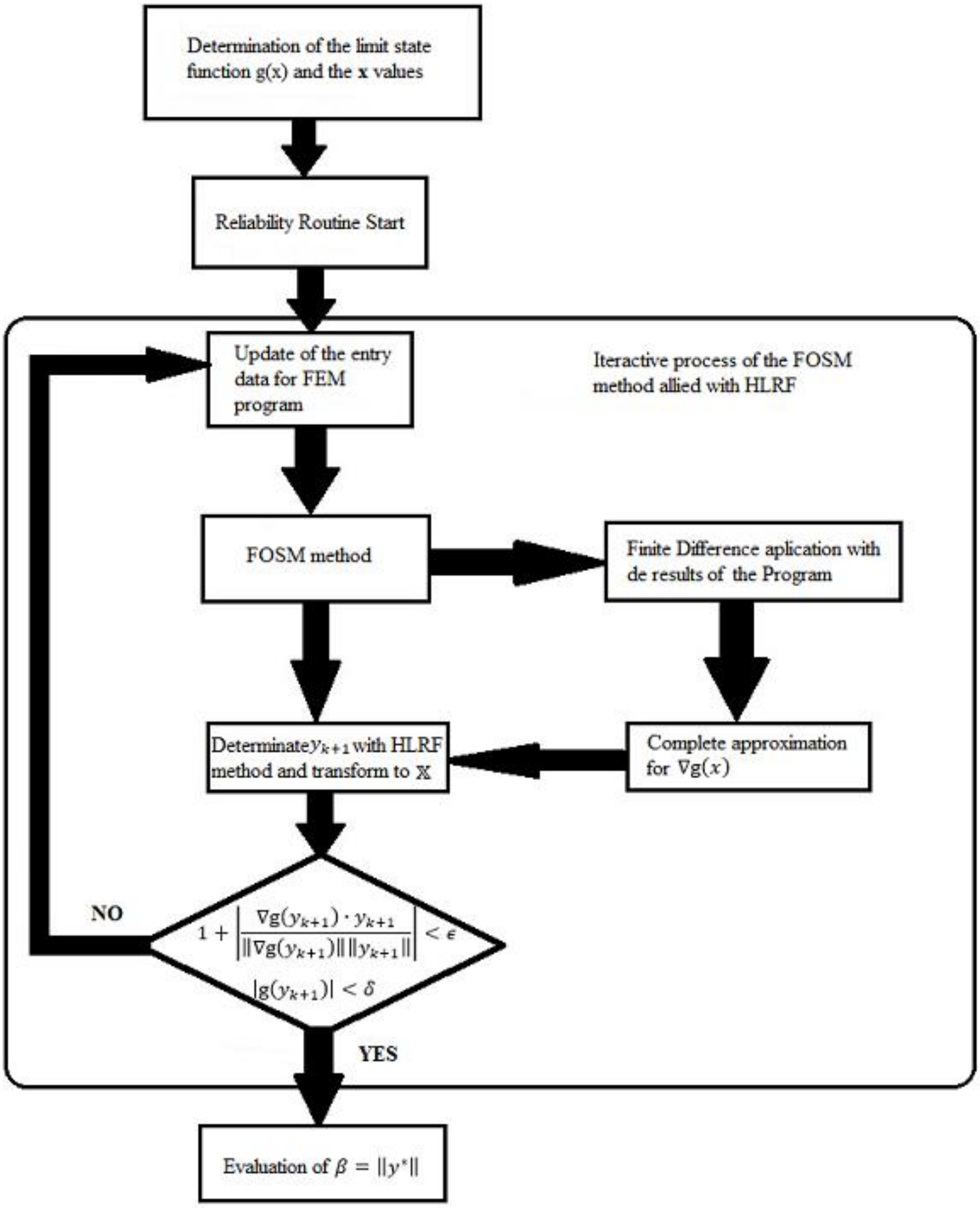

Figure 5. Flowchart of the proposed scheme for the algorithm to be used

4 DIFFERENTIAL SETTLEMENT

The differential settlement is the displacement difference between each different foundation that can generate additional stresses or cracks in the structural elements that possibly will cause pathologies. Those pathologies can be classified according to Hachich *et al*. (1998) as:

Architectural damage or to the building visual appearance. Pathologies that can be visible and cause a discomfort, like cracks on the walls and floor displacements;

Damage for the functionality or usage of the building. Problems as ramp incline inversion and the declivity of the edification;

Structural damage. Pathologies that can occur at the structural components which can compromise safety.

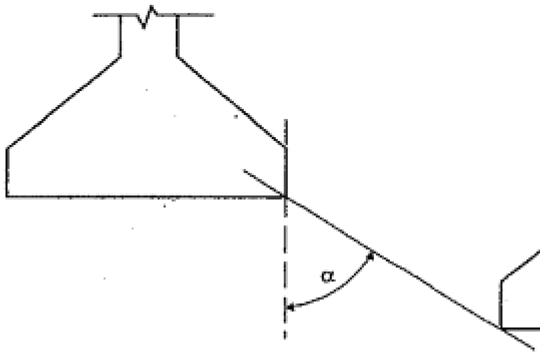

Figure 6. Scheme of a differential settlement (NBR6122:2010)

That phenom is commented at the NBR6122:2010, restricting the values of the differential settlements according to α angle shown in the Fig.6:

Table 1. Differential settlement angles according to NBR6122:2010

Soil Type	α
Weak	≥ 60º
Resistant	45º
Rocks	30º

As a complement about the values determined by the Brazilian standard, the technical complementary standard of São Paulo Subway, NC03 (1980) and Hachich *et al*. (1998), present specific values for differential settlement δ_{max} according to the type of the soil:

Table 2. Differential settlement maximum values δ_{max}:

Soil Type	δ_{max}
Sand	25mm
Clay	40mm

For other types of soil there is no recommendation, only making bearing capacity experiment *in situ*. Those values can be used only for conventional structures as reinforced concrete, not recommended for structural masonry.

5 NUMERICAL SIMULATION

To show the application of the presented algorithm, in this section an hypothetical example of a multi-storey building with shallow foundation designed in Brazil will be presented. The project is about an 8-floor building, with the standard floor with 6x6m and 9 footing foundations of 2x2x1m on sand soil which the SPT (Standard Penetration Test) rounds 2 to 5, beams 20x55cm, columns 70x20cm, C25 concrete for all the structure. The wind is acting at the X direction, according to Fig.7 axis along with the parameters in NBR6123:1988, for the capital of São Paulo. The building model is indicated in Figure 7.

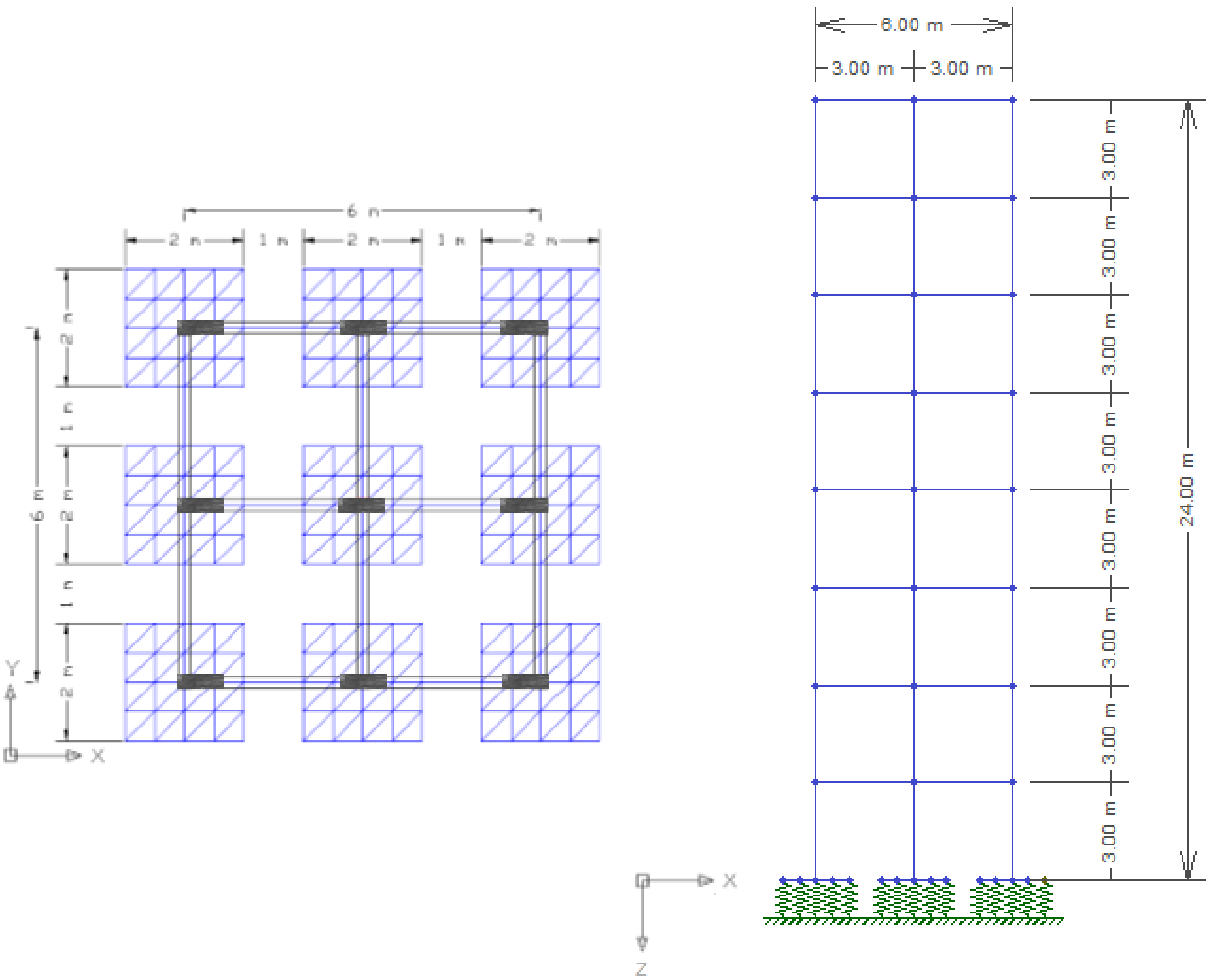

Figure 7. Model scheme of the building

The following analysis contains the differential settlement between the foundation structures, using the Winkler's model for soil-structure interaction, the value $k_{soil} = 3{,}000\ kN/m^3$ was adopted for all the elements, referencing to SPT 2 to 5 according to NC03 (1980) and Hachich *et al*. (1998). For all the other parameters, the JCSS (2001) was the main reference for all the statistical behavior, whereas those standards are based on many others as North-America, Europe and Asia, and the project is designed in Brazil, so an adaptation is done as Ramirez (2016) and Santos *et al*. (2014) allied with Brazilian technical standards.

Table 3. Values adopted for statistical analysis

Variable	Attribution	Symbol	Unit	Mean	Coefficient of variation (C.V)	Standard Deviation
1	Basic Velocity	v_0	m/s	45	0.15	6.75
2	Pressure Coefficient	C_p	$adim.$	1.4	0.15	0.21
3	S1 Factor	$S1$	$adim.$	1	0.15	0.15
4	S2 Factor 1rst floor	$S2_1$	$adim.$	0.72	0.15	0.11
5	S2 Factor 2nd Floor	$S2_2$	$adim.$	0.78	0.15	0.12
6	S2 Factor 3rd Floor	$S2_3$	$adim.$	0.82	0.15	0.12
7	S2 Factor 4th Floor	$S2_4$	$adim.$	0.85	0.15	0.13
8	S2 Factor 5th Floor	$S2_5$	$adim.$	0.88	0.15	0.13
9	S2 Factor 6th Floor	$S2_6$	$adim.$	0.9	0.15	0.14
10	S2 Factor 7th Floor	$S2_7$	$adim.$	0.91	0.15	0.14
11	S2 Factor 8th Floor	$S2_8$	$adim.$	0.93	0.15	0.14
12	Live Load	q	kN/m	3	0.3	0.9
13	Soil stiffness for footing 1	k_1	kN/m^3	3000	0.3	900
14	Soil stiffness for footing 2	k_2	kN/m^3	3000	0.3	900
15	Soil stiffness for footing 3	k_3	kN/m^3	3000	0.3	900
16	Soil stiffness for footing 4	k_4	kN/m^3	3000	0.3	900
17	Soil stiffness for footing 5	k_5	kN/m^3	3000	0.3	900
18	Soil stiffness for footing 6	k_6	kN/m^3	3000	0.3	900
19	Soil stiffness for footing 7	k_7	kN/m^3	3000	0.3	900
20	Soil stiffness for footing 8	k_8	kN/m^3	3000	0.3	900

21	Soil stiffness for footing 9	k_9	kN/m^3	3000	0.3	900
22	Error factor for the settlement	θ_s	$adim.$	1	0.25	0.25

A common analysis for differential settlement was made for this model, which is made with characteristic load values, presented as the mean values. The results are obtained by running the FEM program with the mean presented on Table 3 which are values commonly used at building designs according to Brazilian standards. So, the results provided by the program are in Table 4:

Table 4. Differential settlement results by nodes and coordinates

Node	Coord. X	Coord. Y	$d_z(mm)$
289	0.0	0.0	15.1
290	0.0	3.0	17.8
291	0.0	6.0	15
292	3.0	0.0	20.1
293	3.0	3.0	24.3
294	3.0	6.0	20.1
295	6.0	0.0	22.6
296	6.0	3.0	25.3
297	6.0	6.0	22.6

As it can be seen, the maximum differential settlement value that could be obtained is about $\delta = 7.5\ mm$, thus is below the maximum of $\delta_{máx} = 25\ mm$.

After this simple simulation, now it is shown the statistical simulation with the presented algorithm, to do that de limit state function should be:

$$g(\boldsymbol{x}) = \delta_{máx} - \theta_s \delta \tag{31}$$

So, we can fix the nodes 296 and 290 to determinate δ. The term θ_s in Eq.(31) is about the intrinsic error in the model, once the procedure is an approximation. And $\delta_{máx}$ is the limit value presented in session 4 that represents the deterministic limit value fixed. Although there is no error factor for $\delta_{máx}$, it is important to notice that this value could have one, once it is theoretical.

Afterwards, the probabilistic algorithm is used in the model, and the result for the reliability index and the project point are shown in tables 5 and 6.

Table 5. Reliability index and Probability of Failure

Reliability index β	P_f
2.36	9.14-E3

Table 6. Values for the project point

Variable	Attribution	Symbol	Unit	Project Point
1	Basic Velocity	v_0	m/s	52.7
2	Pressure Coefficient	C_p	$adim.$	1.5
3	S1 Factor	$S1$	$adim.$	1.2
4	S2 Factor 1rst floor	$S2_1$	$adim.$	0.8
5	S2 Factor 2nd Floor	$S2_2$	$adim.$	0.8
6	S2 Factor 3rd Floor	$S2_3$	$adim.$	0.8
7	S2 Factor 4th Floor	$S2_4$	$adim.$	0.9
8	S2 Factor 5th Floor	$S2_5$	$adim.$	0.9
9	S2 Factor 6th Floor	$S2_6$	$adim.$	0.9
10	S2 Factor 7th Floor	$S2_7$	$adim.$	0.9
11	S2 Factor 8th Floor	$S2_8$	$adim.$	0.9
12	Live Load	q	kN/m	3
13	Soil stiffness for footing 1	k_1	kN/m^3	3029
14	Soil stiffness for footing 2	k_2	kN/m^3	3659
15	Soil stiffness for footing 3	k_3	kN/m^3	3286
16	Soil stiffness for footing 4	k_4	kN/m^3	3029
17	Soil stiffness for footing 5	k_5	kN/m^3	3029
18	Soil stiffness for footing 6	k_6	kN/m^3	3000
19	Soil stiffness for footing 7	k_7	kN/m^3	2571

20	Soil stiffness for footing 8	k_8	kN/m^3	2170
21	Soil stiffness for footing 9	k_9	kN/m^3	2656
22	Error factor for the settlement	θ_s	$adim.$	1.2

In order to interpret correctly the meaning of the probability in that context, the JCSS (2001) brings standard values for Ultimate Limit State (ULS) and Service Limit State (SLS). These values represent a range of costs for failure prevention and repair of structures. Even though the fact that differential settlement can provoke ULS at some situations, in this work will be approached what is more likely to occur, and that would be SLS problems:

Table 7. Service Limit State range of values by JCSS (2001)

Relative cost of safety measure	Target values for β (irreversible SLS)
High	1.3 ($P_f \approx 10^{-1}$)
Normal	1.7 ($P_f \approx 5\times10^{-2}$)
Low	2.3 ($P_f \approx 10^{-2}$)

Therefore, the results obtained with the statistic algorithm represents that the structure could present a low cost for any safety measure that may be necessary in its lifetime.

6 CONCLUSION AND DISCUSSION

In the present work, it was made a routine of an iteration process in order to obtain a statistical response, based on structure reliability, applying one of the simplest methods as the First Order Second Moment (FOSM) method. To link the method with a Finite Element-program, the Finite Difference was needed once a direct differentiation is not possible for the limit state function. Besides the routine, there was a difficult to adapt the JCSS (2001) values for Brazilian environment, even with the references.

Allied with the common method, a statistical analysis brings more safety and reliability for the results, once that applying the variability for the variables, the results tend to be more similar to the real phenomena. Despite of the results obtained, the coefficient of variation can be reduced by an adequate statistical study for the variables, what can bring more adequate results.

To future works, an Ultimate Limit State analysis could be made, in order to study the structural reliability for bending beams or collumns with a previously designed multi-storey building.

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