MICROMECHANICAL APPROACH TO DAMAGE PROPAGATION CRITERION IN FRACTURED VISCOELASTIC MATERIALS

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Abstract. *The paper investigates the role of stress transfer along embedded fractures on damage propagation within geomaterials regarded as homogenized viscoelastic media. Unlike cracks, fractures can be viewed as discontinuities able to transfer normal and tangential efforts. The damage propagation approach is developed relying upon a micromechanical reasoning together with macroscopic thermodynamics concepts. The first step of the analysis consists of formulating closed-form expressions for the overall elastic behavior of the fractured material by making use of the Mori-Tanaka homogenization scheme. It is shown that the homogenized behavior can always be represented by means of a generalized Maxwell model. Seeking a simplified rheological representation for the overall behavior that would be more tractable for damage propagation analysis, an approximate Burger model is formulated. At macroscopic scale, thermodynamics arguments indicate that damage propagation criterion depends on the energy release rate, which is computed from the expression of the free energy of the homogenized viscoelastic material. Several numerical applications are finally presented, comparing the values of energy release rate predicted for fractures and for cracks. It stems from this comparison that cracks propagate for strain levels significantly lower than those required for fractures propagation.*

Keywords: *Fracture, Cracks, Viscoelasticity, micromechanics, Damage Propagation*

INTRODUCTION

A main characteristic of many engineering materials such as rocks and more generally geomaterials, is the presence at different scales of discontinuities (cracks or fractures). The term “discontinuities” refers to a zone of small thickness along which the mechanical properties of the matrix material are significantly degraded (Barton et al., 1985 and Maghous et al., 2008). From a constitutive viewpoint, two class of discontinuities should be distinguished: 1) cracks, which usually refer to discontinuities without stress transfer; 2) fractures that are discontinuities with specific mechanical behavior and, as such, are able to transfer normal and tangential efforts. Even cracks lead to easier models, the representation by means of fractures lead to a more precise result. The differences between the results using both kinds of discontinuities must be studded before we decide apply one or the other model.

When a material is subjected to external loading, existing discontinuities are expected to grow and propagate. In this context, most of the theoretical or computational propagation analyses have focused on the instantaneous response (elastic or plastic) of the material. However, in many situations, delayed behavior reveals a fundamental component of the material deformation. In order to extend the classical elastic damage analysis to cracked viscoelastic materials, Nguyen and Dormieux (Nguyen et al., 2010, 2011, 2013 and Nguyen and Dormieux 2014) developed a micromechanical-based formulation to determine the crack propagation criterion. Based on these ideas, the objective of the present paper is to extend the previous analysis to address the situation of fractures. Relying upon a micromechanical reasoning combined with thermodynamics arguments, the modeling is applied to assess the effects on the viscoelastic properties and damage propagation induced at macroscopic scale by the presence of discontinuities at finer scale. Comparison between the cases of cracks and fractures is in particular investigated.

1 HOMOGENIZED VISCOELASTIC BEHAVIOR

As far as the constitutive modeling of geomaterials constituents is concerned, two classes of discontinuities are generally distinguished: cracks and fractures. Geometrically, both discontinuities are usually modeled as interfaces, the fundamental difference lies in their ability to transfer or not stress efforts. Unlike cracks, fractures can transfer normal and tangential stresses and as such, are endowed with a specific mechanical behavior. The latter is described by means of a relationship relating the stress vector $\underline{T} = \underline{\underline{\sigma}} \cdot \underline{n}$ acting on the fracture and the corresponding relative displacement $[\underline{\xi}]$ (Maghous and al., 2011):

$$\underline{T} = \underline{\underline{k}} \cdot [\underline{\xi}] \qquad \text{with} \qquad \underline{\underline{k}} = k_n \, \underline{n} \otimes \underline{n} + k_t \left(\underline{t} \otimes \underline{t} + \underline{t}' \otimes \underline{t}' \right) \tag{1}$$

The stiffness $\underline{\underline{k}}$ is defined by the scalars k_n and k_t referring to the normal and shear stiffness of the fracture, respectively. The vector $\underline{n}$ correspond to the normal-to-plane direction while both $\underline{t}$ and $\underline{t}'$ are vectors in damage plane (see Fig. 1). These three vectors constitute an orthonormal basis for fracture orientation.

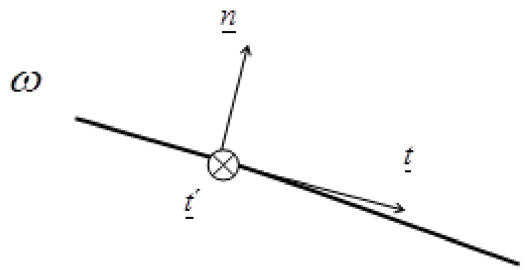

Figure 1. Local frame for fracture ω modeled as an interface

1.1 Homogenized Viscoelastic Behavior in Carson-Laplace Domain

In order to investigate the damage propagation in fractured viscoelastic materials, the mechanical behavior of the medium at macroscopic scale should first be formulated. Indeed, and in anticipation of section 2, the approach developed in the subsequent analysis for the damage propagation criterion requires computing the expression of macroscopic free energy, which explicitly depends on the homogenized viscoelastic relaxation tensor. In the context of non-ageing linear viscoelasticity, the overall behavior of the damaged viscoelastic material can be obtained combining micromechanical arguments along with the correspondence principle between the Carson-Laplace and time domains (Le et al., 2007). The correspondence principle expresses the equivalence between the viscoelastic problem in time domain and the associated elastic problem in the Laplace domain (Bland, 1960 and Salençon, 2009). It is recalled that the Carson-Laplace transform $p \rightarrow \mathrm{u}^*(p)$ of a time-dependent function $t \rightarrow \mathrm{u}(t)$ is defined by:

$$\mathrm{u}^*(p) = \int_{-\infty}^{\infty} \dot{u}(t) e^{-pt} dt \tag{2}$$

Assuming elastic isotropy for the individual constituents (matrix and fractures) and isotropic spatial distribution for the discontinuities at micro-scale, the homogenized elastic stiffness tensor $\mathbb{C}_{\mathrm{hom}}$ was determined from the elastic properties of the matrix and fractures, and from the damage density parameter $\epsilon = \mathcal{N} a^3$, where $\mathcal{N}$ is the damage density expressed by volume unity and a is the mean radius of fractures (Maghous et al., 2014). Combining the homogenized stiffness tensor $\mathbb{C}_{\mathrm{hom}}$ with the correspondence principle, the homogenized relaxation tensor can be determined in the Carson-Laplace Domain (Aguiar, 2016):

$$\mathbb{R}^*_{hom} = \mathbb{C}^*_{hom} = 3\, k^*_{hom}\; \mathbb{J} + 2\, \mu^*_{hom}\; \mathbb{K} \tag{3}$$

where the fourth-order tensors $\mathbb{J}$ and $\mathbb{K}$ are defined as $\mathbb{J} = \frac{1}{3} \underline{\underline{1}} \otimes \underline{\underline{1}}$ and $\mathbb{K} = \mathbb{I} - \mathbb{J}$. Parameters k^*_{hom} and μ^*_{hom} are the Carson-Laplace transform of the homogenized bulk and shear moduli, respectively:

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$$
\begin{aligned}
k_{hom}^* &= 3\, k_s^* \frac{\beta_1^*}{\beta_2^*} \\
\mu_{hom}^* &= \mu_s^* \frac{45\beta_1^*\beta_3^*}{\beta_4^*}
\end{aligned}
\tag{4}
$$

where k_s^* and μ_s^* are respectively the bulk and shear moduli of matrix material in the operational space whereas k_j^* and μ_j^* are the bulk and shear moduli of fractures in the same space. It is noted that to cracks, both k_j^* and μ_j^* moduli are null. Coefficients β_j^* are dependent on the rheological model adopted for the viscoelastic behavior of matrix and fracture material, and on the crack density parameter ϵ :

$$
\begin{aligned}
\beta_1^* &= 9\, k_s^*\, \pi\, \mu_s^* + 9\, k_s^*\, a\, \mathrm{k}_n^* + 12\, \mu_s^*\, a\, \mathrm{k}_n^* + 12\, k_s^*\, a\, k_t^* + 16\, \mu_s^*\, a\, k_t^* + 3\, \pi\, \mu_s^{*2} \\
\beta_2^* &= 12\, k_s^{*2}\epsilon\, \pi + 16\, k_s^*\, \epsilon\, \pi\, \mu_s^* + 3\, \pi\, \mu_s^{*2} + 9\, k_s^*\, \pi\, \mu_s^* + 12\, a\, \mu_s^*\, \mathrm{k}_n^* + 9\, k_s^*\, a\, \mathrm{k}_n^* + 16\, a\, \mu_s^*\, \mathrm{k}_t^* + 12\, k_s^*\, a\, k_t^* \\
\beta_3^* &= 9\, k_s^*\, \pi\, \mu_s^* + 12\, k_s^*\, a\, k_t^* + 16\, \mu_s^*\, a\, k_t^* + 6\, \pi\, \mu_s^{*2} \\
\beta_4^* &= \frac{16}{3}\pi\epsilon\, \mu_s^*\left(3k_s^* + 4\mu_s^*\right)\left(2\beta_1^* + \beta_3^*\right) + 5\beta_1^*\, \beta_3^*
\end{aligned}
\tag{5}
$$

It is noted that the k_s^* and μ_s^* (as well as k_j^* and μ_j^*) values are dependent on the rheological model adopted for the viscoelastic behavior of the material constituents. If the Kelvin-Voigt rheological model (represented in the Fig. 2) is adopted for instance to the matrix, moduli k_s^* and μ_s^* take the fallowing form:

$$
\begin{aligned}
k_s^* &= \frac{k_{m,s}^e\left(p\, k_{K,s}^v + k_{K,s}^e\right)}{p\, k_{K,s}^v + k_{K,s}^e + k_{m,s}^e} \\
\mu_s^* &= \frac{\mu_{m,s}^e\left(p\, \mu_{K,s}^v + \mu_{K,s}^e\right)}{p\, \mu_{K,s}^v + \mu_{K,s}^e + \mu_{m,s}^e}
\end{aligned}
\tag{6}
$$

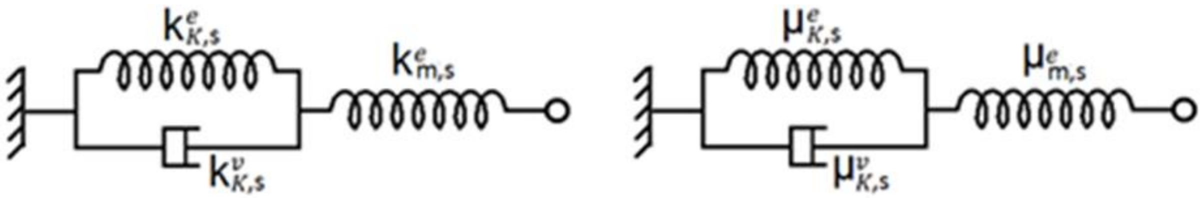

Figure 2: Rheological model of Kelvin-Voigt

Similarly to the laboratory testing classically used for evaluating normal and tangential stiffness of fractures, determination of viscous properties defining the rheological models for fractures should also rely upon experimental procedures.

1.2 Inverse of Carson-Laplace Transform

Equation (3) represents the expression of homogenized relaxation tensor in the Carson-Laplace domain. Assessment of overall viscoelastic properties would thus consist in transforming the above equation from Carson-Laplace to time domain. There are a large number of procedures to compute the inverse transform (Jagerman, 1982; Honig and Hirdes, 1984; Dolonato, 2002 and Ahn et al., 20043), however most methods involve complex operations since they were originally formulated for addressing general mathematical expressions. Then, the idea herein is to develop an analytical procedure that is specifically devised for deriving the inverse Carson-Laplace transform of relationships (3).

The first step consists in observing that the bulk and shear moduli can always be written as a quotient of two polynomials functions of variable p. Referring either to k^*_{hom} or μ^*_{hom} by the generic relaxation function on R^*_{hom}, it follows that:

$$\mathrm{R}^*_{hom} = \frac{A(p)}{B(p)} \tag{7}$$

Polynomials $A(p)$ and $B(p)$ are generally expressed as:

$$\begin{aligned} &\mathrm{A}(\mathrm{p}) = \sum_{k=0}^{n} a_k\, p^k \\ &\mathrm{B}(\mathrm{p}) = \sum_{k=0}^{n} b_k\, p^k = \prod_{k=1}^{n} (p - R_k) \quad ; \quad b_n = 1 \end{aligned} \tag{8}$$

where a_k and b_k are real coefficients depending on the rheological matrix and fractures models, n is the degree of polynomials $A(p)$ and $B(p)$, and scalar R_k is the kth roots of $B(p)$. It should be noted that for usual rheological models, the polynomial $B(p)$ admits only simple roots and b_0 is non-null (i.e., $b_0 \neq 0$).

Using the partial fraction procedure, we can show that the relaxation function $R_{hom}(t)$, which is the inverse of Carson-Laplace transform of R^*_{hom}, can be expressed in the time domain as (Aguiar and Maghous, 2016):

$$R_{hom}(t) = \left[\frac{a_0}{b_0} + \sum_{k=1}^{n} D_k e^{R_k\, t} \right] Y(t) \tag{9}$$

with

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$$D_k = \left[\frac{C(p)}{\frac{\partial B(p)}{\partial p}} \right]_{p=R_k} = \left[\frac{C(p)(p-R_k)}{B(p)} \right]_{p=R_k} \quad ; \quad C(p) = \frac{1}{b_0} \sum_{k=1}^{n} \left[(b_0 a_k - a_0 b_k) p^{k-1} \right] \tag{10}$$

where $Y(t)$ is the Heaviside step function at origin and $R_{hom}(t)$ is the representation of the viscoelastic bulk and shear moduli from the relaxation tensor $\mathbb{R}_{hom}$ in the time domain.

At macroscopic scale, the difference between cracks and fractures can be illustrated by analyzing the relaxation tensor. Adopting the Burger model to describe viscoelasticity of matrix material and an elastic model for fractures (as described at Fig. 3), Figure 4 shows the evolution of the normalized homogenized relaxation function of material with fractured or cracks:

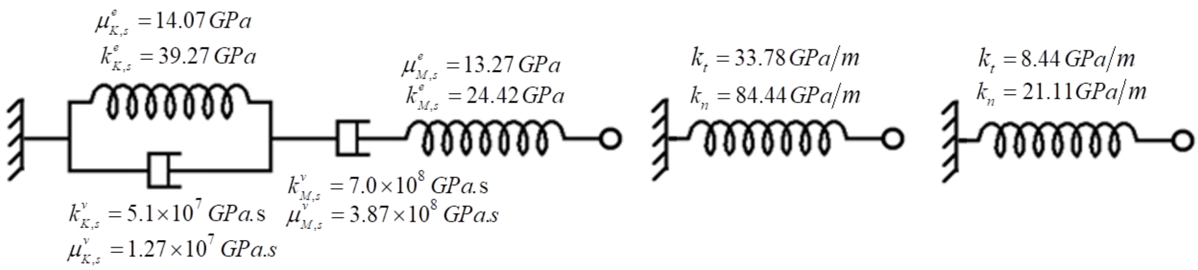

Figure 3: a) Matrix Burger Model b) Stiff Elastic Fracture c) Soft Elastic fracture

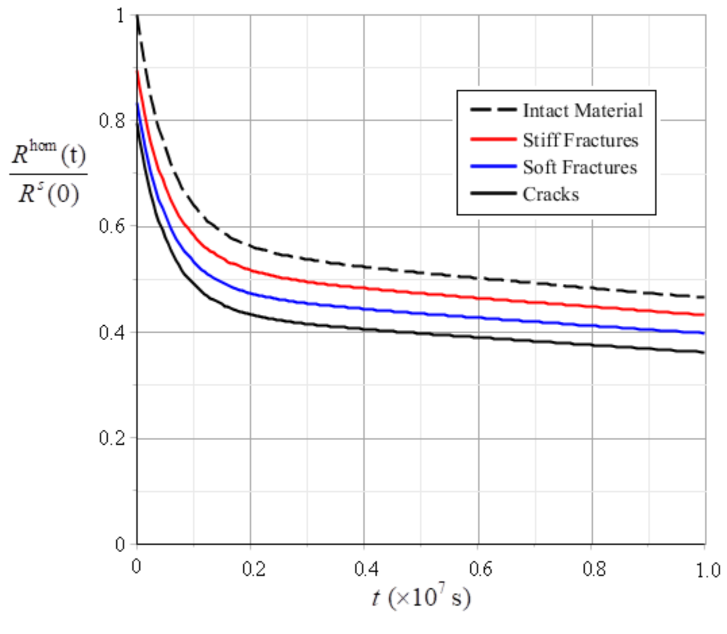

Figure 4: Comparison between cracks and fractures in relaxation function $(\epsilon = 0.1, \mathcal{N} = 1)$

As observed, fractures and cracks induce a decrease in material stiffness, that is, an enhancement of deformability at macroscopic scale. The decrease in macroscopic relaxation

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function becomes significant as the stiffness of fractures is decreasing. The limiting case of cracked material is obtained when the stiffness of fractures is zero. The case of intact matrix material is retrieved for fractures with infinite stiffness.

1.3 Rheological Representation of the Overall Behavior

Equation (9) provides the overall behavior of the bulk and shear moduli of the homogenized viscoelastic material for isotropic distribution of discontinuities. However, in the perspective of developing a damage propagation analysis, it is fundamental to identify a rheological model that can represent the material behavior at macroscopic scale. It is first observed that the behavior described by Eq. (9) is mathematically equivalent to the generalized Maxwell model represented in Fig. 5 (Aguiar and Maghous, 2016) and s expressed by

$$R_{G-Max}(t) = \left[E_0 + \sum_{k=1}^{n} E_k \, e^{\left(-\frac{E_k}{\eta_k} t\right)} \right] Y(t) \tag{11}$$

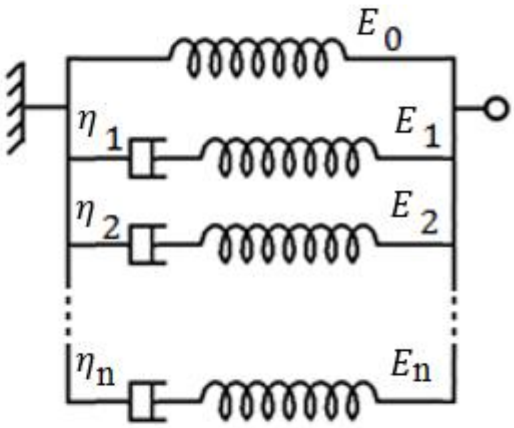

Figure 5: Generalized rheological model of Maxwell

In order to correlate both equations, the following relations should be related:

$$E_0 = \frac{a_0}{b_0} \quad ; \quad E_k = D_k \quad ; \quad -\frac{E_k}{\eta_k} = -\frac{1}{\tau_k} = R_k \Rightarrow \eta_k = -\frac{D_k}{R_k} \tag{11}$$

Roots R_k are all negative and they represent the inverse of macroscopic characteristic relaxation times (Aguiar and Maghous, 2016). This means that in the context of the adopted framework and restrictions related to rheological models considered for of matrix and fractures, an appropriate generalized Maxwell model can always describe exactly the overall viscoelastic behavior of the fractured medium. It is noted that the Eq. (12) express relations for a generic relaxation function, being the bulk or shear moduli considered separately.

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1.4 Simplified Burger Rheological Model

The rheological representation by the generalized Maxwell model may involve a high number of rheological elements (dashpots and springs). Consequently, it turns out rather complex to perform the damage propagation analysis for the fractured material. The idea consists in simplifying the approach by considering an approximate rheological model with fewer elements. Nguyen et al. (2011) have proposed a procedure to approximate the homogenized behavior of a cracked viscoelastic material through a Burger model. However, this procedure have to be adapted for the fracture framework.

Basically, the approach consists in comparing in the Carson-Laplace domain the homogenized behavior of the damaged viscoelastic material (expressed by Eq. (13)) with the Burger model (Eq. (14)). Since both equations cannot be directly compared, the idea is to compare their respective series expansions. The procedure must represent at least a good approximation to short and long times. This achieved by identifying the series expansions at $p=\infty$ (which is mathematically equivalent to $t=0$) and at $p=0$ (i.e., $t=\infty$). Originally this procedure was developed considering the Burger model for the matrix material, but it can be extended to include the Kelvin-Voigt and Maxwell models by considering infinite properties for the appropriate dashpot or spring element in the Burger model.

$$\frac{1}{k_{hom}^*}=\frac{1}{k_s^*}+\frac{4\pi\epsilon\left(3k_s^*+4\mu_s^*\right)}{\beta_1^*}=\frac{1+\epsilon Q^*}{k_s^*} \quad ; \quad Q^*=\frac{4\pi\left(3k_s^*+4\mu_s^*\right)k_s^*}{\beta_1^*} \tag{13}$$

$$\frac{1}{k_{Bur}^*}=\frac{1}{k_{M,Bur}^{\mathrm{e}}}+\frac{1}{p\,k_{M,\mathrm{Bur}}^{\mathrm{v}}}+\frac{1}{k_{K,\mathrm{Bur}}^{\mathrm{e}}+p\,k_{K,Bur}^{\mathrm{v}}} \tag{14}$$

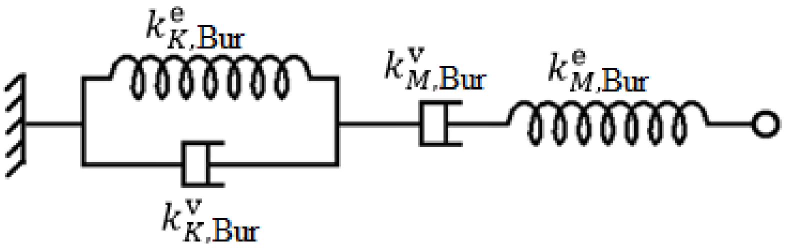

Figure 6: Rheological model of Burger applied to bulk module.

For the sake of simplicity, the approximate model shall be presented for the bulk modulus, the results regarding the shear modulus are present in Aguiar and Maghous (2016). Figure 6 illustrate the Burger model for the bulk modulus. Comparing both series expansions of (13) and (14), the moduli $k_{M,bur}^e$, $k_{K,bur}^e$, $k_{M,bur}^v$, and $k_{K,bur}^v$ in Eq. (14) are defined as:

$$\frac{1}{k_{M,bur}^e}=\frac{1+\epsilon Q_M^e}{k_{M,s}^e} \quad ; \quad \frac{1}{k_{K,bur}^e}=\frac{1+\epsilon Q_K^e}{k_{K,s}^e} \quad ; \quad \frac{1}{k_{M,bur}^v}=\frac{1+\epsilon Q_M^v}{k_{M,s}^v} \quad ; \quad \frac{1}{k_{K,bur}^v}=\frac{1+\epsilon Q_K^v}{k_{K,s}^v} \tag{15}$$

where $k_{M,s}^e$, $k_{K,s}^e$, $k_{M,s}^v$ and $k_{K,s}^v$ are the Burger coefficients of the matrix model. the coefficients Q_M^e, Q_K^e, Q_M^v and Q_K^v are represented as:

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$$Q_M^e = Q_0^\infty \quad ; \quad Q_K^e = Q_0^0 + \frac{k_{K,s}^e}{k_{M,s}^v} Q_1^0 - \frac{k_{K,s}^e}{k_{M,s}^e}\left(Q_0^\infty - Q_0^0\right)$$
$$Q_M^v = Q_0^0 \quad ; \quad Q_K^v = Q_0^\infty + \frac{k_{K,s}^v}{k_{M,s}^e} Q_{-1}^\infty - \frac{k_{K,s}^v}{k_{M,s}^v}\left(Q_0^0 - Q_0^\infty\right) \tag{16}$$

The values of Q_0^∞, Q_{-1}^∞, Q_0^0 and Q_1^0 are dependent on the specific rheological models adopted for the matrix and fractures. When the Burger model for the matrix together with the Maxwell model for fracture elements are adopted for instance, their values take the following form:

$$Q_0^0 = \frac{4\pi\, k_{M,s}^v \left(3k_{M,s}^v + 4\mu_{M,s}^v\right)}{3\pi\mu_{M,s}^v\left(3k_{M,s}^v + \mu_{M,s}^v\right) + a\left(3\mu_{M,s}^v k_{M,j}^v + 3k_{M,s}^v \mu_{M,j}^v + 4\mu_{M,s}^v \mu_{M,j}^v\right)}$$
$$Q_1^0 = \frac{4\pi\, k_{M,s}^v \left(\frac{P_1}{3}\mu_{K,s}^e \mu_{M,s}^e \mu_{M,j}^e \left(3k_{M,s}^v + 4\mu_{M,s}^v\right) P_2 + P_3\right)}{k_{M,s}^e k_{K,s}^e \mu_{M,s}^e \mu_{K,s}^e k_{M,j}^e \mu_{M,j}^e \left(P_1\right)^2}$$
$$Q_0^\infty = \frac{4\pi\, k_{M,s}^e \left(3k_{M,s}^e + 4\mu_{M,s}^e\right)}{3\pi\mu_{M,s}^e\left(3k_{M,s}^e + \mu_{M,s}^e\right) + a\left(3k_{M,s}^e k_{M,j}^e + 12\mu_{M,s}^e k_{M,j}^e + 12k_{M,s}^e \mu_{M,j}^e + 16\mu_{M,s}^e \mu_{M,j}^e\right)} \tag{16}$$
$$Q_{-1}^\infty = \frac{4\pi\, k_{M,s}^e \left(\frac{1}{3}\mu_{K,s}^v \mu_{M,s}^v \mu_{M,j}^v \left(3k_{M,s}^e + 4\mu_{M,s}^e\right) S_1 S_2 + S_3\right)}{k_{M,s}^v k_{K,s}^v \mu_{M,s}^v \mu_{K,s}^v k_{M,j}^v \mu_{M,j}^v \left(S_1\right)^2}$$

where $k_{M,j}^e$ and $k_{M,j}^v$ are the elastic and viscous parts of k_n Maxwell model and $\mu_{M,j}^e$ and $\mu_{M,j}^v$ are the elastic and viscous parts of k_t Maxwell model. The expression of P_i and S_i are summarized in appendix. Similarly, the Carson-Laplace transform of the homogenized shear modulus expresses as:

$$\frac{1}{\mu_{bur}^*} = \frac{1}{\mu_{M,bur}^{\mathrm{e}}} + \frac{1}{p\,\mu_{M,bur}^{\mathrm{v}}} + \frac{1}{\mu_{K,bur}^{\mathrm{e}} + p\,\mu_{K,bur}^{\mathrm{v}}} \tag{17}$$

where

$$\frac{1}{\mu_{M,hom}^e} = \frac{1+\epsilon\, M_M^e}{\mu_{M,s}^e} \; ; \; \frac{1}{\mu_{K,hom}^e} = \frac{1+\epsilon\, M_K^e}{\mu_{K,s}^e} \; ; \; \frac{1}{\mu_{M,hom}^v} = \frac{1+\epsilon\, M_M^v}{\mu_{M,s}^v} \; ; \; \frac{1}{\mu_{K,hom}^v} = \frac{1+\epsilon\, M_K^v}{\mu_{K,s}^v} \tag{18}$$

The expressions of the above coefficients are given in Aguiar and Maghous (2016). It is observed that the approximate Burger model is originally formulated herein for the situation where the matrix material is also described by a Burger model.

2 FRACTURE PROPAGATION IN VISCOELASTIC MEDIA

Applying external loading to a material is expected increase the size and amount of fractures. The analysis of cracks propagation in elastic and plastic media is well-known in the literature (Griffth, 1921; Irwin, 1961 and Bieniawski, 1967). However, the propagation of cracks in viscoelastic materials only gets more featured with works published by Nguyen (Nguyen et al., 2010, 2013 and Nguyen, 2010).

2.1 Thermodynamics Framework

The analysis of fracture/crack propagation is an important field in several engineering areas. Classically the propagation condition is determined for individual fractures. This approach is widely used for large scale fractures that involve a limited number of fractures propagating. In the context of linear fracture mechanics, the criterion of fracture propagation can be established based on the expression of energy dissipation:

$$\dot{\mathcal{D}} = \underline{\underline{\sigma}} : \dot{\underline{\underline{\varepsilon}}} - \frac{\partial \Psi}{\partial \underline{\underline{\varepsilon}}} : \dot{\underline{\underline{\varepsilon}}} - \frac{\partial \Psi}{\partial s} \dot{s} = -\frac{\partial \Psi}{\partial s} \dot{s} \tag{19}$$

where $\dot{\mathcal{D}}$ is the energy dissipation, s is the average length of the damage elements and Ψ is the elastic energy in the material. Thus, it is natural to establish the propagation condition by comparing the energy release rate $\mathcal{F} = -\frac{\partial \Psi}{\partial s}$ with a given critical energy value $\mathcal{G}_c$ (Griffth, 1921).

When the medium involves a large number of small scales fractures, such as common geomaterials, the classical direct approach becomes ineffective. The solution presented by several authors consists in resorting to a micromechanical interpretation of the damage propagation, revealing more suitable. Accordingly, the expression of energy dissipation is macroscopically reformulated, contemplating the overall variables of the system:

$$\frac{\dot{\mathcal{D}}}{|\Omega|} = \underline{\underline{\Sigma}} : \dot{\underline{\underline{\mathrm{E}}}} - \frac{\partial W}{\partial \underline{\underline{\mathrm{E}}}} : \dot{\underline{\underline{\mathrm{E}}}} - \frac{\partial W}{\partial \epsilon} \dot{\epsilon} = \mathcal{F}\, \dot{\epsilon} \qquad \text{with} \quad \mathcal{F} = -\frac{\partial W}{\partial \epsilon} \tag{20}$$

$|\Omega|$ is the volume of the element, $\underline{\underline{\Sigma}}$ and $\underline{\underline{\mathrm{E}}}$ are the macroscopic stress and strain fields, W is the free energy of the homogenized material, and ϵ is the parameter that represents macroscopically the damage. In the case of fractured material, ϵ is exactly the fracture density parameter introduced in section 1. Since the expression is similar to the classical approach, the condition of fracture propagation is once again established by comparing the macroscopic energy release rate with the critical energy $\mathcal{G}_f$ (Dormieux and Kondo, 2016).

The approaches described above exclusively deals with elastic materials. In order to address the crack propagation in viscoelastic materials, Nguyen et al. (Nguyen et al., 2010, 2013 and Nguyen, 2010) proposed a procedure that generalized the elastic approach. The procedure takes advantage of the micromechanical reasoning described above to evaluate the viscoelastic properties of homogenized materials and by introducing of a viscous dissipation term:

$$\frac{\dot{\mathcal{D}}}{|\Omega|} = \underline{\underline{\Sigma}} : \dot{\underline{\underline{\mathrm{E}}}} - \frac{\partial W}{\partial \underline{\underline{\mathrm{E}}}} : \dot{\underline{\underline{\mathrm{E}}}} - \frac{\partial W}{\partial \left\{ \underline{\underline{\varepsilon}}^{v} \right\}} : \left\{ \dot{\underline{\underline{\varepsilon}}}^{v} \right\} - \frac{\partial W}{\partial \epsilon} \dot{\epsilon} = \mathcal{F} \, \dot{\epsilon} - \frac{\partial W}{\partial \left\{ \underline{\underline{\varepsilon}}^{v} \right\}} : \left\{ \dot{\underline{\underline{\varepsilon}}}^{v} \right\} \tag{21}$$

where $\left\{ \dot{\underline{\underline{\varepsilon}}}^{v} \right\}$ represents the viscous strain fields. It is thus possible to split the part of dissipation corresponding to the crack propagation from viscous dissipation:

$$\frac{\dot{\mathcal{D}} - \dot{\mathcal{D}}_{vis}}{|\Omega|} = -\frac{\partial W}{\partial \epsilon} \dot{\epsilon} = \mathcal{F} \, \dot{\epsilon} \tag{22}$$

Following a similar reasoning as in elasticity, the condition of damage propagation is derived from comparison of the viscoelastic macroscopic energy release rate $\mathcal{F}$ with the critical energy $\mathcal{G}_v$ of the material:

$$\mathcal{F} < \mathcal{G}_v \Rightarrow \dot{\epsilon} = 0 \quad ; \quad \mathcal{F} = \mathcal{G}_v \Rightarrow \dot{\epsilon} \geq 0 \tag{23}$$

It is noted, however, that the definition of the energy release rate depends on the macroscopic free energy W of the fractured viscoelastic material:

$$\mathcal{F} = -\frac{\partial W}{\partial \epsilon} \left(\underline{\underline{\mathrm{E}}}, \epsilon, \left\{ \underline{\underline{\varepsilon}}^{v} \right\} \right) \tag{24}$$

This last framework can be applied also to fractures, like demonstrated in (Aguiar, 2016). Then, the next step will consist in determining the macroscopic free energy of fractured viscoelastic materials.

2.2 Expression of Macroscopic Energy Release Rate

As emphasized by Eq. (24), the prelaminar determination the free energy of the macroscopic material is needed to express the energy release rate. Due to the discontinuous nature of the fractured material, it is difficult to determine analytically the free energy starting from a microscopic view point. Therefore, the idea of the approach is to resort to a macroscopic representation of the free energy, regarding the fractured material is homogenized. The fundamental definition of free energy corresponds to the sum of the elastic energy stored in each spring of the system:

$$W = \frac{1}{2} \sum_{k=0}^{n} \underline{\underline{\varepsilon}}_k^e : \mathbb{C}_k^e : \underline{\underline{\varepsilon}}_k^e \tag{25}$$

where $\underline{\underline{\varepsilon}}_k^e$ is the elastic deformation acting in the kth spring of the system, $\mathbb{C}_k^e$ is the elastic stiffness of the kth spring of the system and n is the total number of sprigs in the system. To apply this definition, it is convenient to represent the homogenized behavior by means of a rheological model. The idea consists in adopting a simplified rheological model involving few number of elements, such as the approximate Burger model. In that case, the macroscopic free energy can be represented as:

$$W = \frac{1}{2}\left(\underline{\underline{\mathrm{E}}}_M^e : \mathbb{C}_{M,hom}^e : \underline{\underline{\mathrm{E}}}_M^e + \underline{\underline{\mathrm{E}}}_K^e : \mathbb{C}_{K,hom}^e : \underline{\underline{\mathrm{E}}}_K^e\right) \tag{26}$$

where the strain $\underline{\underline{\mathrm{E}}}_M^e$, $\underline{\underline{\mathrm{E}}}_K^e$ and the stiffness $\mathbb{C}_{M,hom}^e$, $\mathbb{C}_{K,hom}^e$ are indicated in the Fig. 7.

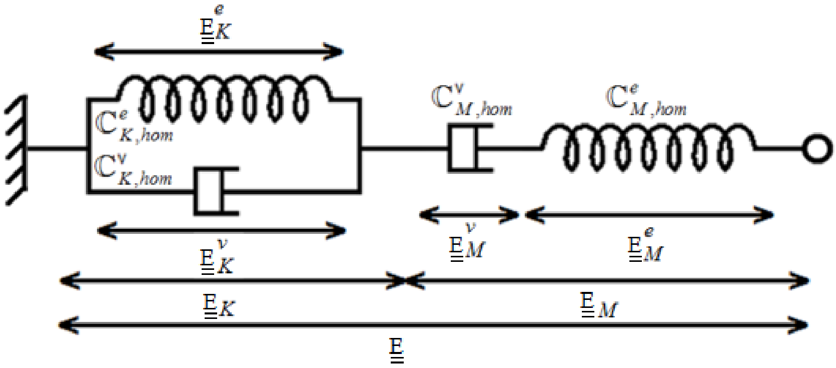

Figure 7: Burger's rheological model

For the sake of simplicity, the following procedure will be described in the one-dimensional setting. Thus, the Eq. (26) takes the form:

$$W = \frac{1}{2}\left(\mathrm{E}_M^{e\ 2} C_{M,hom}^e + \mathrm{E}_K^{e\ 2} C_{K,hom}^e\right) \tag{27}$$

Referring to the Burger model, the unique element with instantaneous behavior is $\mathbb{C}_{M,hom}^e$. It is therefore possible to separate an instantaneous energy from the residual energy:

$$W_{inst} = \frac{1}{2}\mathrm{E}_M^{e\ 2} C_{M,hom}^e \qquad and \qquad W_{res} = \frac{1}{2}\mathrm{E}_K^{e\ 2} C_{K,hom}^e \tag{28}$$

It follows that the instantaneous and the residual energy are clearly defined. However, it is necessary to determine a relation between the total strain E and the strains E_M^e and E_K^e associated with the springs of the Burger model. In the particular case of Burger rheological model, we obtain the following relation:

$$\mathrm{E}_M^e = \frac{R_{\mathrm{hom}}}{C_{M,\mathrm{hom}}^e} \otimes \mathrm{E} \qquad and \qquad \dot{\mathrm{E}}_K^e = \frac{R_{\mathrm{hom}}}{C_{K,\mathrm{hom}}^v} \otimes \mathrm{E} - \frac{C_{K,\mathrm{hom}}^e}{C_{K,\mathrm{hom}}^v}\mathrm{E}_K^e \tag{29}$$

where $\otimes$ denotes the Boltzman product defined as:

$$A(\mathrm{t}_0,\mathrm{t}) \otimes B(t) = A(t_0, t = t_0)B(t) + \int_{t_0}^{t} \frac{dA(t_0, \mathrm{t}-\tau)}{d\tau} B(\tau)\,\mathrm{d}\,\tau \tag{30}$$

Considering the particular case $\mathrm{E}(t)=\mathrm{E}\,Y(t)$, where E is a constant, and solving the above ordinary differential equation with respect to E_K^e, the free energy is represented as:

$$W=\underbrace{\frac{1}{2}\frac{{R_{\mathrm{hom}}}^2(t)}{C^e_{M,\mathrm{hom}}}\mathrm{E}^2}_{W_{ins}}+\underbrace{\frac{1}{2}\frac{C^e_{K,hom}}{{C^v_{K,hom}}^2}e^{-2\frac{C^e_{K,hom}}{C^v_{K,hom}}t}\left(\int_o^t R_{hom}(\tau)e^{\frac{C^e_{K,hom}}{C^v_{K,hom}}\tau}d\tau\right)^2\mathrm{E}^2}_{W_{res}} \tag{31}$$

Applying the Eq. (31), we can compare the free energy using cracks and fractures. Similarly to the relaxation function, hear we can simulate the free energy of an intact material by considering a fracture with infinite stiffness. Figure 8 compares the free energy to these situations:

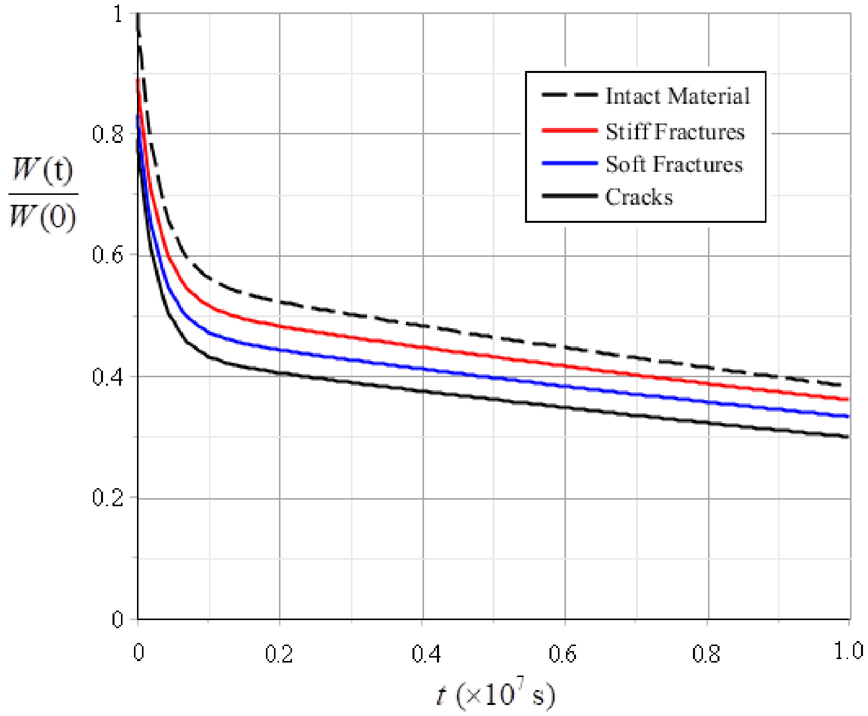

Figure 8: Comparison between cracks and fractures in free energy $(\epsilon=0.1,\mathcal{N}=1)$

We can see from Fig. 8 that the free energy is larger for fractures than for cracks. Stiff fractures provide higher free energy than soft fractures, but for all cases the presence of damage result in a reduction in the material free energy when compared to that of intact materials. In itself, the value of the free energy does not represent a physical quantity which is relevant to fracture propagation, but it is extremely important to compute the energy release rate.

Lastly, in order to determine the energy release rate, we introduce the Eq. (31) in the Eq. (24):

$$\mathcal{F} = -\frac{1}{2}\mathrm{E}^2 \frac{\partial}{\partial \epsilon}\left(\frac{R_{\mathrm{hom}}{}^2(t)}{C^e_{M,\mathrm{hom}}} + \frac{C^e_{K,hom}}{C^v_{K,hom}{}^2} e^{-2\frac{C^e_{K,hom}}{C^v_{K,hom}}t} \left(\int_o^t R_{hom}(\tau) e^{\frac{C^e_{K,hom}}{C^v_{K,hom}}\tau} d\tau \right)^2 \right) \tag{32}$$

It should be kept in mind that this expression was determined in the specific case defined by $\mathrm{E}(t) = \mathrm{E}\,Y(t)$. For a given strain history, Eq. (32) must be solved analytically or numerically to express E^e_M and E^e_K as functions of the strain history. Figure 9 compares the energy release rate for different types of fractures:

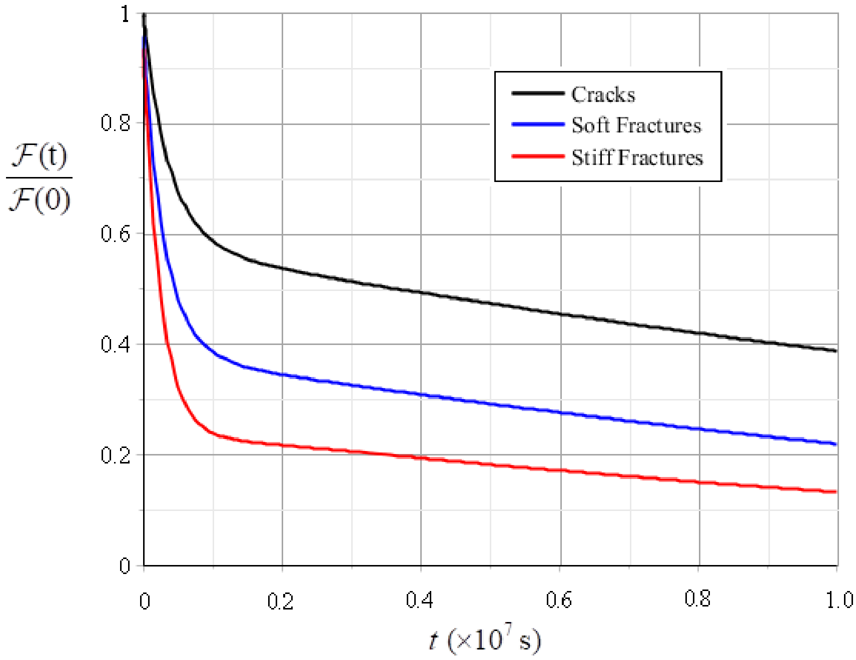

Figure 9: Macroscopic energy release rate computed for cracks and fractures $(\epsilon = 0.1, \mathcal{N} = 1)$

The energy release rate is the fundamental variable in the damage propagation. By the Fig. 9 we can observe that fractures provide a lower energy release rate than cracks, which means a higher difficulty to fracture propagation (see Eq. (23)). The importance of this result concerns directly to the damage propagation. Assuming a specific constant value to the critical energy $\mathcal{G}_v$ (identical to fractured and cracked materials), we observe from Fig. 9 that the fractures can propagate for a short time and then stop its propagation. By the other hand, cracks can propagate indeterminately. In other words, the fractures provide a more steady-to-propagation model, where is more difficult to the damage propagates than using cracks. We observe that the Fig 9 doesn't present the energy release rate of an intact material, it occurs because the used model can't predict the damage propagation when there aren't discontinuities in material.

3 CONCLUSION

Taking advantage of the correspondence principle and the elastic homogenization schemes developed in other works, the exact overall behavior of viscoelastic damaged materials was presented. A comparison between the model considering cracks and fractures has been presented, giving evidence of the effects of discontinuity (crack or fracture) properties on the macroscopic relaxation modulus of the fractures material. In order to perform the damage propagation analysis, a generalized Maxwell rheological model aimed at representing exactly the homogenized behavior has been determined. In addition, an approximate Burger model involving fewer elements has also been formulated.

In the subsequent step, an approach to the criterion of damage propagation depending on energy dissipation was presented. It relies upon an extension of the micromechanical elastic approach to account for the viscous dissipation. It has been showed that the energy release rate is the variable associated with the damage propagation. In order to compute the energy release rate, the macroscopic free energy expression was derived analytically based on thermodynamics and micromechanics concepts. This is achieved by using the approximate Burger model to relate the spring strains to the total strain. The free energy obtained for cracked material has been compared to that of a medium with fractures, showing as expected that cracks induce a decrease in free energy of the material than fractures.

Finally, the energy release rate was analytically derived from the knowledge of the free energy expression. Comparing the energy release rate in the case of cracks and fractures, it is observed that damage is more difficult to propagate in materials with fractures than materials with cracks. Keeping in minds that geomaterial discontinuities generally present a certain degree of roughness, it is expected that the assumption of crack behavior for the discontinuities will overestimate the risk and extension of the damage propagation.

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