A BENCHMARK SYSTEM RBDO EXAMPLE INVOLVING PROGRESSIVE COLLAPSE OF HYPER-STATIC TRUSS

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Abstract. *In conventional Reliability-Based Design Optimization (RBDO), a reliability constraint is specified for each possible failure mode of the structure. System RBDO is a more comprehensive formulation, because a single system failure probability constraint is specified; hence, the individual failure modes can compete with each other, leading to further reductions in the objective function. Unfortunately, not many examples of system RBDO are available in the literature. This paper presents a benchmark system RBDO example, which involves the progressive collapse of a hyper-static truss. Our solution corrects the problems encountered in two prior publications of the same academic example. Given the correct solution, this problem may become a reference benchmark for the solution of system RBDO problems. In the same line, limitations of a system RBDO formulation are discussed. It is shown that conventional system RBDO formulations always lead to isostatic optimal designs. In order for the solutions to include hyper-static structures, the benefits of having a primary failure of a hyper-static member as a warning, before final collapse, have to be quantified. In other words, failure consequences need to be differentiated. This can be accomplished via so-called risk-based optimization, discussed but not shown herein.*

Keywords: *RBDO, System reliability, Progressive collapse, Benchmark example, Risk optimization*

1 INTRODUCTION

Use of single limit state reliability constraints in Reliability-Based Design Optimization (RBDO) is a direct inheritance of deterministic design optimization. However, for problems involving multiple failure modes, hence multiple limit states, it makes more sense to work with system reliability constraints. Imposing a single system reliability constraint warrants design safety without over-constraining the problem with undue component reliability constraints. In a system RBDO problem with single system reliability constraint, the failure modes compete with each other (Beck & Gomes, 2012): a failure mode for which reliability is costlier can be designed with smaller safety margin, to be compensated by greater reliability of other modes. In the same line, a hyper-static member failure which has smaller impact on progressive collapse can be designed with less reliability.

A review of the published literature in RBDO (Beyer & Sendhoff, 2007; Schuëller & Jensen, 2009; Aoues & Chateauneuf, 2010; Lopez & Beck, 2012) reveals that the large majority of methods and benchmark examples address single failure mode reliability constraints (Tu et al., 1999; Youn et al. 2003; Youn & Choi, 2004a; Youn & Choi, 2004b; Du & Chen, 2004; Liang et al., 2004; Yi et al., 2008; Yi & Cheng, 2008). Just recently, greater attention has been given to system-reliability-oriented design (Enevoldsen & Sørensen, 1993; Royset & Der Kiureghian & Polak, 2001; Liang et al., 2007; Aoues & Chateauneuf, 2008; McDonald & Mahadevan, 2008; Nguyen et al., 2010). In this line, benchmark examples involving relevant system RBDO problems are scarce. Examples involving progressive failure are extremely important in civil engineering, considering the current trend to incorporate robustness concepts in design standards. For the community, finding in the published literature examples with unjustified simplifications and wrong solutions is counterproductive.

This paper addresses the solution of an academic but representative system RBDO problem involving the progressive failure of a hyper-static truss. Solution of this problem has been published twice (McDonald & Mahadevan, 2008; Nguyen et al., 2010), containing the same problematic issues identified herein. By removing these issues and presenting a correct solution, we hope that this representative academic problem can become a benchmark solution of system RBDO.

2 PROBLEM DESCRIPTION

This paper addresses the system RBDO problem of determining the cross-section areas of the 6 members of the hyper-static truss illustrated in Fig. 1. The design vector **d** is composed of the cross-sectional areas of the members, $\mathbf{d}=\{A_i\}$ (i=1,...,6), which are considered deterministic, as in (McDonald & Mahadevan, 2008; Nguyen et al., 2010). The applied load F_A is assumed to follow a normal distribution with mean of 4450 kN and standard deviation of 445 kN, while the (ultimate) strengths of the members, S_i (i=1,...,6), are assumed to follow normal distribution with mean of 745 MPa and standard deviation of 62 MPa. All random variables, S_1,...,S_6 and F_A, are assumed statistically independent of each other. The target system failure probability is given as $P_{sys,t} = 10^{-3}$, which corresponds to a target reliability index of $\beta_{sys,t} = 3.09$. In all solutions presented herein, buckling and dynamic load redistribution effects are disregarded. The bars are assumed to fail in a brittle manner once their ultimate strength is reached. The original references (McDonald & Mahadevan, 2008;

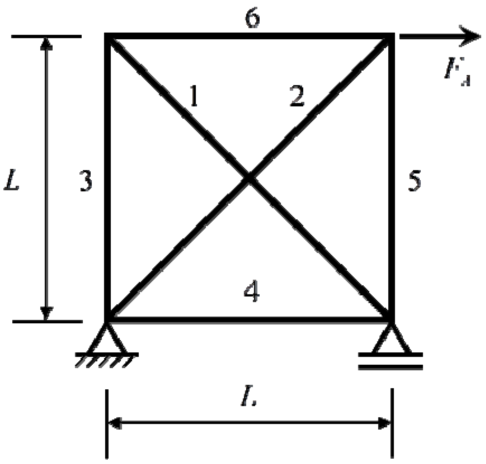

Figure 1: Hyper-static six member truss and bar numbers.

Nguyen et al., 2010) refer to yield failure, but this introduces more conceptual difficulties. In our implementation, it would have been easy to include buckling of compressed members, but we have tried to follow the hypothesis of previous implementations.

Two versions of this problem are solved in McDonald & Mahadevan (2008) and Nguyen et al. (2010). In a simplified version, load redistribution effects are ignored, and system failure is simply characterized by the series combination of the cut-sets resulting from (instantaneous) failure of two members. The probability of each cut-set was calculated by McDonald & Mahadevan (2008) as a parallel system, using the product of conditional marginals. A first-order bounding formula was employed to evaluate the series system failure probability. Later on, Nguyen et al. (2010) compared the solution obtained by McDonald & Mahadevan (2008) to a more "precise" solution, which was obtained using system matrix multiplication (Song & Kang, 2009).

In this technical note, we focus on the most relevant version of the problem, that is: when load redistribution after a primary member failure is considered. Load redistribution is different for each of the six possible "first member" failure events. The complexity in solving the system reliability, in this case, arises from the fact that failure of the remaining members should be described as new component events. Figure 2 shows the numbering choice by Nguyen et al. (2010), for the component failure events in the original structure, and for all possible secondary member failures.

The structure survives if (1) no member fails in the original configuration or (2) one member fails but no further member failures take place. Hence, system failure is characterized by seven link-sets composed of a total of 36 component events. However, due to mutual exclusiveness of the seven link-sets, system probability can be computed as the sum of the probabilities of the individual link-sets, which reduces the number of components from 36 to 12:

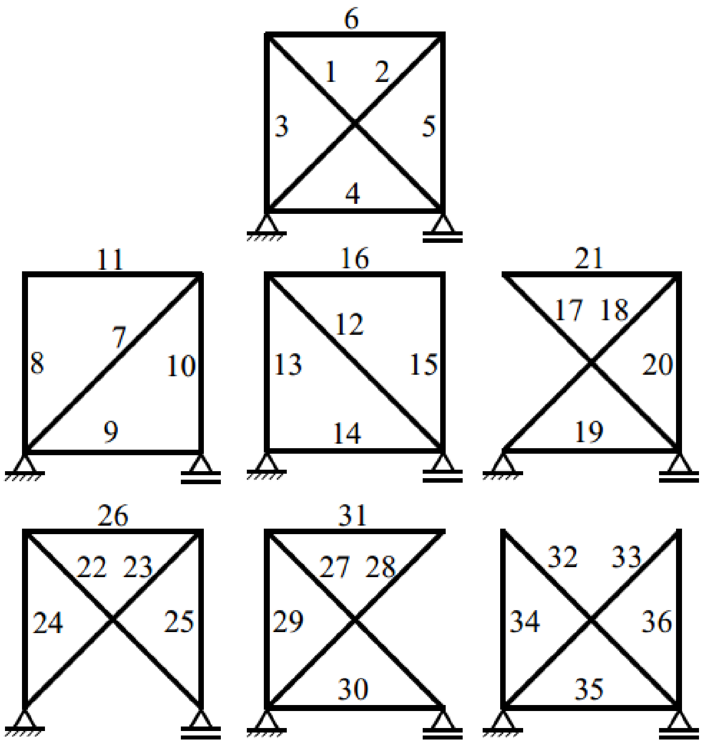

Figure 2: Component failure events for progressive collapse analysis (after Nguyen et al., 2010).

$$\begin{aligned} P\left(\bar{E}_{sys}\right) = \; & P\left(\bar{E}_1\bar{E}_2\bar{E}_3\bar{E}_4\bar{E}_5\bar{E}_6\right) + P\left(E_1\bar{E}_3\bar{E}_4\bar{E}_6\bar{E}_7\bar{E}_{10}\right) + \\ & + P\left(E_2\bar{E}_5\bar{E}_{12}\bar{E}_{13}\bar{E}_{14}\bar{E}_{16}\right) + P\left(\bar{E}_1E_3\bar{E}_4\bar{E}_6\bar{E}_7\bar{E}_{10}\right) \\ & + P\left(\bar{E}_1\bar{E}_3E_4\bar{E}_6\bar{E}_7\bar{E}_{10}\right) + P\left(\bar{E}_2E_5\bar{E}_{12}\bar{E}_{13}\bar{E}_{14}\bar{E}_{16}\right) \\ & + P\left(\bar{E}_1\bar{E}_3\bar{E}_4E_6\bar{E}_7\bar{E}_{10}\right) \end{aligned} \tag{1}$$

where E_i and $\bar{E}_i$ denote the failure and survival event of the i-th component, respectively. This solution was employed by Nguyen et al. (2010) as follows.

2.1 Problematic solutions

The 6-bar truss problem presented herein is a hyper-static problem, which cannot be solved by equilibrium equations alone. A proper solution to this problem is obtained by considering the compatibility conditions in terms of the strain of all members. This solution is further addressed in Section 3.

An undue simplification, taken in both references (McDonald & Mahadevan, 2008; Nguyen et al., 2010), was to consider the forces, and the stresses, in both diagonal members as being the same. This leads to the incorrect load factors that multiplies the applied load F_A, presented in Eq. (2):

$$
\begin{aligned}
&\min_{\mathbf{d}=\{A_1,\ldots,A_6\}} f(\mathbf{d}) = \sqrt{2}\left(A_1 + A_2\right) + A_3 + A_4 + A_5 + A_6 \\
&\text{s.t.} \quad P_{sys} = 1 - P\left(\bar{E}_{sys}\right) = 1 - \sum_{k=1}^{7} P\left[\bigcap_{i \in L_k} g_i\left(\mathbf{d},\mathbf{X}\right) \le 0\right] \le P_{sys}^{t} = 0.001 \\
&\quad g_i\left(\mathbf{d},\mathbf{X}\right) = \begin{array}{ll}
A_i F_i - \left(\sqrt{2}/2\right) F_A, & i = 1,2 \\
A_i F_i - \left(1/2\right) F_A, & i = 3,\ldots,6 \\
A_2 F_2 - \sqrt{2} F_A, & i = 7 \\
A_5 F_5 - 1.0 F_A, & i = 10 \\
A_1 F_1 - \sqrt{2} F_A, & i = 12 \\
A_3 F_3 - 1.0 F_A, & i = 13 \\
A_3 F_4 - 1.0 F_A, & i = 14 \\
A_6 F_6 - 1.0 F_A, & i = 16 \\
A_1, A_2, A_3, A_4, A_5, A_6 & \ge 0
\end{array}
\end{aligned}
\tag{2}
$$

where $L_k, k = 1,\ldots,7$ is one of the seven link-sets presented in Eq. (1).

Surprisingly, even for the optimal truss designs found in McDonald & Mahadevan (2008) and Nguyen et al. (2010), these forces are not the same! The optimal truss layout obtained in the latter work is a hyper-static truss, with member areas of 19,940 mm^2, for diagonal bars 1 and 2, and 14,440 mm^2 for the remaining bars. This hyper-static result is a clear consequence of the unverified "same force at diagonals" hypothesis. The individual bar failure reliability index is 3.48 for the diagonal bars and 3.65 for the remaining bars. The reliability index for the conditional second-bar failures are all negative, indicating a probability of failure larger than 50%. The objective function, sum of all member cross-section areas, is 114.13x10^3 mm^2. A simple static analysis, for these member areas, shows that the forces in both diagonal members are not the same, nullifying the initial hypothesis: F_1 = –2369.35 kN, whereas F_2 = +3923.90 kN.

A single hand calculation shows that the solution found by Nguyen et al. (2010) is strongly in error. Since the limit state functions (Eq. 2) are linear in two Gaussian random variables, the reliability index for diagonal bar 1 is obtained in exact form as:

$$
\beta_1 = \frac{E[g_1(S_1,F_A)]}{Var[g_1(S_1,F_A)]} = \frac{A_1\mu_{S_1} - \sqrt{2}\mu_{F_A}/2}{\sqrt{A_1^2\sigma_{S_1}^2 + 2\sigma_{F_A}^2/4}} = 10.0548 \tag{3}
$$

This result is much larger than the reported reliability indexes of β_1=2.89 (McDonald & Mahadevan, 2008) and β_1=3.48 (Nguyen et al., 2010). We speculate that such gross errors may be consequence of units or unit conversion problems, but we could not spot the source.

3 SOLUTION OF REFERENCE PROBLEM

3.1 Consistent formulation

A consistent solution to the problem is obtained by imposing the compatibility conditions for the strain of the six bars. The elongation of each bar is given by:

$$\varepsilon_i = \frac{N_i L_i}{E_i A_i} \tag{4}$$

where N_i is the axial force acting on bar i, L_i and A_i are the bar's length and cross-section area, and E_i is the Young Modulus of the material, which is assumed to be the same for all bars in order to not influence the analysis. This can be done by means of a finite element formulation:

$$\mathbf{K}(\mathbf{d})\cdot\mathbf{U}=\mathbf{F} \tag{5}$$

where $\mathbf{K}(\mathbf{d})$ the stiffness matrix, $\mathbf{U}$ is a displacement vector and $\mathbf{F}$ is the force vector. Nodes are numbered as follows (Fig. 1): top left (1), top right (2), bottom left (3) and bottom right (4). After imposing the zero displacements at nodes 3 and 4, Eq. (5) becomes:

$$\begin{bmatrix} \frac{A1}{2\sqrt{2}}+A6 & -\frac{A1}{2\sqrt{2}} & -A6 & 0 & -\frac{A1}{2\sqrt{2}} \\ -\frac{A1}{2\sqrt{2}} & \frac{A1}{2\sqrt{2}}+A3 & 0 & 0 & \frac{A1}{2\sqrt{2}} \\ -A6 & 0 & \frac{A2}{2\sqrt{2}}+A6 & \frac{A2}{2\sqrt{2}} & 0 \\ 0 & 0 & \frac{A2}{2\sqrt{2}} & \frac{A2}{2\sqrt{2}}+A5 & 0 \\ -\frac{A1}{2\sqrt{2}} & \frac{A1}{2\sqrt{2}} & 0 & 0 & \frac{A1}{2\sqrt{2}}+A4 \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_{H1} \\ F_{V1} \\ F_{H2} \\ F_{V2} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F_A\cdot\cos(\theta) \\ F_A\cdot\sin(\theta) \\ 0 \end{Bmatrix} \tag{6}$$

By solving Eq. (6) for the unknown displacements, for a unitary load F_A=1, the following hyper-static (H) load factors $\boldsymbol{\alpha}^H$ are obtained:

$$\boldsymbol{\alpha}^H = \frac{1}{\kappa}\begin{Bmatrix} -2A_1A_3A_4A_6\left(A_2+2\sqrt{2}A_5\right) \\ 2A_2A_5\left(2\sqrt{2}A_3A_4A_6+A_1\left(A_4A_6+A_3\left(A_4+A_6\right)\right)\right) \\ A_1A_3A_4A_6\left(\sqrt{2}A_2+4A_5\right) \\ A_1A_3A_4A_6\left(\sqrt{2}A_2+4A_5\right) \\ -A_2A_5\left(4A_3A_4A_6+\sqrt{2}A_1\left(A_4A_6+A_3\left(A_4+A_6\right)\right)\right) \\ A_1A_3A_4A_6\left(\sqrt{2}A_2+4A_5\right) \end{Bmatrix} \tag{7}$$

where α_i^H is the fraction of load F_A taken by the i[th] bar, and κ is the common denominator:

$$\kappa = 4A_2A_3A_4A_5A_6 + A_1\left(4A_3A_4A_5A_6+\sqrt{2}A_2\left(A_4A_5A_6+A_3\left(A_5A_6+A_4\left(A_5+A_6\right)\right)\right)\right) \tag{8}$$

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Load factors in Eq. (7) are valid for the hyper-static 6-bar structure only. For isostatic structures derived from the 6-bar truss, load factors are obtained by nullifying the corresponding member area A_i in Eq. (6), and solving it for unitary load F_A=1. For the isostatic configurations appearing in Fig. 2, for instance, load factors are obtained as:

$$\boldsymbol{\alpha}^{I_1}=\begin{Bmatrix}0\\ \sqrt{2}\\ 0\\ 0\\ -1\\ 0\end{Bmatrix};\ \boldsymbol{\alpha}^{I_2}=\begin{Bmatrix}-\sqrt{2}\\ 0\\ 1\\ 1\\ 0\\ 1\end{Bmatrix};\ \boldsymbol{\alpha}^{I_3}=\begin{Bmatrix}0\\ \sqrt{2}\\ 0\\ 0\\ -1\\ 0\end{Bmatrix};\ \boldsymbol{\alpha}^{I_4}=\begin{Bmatrix}0\\ \sqrt{2}\\ 0\\ 0\\ -1\\ 0\end{Bmatrix};\ \boldsymbol{\alpha}^{I_5}=\begin{Bmatrix}-\sqrt{2}\\ 0\\ 1\\ 1\\ 0\\ 1\end{Bmatrix};\ \boldsymbol{\alpha}^{I_6}=\begin{Bmatrix}0\\ \sqrt{2}\\ 0\\ 0\\ -1\\ 0\end{Bmatrix}. \tag{9}$$

where $\alpha_i^{I_j}$ is the fraction of load F_A taken by the ith bar, given failure of the jth bar.

With these preliminaries, limit state equations for hyper-static and for isostatic single bar failures are written as:

$$\begin{aligned} h_i(S_i,F_A)&=A_iS_i-\alpha_i^H F_A, \quad i=1,...,6;\\ g_i(S_i,F_A)&=A_iS_i-\alpha_i^I F_A, \quad i=1,...,6. \end{aligned} \tag{10}$$

For the hyper-static truss, system failure is characterized by failure of an isostatic member $\{g_j \le 0\}$, given primary failure of a hyper-static member, $\{h_i \le 0\}$. Each hyper-static member failure starts a mutually exclusive failure path. Hence, system failure probability is evaluated as:

$$P_{sys}=\sum_{i=1}^{6}\left(P[\{h_i\le 0\}]P\left[\bigcup_{j=1,j\ne i}^{6}\{g_j\le 0\}\right]\right) \tag{11}$$

In this work, straightforward Monte Carlo simulations are employed to evaluate system failure probabilities. This is done mainly to avoid any approximations from FORM when calculating bi-modal bounds from conditional failure probabilities. With these preliminaries, the system RBDO problem is stated as:

$$\begin{aligned} &\min_{\mathbf{d}=\{A_1,\dots,A_6\}} f(\mathbf{d})=\sqrt{2}\left(A_1+A_2\right)+A_3+A_4+A_5+A_6\\ &\text{s.t.:}\ \begin{cases} P_{sys}\le 10^{-3}\\ A_1,\dots,A_6\ge 0.01 \end{cases} \end{aligned} \tag{12}$$

3.2 Solution algorithm

In this paper, we are not concerned with efficiency, but with accuracy. We are focusing on the solution, and not on the solution method. It is known that the system RBDO formulation in Eq. (13) is a non-convex problem, with multiple local minima. Hence, global optimization methods are needed. The solutions reported below were obtained using the Particle Swarm Optimization (PSO) algorithm. Ranked Weighted Average (RWA) Monte Carlo simulation (Okasha, 2016) was employed to solve the system reliability constraint. These solutions are robust and precise, yet computationally intensive. Each solution was obtained with several runs of the algorithm: means and coefficient of variation (COV) of relevant quantities are presented in the sequence. Unless otherwise stated, 25 runs were performed for each solution.

3.3 Consistent results for reference configuration

Results obtained for the consistent solution of the reference problem are presented in Table 1 and Figure 3. It is observed that, in the optimal configuration, constraints for minimal cross-section area of bars 1, 3, 4 and 6 became active, showing that these bars would not be necessary to warrant (unstable) equilibrium[1] and the required system safety. Hence, the optimal solution for this problem is a two bar (hipo-static) structure, as shown in Fig. 3. It is also observed that, in our solution, the system reliability constraint is not attended in exact form, but within the 5% confidence interval of the sampling error.

The two bar solution presented herein is the obvious (in retrospect) solution to this problem! This is very different than the hyper-static solutions presented in McDonald & Mahadevan (2008) and Nguyen et al. (2010). In the system RBDO formulation, there is no gain in having

Table 1: Cross-section areas, objective function and system reliability index for optimum solution.

Design variables	Mean (10^3 mm^2)	COV (%)
A_1	0.010	---
A_2	12.577	1.29
A_3	0.010	---
A_4	0.010	---
A_5	9.256	3.03
A_6	0.010	---
Obj. Function $f(\mathbf{d})$	**27.087**	0.85
Reliability index β_{sys}	**3.010**	2.62

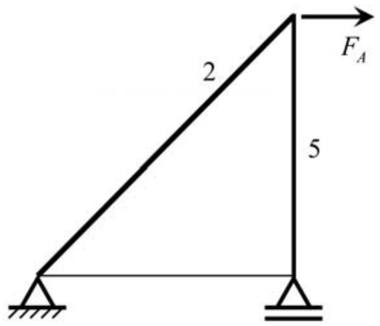

Figure 3: Optimum solution of reference problem.

[1] In fact, basic statics says that if bar 4, of minimal area, is not removed, the structure is isostatic. In the text, we call this a two-bar solution, but in the figures, we keep bar 4 of minimal area, for the illustrated structures to be isostatic. Also, a non-sliding right support could be included; the solution for a non-sliding support is identical to the one presented in Table 1.

hyper-static bars in this structure, as system reliability can be warranted by larger cross-section areas of isostatic bars. The problem formulation does not offer any benefits for having a (hypothetical) warning after first member failure or for having progressive collapse instead of instantaneous failures. So, adding redundant bars increases substantially the construction cost, while does not provide reduction of the system failure probability at a bigger (or at least equal) rate.

4 SOLUTION OF RBDO PROBLEM VARIANTS

4.1 Uncertain load direction

The consistent solution described in Section 3 allows us to investigate several problem variants, which could be of interest to research in system RBDO. The first variant is to assume uncertainty in the load direction, as illustrated in Figure 4. The angle θ is assumed to follow a normal distribution, with zero mean and standard deviation of 15º. We assumed that this could yield hyper-static solutions, but this was not the case. Table 2 shows the results obtained for this variant: the same (hipo)static structure was obtained (Fig. 3), but cross-section areas slightly increased due to loading direction uncertainty.

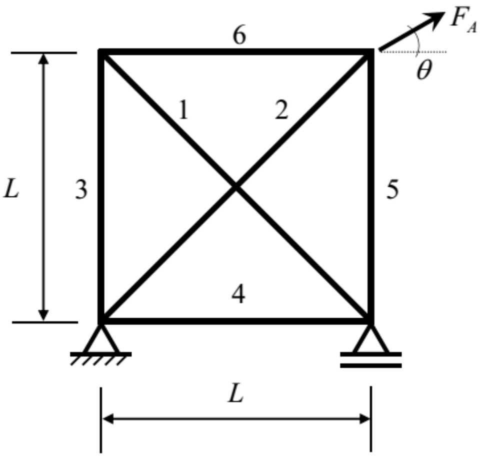

Figure 4: Variant of hyper-static six member truss problem with random force direction.

Table 2: Results for problem variant with θ ~ N(0º;15º).

Design variables	Mean (10^3 mm^2)	COV (%)
A_1	0.010	---
A_2	12.638	1.25%
A_3	0.010	---
A_4	0.010	---
A_5	11.109	1.53%
A_6	0.010	---
Obj. Function $f(\mathbf{d})$	**29.026**	1.29%
Reliability index β_{sys}	**3.041**	3.50%

4.2 Additional load

The next variant was to include an additional horizontal load at the upper-right node, same value as F_A. For this case, both load directions are assumed deterministic. Results are shown in Figure 5 and Table 3. Again, the optimal solution is a hipostatic truss, with only three bars, required to warrant (unstable) equilibrium (Fig. 5, left). For this problem, solution eventually converged to a local minimum, formed by the isostatic structure shown in Fig. 5 (right).

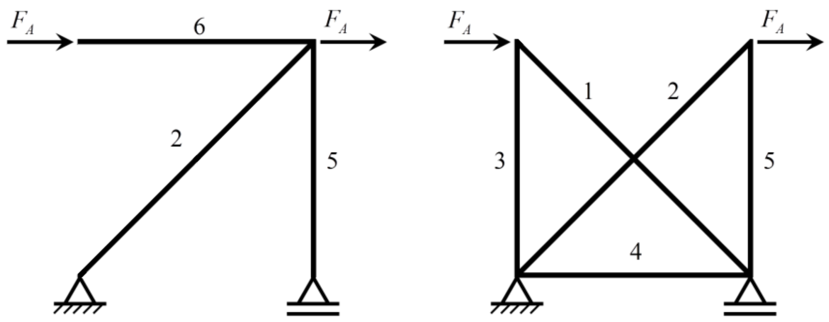

Figure 5: Optimum (hipostatic) solution of two-load variant (left) and isostatic local minimum solution (right).

Table 3: Results for problem variant with two horizontal loads.

Design variables	Global minimum (hipo-static)		Local minimum (isostatic)
	Mean (10^3 mm^2)	**CV**	**Mean (10^3 mm^2)**
A_1	0.010	---	12.671
A_2	25.351	4.07%	13.897
A_3	0.010	---	10.000
A_4	0.010	---	10.078
A_5	17.993	3.52%	9.966
A_6	9.364	5.90%	0.010
Obj. Function $f(\mathbf{d})$	**63.242**	2.92%	**67.626**
Reliability index β_{sys}	**3.107**	7.71%	**3.126**

4.3 Two loads with uncertain direction

One last variant of the reference problem described was solved, in order to check if the optimal structural configuration is always isostatic. Results for the problem variant with two loads of random direction are shown in Figure 6 and Table 4. Two different solutions are obtained as “global minimum”, with very similar objective function values. These solutions correspond to isostatic structures of four and five relevant bars, as shown in Fig. 6. Solution A appeared twice as often as solution B. The local minimum (hyper-static) solution appeared only twice in 50 runs of the algorithm.

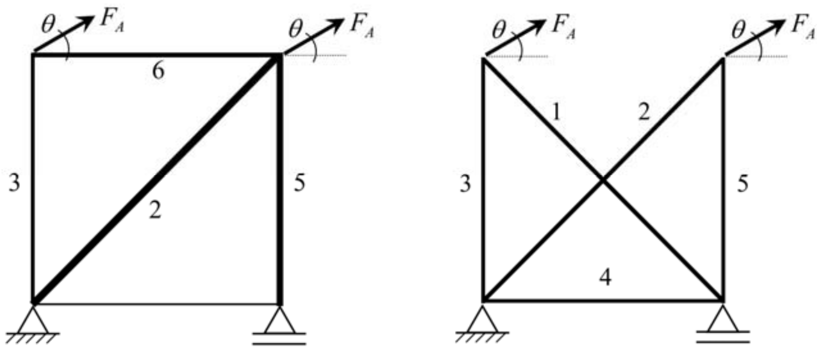

Figure 6: Optimum isostatic solutions for problem with two loads and θ ~ N(0º;15º): solution A (left) and solution B (right).

Table 4: Results for problem variant with two loads and θ ~ N(0º;15º).

Design variables	**Solution A (4-bar solution)**		**Solution B (5-bar solution)**		**Local minimum (6-bar solution)**	
	Mean (10^3 mm^2)	**C.V.**	**Mean (10^3 mm^2)**	**C.V.**	**Mean (10^3 mm^2)**	**C.V.**
A_1	0.010	---	13.260	2.37%	4.4952	31.8%
A_2	25.384	4.37%	12.4485	10.9%	21.6763	5.43%
A_3	5.646	9.67%	11.5822	4.52%	6.4430	28.9%
A_4	0.010	---	10.1662	24.0%	3.3846	29.2%
A_5	19.805	5.59%	12.5233	13.4%	16.7553	7.49%
A_6	9.793	5.07%	0.1000	---	10.3312	27.4%
Obj. Function, $f(\mathbf{d})$	**71.167**	2.64%	**71.920**	4.29%	**73.926**	6.64%
Rel. index, β_{sys}	**3.095**	---	**3.083**	---	**3.099**	---

5 DISCUSSION OF RISK-BASED OPTIMIZATION

Unlike the solutions presented by McDonald & Mahadevan (2008) and Nguyen et al. (2010) lead us to believe, the system RBDO problem formulation in Eq. (12) does not contain any incentive for the permanence of hyper-static bars; hence the isostatic solutions found in Sections 3 and 4. This formulation is limited, in the sense that it does not account for the costs (or consequences) of failure. Hence, it cannot properly model the effects of progressive collapse of the hyper-static truss.

The risk optimization formulation increases the scope of the RBDO problem by including the expected costs of failure. For each failure mode, expected costs of failure (C_{EF}) are obtained by multiplying known (or anticipated) failure costs (C_F) by failure probabilities (P_f):

$$C_{EF} = C_F \cdot P_f \tag{13}$$

The objective function is assembled by adding all costs over the life-cycle of the structure. This can include costs of operation, inspection, maintenance, and cost of disposal; all of

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which are very application-dependent. Furthermore, a distinction can be made between costs for a service failure and collapse. One reasonable hypothesis would be to assume that the failure of a hyper-static member is a service failure, since it does not necessarily lead to progressive collapse. The failure of an isostatic member, however, always leads to full collapse. The distinction between collapse and service failure costs can be justified, for instance, by the warning that a primary failure would provide: theoretically, this would allow the structure to be evacuated, before final collapse. Evacuation would prevent the payment of compensations for injury and death. Early warning could also reduce or eliminate environmental damage.

Hence, the objective function for risk optimization can be written as:

$$C_{ET}(\mathbf{d}) = C_C(\mathbf{d}) + C_{SF} \cdot P_{Sf}(\mathbf{d}) + C_{UF} \cdot P_{Uf}(\mathbf{d}) + C_{SPEC}(\mathbf{d}) \tag{14}$$

where C_{ET} is the total expected cost, C_C is the construction cost, C_{SF} and C_{UF} are the cost of a service and ultimate failure, respectively, P_{Sf} and P_{Uf} are the probabilities of service and ultimate failures, respectively, and C_{SPEC} is the remaining problem specific costs.

Finally, the risk optimization problem is formulated as:

$$\begin{aligned} &\min_{\mathbf{d}=\{A_1,\ldots,A_6\}} C_{ET}(\mathbf{d}) \\ &\text{s.t.: } \{A_1,\ldots,A_6\} \geq 0.01 \end{aligned} \tag{15}$$

It is important to observe that the risk optimization formulation does not require a constraint in the system reliability: solution of this problem actually provides the optimal point of compromise between safety and economy, as well as between competing failure modes, for instance, between service and ultimate failure modes (Beck & Gomes, 2012; Beck & Verzenhassi, 2008; Beck et al., 2012; Gomes & Beck, 2013; Beck et al., 2015). The risk optimization solution actually provides the optimal system reliability, for the assumed cost-of-failure scenario.

6 CONCLUDING REMARKS

This paper addressed the optimal design of hyper-static trusses, considering the load-re-distribution effects of progressive collapse. Solutions were obtained based on system RBDO formulation. The primary goal of the system RBDO solution was to address and circumvent problems encountered in published solutions of the reference hyper-static truss optimization. Then, some problem variants were considered and it was shown that all system RBDO solutions lead to isostatic optimal designs, because the advantages of having hyper-static members are not properly represented in this formulation. In the risk optimization formulation, the benefits of having a primary hyper-static member failure as a warning, before final collapse, can be better quantified. Solutions presented herein are relevant within the modern trend for robust design considering progressive collapse and are intended to become benchmark examples of system RBDO problems.

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