COMPARATIVE ANALYSIS OF THE DYNAMIC RESPONSE OF AN ALTERNATIVE TIRE MODEL

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Abstract. *The tire models have a great importance in the terrestrial vehicle modeling. By means of the multibody modeling, the amplitudes, frequencies, and other parameters relative to forces and displacements necessary to the design in fatigue of the mechanical components are estimated. Part of the tire model is the terrain envelope, that is about a terrain signal modifier filter that has as goal reproduce the effect of the tire contact patch. A necessity evaluation of some current models will be done in the context of different degrees of irregularities, different terrains will be filtered by the envelope models known as rigid wheel, and double elliptic cam. Comparing the response obtained in each model for each terrain by power spectral density, will be verified the importance of the envelope models as part of the tire model in distinct contexts.*

Keywords: *Tire model, Terrain envelope, Vehicle modeling, Multibody dynamics*

1 INTRODUCTION

In the context of automotive structural numeric simulation, one necessary step for a complete definition of the problem is the multi-body simulation. In multi-body simulation the vehicle is represented in a simplified way, generally by concentrated masses, plus springs and dampers that connect them. Then is possible to estimate in a effective way the forces that act on specific parts of the subsystem. That estimate is used after as input in other simulation contexts and fatigue estimation.

Inside the multi-body simulation of a terrestrial vehicle, one of the most complex and relevant components to be represented with high fidelity is the tire. This model is normally represented by a set of sub-models that couple or superpose each other. Being the terrain envelope model responsible in some models to represent part of the interaction with the terrain, in this case, it transmits in the model response where occurs the interaction, as well promoting an attenuation on the original excitation, effect verified in reality.

In the literature is possible to find various publications that present this topic. The tire that acts as a geometric filter due to their finite dimensions, smoothen the sharp edges from terrain irregularities. If we wish to simulate the tire dynamics in uneven roads we should take in account this effect (Zegelaar, 1998).

The model of single contact point is the most used, this model is valid for large wavelength (greater then 3 meters), this model can be used in surfaces with random irregularities generated by filtered white noise. Passing through discrete obstacles (cleats) results in very high accelerations from the tire with the single contact point model (Badalamenti e Doyle Jr., 1998).

Yet the rigid wheel model, it consists of a rigid wheel rolling over obstacles. There is only one contact point, neglecting special cases where the rigid circle has more then one contact point with the surface. The contact point isn't restrict to be above the wheel axis (Zegelaar, 1998). Obstacles of short wavelength are filtered by this model (Guo, 1993). The models of rigid ring cover the first mode of vibration of a tire, and they are capable of running in real time embedded systems (Rampalli, 2011).

It was observed yet, comparing the approach of the rigid circle with a flexible circle that for the flexible circle the axis position seems to change position in accord with a superposition from the displacement of two sine functions (figure 1)(Zegelaar, 1998). This observation was used to develop a more pragmatic approach to derive the effective terrain surface (Pauwelussen, 2014).

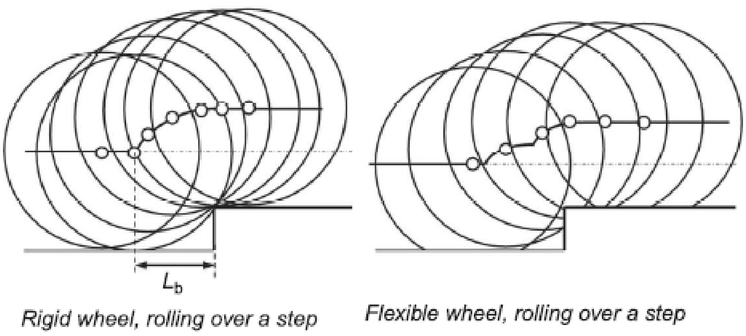

Figure 1: Comparison between rigid and flexible wheel

This approach (figure 2), consists in the following steps:

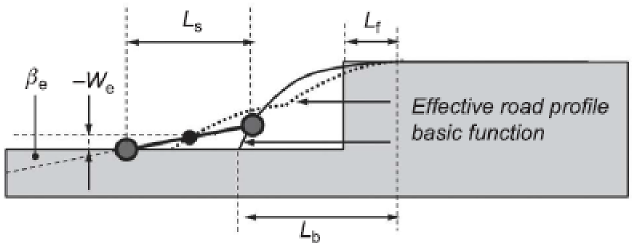

Figure 2: Effective profile developed model

1. Use the step input to derive a quarter sine wave type basic road function. This basic function is defined for an X interval of length L_b. It is found that this function is shifted over a distance L_f in the direction of the step obstacle.
2. Use a two-point follower model with length L_s, moving along this basic function.
3. The midpoint of this two-point follower describes the height w_e of the effective road input.
4. The orientation of the two-point follower describes the effective road slope β_e.

(Pauwelussen, 2014).

The follower length L_s, appears to be on the order of 80% of the contact length. It was shown that the shift L_f and the length L_b only depend on the step height and on the tire, not on the tire load. This approach allowed a distinction between the road-dependent parameters and the tire load-dependent parameters (Zegelaar, 1998).

We should consider three types of terrain profile:

1. The original road profile
2. The effective road profile: the position of the wheel axis, that will be used as input for the dynamic analysis, using the simple contact point model.
3. The basic road function: a road profile, defined previously by a unitary obstacle, to be used as intermediary step between the original road profile and the effective road profile.

This approach brought to surface a series of considerations, which made this not an ideal approach for an effective road profile tool:

- The shift L_f was difficult to interpret in terms of physical phenomena, and an explicit expression for the basic road function that would end at the step obstacle is preferred.
- A sine function for the description of the basic road function appeared not to be a very accurate approach, also with the shift L_f in mind.
- The superposition of steps to build an arbitrary road shape appeared to be cumbersome, specially in case of a downward slope. (Pauwelussen,2014)

It was proposed a alternative for the sine function, introducing elliptical cams. Also was verified that the basic function can be described by an elliptical cam, with half axis length of a_e

and b_e plus a exponent c_e. (Schmeitz, 2004)

$$(\frac{x}{a_e})^{c_e} + (\frac{y}{a_e})^{c_e} \tag{1}$$

With the cam following the road profile (figure 3). This approach can be justified by observation of the tire close to the contact zone it has an elliptical form.

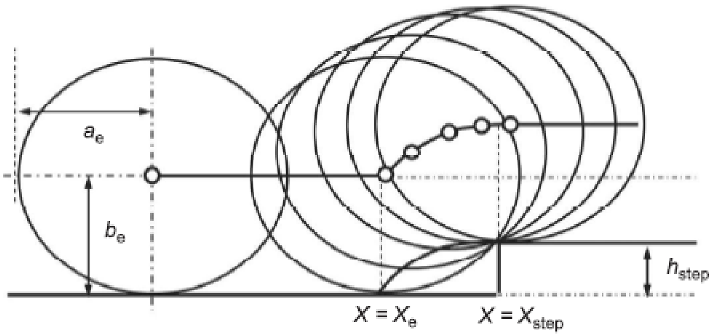

Figure 3: Elliptical cam crossing step obstacle

A two-point follower still utilized, but this time with two elliptical cam's connected by its centres, or equivalently represented by their contact points immediately above the center, as observed in the (figure 4). (Schmeitz, 2004)

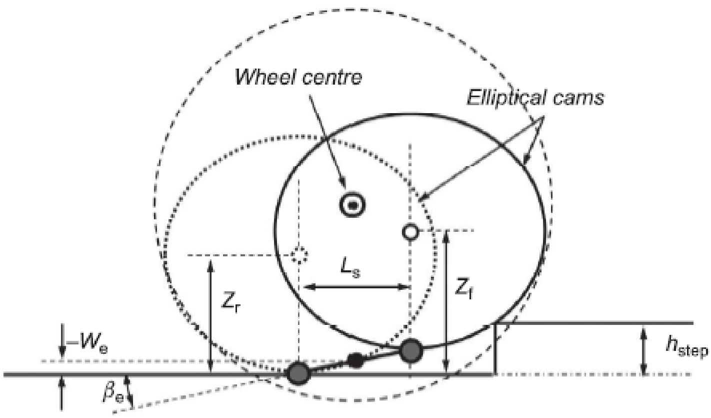

Figure 4: Double elliptical cam model detailed

The actual work has the goal of verifying in different contexts the significance from the use of the road envelope models. This work has in addition a goal, to sum up to the previous works and to the package of automotive simulation developed continuously in the research group. The comparison of the response in a simple contact point model with the rigid ring model, and with a double elliptical cam follower (figure 5), applied to diverse classified road conditions, should allow a use evaluation for different models in diverse applications.

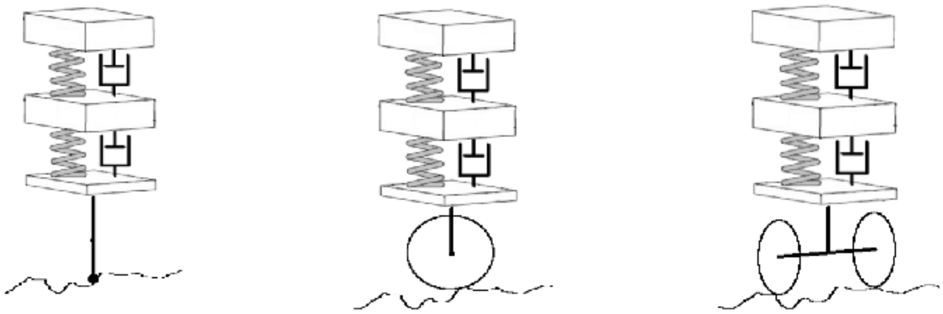

Figure 5: Illustration of tire envelope models of this work

2 NUMERICAL METHODS FOR TIRE MODELING

2.1 Rigid ring tire model

First it was implemented a multi-body tire model, known as the rigid ring, this model is based in the premise that the tire belt (the mass that don't touch the ground) is a rigid ring connected to the axis through a set of springs and dampers, in the horizontal, vertical an angular directions (figure 6) (Frey, 2009). In this case it was adapted, taking in account only the vertical contribution and implemented in a commercial simulation software. This simplification was made in order to make the model more stable and compatible with the desired evaluation. By this model it will be possible to evaluate the response due to excitation coming from the terrain envelope models.

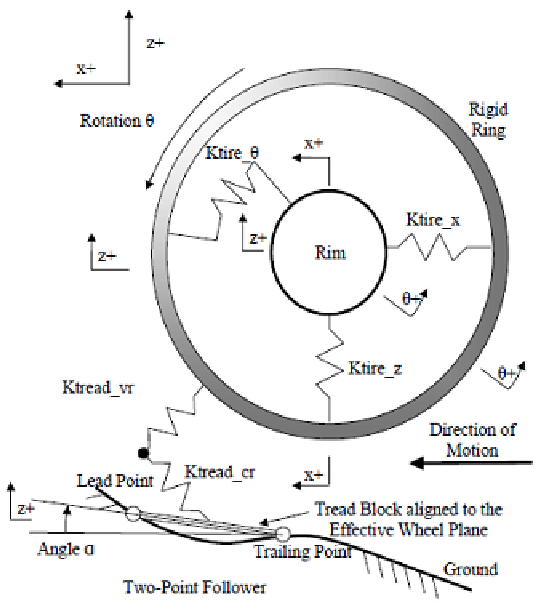

Figure 6: Rigid ring complete model

There were derived the equations of motion, using the Newton method, for the degrees of freedom from the wheel:

$$-k_{zaxis}(z_{wheel} - z_{car}) + k_{ztire}(-z_{wheel} + z_{ring}) = M_{wheel}\ddot{Z}_{wheel} \tag{2}$$

For the rigid ring:

$$-k_{ztire}(z_{ring} - z_{wheel}) + k_{vr}(z_{ring} - z_{tread})\cos^2\alpha - k_{cr}(z_{ring} - z_{tread})\sin^2\alpha \\ = M_{ring}\ddot{Z}_{ring} \quad (3)$$

For the tire thread:

$$F_{norm}\cos\alpha - k_{vr}(z_{tread})\cos^2\alpha - k_{cr}(z_{tread} - z_{ring})\sin^2\alpha = M_{tread}\ddot{Z}_{tread} \quad (4)$$

The newton method was used to derive these equations for the kinetic and potential energies, for the non conservative energy they can be obtained by inspection because they are analogous to equations from potential energy, damping terms are included later with the only difference that they are related with velocities (Jazar, 2008). The natural frequency from a loaded tire is around 80 hz (Pacejka 2002), it was calculated the natural frequency from the rigid ring with the adopted constants, by the spectral decomposition method (Jazar, 2008), and it resulted in 77.7 hz, a frequency close to this can be observed in the spectral analysis later.

2.2 Terrain envelope tire models

Single contact point follower

Considered the most basic and popular, the single contact follower doesn't modify the original road profile. This model is used as a base model for comparison, and don't require any numerical implementation. The road profile is simply used directly at the rigid ring model.

Rigid wheel follower

The rigid wheel follower can be considered the abstraction immediately after the single contact follower, it takes the tire as a rigid circular body, that only touch the road profile. This model was implemented as a filter of the original road profile, for this implementation, first is made a local scan below the tire, considering it a perfect circle, so its made a comparison between the circle height and the road profile as the following equation:

$$z_{circle_j} = -\sqrt{R^2 - x_j^2} + R \quad (5)$$

The z_{circle} represents the height of the circle at the local point j , R the tire radius, and x_j the local horizontal coordinate (ranging from $-R$ to R).

After this comparison the center of the circle is repositioned in a way that the circle only touch the road profile, as in the following equation:

$$z_{axis_i} = \sqrt{R^2 - x_j^2} + Z_i \quad (6)$$

Where the i index represent the global position of the road profile Z_i. The corollary is that the effective road profile is given by:

$$z_{erp_i} = max(z_{axis_i}) - R \quad (7)$$

Double elliptical cam

The double elliptical cam is a modern tire envelope model, evolved as a result of a continuous research developed at T.U. Delft, applied in vehicle dynamics commercial tools. It is a direct evolution of the dual follower, this model brings a adjustment by a visual approach, verifying that the tyre has the shape of half elliptical cam at his entrance and exit zones. The cam, a modified ellipse with equation defined by the parameters a_e , b_e and c_e, the ellipse length, height and exponent Eq. (1).

The parameter a_e is given by a function of the half contact length a that is approximated by a parabolic function based in measurements (Allen, 2013).

$$a = -1.26 * 10^{-11} F_z^2 + 2.98 * 10^{-6} F_z + 0.0317 \tag{8}$$

$$a_e = R_0 - \zeta a \tag{9}$$

In this case it was adopted a constant force F_z of 2500 N. The parameter a_e is function as well of the free radius of the tire R_0 and from the footprint fraction ζ , that was assumed constant $\zeta = 0.7$ (Allen,2013).

The Eq. (1) is reorganized, and a scan is made in the local x coordinate from $-a_e$ to a_e, in a way to obtain the elliptical cam envelope given by Eq. (10).

$$Z_{tire_i} = -b_e(1 - (\frac{-x_i}{a_e})^{c_e})^{\frac{1}{ce}} + R \tag{10}$$

After being defined the envelope, this is used to the filtering process with the road profile. The road profile filtering is made doing a comparison of the terrain height that is above the elliptical cam, and the scan proceeds to each x_{ij}, the index i in reference to the global coordinate, and the j for the local (above the cam) scan. Then is calculated the cam center height by eq. (11).

$$Z_{cam_i} = b_e(1 - (\frac{-x_{ij}}{a_e})^{c_e})^{\frac{1}{ce}} + Z_i \tag{11}$$

Similarly to the procedure for the rigid wheel, now the effective road profile is calculated by eq. (12).

$$Z_{erp_i} = max(z_{cam_i}) - b_e \tag{12}$$

After this first scan of the filter, we consider a horizontal shift equivalent to the connection length l_s function of the footprint fraction and from the half contact length:

$$l_s = 2a\zeta \tag{13}$$

From l_s is calculated the effective road profile Z_{tandem}, with the tandem configuration, and the effective road slope β_i.

$$Z_{tandem} = \frac{Z_{erp_{i-shift}} + Z_{erp_i}}{2} \tag{14}$$

$$\beta_i = \frac{Z_{erp_{i-shift}} + Z_{erp_i}}{l_s} \tag{15}$$

3 RESULTS

First a verification of the models was made with a unitary step obstacle, and sinusoidal excitations, that made possible to verify which frequency bands where filtered by each model figure (7). The base procedure steps were: the road profile was filtered by each envelope model, then this ”effective road profile” is used as input in the rigid ring model. The results from this were then analysed in frequency domain. For those analysis the models will be referred as: ”RP” road profile, ”SCP” single contact follower, ”RWF” rigid wheel follower, ”DEC” Double elliptical cam.

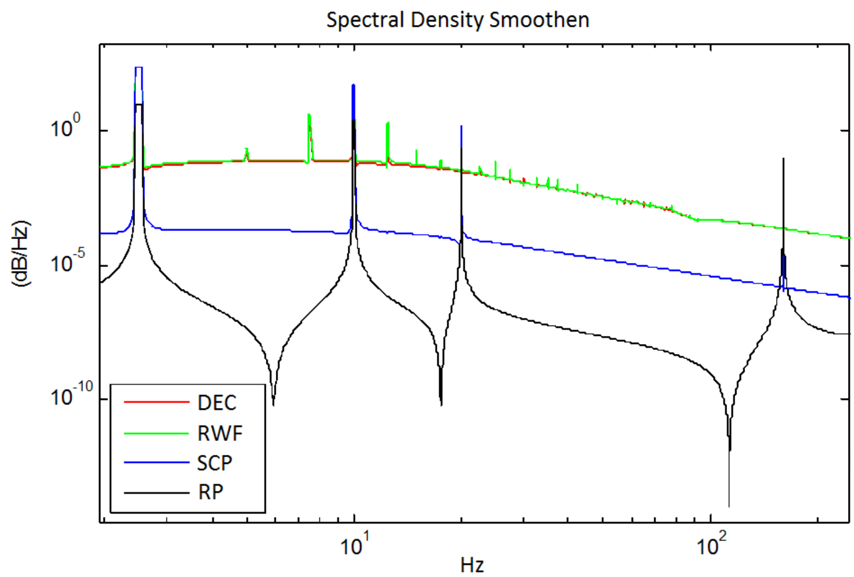

Figure 7: Spectral density, sinusoidal profile response

The next analysis requires the filtering of road profiles computationally generated, classified as ISO B, ISO D and ISO F [Santos, 2014]. In figure (8) a brief evaluation of force response in time domain and a length of 0.5m, where is possible to verify a great difference in the response amplitudes, the filter ”DEC” always providing greater smoothing in the road excitation, followed by ”RWF”, and as expected ”SCP” presents the most accentuated peaks.

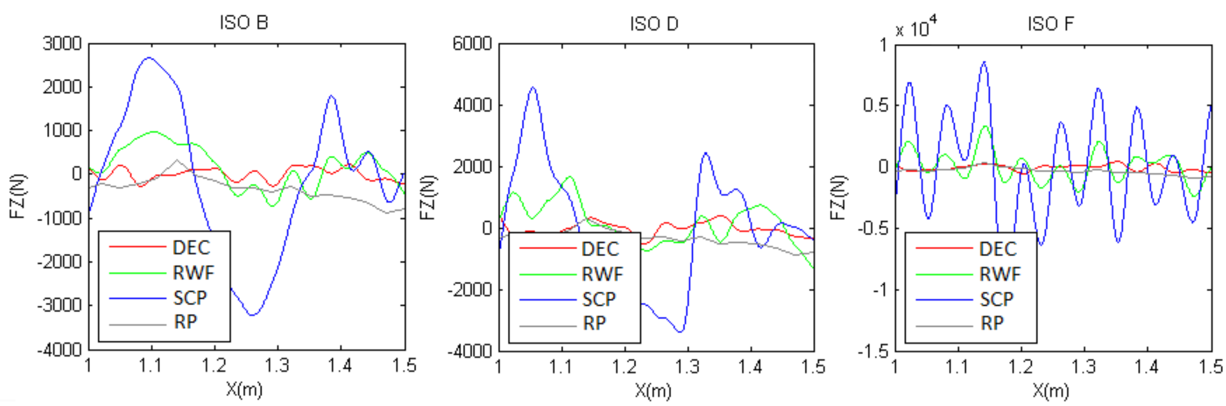

Figure 8: Spectral density, response to a sinusoidal profile

Observing the subset of ISO F road profile, now at frequency domain, with power spectral density obtained by fast Fourier transform, figure (9), is noted steeper peaks showing the frequencies provided by the road profile excitation. Is possible to verify that under 10 hz, the models don't have any significant difference in range, after 19 hz is visible that the model "SCP" starts to exhibit significantly higher amplitudes , representing acceleration peaks. Is noted also in the models "DEC" and "RWF" a large length peak, close to 80 hz, this peak represents the first vibration mode of the tire. Comparing the three models "SCP" and "RWF" seems to have concordance in frequency, while "DEC" presents a phase shift with a lower frequency.

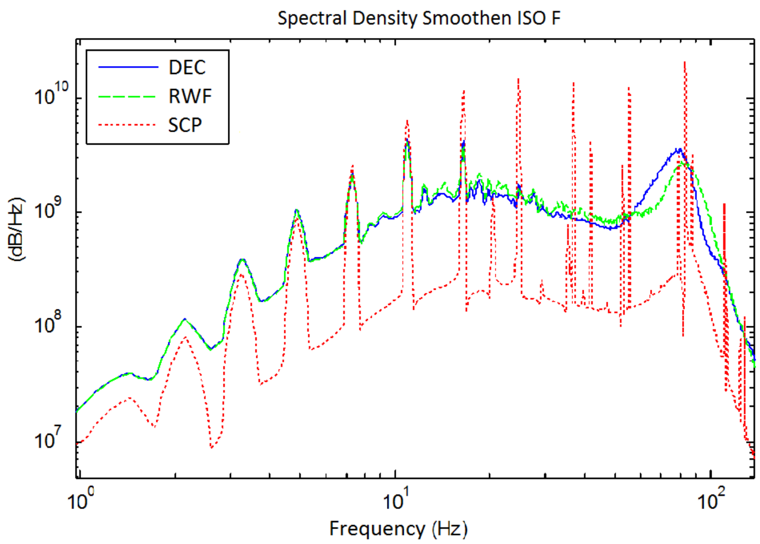

Figure 9: Force response, frequency domain, ISO F profile

Applying the same analysis for a subset of the ISO B profile, figure (10), until 10 hz the observation is that: "SCP" don't present significant peaks, above this frequency they are perceived. Is visible as well that the "DEC" and "RWF" agree in all observed range, including the natural frequency, differing from what was observed in ISO F profile.

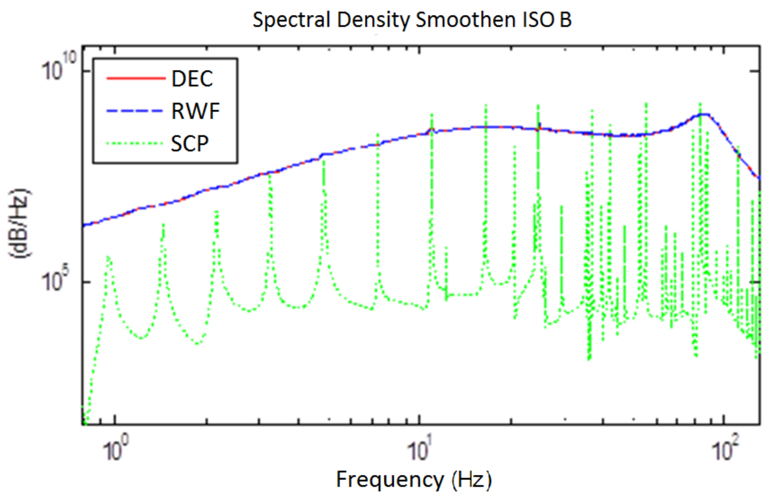

Figure 10: Force response, frequency domain, ISO B profile

4 CONCLUSIONS

The use of the tyre envelope models showed some clear differences from using a road profile without filtering. The attenuation in peaks of acceleration, without loosing definition in the excited frequencies is a important result.

In comparison between "DEC" and "RWF", in terrains of good and average quality classified as "ISO B" and "ISO D", isn't visible in the frequency domain a significant difference in phase or amplitude. This is an important result because it, with the required caution, mean that the simplest, purely theoretical model (it doesn't require any experimental data to run), gives the same response up to average terrains. But in bad quality "ISO F" classified road profile, it is visible a difference of about 6.7% in phase and 25% in amplitude, figure (11),being this difference significant in the after analysis.

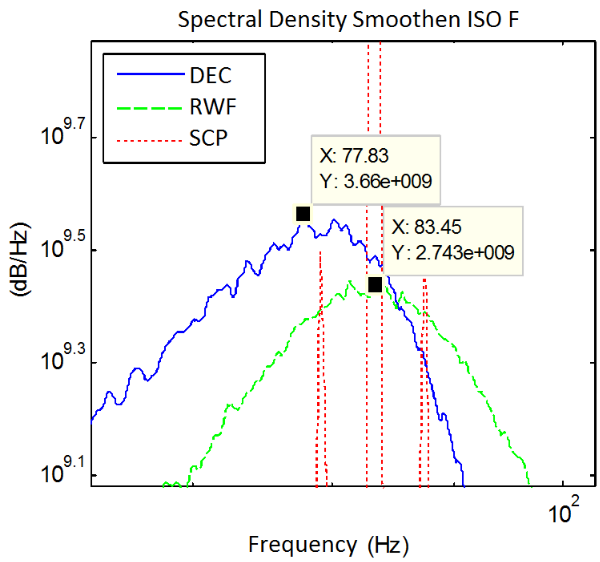

Figure 11: Force response, frequency domain, ISO F profile

Another relevant difference between the "DEC" and "RWF" model is in the time domain, where the model "DEC" always provides a greater attenuation, in any time window observed, and in any road profile, in forces and in displacements. Yet the reduced amplitudes in the time domain analysis, in the frequency domain getting closer to the natural frequency the amplitudes were bigger, that shows the model "DEC" is more sensible to excitation in higher up to the resonant frequency, than the "RWF" model.

The analysis developed in this work provided a mean to observe the diverse effects between models, the "SCP" model the most used due its simplicity is the most deficient. Then models "DEC" and "RWF" provided equivalent response in frequency domain, what gives an advantage for the purely theoretical model "RWF", but in time domain they all differ in every analysis. This bring the conclusion that the "RWF" model less costly, can bring good results up to average road profile evaluations in frequency domain, but the most advanced model "DEC" show significant difference both in all time domain analysis, and road profiles classified as bad condition in frequency domain, but this differentiation comes with a higher cost, due to a requirement of experimental data.

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