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CHARACTERIZATION OF VISCOELASTIC MATERIALS USING THE FRACTIONAL DERIVATIVE MODEL AND CONSIDERING FREQUENCY, TEMPERATURE AND STATIC DEFORMATION

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Abstract. *The present work presents a methodology for a dynamic integrated characterization of thermorheologically simple viscoelastic materials, considering simultaneously, the influence of frequency, temperature, and static deformation, the latter due to the preload. To that end, a technique based on a four-parameter fractional derivative model, WLF and Nashif's static deformation models is proposed. The influence of static deformation is introduced by means of standardized tests, performed at different percentages of deformation. For the predetermined ranges of frequency, temperature, and static deformation values, the complex Young's modulus is measured maintaining the constant low level dynamic amplitude of excitation. The experimental data of a virtual material, created from the butyl rubber parameters – obtained from an adjustment with frequency and temperature influences are used for the functional verification of the method. The proposed methodology aims at identifying the fundamental parameters of the fractional derivative model for the complex Young's modulus - which includes the frequency effect - as well as identifying additional parameters which also depict the effects of temperature and deformation. The identification follows the inverse problem* *concept, minimizing the objective function constructed from the mean square error between the experimental Young's modulus and its analytical equivalent obtained through the adopted model. At the end of the optimization process, all desired parameters are obtained, allowing – among other things – the construction of standardized nomograms for the complex Young's modulus of the material under analysis. The introduction of the preload effect into the nomogram provides an extra support for vibration treatment projects.*

Key words: *viscoelastic materials, integrated dynamic characterization, fractional derivatives, nonlinear optimization techniques.*

1 INTRODUCTION

The four-parameter fractional model is a well-established model in identifying the dynamic characteristics of thermorheologically simple viscoelastic materials and commonly applied in problem solving projects related to acoustic insulation and reduction of mechanical vibrations. This model is recognized for its applicability regarding the small quantity of parameters it needs to describe the dynamic behavior of those materials, since it is able to include the memory characteristics of the viscoelastic material (VEM) behavior regarding deformation.

On the line of VEMs characterization, the first contributions were given by Gemant (1936, 1938), which later – in 1950 - culminated in the publication of a book entitled *Frictional Phenomena in Journal of Applied Physics.* That autor justified the need to use differential operators for analyzing the profile of relaxation curves of some viscoelastic fluids. Using developments by Nutting (1921, 1943, 1946), Scott-Blair (1944, 1947, and 1949) used the fractional calculus approach to demonstrate that the relaxation tension phenomenon could be described by fractional powers over time. Was realized that the fractional order derivatives modeled both Nutting's observations for tension relaxation and those of Germant in function of frequency simultaneously. Gerasimov (1948) proposed to use fractional derivatives to define his viscoelasticity model. Rabotnov (1948) used Volterra's integral operators with weakly singular nuclei which could be interpreted in terms of integrals and fractional derivatives. The beginning of modern applications of fractional calculus in linear viscoelasticity is attributed to Bagley (1979), followed by works by Bagley and Torvik (1979, 1983a, 1983b, 1985), and Torvik and Bagley (1984). Recently, Arikoglu (2014) proposed a new model based on ten-parameter fractional derivatives that are determined through genetic algorithm. The author makes a comparison with the preexisting models showing good results in the comparison with experimentally obtained data.

In the frequency domain, Hashin (1970) presented a theoretical and experimental development to obtain the complex model of elasticity of particulate isotropic viscoelastic composite materials. Willis *et al.* (2001) propose an experimental-numeric methodology in which the Young's modulus and those of complex shearing are obtained by exciting a test body connected to a base coupled to a piezoelectric actuator in a chamber with controlled pressure and temperature. The parameters of the model are estimated through a computational code which minimizes the difference between the data and the prediction obtained through finite elements. Lopes *et al.* (2004) presented a methodology for characterizing VEMs, based on the fractional derivative model, considering the influences of temperature and frequency. Experimentally, those authors use a one-degree-of-freedom arrangement in which transmissibility is measured. Using the inverse problem technique of identification combined with nonlinear optimization, those authors obtain the parameters of the fractional derivative model for the complex shear modulus. Espíndola *et al.* (2005) present a variation of the work by Lopes *et al.* (2004), in which the displacement factor due to temperature is treated differently, replacing the Willians-Landel-Ferry equation by another form of displacement factor. Pagnacco *et al*. (2007) proposes a technique to determine the identification parameters for rheologically simple VEMs with a numerical-experimental hybrid approach using complete field measurements. In that work, the autors developed a generic method capable of treating power and displacement measures based on finite-element concepts. Zopf *et al*. (2014) present two approaches on the use of fractional derivatives using Zener's solid model. The fractional model was developed in accordance with the finite-deformation theory. In addition, constitutive equations are derived based on two algorithms tackling the fractional integration over time to choose the material model. Xiao *et al* (2014) propose a numerical model to identify the constitutive parameters of the fractional model for VEM's. An explicit semi-analytical numerical model and the finite-difference method are derived in order to solve VEM problems. A method based on the inverse problem model of identification is applied by the autors. The feasibility of the model is demonstrated using a numerical verification of a flat problem of identification formulated using Kelvin-Voigt's fractional model. Bonfiglio *et al.* (2016) introduce a new method for determining the values of Young's complex modulus – for isotropic and homogeneous VEMs – in the frequency domain, using a simplified transfer matrix approach. The method can be used for simulations and optimizations of VEM performance in vibration or acoustic insulation applications. Sousa *et al.* (2017) present a methodology to obtain the parameters of the fractional model and of the temperature influence for viscoelastic materials in which the experimental data are obtained by digitalizing the nomogram of the material provided by the manufacturer.

The present work proposes to approach the identification of the dynamic properties of thermorheologically simple VEMs in which, besides the influence of frequency and temperature, a new factor is added: the influence of static deformation in the form of a preload. The experiment is realized considering a constant low level dynamic load due to the vibratory excitation.

First, the analysis is performed experimentally with various temperatures and preloads in a frequency spectrum, so that, for each pair of previously established temperatures and preloads, one obtains - in separate experiments - the values of the elasticity modulus. Finally, the results for all combinations of temperature and preload are gathered together for an integrated identification process, in which all results are analyzed simultaneously in a single processing. At the end of the identification process, the fractional model parameters are obtained together with the temperature and preload influence factors. Such identification will allow the construction of nomograms for each condition of static loading (preload).

2 DYNAMIC CHARACTERIZATION OF VISCOELASTIC MATERIALS

2.1 Classic Methodology

The usual classic way of describing the dynamic behavior of VEMs is through a representation using the complex modulus elasticity or shearing (Snowdon, 1968). In that representation, while the real part responds for the elastic characteristics, the imaginary part responds for the ability the material has to dissipate vibratory energy.
Thus, the general representation of Young's complex modulus is:

$$E_c = E_R + i.E_I\,, \qquad (1)$$

where E_c is Young's complex modulus, E_r is the real part, called dynamic modulus of elasticity and E_I is the imaginary part, called lost modulus and $i = \sqrt{-1}$. From this definition, the loss factor η_E is obtained as follows:

$$\eta_E = \frac{E_I}{E_R}\,, \qquad (2)$$

from which, Eq. (1) can be rewritten as:

$$E_C = E_R\left(1 + i.\eta_E\right). \qquad (3)$$

Considering the frequency effect, Eq. (3) can be written the following way:

$$E_C(\Omega) = E_R(\Omega)\left[1 + i.\eta(\Omega)\right], \qquad (4)$$

where Ω stands for frequency.

For Zener´s mechanical model, the four parameter fractional model can be represented by

$$E_C(\Omega) = \frac{E_L + E_H b_o [i.\Omega]^\beta}{1 + b_o [i.\Omega]^\beta}\,, \qquad (5)$$

where E_L is the lower asymptote of dynamic Young modulus, E_H is the higher asymptote of dynamic Young modulus, b_o is an empirical real parameter, Ω is the frequency, β is the fractional derivative order and $i = \sqrt{-1}$.

For thermorheologically simple VEMs, the influence of temperature was modeled by Ferry(1980) using the time-temperature superposition concept (TTS), from a reference temperature. Thus, its influence may be modeled by an empirical relation named 'WLF equation' (Williams-Landel-Ferry):

$$\log_{10} \alpha_T(T) = \frac{-\theta_1 (T - T_o)}{\theta_2 + T - T_o}\,, \qquad (6)$$

where θ_1 and θ_2 are parameters to be determined for the material.

Presenting the results in two single curves - dynamic modulus and the loss factor – it is possible to obtain a nomogram developed by Jones (1978), named 'reduced frequency nomogram', (Fig. 1) according to norm ISO 10112, 1991.

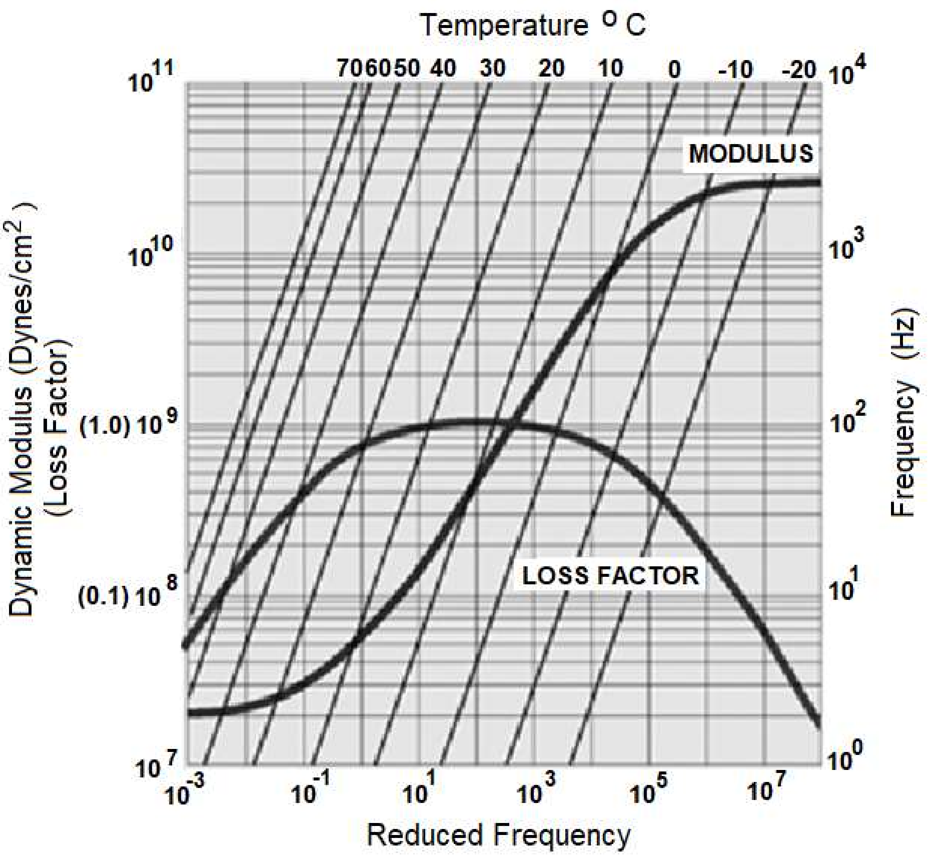

Figure 1: Reduced Frequency Nomogram – C-1002.

Source: Sousa *et al.* (2017)

Introducing the temperature factor through Eq. (6), Eq. (5) becomes:

$$E_C(\Omega,T)=\frac{E_L+E_H b_o\left[i.\alpha_T(T)\Omega\right]^{\beta}}{1+b_o\left[i.\alpha_T(T)\Omega\right]^{\beta}}, \tag{7}$$

where $\Omega_R=\alpha_T(T).\Omega$ is called reduced frequency.

In the classical approach, the parameters that compose Eq. (7) are obtained through a graphic constructed from the experimental data of the modulus for the tested frequencies and temperatures. Such graphic is named ‘wicket-plot’ (ASTM E-756-98, 1998) and it defines single values for E_L and E_H, the upper and the lower limits of the real modulus, respectively, together with the curve with the biggest loss factor which defines a single β value and the reference temperature.

2.2 Integrated Methodology

Lopes *et al.* (2004) introduced the methodology for VEMs dynamic identification named ‘integrated dynamic methodology’, as already described in the introduction of the present work.

Focusing on Young's complex modulus, some redefinitions are necessary in order to apply the integrated methodology to the present study. Error is calculated through the difference between mathematically obtained values for modulus (Ec) and its equivalent experimental values (E_E) by $\mathbf{e}_{jk}$,

$$\mathbf{e}_{jk}(\Omega_j, T_k) = \mathbf{E}_\mathbf{C}(\Omega_j, T_k) - \mathbf{E}_E(\Omega_j, T_k) \quad \text{with j=1..p, k=1..q,} \tag{8}$$

where p represents the number of frequencies measured and q is the number of temperatures. The terms that compose Eq. (8) constitute a matrix of the p x q order.
With that, in the optimization problem, the objective function is defined as follows:

$$f(x) = \sum_{k=1}^{q} \left[\sum_{j=1}^{p} (\mathbf{e}_{jk})(\mathbf{e}_{jk})^C \right]. \tag{9}$$

where index C denotes the complex conjugate value, and the minimization of f(x) will result in the identification of the parameters that is composed by:

$$\mathbf{x}^T = [E_L, E_H, b_o, \beta, \theta_1, \theta_2, T_o]. \tag{10}$$

The parameters identified in Eq. (10) allow constructing the nomogram for the elastometer in study.

2.3 Effect of the Dynamic Load/Dynamic Deformation

The methodologies presented in items 2.1 and 2.2 have in common the fact they consider the frequency and temperature effects on the dynamic behavior of VEMs. In certain situations, however, the effects of the dynamic deformation amplitude imposed by the cyclic leading, particularly in temperatures in which the VEM presents a rubbery behavior, the dynamic modulus and the loss factor vary more slowly with the temperature than with the deformation amplitude. Thus, dynamic deformation becomes more important than temperature in those conditions. Generally speaking, the damping properties vary with the amplitude of the dynamic deformation similarly to that of temperature (Nashif *et al.*, 1985).

By observing Figs. 2 and 3, it is possible to notice that the displacement factor due to the dynamic deformation assumes a similar form to the displacement factor of temperature:

$$\log \alpha_\varepsilon = \frac{-C_1(\varepsilon - \varepsilon_0)}{C_2 + \varepsilon - \varepsilon_0} \tag{11}$$

Constants C_1 and C_2 are material constants and can be obtained from the experimental data, whereas ε is a measured deformation and ε_0 is one reference deformation, taken in an analogous way to that of the reference temperature. Then, the Eq. (5) would take the form of Eq. (12):

$$E_C(\Omega, \varepsilon) = \frac{E_L + E_H b_o [i.\alpha_\varepsilon(\varepsilon)\Omega]^\beta}{1 + b_o[i.\alpha_\varepsilon(\varepsilon)\Omega]^\beta} \tag{12}$$

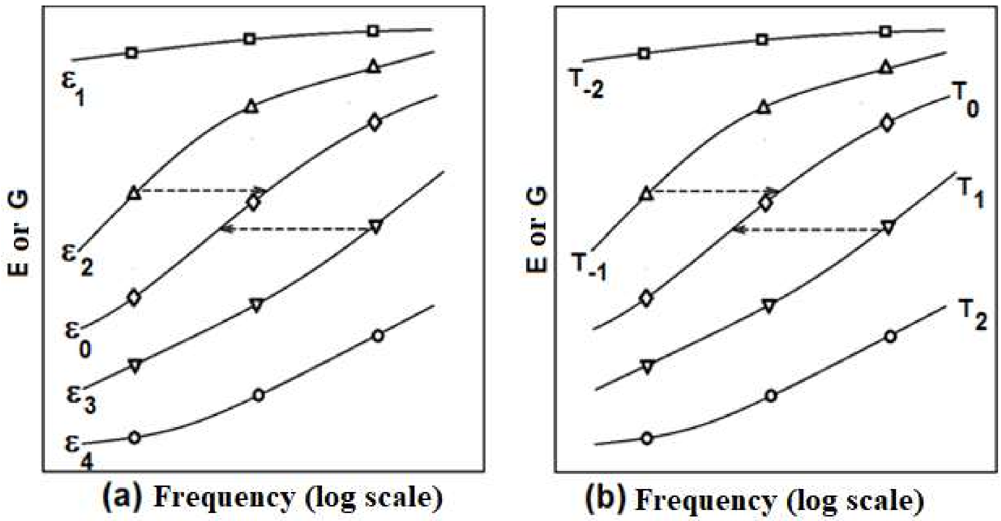

Figure 2: Dynamic Modulus Variation with frequency and (a) Deformation (b) Temperature. Adapted from Nashif *et al.* (1985)

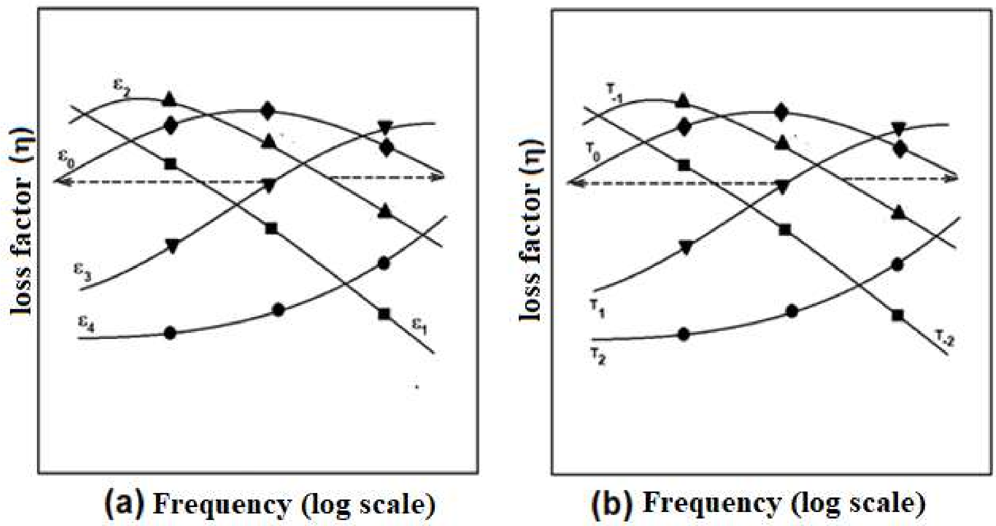

Figure 3: Loss Factor Variation with frequency and (a) Deformation (b) Temperature. Adapted from Nashif *et al*. (1985)

In Eq. (12), it was considered the environment temperature is constant or reference temperature. $[\alpha_T(T)=1]$. When the temperature is not constant, eq. 12 is best represented as follows:

$$E_C(\Omega,T,\varepsilon)=\frac{E_L+E_H b_o[i.\alpha_T(T).\alpha_\varepsilon(\varepsilon)\Omega]^\beta}{1+b_o[i.\alpha_T(T).\alpha_\varepsilon(\varepsilon)\Omega]^\beta} \tag{12a}$$

Under certain conditions, for example, when $\varepsilon=\varepsilon_o$, in which only one deformation is considered, $\alpha_\varepsilon(\varepsilon)$ tends to 1.

2.4 Preload effect (Static Load)/Static Deformation

As applied to isolation of vibrations, static deformation is, in most cases, a consequence of applying a constant force, such as the weight of a machine whose vibrations one wishes to isolate from the base. This condition is particularly important in cases of high frequency vibrations.

In practical terms, the analysis of each of the effects – temperature, dynamic load, and static load – must be carried out separately, since the analysis involves effects that will add up and perhaps cause mutual intensification. It is the case, for example, of the dynamic deformation that tends to produce, beyond certain limits, a considerable increase in the inner temperature of the VEM, Nashif et al. (1985). It is also important to emphasize that the effect on the dynamic characteristics (elasticity modulus) of the preload variation differs greatly from those of temperature and dynamic-load, in the sense that it causes an increase or decrease in the values of the modulus for the same frequencies thus producing an 'orthogonal' effect in relation to those effects of temperature and dynamic load. (Fig. 4)

In isolation, the preload effect presented here has its theoretical base in Mooney-Rivlin's equation. This equation was initially developed by Mooney (1940) and later improved by Rivlin (1947).

For small deformations, Nashif *et al.* (1985) provide

$$\sigma=2(C_3.\lambda+C_4)\left(\lambda-\frac{1}{\lambda^2}\right)\quad with\quad \lambda<3, \tag{13}$$

where σ represents the normal tension, C_3 and C_4 are Mooney-Rivlin's material constants and $\lambda=\varepsilon+1$, in which ε is the engineering deformation. Replacing λ for ε in Eq. (13), and considering ε as very small, one has:

$$\sigma=2[C_3(\varepsilon+1)+C_4]\left[\frac{\varepsilon(\varepsilon^2+3\varepsilon+3)}{(\varepsilon+1)^2}\right]. \tag{14}$$

Thus,

$$\frac{\sigma}{\varepsilon}=2[C_3(\varepsilon+1)+C_4]\left[\frac{(\varepsilon^2+3\varepsilon+3)}{(\varepsilon+1)^2}\right]. \tag{15}$$

With a very small ε, Eq. (15) becomes:

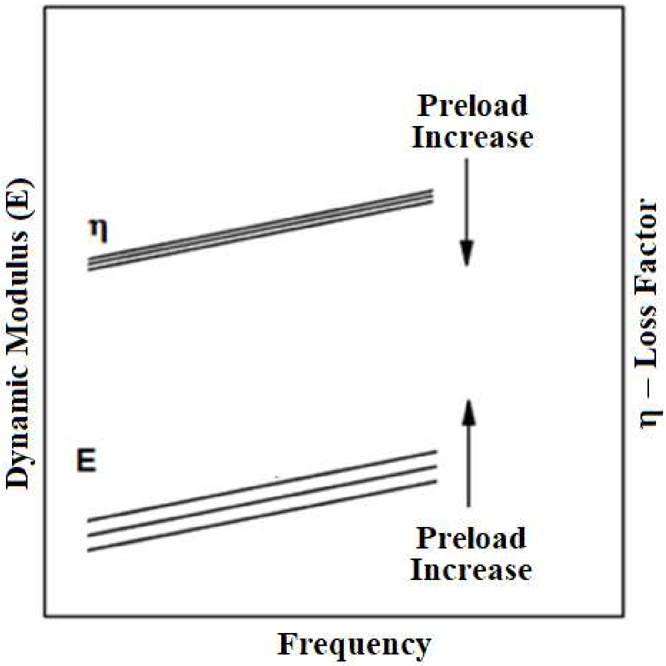

Figure 4 – Dynamic Young Modulus and Loss Factor variation x Preload
Adapted from Nashif *et al.* (1985)

$$\frac{\sigma}{\varepsilon} = 6(C_3 + C_4), \tag{16}$$

that is,

$$E_o = 6(C_3 + C_4). \tag{17}$$

Equation (17) represents the static modulus, keeping in mind that, in this case, the dynamics effect is not considered

2.5 Static Load and Dynamic Load Combined Effect

According to Nashif *et al.* (1985), the representation of the combined effect of materials like rubber, subjected to dynamics load superposition with static preload, through tension expression regarding frequency and deformation can be done by:

$$\sigma(\Omega, \lambda) = F(\lambda)E(\Omega). \tag{18}$$

Using Eq. (13) in place of F(λ) in Eq. (18),

$$\sigma(\Omega, \lambda) = 2(C_3.\lambda + C_4)\left(\lambda - \frac{1}{\lambda^2}\right)E(\Omega). \tag{19}$$

The static and dynamic combined effect of Eq. (19) may be used in the expression:

$$E(\Omega,\lambda)=\lambda\frac{d\sigma}{d\lambda}, \tag{20}$$

which would become:

$$E(\Omega,\lambda)=[C_3F_1(\lambda)+C_4F_2(\lambda)]E(\Omega), \tag{21}$$

with:

$$F_1(\lambda)=2\left[2\lambda^2+\frac{1}{\lambda}\right], \tag{22}$$

$$F_2(\lambda)=2\left[\lambda+\frac{2}{\lambda^2}\right]. \tag{23}$$

Using the complex approach of the modulus, Eq. (21) becomes:

$$E(\Omega,\lambda)[1+i.\eta(\Omega,\lambda)]=\{C_3[1+i.\eta_1(\Omega)]F_1(\lambda)+C_4[1+i.\eta_2(\Omega)]F_2(\lambda)E(\Omega)\}. \tag{24}$$

The real and the imaginary parts of the modulus can be obtained from Eq. (24):

$$E(\Omega,\lambda)=[C_3F_1(\lambda)+C_4F_2(\lambda)]E(\Omega), \tag{25}$$

$$\eta(\Omega,\lambda)=[C_3F_1(\lambda)\eta_1+C_4F_2(\lambda)\eta_2]E(\Omega). \tag{26}$$

Additionally, η_1 can be assumed to be equal to zero and with a very small ε, $\lambda\rightarrow 1$, equations (25) and (26) can be written as follows (Ferry, 1980):

$$E(\Omega,\lambda)=[C_3.F_1(\lambda)+C_4.F_2(\lambda)]\frac{E(\Omega)}{6(C_3+C_4)}, \tag{27}$$

$$\eta(\Omega,\lambda)=\frac{(C_3+C_4)F_2(\lambda)}{C_3.F_1(\lambda)+C_{4.}F_2(\lambda)}\eta(\Omega). \tag{28}$$

In equations (27) and (28), E(Ω) and η(Ω) are the dynamic modulus and the loss factor, respectively, obtained through equation (5) when put in the form of Eq. (3). E(Ω,λ) and η(Ω,λ) are the dynamic modulus and the loss factor, respectively, of the complex Young modulus, as follows:

$$E_C(\Omega,\lambda) = E(\Omega,\lambda)[1 + i.\eta(\Omega,\lambda)]. \tag{29}$$

With equations (27) and (28), it is possible to produce a general equation that may include the effects of frequency, temperature, dynamic amplitude, and deformation due to the preload [Nashif *et al.* (1985)]:

$$E(\Omega,T,\varepsilon,\lambda) = [C_3.F_1(\lambda) + C_4.F_2(\lambda)]\frac{E(\Omega,T,\varepsilon)}{6(C_3 + C_4)}, \tag{30}$$

$$\eta(\Omega,T,\varepsilon,\lambda) = \frac{(C_3 + C_4)F_2(\lambda)}{C_3.F_1(\lambda) + C_{4.}F_2(\lambda)}\eta(\Omega,T,\varepsilon). \tag{31}$$

In equations (30) and (31), terms $E(\Omega,T,\varepsilon)$ and $\eta(\Omega,T,\varepsilon)$ are obtained from Eq. (12a), the same way equations (27) and (28) are obtained with the aid of Eq. (6).

3 APPLIED METHODOLOGY

The dynamic identification of viscoelastic materials considering the influence of frequency, temperature, and static deformation - as a constant preload in time - is carried out in two different stages: the first is the experimental stage, in which data that will constitute the base for obtaining the adjustment parameters of the model are collected; and the second is the computational stage.

3.1 Integrated Dynamic Characterization

Considering the integrated methodology proposed by Lopes *et al*. (2004) and used in the present work - and presented in section 2.2 - for Young's complex modulus, the mathematical model that will be the base for the objective function is the one presented by equations (30) and (31). The present proposal verifies the influence of the preload together with the temperature in the frequency domain. The vibration amplitude is controlled and kept the same in all temperatures and preloads tested. As a result of this, factor $\alpha_\varepsilon(\varepsilon)$ of the dynamic deformation – vibration amplitude - Eq. (11) is assumed to be equal to 1. The complex modulus equation that will allow obtaining the real and imaginary portions for equations (30) and (31) will be equal to (7):

$$E_C(\Omega,T) = \frac{E_L + E_H b_o[i.\alpha_T(T)\Omega]^\beta}{1 + b_o[i.\alpha_T(T)\Omega]^\beta}. \tag{7}$$

Thus:

$$E(\Omega,T)=\Re[E_C(\Omega,T)], \tag{32}$$

$$\eta(\Omega,T)=\frac{\Im[E_C(\Omega,T)]}{\Re[E_C(\Omega,T)]}. \tag{33}$$

Equations (30) and (31) will be as follows:

$$E(\Omega,T,\lambda)=[C_3.F_1(\lambda)+C_4.F_2(\lambda)]\frac{\Re[E_C(\Omega,T)]}{6(C_3+C_4)}, \tag{34}$$

$$\eta(\Omega,T,\lambda)=\frac{(C_3+C_4)F_2(\lambda)}{C_3.F_1(\lambda)+C_{4.}F_2(\lambda)}\frac{\Im[E_C(\Omega,T)]}{\Re[E_C(\Omega,T)]}. \tag{35}$$

With the influence of the preload, the expression of Young's complex modulus will be written as follows:

$$E_C(\Omega,T,\lambda)=E(\Omega,T,\lambda)[1+i.\eta(\Omega,T,\lambda)]. \tag{36}$$

In optimization, equations (22), (23), (32), (33), (34) and (35) lead to the following error equation:

$$\mathbf{e}_{jkl}(\Omega_j,T_k,\lambda_l)=\mathbf{E_C}(\Omega_j,T_k,\lambda_l)-\mathbf{E_E}(\Omega_j,T_k,\lambda_l) \quad \text{with j=1..p, k=1..q, l=1..r,} \tag{37}$$

where $\mathbf{E_E}$ represents the experimental value of the complex modulus, p, the number of measured frequencies, q, the number of temperatures and r, the number of measured preload values. The objective function is then represented as:

$$f(x)=\sum_1^r\left\{\sum_1^q\left[\sum_1^p(\mathbf{e}_{jkl})(\mathbf{e}_{jkl})^C\right]\right\}. \tag{38}$$

In Eq. (38), index C denotes the complex conjugate value. Optimization will lead to the solution vector:

$$\mathbf{x}^T=[E_L,E_H,b_o,\beta,\theta_1,\theta_2,T_o,C_3,C_4]. \tag{39}$$

The values of $F_1(\lambda)$ and $F_2(\lambda)$ are not included in the solution vector because parameter λ of the preload is known and applied in equations (22) and (23).

The process of identification is conducted using a computational code in MATLAB, based on the FMINSEARCH function that can be previously helped by using genetic algorithms - GA, in MATLAB – for an initial approximation. Using the FMINSEARCH function may be more efficient than the FMINCON function, considering that the parameters of temperature and preload parameters θ_1, θ_2, C_3, and C_4 have unknown values or intervals. This justifies a combined use of genetic algorithms at first for an initial approximation that will be followed by a more precise definition through non-linear optimization, in a single program. The process is concluded with the definitions of the necessary parameters for the construction of nomograms. For the sake of having a clear reading, it is interesting to construct a nomogram for each preload deformation value.

3.2 Performing the Experiments

One assumes that the tests must be conducted in a test machine of the Universal type, for polymers, equipped with a controlled temperature chamber. The procedure for carrying out the tests must follow a specific norm – the recommendation is for norm ASTM D-5992-96 or a revised version of it. Each specimen of the sample will be used only once, for each predetermined temperature and preload, until all combinations have been experimented. For a predetermined number n of temperatures and p of preload deformation, n x p tests will be necessary to achieve the experimental objective.

4 RESULTS

4.1 Experimental Data

In the present work, the experimental data were obtained by numerical simulation. The equations (34) and (35) are used to find the "experimental data" and the values were corrected by Eq. (40) for more realistic condition. Then, the "experimental data" are

$$E_{EXP} = E_C + 0{,}025.(RAND - 0{,}5).E_C\,. \tag{40}$$

where RAND is the generating force of the random number in the MATLAB, and Ec is the complex Young modulus value obtained from Eq. (36). The multiplication factor in the second member of the equation represents the order of magnitude of the dispersion in the experimental data - in this case, plus or minus 2.5% the exact value.

The parameters used to generate the experimental database were defined according to Table 1.

Considering the number of tested temperatures and preloads, 56 samples and tests would be necessary for obtaining the necessary data to perform the dynamic characterization. For an adequate characterization, one that makes it feasible to construct a nomogram of the material, it is fundamental that the temperature spectrum/range be broad enough, from negative to positive temperatures. For cost reduction, it is interesting to use only the preload values that are essential for the identification.

The construction of variation graphics of the dynamic modulus and loss factor with the preload, as well as of the wicket plot, indicate whether the material has the expected behavior, that is, whether it is thermorheologically simple.

Fig. 5 shows the growth of the dynamic Young modulus with the increased preload.

Fig. 6 shows the inverse variation of the loss factor with the increased preload, as foreseen by Nashif *et al.* (1985).

Fig. 7 presents the wicket plot, where the characteristic form of a thermorheologically simple material is verified - Rouleau *et al.*(2013). Obviously, the graphic presents an optimal condition which will practically be impossible to find in a real material. However, for validation of the results, the certification that the experimental results comply is important.

Table 1: Values of the parameters of the proposed model for generating experimental data.

TEMPERATURES (°C)	-60	-40	-20	0	20	40	60	70
PRELOAD DEFORMATION	(%)	-0.5	-1	-2	-3	-4	-5	-7
Obs: $\varepsilon = \Delta L/Lo$ $\% = \varepsilon \times 100$	ε	-0.005	-0.01	-0.02	-0.03	-0.04	-0.05	-0.07
$\lambda = 1 + \varepsilon$ (+) traction (-) compression	λ	0.995	0.99	0.98	0.97	0.96	0.95	0.93
DYNAMIC AMPLITUDE (ε_d)		0.001						
MEASURED FREQUENCIES (Hz)	1	2	4	6	10	15	25	50
	100	150	200	250	300	350	400	450
	500	600	700	800	900	1000	-	-
PARAMETERS OF FRACTIONARY MODEL Obs.: Pa = Pascal (N/m^2)		Eo	8.093×10^6		Pa			
		E_∞	9.089×10^8		Pa			
		b_o	0.0024					
		β	0.3018					
		T_o	10		°C			
		θ_1	10.33					
		θ_2	163.11					
		C_3	-8					
		C_4	35					

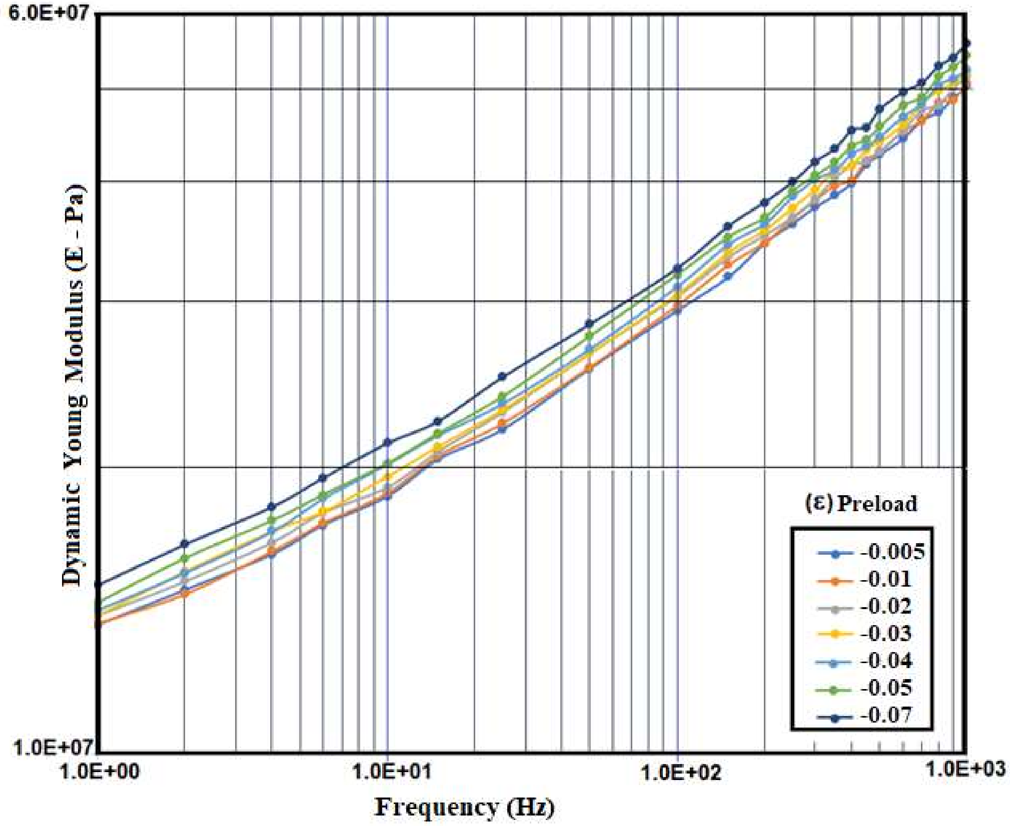

Figure 5 - Variation of Young's modulus with preloads from ε=0.005 to ε=0.07

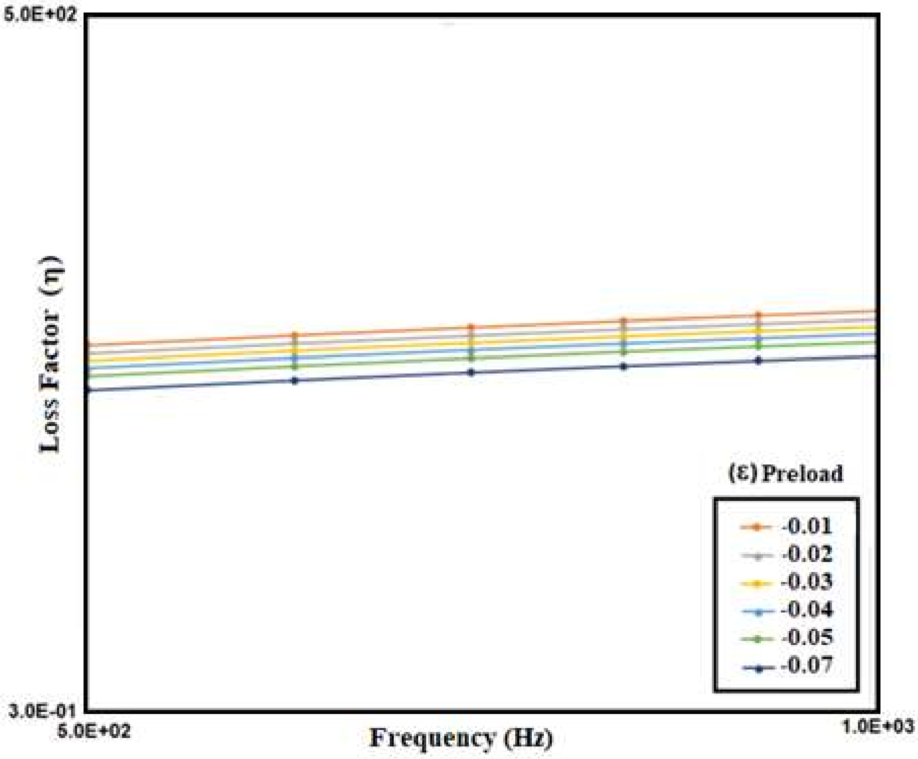

Figure 6 - Variation of the Loss Modulus from ε=0.01 to ε= 0.07

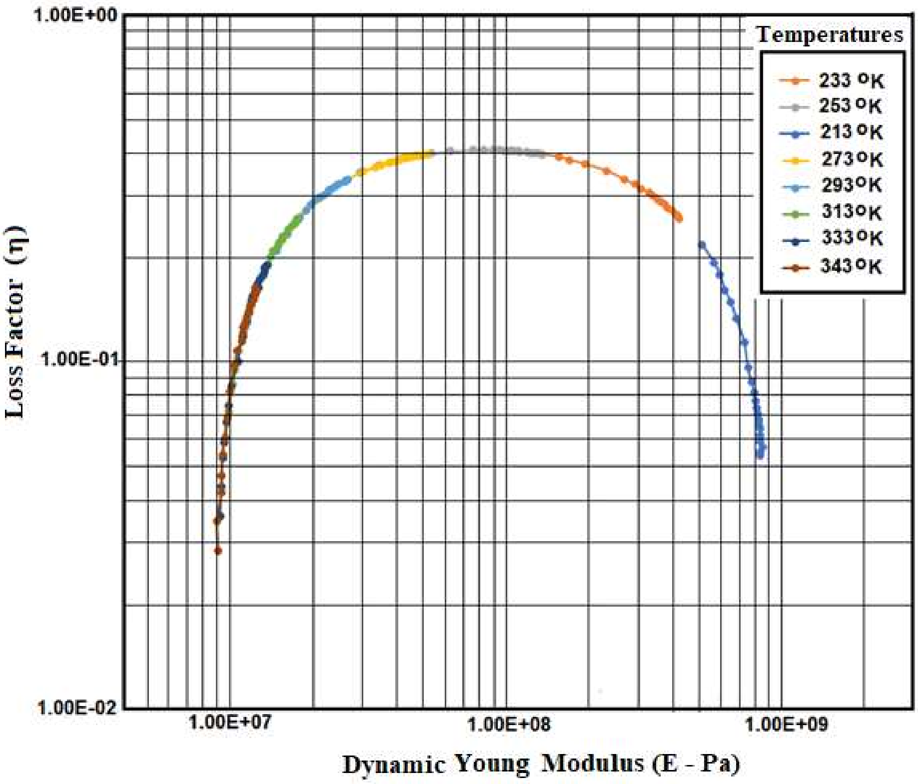

Figure 7 – Wicket Plot

4.2 Integrated Identification Results

The 'experimental' data were loaded in a program constructed in MATLAB. Two optimization techniques were used: genetic (GA) and non-linear (FMINSEARCH) algorithms.

Table 2 presents a comparison between the values used for generating the experimental data and the values obtained from the optimization procedure. The quadratic error at the end seems high, but, as it will be possible to see further on, that was already expected, in view of the randomly introduced dispersion.

When the results are examined in their graphic form, it is possible to notice the origin of the final quadratic error of the identification process. For the case analyzed here, graphically speaking, 56 graphics are necessary like those 4 in Fig. 8. The quadratic error in the integrated process will be the result of the sum of the differences observed between the line of the experimental result (green line) and the adjustment (blue line).

RESULTS AND DISCUSSION

The integrated identification method proved to be effective for a thermorheologically simple material in which frequency, temperature, and static deformation originating from static preload are present.

The addition of parameters C_3 and C_4 of the preload – in the fractional model – will implicate the need for the construction of different nomograms for each preload value measured, which has an important and unprecedented value for a vibration isolation project.

Table 2 – Comparison between the Optimization Results and the Expected Values

Parameter	Proposed Values		Results		Error (%)
E_o	8.093×10^6	Pa	8.0922×10^6	Pa	0.1
E_∞	9.089×10^8	Pa	9.1038×10^8	Pa	0.16
β	0.3018	-	0.3015	-	0.1
b_o	0.0024	-	0.0024	-	-
θ_1	10.33	-	10.3457	-	0.15
θ_2	163.11	-	163.2094	-	0.06
C_3	-8	-	-8.0161	-	0.2
C_4	35	-	36.0949	-	3.12
Total Quadratic Error			0,0611		(Final)

One must notice that the main difficulties in the identification process with preload weighing lie basically in carrying out the tests. As temperature and preload must be analyzed individually, the number of experiments is very high and, therefore, each additional temperature or preload result in a considerable increase in costs. This aspect must be analyzed with care in order that temperatures and preloads are not unnecessarily included in the process.

Another aspect to be considered is related to the final quadratic error and the influence of temperature in the measured property. As can be observed in Fig.8, among the limit temperature in the identification, there is a great difference that can be up to 1000 times among the values of the modulus in both conditions. This represents a big difference in the minimization process, since it is the value of the difference between the calculated and the experimental modulus that will determine the error to be optimized.

In the nonlinear optimization process, disregarding any weighing, will tend to reduce to the maximum the error in low temperatures, in which the modulus is much larger than in high temperatures. This can be observed in Fig. 8, comparing the graphics by temperature. A weighing solution can be to work with a relative error, in which the error is compared to the value of the experimental measurement for the same frequency. The absolute error will be higher, but there will be an improvement in the higher temperatures, even at the cost of a slight reduction in the accuracy at low temperatures.

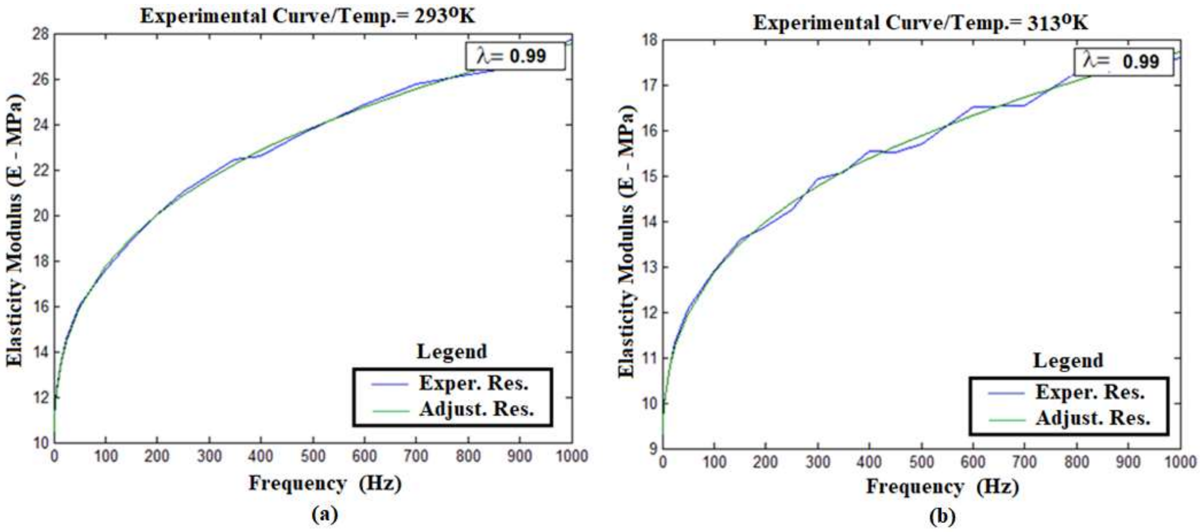

Figure 8 – Graphics of the integrated identification process for preloads from 1% to different temperatures: (a) 1% preload deformation (l=0.99) at 293ºK ;(b) 1% preload deformation (l=0.99) at 313ºK

CONCLUSION

It was presented a procedure to determine the characteristic parameters of thermoreologically simple elastomers in the fractional four - parameter model derived from the application of the fractional derivative concept on the Zener mechanical model. The technique, based on hybrid optimization with the combination of genetic algorithms and nonlinear optimization, is validated with fictitious experimental data created from a real reference material. The results presented consistency and pointed to the viability of the new procedure.

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REFERENCES

Arikoglu A., 2014. A new fractional derivative model for linearly viscoelastic materials and parameter identification via genetic algorithms, *Rheological Acta*, vol. 53, pp. 219-233.

ASTM E 756-98, 1998. Standard Test Method for Measuring Vibration Damping Properties of Materials, *ASTM.*

ASTM D 5992-96, 2011. Standard Guide for Dynamic Testing of Vulcanized Rubber and Rubber-Like Materials Using Vibratory Methods, *ASTM.*

Bagley, R. L., 1979. Applications of generalized derivatives to viscoelasticity, Ph. D. Thesis, *Air Force Institute of Technology*.

Bagley, R. L.; Torvik, P. J., 1979. A generalized derivative model for an elastomer damper; *The Shock and Vibration Bulletin,* vol. 49, n. 2, pp. 135-143.

Bagley, R. L.; Torvik, P. J., 1983a. A theoretical basis for the application of fractional calculus to viscoelasticity; *Journal of Rheology*, vol. 27 pp. 201-210.

Bagley, R. L.; Torvik, P. J., 1983b. Fractional calculus – A different approach to the finite element analysis of viscoelastically damped structures, *AIAA Journal*. vol. 21, n. 5, pp. 741-748.

Bonfiglio, P.; Pompoli, F.; Horoshenkov, K. V.; Rahim, M. I. B. S., 2016 A simplified transfer matrix approach for the determination of the complex modulus of viscoelastic materials, *Polymer Testing*, vol. 53, pp.180-187.

Espíndola, J. J.; Silva Neto, J. M.; Lopes, E. M. O., 2005. A generalized fractional derivative approach to viscoelastic material properties measurement, *Applied Mathematics and Computation*, vol. 164, pp. 493-506.

Ferry, J. D., 1980. *Viscoelastic Properties of Polymers, New York*, NY, USA John Wiley & Sons.

Gemant, A., 1936. A method of analyzing experimental results obtained from elasto-viscous bodies, *Physics,* vol. 7, pp. 311-317.

Gemant, A., 1938. On fractional differentials, *Philosophical Magazine*. (Series 7). vol. 25, pp. 540-549.

Gerasimov, A., 1948. A generalization of linear laws of deformation and its applications to problems of internal friction. *Prikl. Matem. I Mekh (PMM)* vol. 12, n. 3, pp. 251-260.

Hashin, Z., 1970. Complex moduli of viscoelastic composites I – general theory and application to particulate composites, *International Journal of Solids and Structures*, vol. 6, pp. 539-552.

ISO, 1991. *ISO 10112-1991: Damping Materials – Graphical Presentation of the Complex Modulus.*

Jones, D. I. G., 1978. A Reduced Temperature Nomogram for characterization of Damping Material Behavior, *Shock and Vibration Bulletin*, vol. 48, n. 2, pp. 13-22.

Lopes, E. M. O.; Bavastri, C. A.; Silva Neto, J. M.; Espíndola, J. J., 2004. Caracterização dinâmica integrada de elastômeros por derivadas generalizadas. Belém, *III Congresso Nacional de Engenharia Mecânica.*

Mooney, M., 1940. A theory of large elastic deformations, *Journal of Applied Physics*, vol. 11, pp. 582-592.

Nashif, A. D.; Jones, D. I. G.; Henderson, J. P., 1985. *Vibration Damping,* New York, NY, USA, John Wiley & Sons.

Nutting, P. G., 1921. A new general law of deformations, *Journal of the Franklin Institute*, vol. 191, pp. 679-684.

Nutting, P.G., 1943. A general stress-strain time formula. *Journal of the Franklin Institute*. vol. 235, pp. 513-524.

Nutting, P.G., 1946. Deformation in relation to time, pressure and temperature. *Journal of the Franklin Institute*. vol. 242, pp 449-458.

Pagnacco, E.; Moreau, A.; Lemosse, D., 2007. Inverse strategies for the identification of elastic and viscoelastic material parameters using full-field measurements, *Materials Science and Engineering A*, vol. 452-453, pp. 737-745.

Rabotnov, Yu. N., 1948. Equilibrium of an elastic medium wth after effect, *Prikl. Matem. i Mekh. (PMM)* vol. 12, n. 1, pp. 81-91.

Rivlin, R. S., 1948. Large Elastic deformations of isotropic materials IV. Further developments of the general theory, *Phil. Trans. of the Royal Soc. A*, vol. 241, n. 835, pp. 379-397.

Rouleau, L.; Deü, J. F.; Legay, A.; Le Lay, F., 2013. Application of Kramers-Kronig relations to time-temperature superposition for viscoelastic materials, *Mechanics of Materials,* vol. 65, pp. 66-75.

Scott-Blair, G. W., 1944. Analytical and integrative aspects of the stress-strain-time problem. *Journal Scientific Instruments*. vol. 21, pp. 80-84.

Scott-Blair, G. W., 1947. The role of psychophysics in rheology. *Journal of. Colloid Science.* vol. 2, pp. 21-32.

Scott- Blair, G. W., 1949. *A Survey of General and Applied Rheology*, Pitman, London.

Snowdown, J. C., 1968. *Vibration and shock in damped mechanical systems*, New York, NY, USA, John Wiley & Sons.

Sousa, T. L.; Kanke, F.; Pereira, J. T.; Bavastri, C. A., 2017. Property Identification of Viscoelastic Solid Materials in Nomograms Using Optimization Techniques; *Journal of Theoretical and Applied Mechanics*, in press.

Torvik, P. J.; Bagley, R. L., 1984 On the appearance of the fractional derivative in the behaviour of the real materials. *American Society of Mechanical Engineers Journal of Applied Mechanics*. vol. 51, pp. 294-298.

Xiao, Z.; Haitian, Y.; Yiqian, H., 2014. Identification of constitutive parameters of fractional viscoelasticity, *Comunications in Nonlinear Science and Numerical Simulation,* vol. 19, pp. 311-322.

Willis, R. L.; Wu, L.; Berthelot, Y. H., 2001. Determination of Complex Young and shear dynamic moduli of viscoelastic materials, *Journal of the Acoustical Society of America,* vol. 109, pp. 611-621.

Zopf, C.; Hoque, S. E.; Kaliske, M., 2015. Comparison of approaches to model viscoelasticity based on frictional time derivatives, *Computational Materials Science,* vol. 98, pp. 287-296.