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SETTLEMENTS IN SHALLOW FOUNDATIONS: COMPARISON BETWEEN ANALYTICAL AND NUMERICAL FORECASTS

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Abstract. *The shallow foundations are normally supported on soils, this leads in settlements that must be evaluated according to the work importance. Excessive settlements can produce undesirable structural pathologies. The elastic settlement, or immediate, can be calculated by innumerous analytical formulations, regarding systems with different soil layers in some cases. In this paper is presented the settlement calculation of a hypothetical shallow foundation (foundation supported on residual soil) by means of numerical method using Abaqus v.6.3 computer program (finite element method) and also by means of two analytical formulations: Steinbrenner (1934) generalized by Rosendiz (1967) and Davis and Poulos (1968). The hypothetical shallow foundation (footing) is square with 1.50m side and height equal to 0.60m. The geotechnical profile is formed by four different layers with variable thickness. Also it was varied the natural apparent unit weight of soil and Young´s modulus of each layer. The soil was modeled as a homogeneous, isotropic and as linear elastic material in the first approach and as an elastic-plastic material in the second approach. Over the foundation were applied variable and growing loads, i.e., 100, 200, 300, 400, 500 and 600 kPa. The comparison obtained between numerical (finite element method) and analytical (Steinbrenner, 1934 and Davis e Poulos, 1968) methodologies showed that there is a satisfactory concordance between the numerical forecast and analytical calculations when the soil is considered as purely elastic. The differences between analytical and numerical solutions increase when the soil is considered as elasti-plastic material.*

Keywords: *shallow foundation, settlements, numerical analysis, analytical analysis*

INTRODUCTION

In Brazil the shallow foundations are normally supported on residual soils and the estimation of settlements is an important part of the project to prevent undesirable pathologies in the structure, mainly due to the differential settlements. Sallow foundations over residual soils are subjected to the instantaneous settlement under drained condition. In Brazil the following standards area use for deign of foundations: NBR 6118 “Projeto de estruturas de concreto” and NBR 6122 “Projeto e execução de fundações”.

In this paper was verified the vertical displacement (settlement) occurring just below the foundation, and in its centre, due an external loading. The foundation is square with 1.50m side and 0.60m height. The soil is composed by four different layers varying its Young modulus and apparent unit weight of soil. The foundation has high rigidity and the soil was considered in the first approach as an elastic material and in second approach as eslastic-platic material. The vertical stress due loading and the plastic points are also shown. The settlement evaluated by analytical solutions and numerical solution (FEM: finite element method) were compared. For analytical calculation it were used the formulations of Rosendiz (1967), that is a generalization, for multi-layers of soil, of the formulation provided by Steinbrenner (1934) adopted for only one soil layer. Also, the analytical forecasts were performed using the formulation of Davis and Poulos (1967). The evaluation of settlements by means FEM was done use the computer program Abaqus v6.13.

The differences obtained when the calculation is done by FEM considering the soil as elastic material and elastic-plastic material are discussed. The best concordance between analytical solution and FEM occur when the soil is considered as elastic, because the analytical solutions consider this mechanical behavior for the soil.

1 ANALYTICAL FORMULATIONS

In this section is presented the basic concept of analytical solutions used for evaluation of the settlement of the foundation. The analytical formulations are the following: Rosendiz (1967), generalization of Steibrenner (1934) and of Davis and Poulos (1967).

1.1 Rosendiz (1967)

The formulation of Rosendiz (1967) can be used for a problem involving multi-layers of soil. According to this formulation the soil mass lies on an existing rigid stratum and at this point the settlements are null. In the upper part of rigid stratum until the surface the settlements takes place, because the soil is not indeformable. Considering the existence of indeformable stratum Steinbrenner (1934) proposed an artifice in which the settlement δ below a loaded surface can be calculated by:

$$\delta = \delta' - \delta'' \tag{1}$$

Where:

δ' – Settlement of semi-infinite soil mass at the level of load application;

δ'' – Settlement of semi-infinite soil mass in the depth where exist the indeformable stratum (Alonso, 2009).

The Steinbrenner (1934) approach (Figure 1) was generalized by Rosendiz (1967) that propose a calculation procedure for use in multi soil-layer system (Figure 2). In this case the calculation is done from bottom to top, starting from the indeformable layer. In Figures 1 and 2 the E is the young modulus of soil, ν is the Poisson coefficient and q is the imposed loading.

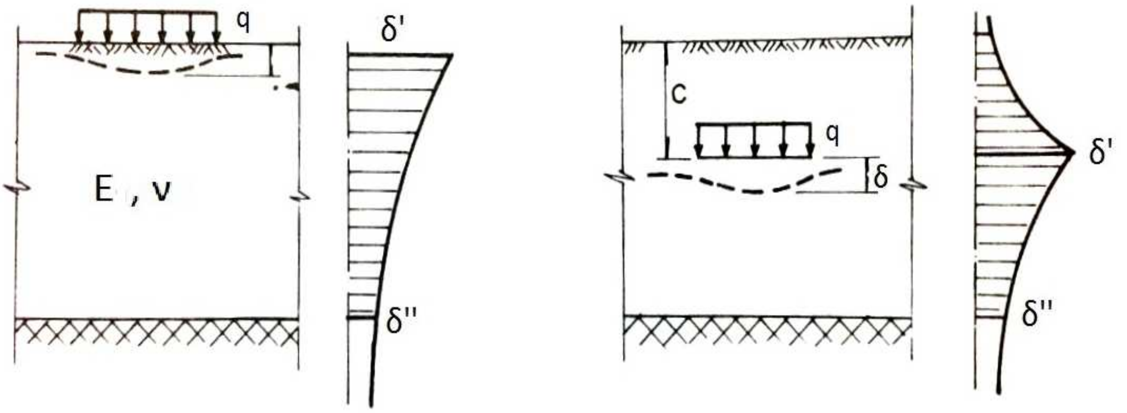


Figure 1. Proposição de Streinbrenner, 1934 (adaptado de ALONSO, 2009).

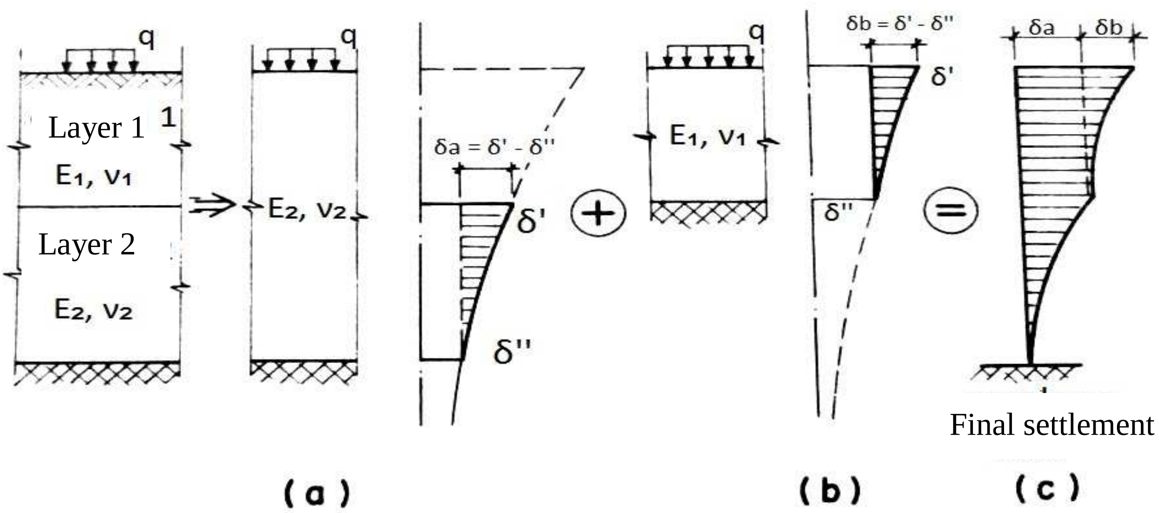


Figure 2. Generalização de Rosendiz, 1967 (adaptado de ALONSO, 2009).

Based on Figure 2 is considered that all soil (from indeformable bottom to the top) is the same as soil layer 2. So, the settlements δ' and δ'' are evaluated in the depth of indeformable and in the top of layer 2 (Figure 2a). So, the settlement for this layer, δ, is provided by Eq. (1). The procedure is repeated, by moving the indeformable to the top of the already calculated layer (Figure 2b). As the indeformable goes up, the graph of settlements is going overlapped (Figure 2c). This process is done for all layers util reach the level of load application (Alonso, 2009).

For determination of the settlement in each layer is used the Eq. (2), as follow:

$$\delta_0 = q \cdot a \cdot \frac{1-\nu^2}{E} \cdot f_0 \qquad (2)$$

Where:

q = Applied load over the footing, (kPa);

a = Width of the footing (m);

f_0 = Factor based on soil layer thickness and dimensions of footing, obtained by using appropriated abacus (Kany *apud* Velloso and Lopes, 2010);

E = Young (elastic) modulus of soil;

ν = Poisson coefficient of soil.

1.2 Davis and Poulos (1968)

The methodology proposed by Poulos and Davis (1968) to estimation of instantaneous settlements in a soil system composed by multi-layers consists in do the summation of the vertical displacements of each layer (Simons and Menzies, 1981). However, unlike Steinbrenner (1934) generalization, the summation of vertical deformation of each layer is not obtained by the difference of the settlements related to the level of load application and the settlements related to the indeformable stratum. In this method, the deformations are calculated based on the centre of each layer and after that these ones are integrated. Another difference is that in the Davis and Poulos (1968) approach the horizontal stresses, σ_x and σ_z are considered in that calculation. The final settlement is obtained using the Eq. (3), as follow:

$$\delta = \sum_{i=1}^{n} \frac{1}{E_i} \cdot \left(\sigma_z - \nu_i \cdot \sigma_x - \nu_i \cdot \sigma_y\right) \cdot h_i \tag{3}$$

Where:

E_i = Young (elastic) modulus of soil of each layer;

ν_i = Possion coefficient of soil for each layer;

σ_z = is the vertical stress due to the imposed load at the considered depth;

σ_x and σ_y = the horizontal stress due to the imposed load at the considered depth.

The value of σ_z is obtained by the Eq. (4), as follow:

$$\sigma_z = I \cdot q \tag{4}$$

Where:

I = Influence factor obtained by means of abacus provided by Fadum *apud* Velloso and Lopes (2010);

q = Applied load by the footing (kPa).

The values of σ_x and σ_y is determined by Eq. (5), considering a normally consolidated condition and at rest condition for the residual soil.

$$\sigma_x = \sigma_y = K_0 \cdot \sigma_z \tag{5}$$

The value of K_0 is given by Eq. (6):

$$K_0 = 1 - sen\phi' \tag{6}$$

Where:

ϕ' = friction angle of the soil.

2 HIPOTHETICAL CASE STUDIED

In this section is presented the model of a footing over a residual soil. The cross view of the model is shown in Figure 3 where is possible to see the multi-layers soil system and the parameters used for the soil. The footing has 1.50m side and 0.60m height. Over the footing were applied the loads, varying from 100 kPa to 600 kPa in steps of 100 kPa.

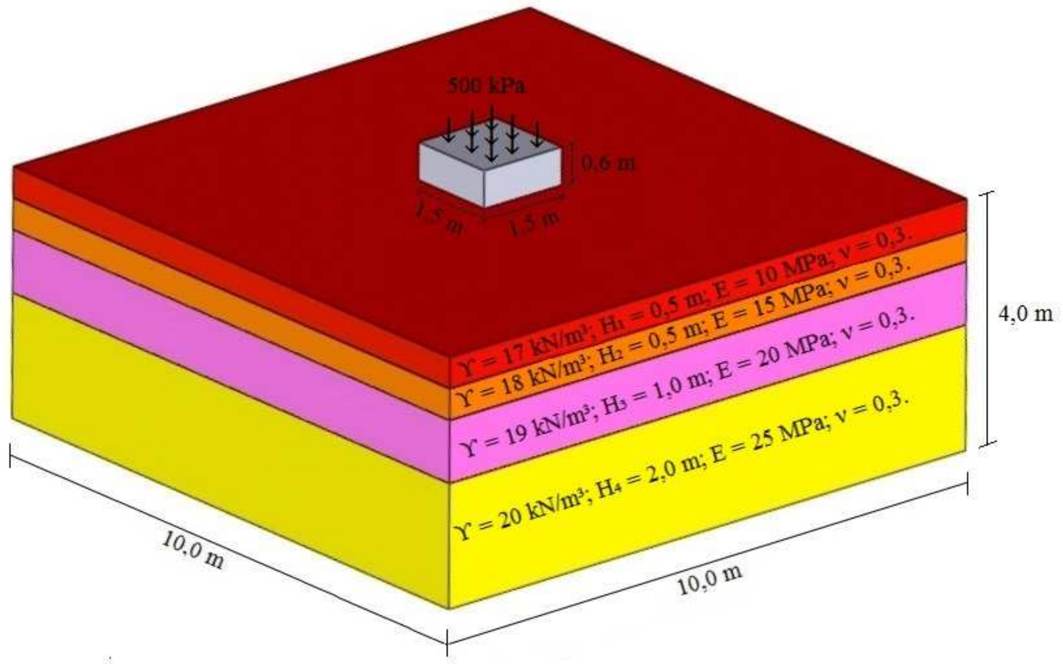


Figure 3. Model utilized in the analysis.

The Table 1 summarizes the geometry and parameters adopted for the four soil layers. The total height of soil is 4.0m and below this an indeformable material is considered.

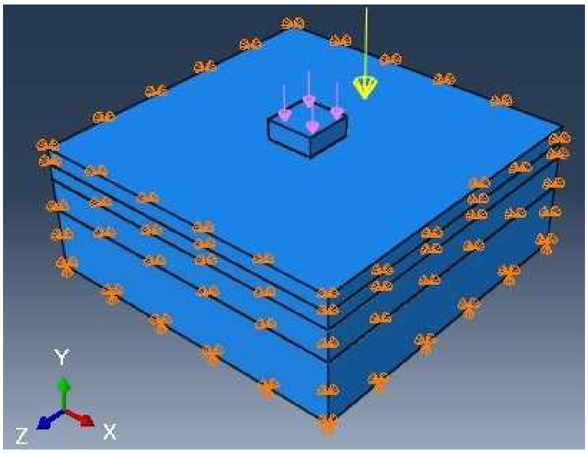
The footing was modeled as an elastic material with Young modulus equal to $21 \cdot 10^6$ kPa and the specific unit weight equal to 25 kN/m³ in order to simulate the concrete. The stress-strain behavior of footing was considered linear-elastic.

The boundary conditions are the following: the soil has freedom to deform vertically and the horizontal deformation is prevented.

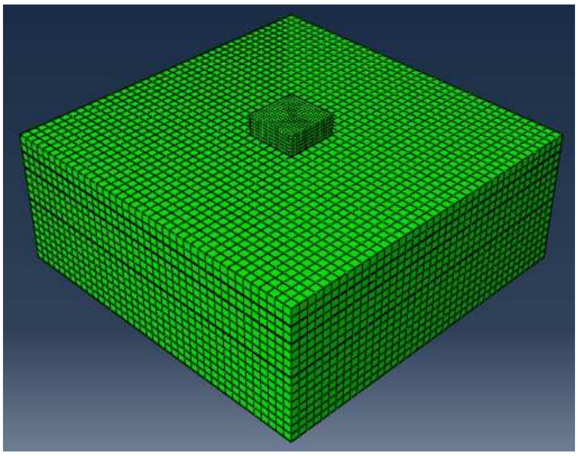
Table 3. Parameters of the soil layers.

Geometry * Parameter** Soil Model**	Soil Layer 1	Soil Layer 2	Soil Layer 3	Soil Layer 4
* Thickness (m)	0.50	0.50	1.0	2.0
** γ_n (kN/m³)	17 kN/m³	18 kN/m³	19 kN/m³	20 kN/m³
** E (kPa)	$10 \cdot 10^3$	$15 \cdot 10^3$	$20 \cdot 10^3$	$25 \cdot 10^3$
** ν [-]	0.30	0.30	0.30	0.30
** ϕ (°)	30	30	30	30
*** Model (analysis 1)	Linear-elastic	Linear-elastic	Linear-elastic	Linear-elastic
*** Model (analysis 2)	Elastic-plastic	Elastic-plastic	Elastic-plastic	Elastic-plastic

The Figure 4a and 4b shows the model with the boundary conditions and external load (variable) activated and the model with the mesh adopted, respectively. The mesh used a square elements and the spacing in region elements was set as follow: for the footing 0.10 m, for the layer 1 0.25 m, for the layer 2 0.25 m, for the layer 3 WW m and for layer 4 0.25 m.



(a)



(b)

Figure 4. Model: a) boundary conditions and external load; b) mesh adopted.

As shown in Table 1 it was performed two kinds of analysis: 1) considering the soil behavior as elastic linear and 2) considering the soil behavior as elastic-plastic. The Figure 5 presents typical curves regarding these two kinds of behavior. In the first analysis yield have not occurred, independently of the external load applied. On the other hand, plastification was observed depending of the magnitude of the applied load, as expected.

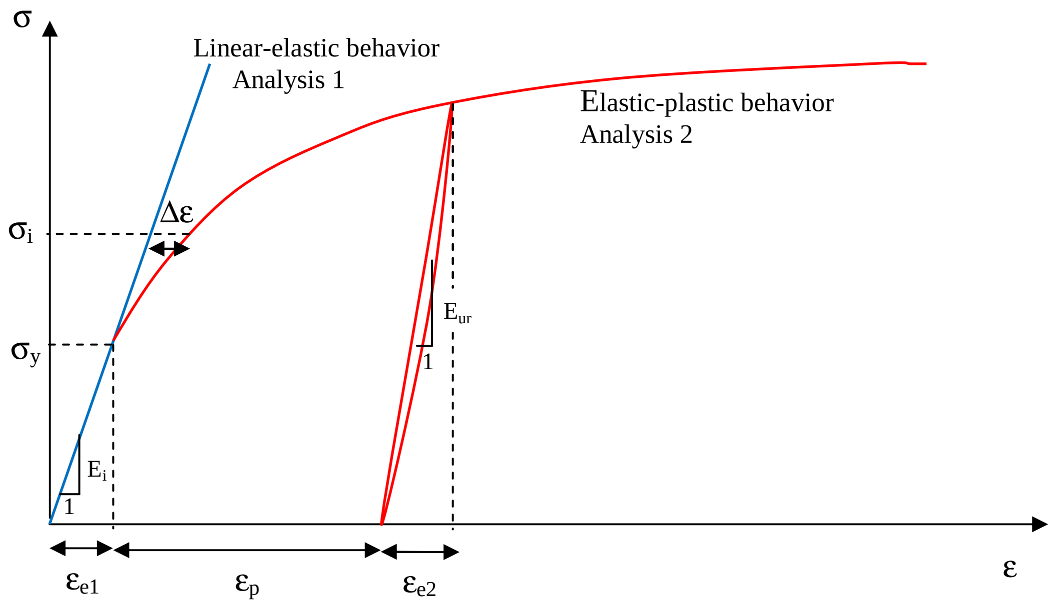


Figure 5. Soil behavior considered: Anlalysis 1, Linear-elastic and analysis 2, eslastic-plastic.

In the Figure 5 the initial tangent modulus of the soil is E_i and the unloading-reloading modulus of the soil is E_{ur}. Values of E_{ur} for soils ranging normally from 1.5 to 3.0 times the value of E_i. The ε_1 and ε_2 are elastic deformations and ε_p is the plastic deformation that can takes place in the case of analysis using the elastic-plastic soil behavior, i.e., if the stress in the soil is greater than σ_y (200 kPa). The stress strain curves for soils exhibit non –linearity when the applied stress is greater than σ_y. To account the non linearity of soil in the Analysis 2 four stress strain curves were considered, as illustrated in Figure 6.

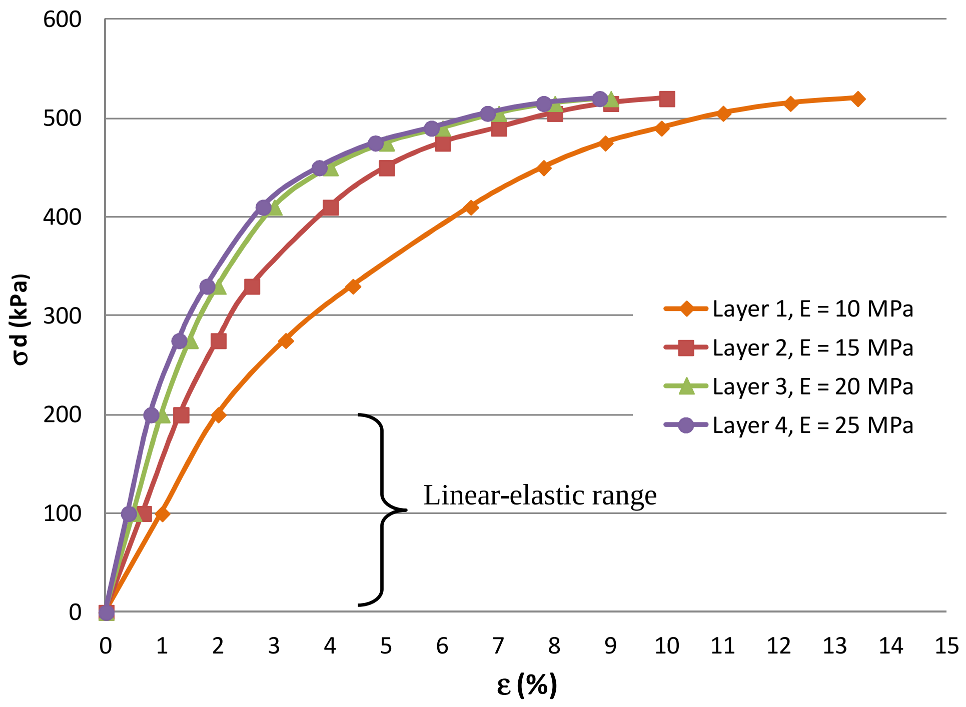


Figure 6. Stress-strain curves considered for soil layers and Analysis 2.

3 RESULTS: ANALYSES 1

In this item the results of analyses 1 are presented. The analysis 1 comprises the assumption that the soil behaves as a liner elastic material. So, the results regarding settlement due footing loading ranging from 0 to 600 kPa (in steps of 100 kPa) are compared. Three results are available: using analytical formulation of Rosendiz (1967) and Davis and Poulos (1968) and using numerical calculation (Abaqus v6.13). The Figure 7 exhibits these results in terms of settlement and applied load.

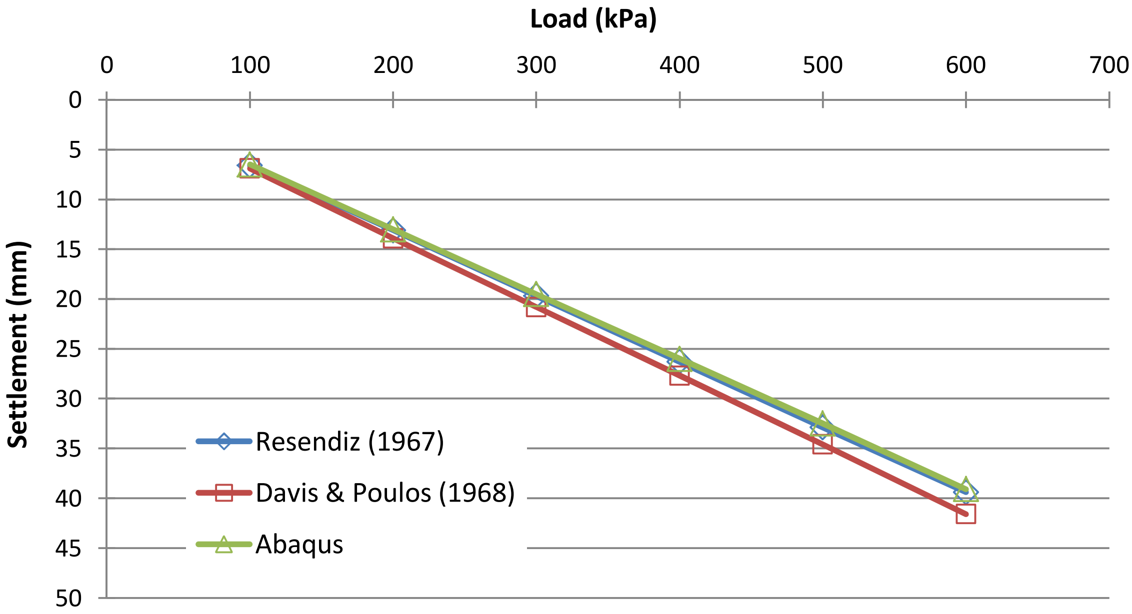


Figure 7. Results for analysis 1: Rosendiz (1967), Davis and Poulos (1968) and Abaqus.

From Figure 7 is possible to observe good concordance between the calculations (Rosendiz, 1967; Davis and Poulos, 1968 and Abaqus). Little divergence is observed in the case of Davis and Poulos (1968) and the higher is the load the higher is the divergence. However, for practical purposes the differences are negligible. So, if the imposed load do not cause yield in soil the approaches of Rosendiz (1967) and Davis and Poulos (1968) can be applied to estimation of footing settlements. The Figure 8 shows the shadings of settlements when the load equal to 600 kPa is applied.

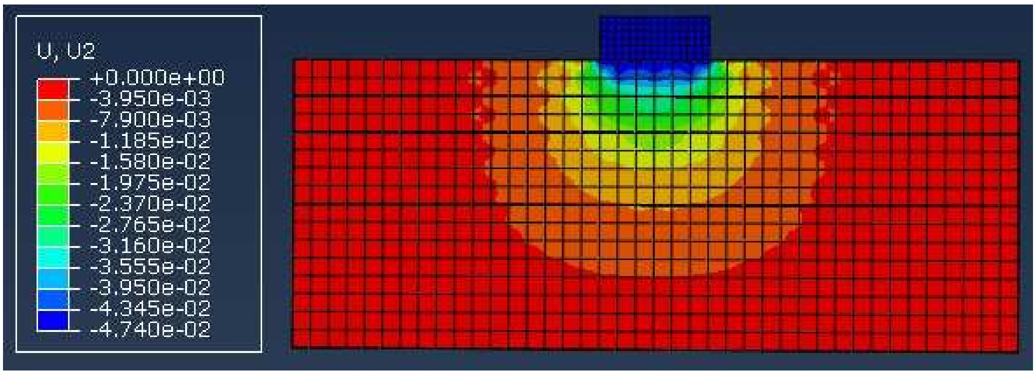


Figure 8. Shadings of settlements for analysis 1 and load equal to 600 kPa.

The Figure 9 shows the results of shadings of vertical stress considering the load of 600 kPa. It is possible to observe that the highest vertical stress occurs near the base of footing.

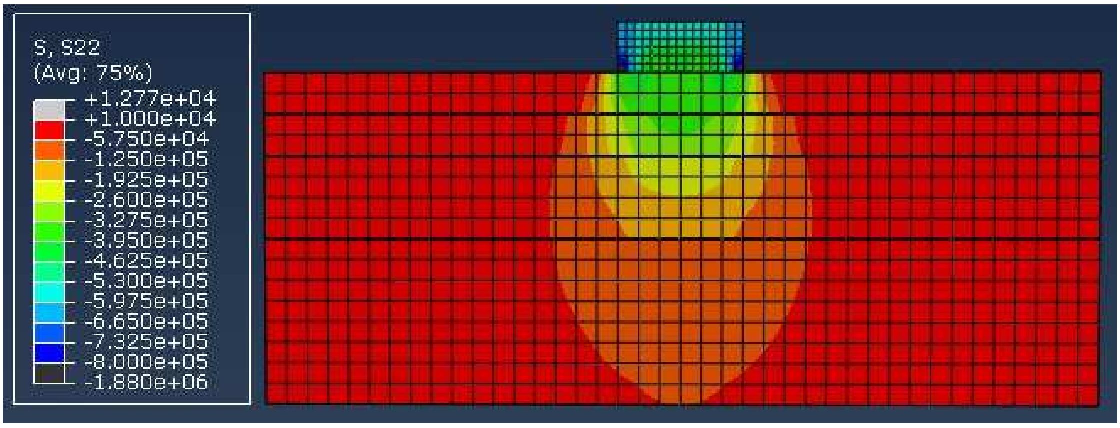


Figure 9. Shadings of vertical stress for analysis 1 and load equal to 600 kPa.

4 RESULTS: ANALYSES 2

In this item the results of analyses 2 are presented. The analyses 2 was performed considering that the soil behaves as an elastic-plastic material according to the stress-strain curves showed, for each layer, in the Figure 6. The linear elastic behavior of the soil occurs in the range of 0 to 200 kPa of vertical stress. As shown in Figure 6 the value of 200 kPa is the limit for the elastic range. For vertical stress over 200 kPa the plastic deformations takes place. The results of settlement as a function of applied load are presented in the Figure 10, using the analytical formulations of Rosendiz (1967) and Davis and Poulos (1968) and numerical calculation (Abaqus v6.13).

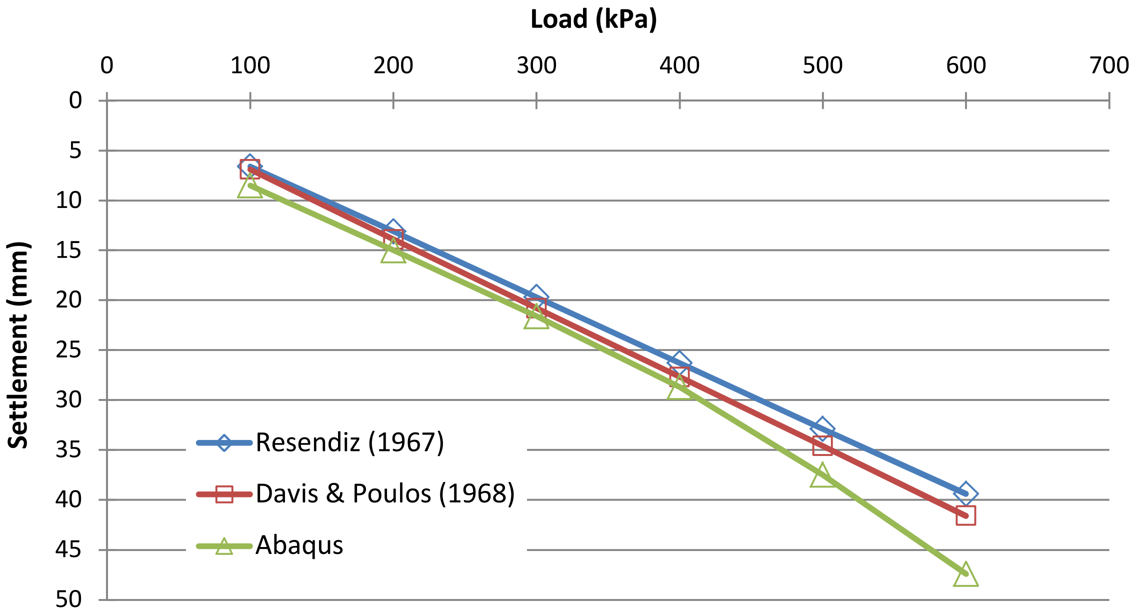


Figure 10. Results for analysis 2: Rosendiz (1967), Davis and Poulos (1968) and Abaqus.

From Figure 10 is possible to observe good concordance between the calculations (Rosendiz, 1967; Davis and Poulos, 1968 and Abaqus) in the range of o to 400 kPa of load. For imposed loads higher than 400 kPa it is observed that the settlements evaluated by numerical analysis present divergence compared to the analytical methods. The higher is the applied load, the higher is the divergence. The difference is represented by Δε, as shown in Figure 5. The yield of soil is the responsible for the higher value of settlement found when the elastic-plastic model is used. The Figure 11 shows the shadings of settlements when the load equal to 600 kPa is applied.

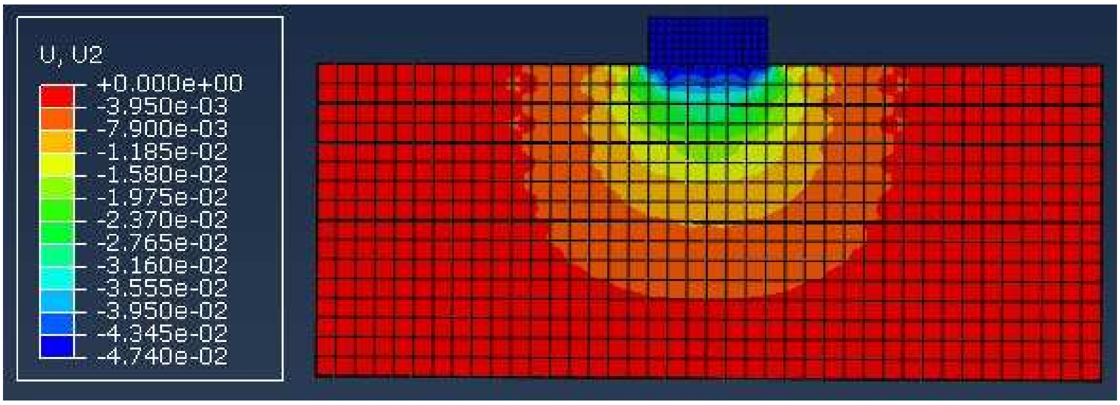


Figure 11. Shadings of settlements for analysis 2 and load equal to 600 kPa.

The Figure 12 shows the results of shadings of vertical stress considering the load of 600 kPa. It is possible to observe that the highest vertical stress occurs near the base of footing.

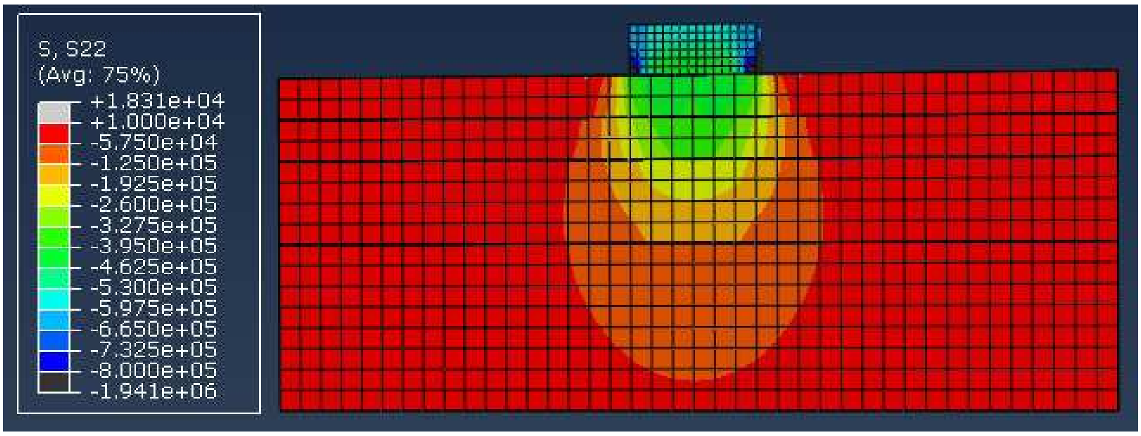
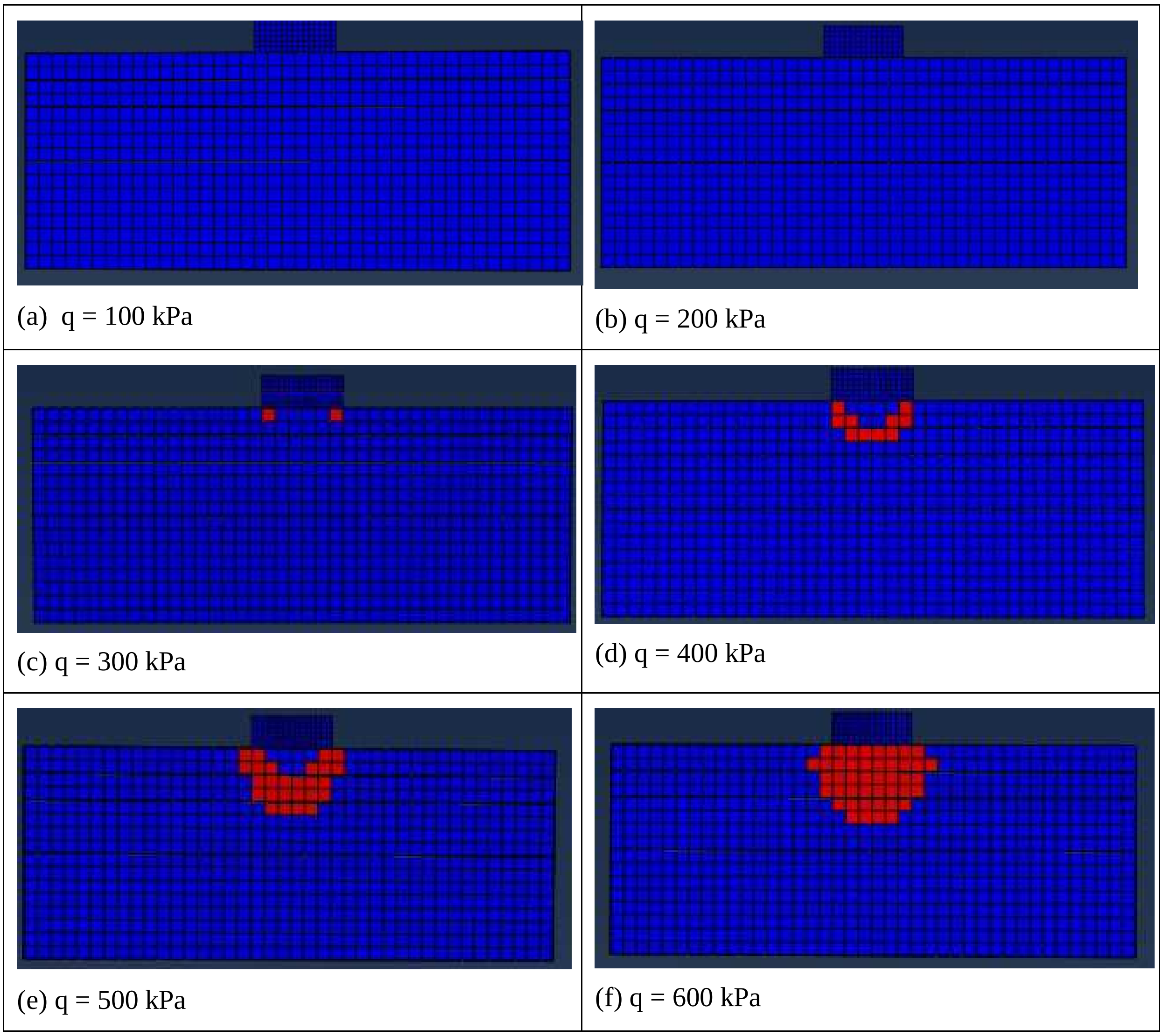


Figure 12. Shadings of vertical stress for analysis 2 and load equal to 600 kPa.

The figure 13 shows the evolution of plastic points from loadings varying from 100 to 600 kPa in steps of 600 kPa. From Figure 13 is possible to observe that, from 100 to 200 kPa of loading, there are not plastic points in the soil mass below the foundation. Increasing the load to 300 kPa appear plastification points in the border of footing. The progressive increase of loading (q) from 400 to 600 kPa leads an enlargement of the zone where the soil is out of linear-elastic behavior (plastic zone). So, in the plastic zone the soil has an additional deformation (Δε, as shown in Figure 5) that increase the final settlement of the footing. Indeed, the settlement obtained by numerical analysis is greater when the soil is considered as an elastic-plastic material. The analytical formulation of Rosendiz (1967) and Davis and Poulos (1968) consider the soil as a purely elastic material and for this reason the additional deformation Δε (see, Figure 6) is not considered in these formulations.



(a) q = 100 kPa

(b) q = 200 kPa

(c) q = 300 kPa

(d) q = 400 kPa

(e) q = 500 kPa

(f) q = 600 kPa

Figure 13. Plastic points in the soil for different values of loading (q) over the footing.

According to the results presented the estimation of settlements caused by a loaded footing can be done more accurately considering the soil as an elastic-plastic material. The consideration that the soil has behaves as a purely elastic material is a simplification. However, if the load imposed is equal or smaller than the yield stress of the soil the prediction will be satisfactory.

5 CONCLUSIONS

This paper has presented the prediction of settlements in a hypothetic case of footing subject to an external load. To perform the predictions were used two analytical formulations (Rosendiz, 1967 and Davis and Poulos, 1968) that consider the soil as elastic material. Also, numerical analyses were performed considering two conditions: a) Analysis 1: the soil has elastic-linear behavior and b) the soil has an elastic-plastic behavior. The main conclusions are:

- Considering the soil as a purely elastic material, the predictions made by the analytical formulations (Rosendiz, 1967 and Davis and Poulos, 1968) agreed fairly well with the numerical (FEM) predictions using Abaqus v6.13;
- Considering the soil as an elastic-plastic material, the predictions made by the analytical formulations (Rosendiz, 1967 and Davis and Poulos, 1968) showed divergence from the numerial (FEM) predictions. The divergences between analytical predictions and numerical predictions are more pronounced, in the studied case, for external loads greater than 400 kPa;
- Considering the soil as an elastic-plastic material the numerical analysis exhibits plastic points in the soil mass for vertical stresses greater than the vertical stress associated with the soil yield (σ_y);
- The settlements obtained when the soil is considered as an elastic-plastic material were greater than of the settlements related to the soil modeled as a purely elastic-material. The differences occur when the vertical stress is greater than the yield stress of the soil, as expected.

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