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OPTIMAL LAYOUT OF TRUSSES CONSIDERING EXPECTED CONSEQUENCES OF FAILURE

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Abstract. *Failure of engineering structures is, to a large extent, controlled by material ductility. However, for truss-like structures, topology also has a strong impact on failure modes, load redistribution and eventual progressive collapse. This paper addresses optimal layout of truss-like structures, considering the effects of uncertainties and expected consequences of failure. By quantifying consequences of primary failures, of progressive and final collapse, and failure costs, different optimal topologies are achieved. The study considers non-linear geometrical behavior, which leads to highly non-linear objective functions. Optimization for minimal total expected life-cycle costs, including the expected costs of failure, also leads to non-convex objective functions, requiring global optimization algorithms. The study addresses optimal design of structures such as steel-frame towers, antennas, domes and shells.*

Keywords: *risk optimization, ground structure approach, progressive collapse*

1 INTRODUCTION

Defying failure is the primary challenge of the structural engineer. It sounds paradoxical, but in order to achieve a successful design, the structural engineer must think about and account for all possible failure modes of a structure. A linear mechanical model is insufficient for correctly modelling such events of failure.

This study addresses optimal layout of trusses, considering progressive collapse and the expected consequences of failure. Trusses are chosen as study object because it is easier to characterize failures, in comparison to continuum structures. Failure of individual elements is considered as discrete events: each element of the structure either fails or survives. Non-linear analysis is employed to properly characterize element failures. Rigid portals and articulated trusses are similarly modeled by the finite element method, but optimizing truss topologies is more difficult than optimizing portals due to joint instability (Ohsaki 2011).

2 METHODOLOGY

This section presents the three fundamental concepts of risk topology optimization: structural optimization (topology), mechanical problem and structural reliability (**Erro! Fonte de referência não encontrada.**). These concepts merge as the objective function is composed of direct fabrication costs (structural volume) and expected costs of failure (probability of failure). The geometric characteristics are determined by the optimization algorithm and the evaluation of the failure probability by the structural reliability algorithm. The optimization and reliability analyses are subject to successive evaluations of the mechanical equilibrium, carried out by a finite element formulation.

Consider a vector of deterministic design variables $\mathbf{d}$ and a vector of random parameters $\mathbf{X}$. The optimization module defines the initial values for the design variables. It calls the reliability module, which evaluates failure probabilities, given $\mathbf{d}$:

$$P_f(\boldsymbol{d}) = \int_{g(\boldsymbol{d},\boldsymbol{x})\leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})d\boldsymbol{x} \tag{1}$$

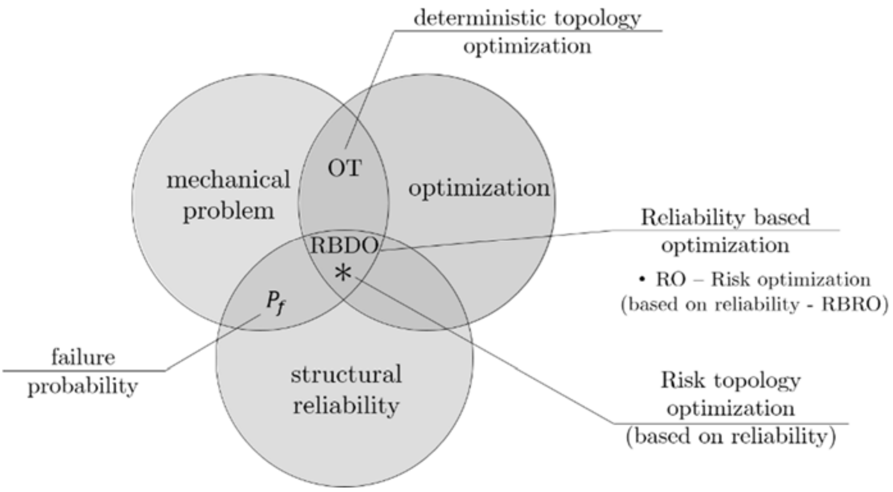

Figure 1. Composition of risk-based optimization problems.

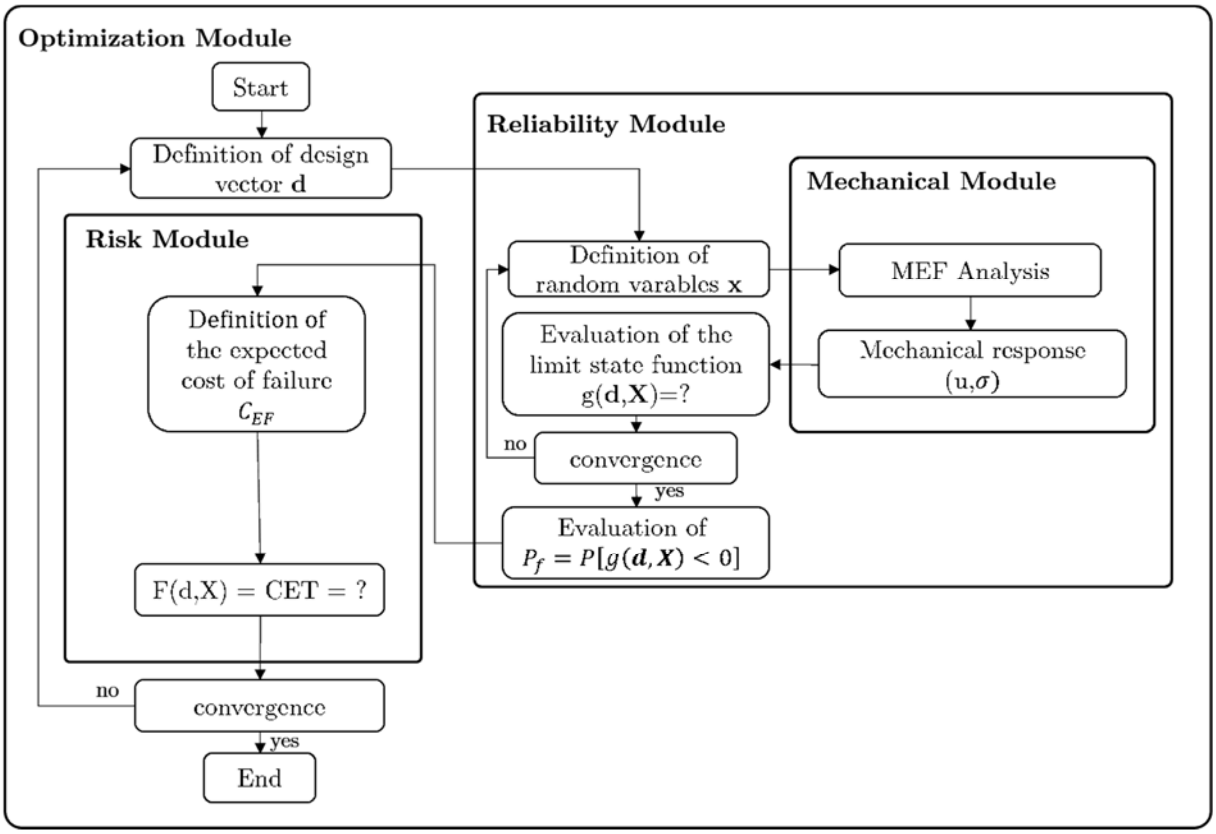

Figure 2. Coupling of modules.

where $g(\mathbf{d}, \boldsymbol{x}) \leq 0$ is the limit state function. The reliability module establishes the current value of the random variable vector $\boldsymbol{X} = \boldsymbol{x}$ for which the limit state function should be evaluated, and then calls the mechanical module. After several runs of the mechanical module, the failure probability can be evaluated. With the failure probability, the optimization module up-dates the design vector ($\mathbf{d}_k \rightarrow \mathbf{d}_{k+1}$), and the process is re-started, until convergence, following Figure 2.

Assume that n mechanical model solutions are needed for convergence of a non-linear FE analysis, and m mechanical responses are needed for failure probability evaluation. If the optimization requires p simulation iterations, solution involves $p * m * n$ mechanical evaluations. Here, p depends mainly on the number of design variables, where 10^2 is a typical number; m depends on the target P_f and the number of MCS samples (10^3 is typical) and n is the number of load steps, typically 10. Hence, a simple solution requires of the order of 10^6 solutions of the mechanical model.

2.1 Mechanical Model

The structure is modelled by positional finite elements. The positional FE is an alternative to describe non-linear geometrical problems since equilibrium is estimated in the deformed or displaced configuration. This formulation has proved to be efficient when compared to classic displacement-based FE. The implementation details can be seen in (Coda; Greco, 2004). For our problem we use the non-linear deformation of Cauchy-Green (E_g) to deal with geometrical nonlinearities and the linear deformation ε for linear cases. The deformations are given in function of the initial length L_0 and the current or final length L, according to Eq. (2).

$$E_g = \frac{1}{2}\left(\frac{L^2}{L_0^2} - 1\right); \varepsilon = \frac{L}{L_0} - 1 \quad (2)$$

During the change of configuration, the total energy Π of a static mechanical system is given by its internal deformation energy U plus the potential energy of external forces W. As the forces are conservative, the minimum total potential energy principle is applied to solve the equilibrium condition. An incremental procedure is used, with the Newton-Raphson method to trace the equilibrium path of the structure. The convergence of incremental-iterative process is better when the load is divided into steps, since the final position of the structure is closer to the initial position.

2.2 Structural Optimization

The structural optimization can be classified according to the design variables used to define both the objective function and the constraints. The three main types are size, shape and topological optimization. The choice for layout optimization is based on the principle of simultaneous optimization of topology, shape and size. That is, optimization of the nodal coordinates and the number and/or arrangement of elements and nodes, considering the variation of the cross-sectional area. Despite the simplicity of the term, the layout optimization is the most complete since it comprises all the design variables of the three basic types of optimization.

A deterministic structural topology optimization problem can be defined as:

$$\boldsymbol{d}^* = arg\, min[C(\boldsymbol{d}): \boldsymbol{d} \in S] \quad (3)$$

where $\mathrm{C}()$ is the objective function, **d** is the design variables vector (nodal coordinates, area of elements, element density) and $\mathrm{S} = \{\mathrm{g_j}(\mathbf{d}) \leq 0,\ \mathrm{j}=1,\ldots,\mathrm{m};\ \mathrm{d_i^{min}} \leq \mathrm{d_i} \leq \mathrm{d_i^{max}},\ \mathrm{i}=1,\ldots,\mathrm{n}\}$ is the design space formed by the inequality constraints $\mathrm{g_j}(\mathbf{d})$ and limits imposed on each variable $\mathrm{d_i}$. The vector $\mathbf{d}$ contains those variables whose value is to be determined in order to obtain the lowest possible value of the objective function.

Truss Optimization. Much of the literature covers truss topology problems with minimal strain energy formulation or formulations in terms of displacements only or stress only. These formulations are known to be unable to correctly predict the actual behavior of the structure. Furthermore, there are few topological optimization problems that include nonlinear description. It can be explained by the increase of degenerate optimal topologies in comparison to the linear analysis, or by the similar behavior presented by the two formulations when there is no consideration of failures which depends on the equilibrium analysis in the displaced position (Buhl; Pedersen; Sigmund, 2000).

The optimization of finite dimensional structures is dealt in (Bendsøe, Martin Philip; Sigmund, 2000) and later by (Ohsaki, 2011). The widely used numerical approach to truss topology optimization is to remove unnecessary members from a highly connected ground structure while nodal locations are fixed. The ground structure approach by (Dorn; Gomory; Greenberg, 1964) is done using the continuous variation of the cross-sectional area of the bars as a design variable and including the possibility of null areas. The optimal structure is then a subset of all elements initially chosen by the connections between the nodal points or the finite element set that connects the boundary conditions imposed and the external loads.

2.3 Reliability and Risk based Optimization

In this paper, we are interested in optimization problemd with consideration of expected consequences of failure. However, this term is often overlooked in classical RBDO problems. In RBDO problems, uncertainty in loading and strength is addressed objectively and modelled probabilistically. In addition, failure probabilities are normally used as design constraints:

$$\boldsymbol{d}^* = arg\,min\left[C(\boldsymbol{d}) : \boldsymbol{d} \in \boldsymbol{S}, P_f \leq P_{f_{adm}}\right] \tag{4}$$

where $C(\boldsymbol{d})$ is the initial or production cost, which may include cost of materials (calculated from the volume) and labor costs, and $P_{f_{adm}}$ is the constraint in terms of an admissible failure probability. The design space of the optimization problem becomes $S=\left\{P_f^{(i)}(\mathbf{d}) \leq P_{f_{adm}}^{(i)}, 1 \leq i \leq n_c\right\}$ where each limit state equation gives a constraint in terms of admissible failure probabilities.

On the other hand, the Risk Optimization is distinguished from other structural optimization formulations by taking into account the effect of uncertainties in terms of probabilities and the expected costs of failure (Beck & Gomes 2012). The total expected costs (C_{ET}) is a sum of initial costs and expected costs of failure.

$$C_{ET}(\boldsymbol{d}) = C(\boldsymbol{d}) + \sum_{i=1}^{n_c} C_{Fi} P_{fi}\,(\boldsymbol{d}) \tag{5}$$

where C_F is the cost of failure for the ith failure mode. We solve a risk optimization problem:

$$\boldsymbol{d}^* = arg\,min[C_{ET}(\boldsymbol{d}) : \boldsymbol{d} \in S] \tag{6}$$

where $S = \{\boldsymbol{d}_{min} \leq \boldsymbol{d} \leq \boldsymbol{d}_{max}\}$ is the admissible design space and $\boldsymbol{d}_{min}$ and $\boldsymbol{d}_{max}$ are the lower and upper bounds of design variables (nodal coordinates, area of elements, element density). The random variable vector $\boldsymbol{X}$ describes the uncertain parameters of the problem. It is important to note that in this formulation the failure probabilities are not used as constraints but rather included in the objective function. Details of this approach can be found in (Gomes & Beck 2013)

3 PROBLEM FORMULATION

We consider a steel structure, where the initial cost is defined only by the construction cost C_C. The C_C is equivalent to the unitary cost of material multiplied by the volume, given by cross-sectional area A_i multiplied by length of the members L_i, Eq. (7). We also define the reference cost C_{REF} which is constant and evaluated from the initial configuration of the structure $(\boldsymbol{d_0})$.

$$C_C(\boldsymbol{d}) = \sum_{i=1}^{m} c_u\, A_i(L_i) \;\; and\, C_{REF} = C_C(\boldsymbol{d_0}) \tag{7}$$

The failure cost C_f can be defined as a function of the reference cost using a proportional factor (Aoues, 2008):

$$C_{EF_n} = C_{f_n} P_{f_n} \quad for \quad C_f = \alpha\, C_{REF} \tag{8}$$

With regard to service costs, we use $\alpha = 1$ in relation to the volume of broken bars. This represents the work and material cost of replacement of each member. Since the probability of failure is specific to each element, the cost of service is given by the sum of the expected costs of failure of each bar.

$$C_{EF_{SF}}(\boldsymbol{d}) = \sum_{i=1}^{m} C_{c_i}(\boldsymbol{d})\, P_{f_i}(\boldsymbol{d}) \tag{9}$$

For ultimate failures, a distinction is made between direct failures of isostatic structures and progressive failure of hyper-static structures. In progressive failure, when there is collapse due to a failure of a second member after failure of a hyper static member the cost is multiplied by k. However if the structure fails due to an isostatic member failure, the failure is direct and the cost is multiplied by *10 k*. This distinction represents compensation resources for injuries and human losses that are not observed in progressive failure, as the structure can be evacuated.

$$C_{UF} = \begin{cases} k\, C_{REF} \rightarrow progressive\ colapse \\ 10\, k\, C_{REF} \rightarrow direct\ colapse \end{cases} \tag{10}$$

The choice of "k" determines different scenarios to be analyzed. The expected cost of the ultimate failure is given by the linear relationship with the system failure probability:

$$C_{EF_{UF}}(\boldsymbol{d}) = C_{UF}\, P_{f_{sys}}(\boldsymbol{d}) \tag{11}$$

The individual probabilities of failure, Eq.(1), are computed by well know reliability methods as FORM (Melchers 1999) or Monte Carlo simulation. In this paper, we use MCS due to its numerical precision. Combined with the Ranked Weighted Average (RWA) by (Okasha 2016) to improve efficiency without significant changes on the P_f value. The RWA is better when used with random design variables, so we admit small standard deviation (10^{-4}) to our vector **d**. To solve the non-convex optimization problem in Eq. (6) we use global optimization methods such as the Particle Swarm Optimization (PSO) algorithm.

4 EXAMPLES

We analyze a simple truss to verify the nonlinear mechanical resolution, the use of risk analysis of progressive collapse and the influence of failure cost scenarios in the optimal design. We use elements with circular cross-section areas as design variables. The global heuristic optimization algorithm had its parameters chosen appropriately according to the test functions available in the literature. However, the algorithm together with the presence of uncertainties does not guarantee convergence for global minima and single solution. The results will be presented through the mean and standard deviation of at least 10 simulations or the minimum value found in this set of simulations.

4.1 Six bar truss

The six bar hyper static truss in Figure 3 is a very important example for dealing with progressive collapse. The initial parameters are: cross-section areas of $2{,}0 \cdot 10^{-2}\ m^2$, length L = 1m and connections and external forces following Figure 3. The material strength properties and load intensity are described as normal distribution variables. The parameters are shown

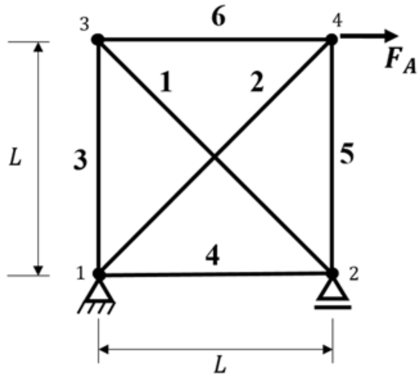

Figure 3. Hyper-static six member truss and bar numbers.

Table 1. Random variables and their parameters.

Variable	Distribution	Nominal Value	Stand. Deviation
Elasticity modulus E	Normal	$2.050\ 10^{11}$	$1.5\ 10^{10}$
Ultimate stress σ_{u_i}	Normal	$7.45\ 10^{8}$	$6.2\ 10^{7}$
Applied Force F_A	Normal	$4.45\ 10^{6}$	$4.45\ 10^{5}$

in Table 1. The default units are meter and Newton. Only failures related exclusively to elements are considered. All random variables are assumed to be statistically independent, and each bar is assigned an ultimate stress variable

Mechanical Model Comparison. The linear and non-linear mechanical resolutions are done with the same MEF algorithm by altering the deformation measurement. The simulations on

Table 2 were performed for the failure consequence scenario with k = 1.

The optimal configuration concentrates material in bars 2 and 5, characterizing a structure that tends to the isostatic configuration. The large coefficients of variation indicate that not always the other bars (1,3,4 and 6) have minimum area at the end of the optimization. In fact, the best of all linear solutions has f.obj = 18.784 and d = {0.01723, 8.497, 0.01, 0.6875, 5.987, 0.01}. The best nonlinear solution follows the same pattern: f.obj = 18.360 and d = {0.05565, 8.454, 0.08783, 0.07185, 6.136, 0.01092). The cost of the best optimal configurations observed is around 13.37% and 13.67%, respectively, of the initial total expected cost (C_{ET}).

Figure 4 shows the optimal configuration with the median values of the analyses considering the most probable configuration. The nonlinear case is shown to be more robust presenting larger cross sections in the secondary elements and slightly higher cost. The fraction of total expected costs, in comparison to the initial configuration, was 16.65% for the linear and 18.16% for the non-linear case.

Influence of failure cost scenarios. With respect to the scenarios, small k values (1 or 5) allow a greater number of optimal structures with minimum area bars such as Figure 4a which can be considered an isostatic structure. This is because both the probability of failure of these structures and the cost of ultimate failure have small intensity. The k =10 scenario results in some hyper static structures. In this scenario, the expected cost of ultimate failure will exert a greater influence on the objective function although the probability of failure remains small.

Table 2. Results obtained for the hyper-static 6-bar truss with unit scenario cost

	Linear		Non Linear	
	Mean $[10^3 mm^2]$	Cov %	Mean $[10^3 mm^2]$	Cov %
A1	0.059294	245.2	0.947859	296.2
A2	**9.2184**	**10.2**	**8.69455**	**29.6**
A3	0.64667	161.8	1.4375	171.5
A4	0.40341	162.0	1.16188	100.0
A5	**7.6031**	**48.0**	**7.88464**	**30.6**
A6	1.03399	128.1	0.51042	290.4
$C_{ET}(d)$	22.8623	17.2	24.9351	21.7
β	3.81541	0	3.815	0

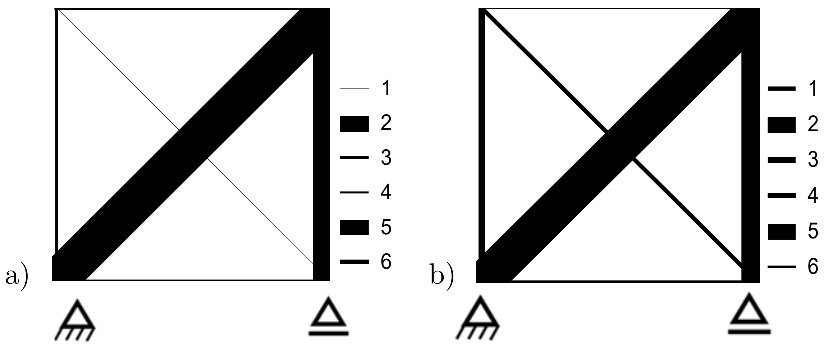

Figure 4. Optimal layout a) linear e; b) nonlinear.

The influence of k in the objective function is even greater for the cases of ultimate failure by direct collapse, Eq. (10). This weight restricts the "removal" of bars from the initial structure resulting in a hyper static structure.

Convergence. The average number of iterations required for convergence is 78 for the linear solution, 95 for the non-linear solution. The linear case, Figure 5a, besides having lower computational cost, converges in a smaller number of iterations, being less disperse. The nonlinear response, Figure 5b, has noticeably great influence on the convergence of the algorithm.

The lowest cost simulation among the 10 simulations for the scenario k = 1 was used to show the convergence process for the nonlinear response. It is observed in Figure 6 that the removal of bars occurs progressively. The algorithm allows the recovery of bar areas, since they are not removed the bar from the structure. The thickness of the bars represents the cross-section area, according to the scale on the right.

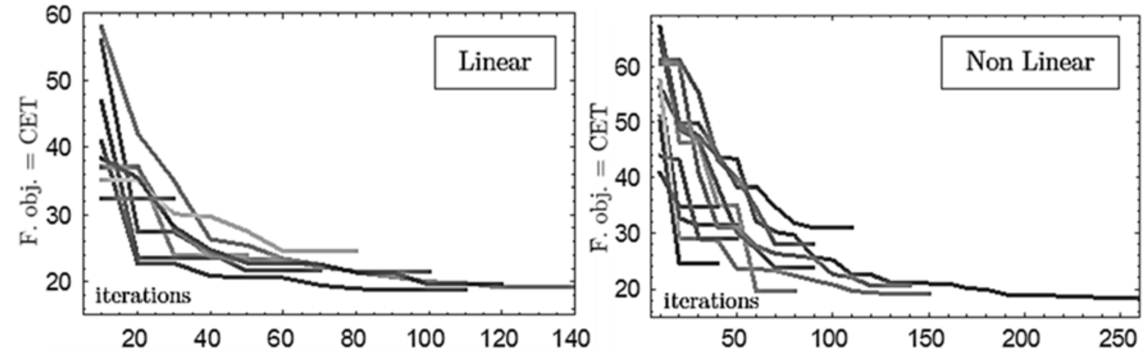

Figure 5. Evolution of the objective function

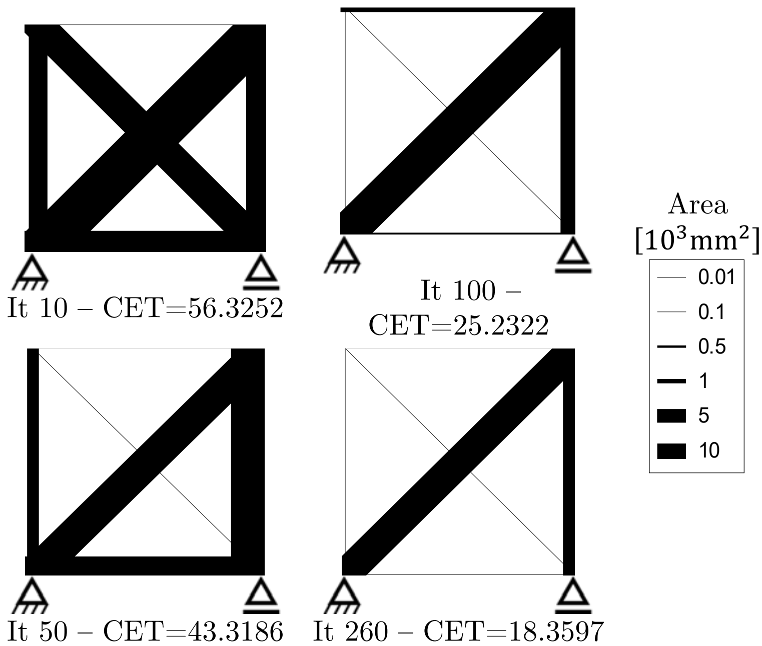

Figure 6. Evolution of the optimal projects for k = 1, non-linear case

5 CONCLUSIONS

The proposed method is able to correctly represent stress failures, which increases the accuracy of the expected consequences of failure, essential for the optimization of risk. The consideration of non-linearity has moderately increased costs tending to slightly weighted structures. The progressive failure prediction algorithm was able to identify relevant failures events and redistribute the loads after the first fail. The use of the ground structure with the lower bound area does not restrict the increase of area of vanished area elements.

Initial analyzes show that the vulnerability of the structure elements varies with its topology. The progressive failure prediction algorithm is extremely important to evaluate the consequences of removing alternative load paths. We advise the use of the algorithm as it allows consideration of the progressive failure in larger problems in which the description of the one-to-one failure is infeasible.

In reliability analysis, the reliability calculation efficiency using RWA and random design variables is not sufficient to improve program performance. The mechanical resolution and iterations of the PSO are still necessary. To improve efficiency, a method that would reduce the number of PSO iterations would be necessary, reducing the number of mechanical evaluations and reliability.

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