



ANALYSIS OF DESIGN METHODOLOGIES OF SUBSEA STRUCTURE FOUNDATIONS

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Abstract.

The aim of this paper is to show that analytical methods tend to yield conservative results for geotechnical mudmat design. This is consistent with the structure of these methods, which are based on exact solutions obtained from the Theory of Plasticity to simplified basic cases of geometry and loading conditions. In order to adapt this basic configuration to real cases, a few correction factors need to be applied. As should be expected, the cumulative effect of these corrections produces a final conservative result. On the other hand, the finite element methods are able to model almost any situation/problem without requiring correction factors. Since they are based on equations of Elasticity and Plasticity Theories, finite element methods can produce highly accurate solutions to highly complex geometry, material behavior and boundary problems. This paper suggests how the analytical and numerical methodologies can be used to bridge this gap and ensure the effective design of shallow foundation of subsea structures. The comparison of the finite element case results, performed with the commercial finite element program, with the analytical methodology, recommended by the API RP 2GEO, has shown that the prediction of the safety factors and soil short- and long-term settlements are in good agreement and of the same order of magnitude; however, the former methodology provides results closer to the real physical values.

Keywords: *shallow foundations, subsea manifold, finite element analysis, offshore engineering.*

INTRODUCTION

The search for oil has extended the frontiers of prospecting and exploration to ever deeper waters. The accelerated pace in this direction requires compatible technological development for the construction of today's complex subsea systems. In the current offshore engineering scenario, the area of structural engineering has sought to design in an increasingly sophisticated way. Specific computational tools for geotechnical analysis are becoming increasingly more accessible and powerful. As a result, they started to have a key role in the day-to-day of the engineering professionals who look for greater efficiency in the execution of their projects; however, the bodies that define the standards that regulate the use and adoption of new upcoming technologies in this area hardly keep pace with the market, becoming outdated as a result. Emphasis has been placed on analytical methods that were initially developed for very simple mechanical and geometric configurations.

The design of foundations is one of the most common and enduring problems in geotechnical engineering. Shallow foundation design utilizes rational design methods, based on Soil Mechanics principles established over 60 years ago. The analytical solutions utilized contain idealizations, either of the problem geometry or of the soil response. They still provide, however, a framework that links the outcome to the various input parameters. The bearing capacity theory relates the vertical, horizontal, and moment loads with the foundation footprint and soil properties. The shear strength of a soil can be defined as the maximum shear stress that the soil can withstand without failing or the shear stress in soil in the plane that the failure is occurring (Pinto, 1998).

More recently, general evaluation of bearing capacity of shallow foundations has been addressed with both analytical and numerical methods (e.g. Bransby and Randolph, 1998; Randolph and Puzrin, 2003; Gourvenec and Randolph, 2003). The interaction of complex loads is still largely undefined though. Design guidelines such as API (API RP 2 GEO (2014)), ISO (ISO 19901-4 (2016)) and DNV (DNVGL-OS-C-101 (2016)) have been slow to evolve and in many places rely on field proven, but somewhat dated recommendations, either empirical or quasi-analytical design methodologies.

In an era where virtually any geotechnical application can be modeled numerically, it is tempting to wonder whether true analytical solutions still have a role (Randolph, 2013). The fact is that although the analytical and numerical approaches come from different theoretical backgrounds, there is nothing that prevents their combination. Actually, the inherent strengths and limitations of each methodology suit them to complement one another.

Our aim in this paper is to show that analytical methods tend to be conservative when applied to geotechnical mudmat design. This is consistent with the structure of these methods, which are based on exact solutions obtained from the Theory of Plasticity (i.e. the basic case of a rigid strip footing on a horizontal surface of a weightless soil, subjected to a central vertical load). In order to adapt this basic configuration to real cases, a few correction factors need to be applied. As should be expected, the cumulative effect of these corrections produces a final conservative result. On the other hand, the finite element methods are able to model almost any situation/problem without requiring correction factors. Since they are based on equations of Elasticity and Plasticity Theories, finite element methods can produce highly accurate solutions to highly complex geometry, material behavior and boundary problems.

1 SUBSEA MANIFOLD WITH SKIRTED MUDMAT FOUNDATIONS

Subsea systems are composed of heavy pieces of equipment (e.g. manifolds, UTAs, PLEMs, PLETs, etc.) that are installed on the seabed. These pieces of equipment are subjected to forces, moments, and torsions that sometimes exceed the bearing capacity of the soil beneath them. Mudmat is a type of shallow foundation, which is composed of thin reinforced steel plates installed on the base of the subsea structures. They “enlarge” the equipment footprint in contact with the soil, which provides additional bearing support by redistributing the stresses to the soil, also reducing the settlements to tolerable values.

Skirted mudmat is another type of shallow foundation, in which the structure loads are transferred to deeper soil layers by vertical slender elements that confine the soil beneath the foundation footprint (skirts). They also increase the foundation bearing capacity to withstand horizontal loads (Fig. 1).

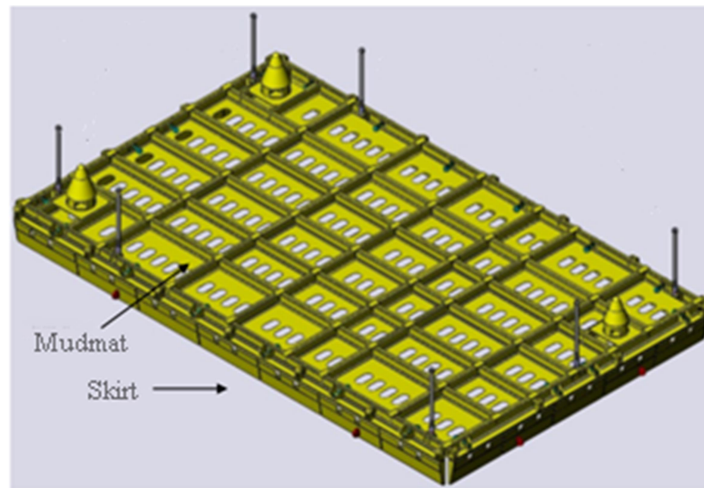


Figure 1. Skirted mudmat (Aker Solutions, 2016)

2 SHALLOW FOUNDATION DESIGN

Despite the huge volume of studies performed, many oil industry regulating bodies still recommend the traditional field-proven design methods based on Terzaghi's theory proposed in 1943. Since then, several researchers have made improvements to adapt his theory to actual day-to-day loading situations (Fig. 2).

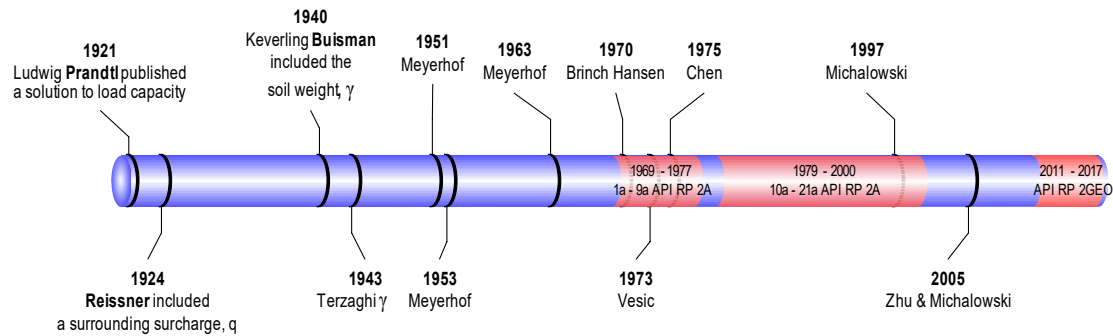


Figure 2. Soil Mechanics development timeline

The Analytical methods were initially developed for very simple mechanical and geometric configurations. The actual cases of foundations, for the most part, depart considerably from those configurations, requiring the introduction of various correction factors. These factors, in turn, introduce errors that will cumulatively affect the accuracy of the results.

Several oil and gas industry normative documents, DNV CN 30.4 (1992), DNVGL-OS-C101 (2016), ISO 19901-4 (2016), API RP 2A-WSD (2014) and more recently, API RP 2GEO (2014), recommend expressions and methodologies to calculate the bearing capacity of shallow foundations; however, more advanced computational tools for geotechnical design are only superficially described in those documents. The lack of more comprehensive regulation on the utilization of advanced numerical methodologies goes against the current trend, which shows that computational tools started to have a key role in the day-to-day of the engineering professionals, who look for greater efficiency in the execution of their projects. All in all, the design of shallow foundations can be carried out by using two different methodologies (Fig. 3).

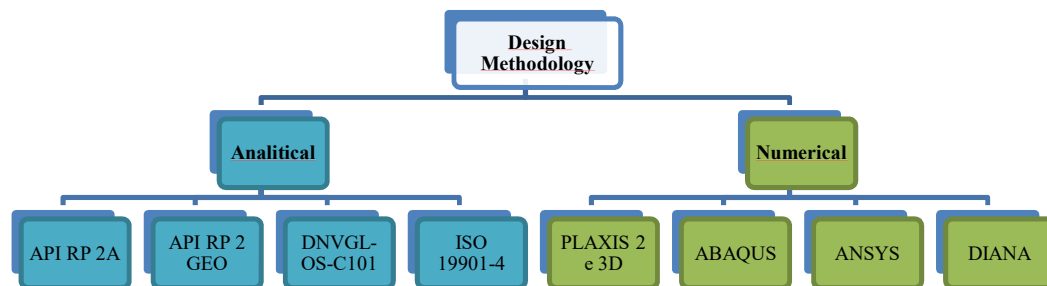


Figure 3. Shallow foundation design methodologies

As of 2011, API RP 2 GEO, Geotechnical and Foundation Design Considerations, has become the API reference on shallow foundation design, superseding API RP 2A in this regard.

2.1 Analysis Procedure

The analysis of the manifold foundation was split into 15 cases (Tab. 1). The initial case corresponds to its self-weight as well as the all MCV cap weights. The first 12 cases address the retrieval of the individual MCV caps and the subsequent installation of the respective MCV and jumper. The installation of the MTUs does not require hub cap removals. The last 3 cases address the removal of each of one of the modules (i.e. choke and crossover) from the manifold. This is necessary to simulate maintenance activities during the project lifespan.

Table 1. Analysis Cases

Analytical	Numerical	Loading
Initial	1	Self-weight and caps
7	3	Self-weight + MCV #7
6	5	Self-weight + MCV #7 to #6
3	7	Self-weight + MCV #7 to #3
4	9	Self-weight + MCV #7 to #4
2	11	Self-weight + MCV #7 to #2
9	13	Self-weight + MCV #7 to #9
8	15	Self-weight + MCV #7 to #8
11	17	Self-weight + MCV #7 to #11
14	19	Self-weight + MCV #7 to #14
10	21	Self-weight + MCV #7 to #10
13	23	Self-weight + MCV #7 to #13
1	25	Self-weight + MCV #7 to #13 - #1
5	27	Self-weight + MCV #7 to #13 - #1 and #5
12	29	Self-weight + MCV #7 to #13 - #1, #5 and #12

The case sequence above has been defined to unbalance the manifold by applying eccentric loads, which create moments, and thus reduce its bearing capacity. Table 2 provides the manifold main features.

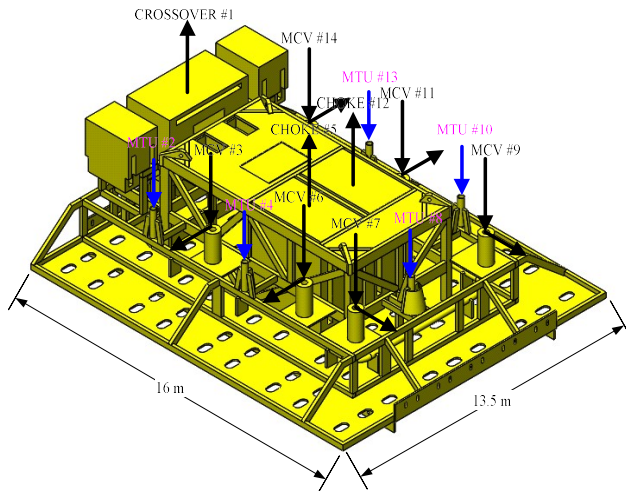


Table 2. Manifold features

Parameter	Value
Weight in water (kN)	2151.2
Length (m)	16
Width (m)	13.5
Skirt length (m)	16
Skirt width (m)	8
Skirt height (m)	0.5

Figure 4. Skirted Manifold (Aker Solutions, 2016)

The resulting forces and moments due to the hoisting / installation of retrievable components are reported in Table 3 below.

Table 3. Forces and moments applied to the manifold retrievable components

Case	Pos.	Force on local axes (kN)			Moment on local axes (kNm)		
		X	Y	Z	X	Y	Z
Initial	NA	12.9	0	-164.6	-23.6	123.1	-0.49
7	SB	23.6	0	-109.7	-29.9	226.8	-2.62
6	SB	23.6	0	-109.7	-29.9	226.8	-2.62
3	SB	0	0	-10.6	0	0	0
4	SB	0	0	-10.6	0	0	0
2	SB	12.9	0	-164.6	-23.6	123.1	-0.49
9	PS	0	0	-12.9	0	0	0
8	Bow	23.6	0	-109.7	-29.9	226.8	-2.62
11	PS	23.6	0	-109.7	-29.9	226.8	-2.62
14	PS	0	0	-10.6	0	0	0
10	PS	0	0	-10.6	0	0	0
13	PS	0	0	-200.8	0	0	0
1	Stern	0	0	-270.7	0	0	0
5	SB	0	0	-270.7	0	0	0
12	PS	12.9	0	-164.6	-23.6	123.1	-0.49

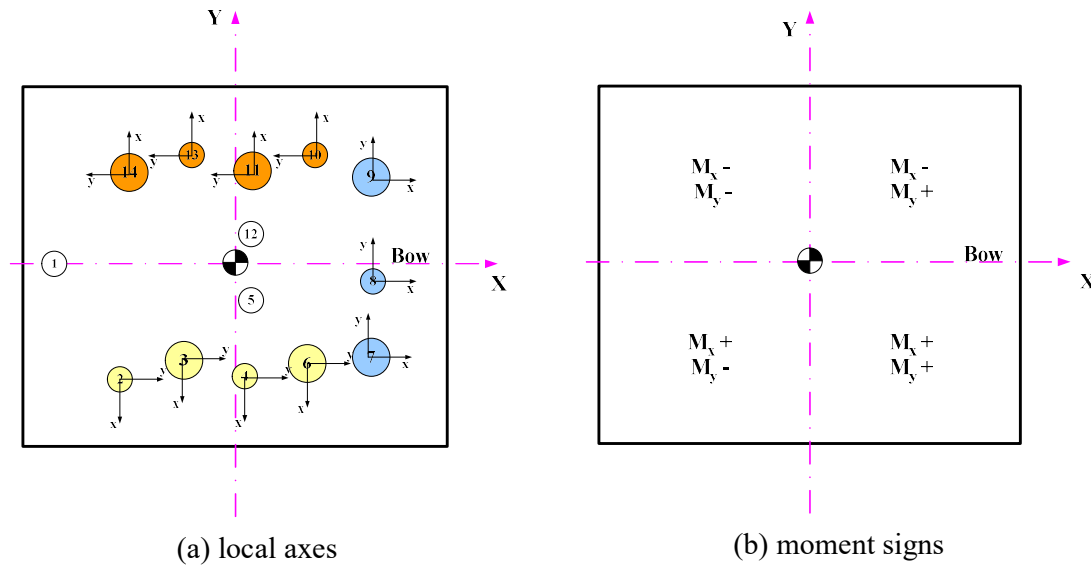


Figure 5. Main retrievable component axes

3 ANALYTICAL SOLUTION

The Analytical calculation was performed in accordance with the API RP 2GEO methodology. The soil parameters were obtained from Fagundes et al. (2012) paper. The soil properties were inferred from the reconstituted soil sample properties utilized in his research to obtain the soil parameters (Tab. 4).

Table 4. Soil Parameters (Inferred from Fagundes et al. (2012))

Parameter	Nomenclature	Value
Submerged unit weight of the soil	γ_{sat} (kN/m ³)	6.1
Undrained shear strength	s_u (kPa)	$su(h) = (2 + 1.5 \times h) *$
Young's modulus	E (kPa)	4339
Elastic shear modulus	G (kPa)	1446
Poisson's ratio	ν	0.5
Ground inclination angle	β (°)	0

Obs.: *h = depth of soil

3.1 Results

The load capacity and safety factor calculations were performed according to the following sequence of installation of MCVs and MTUs: 7, 6, 3, 4, 2, 9, 8, 11, 14, 10 and 13 (Tab. 4). The sequence has been defined so as to overload one manifold side (i.e. starboard) and cause moments that will reduce the effectiveness of the foundation design. The purpose of doing so is to check the foundation's settlements and safety factors in a completely unfavorable installation and/or operating conditions. It should be noted that the eccentricities due to the layout of the removable components in the manifold contributed to a small reduction of the effective area. The results of soil bearing capacity calculations for undrained loading are summarized below (Tab. 5):

Table 5. Analytical Results (API RP 2GEO)

Case	A' (m ²)	Q _{ult} + Q _u (kN)	SF	u _v (mm)	u _h (mm)	θ _r (rad)	θ _t (rad)
Initial	212.9	5062	2.35	22.4	0.0	0	0
7	197.1	4591	2.00	23.9	0.2	0	0
6	186.7	4280	1.78	25	0.4	0	0
3	182.4	4140	1.65	26.2	0.4	0	0
4	181.9	4189	1.66	26.3	0.0	0	0
2	182.0	4188	1.65	26.4	0.0	0	0
9	181.2	4172	1.56	27.8	0.2	0	0
8	180.6	4193	1.56	28	0.0	0	0
11	187.8	4345	1.56	29.1	0.4	0	0
14	199.3	4653	1.60	30.2	0.4	0	0
10	199.5	4723	1.62	30.4	0.0	0	0
13	199.4	4718	1.61	30.5	0.0	0	0
1	184.6	4370	1.58	28.9	0	0	0
5	179.9	4239	1.53	29	0	0	0
12	182.3	4319	1.64	27.4	0	0	0

Obs.: the order of magnitude of the manifold rotation angles was so low (i.e. 10^{-4}) that they were considered zero.

The envelopes of the foundation bearing capacity for the highest and lowest safety factor cases are shown below (Fig. 6).

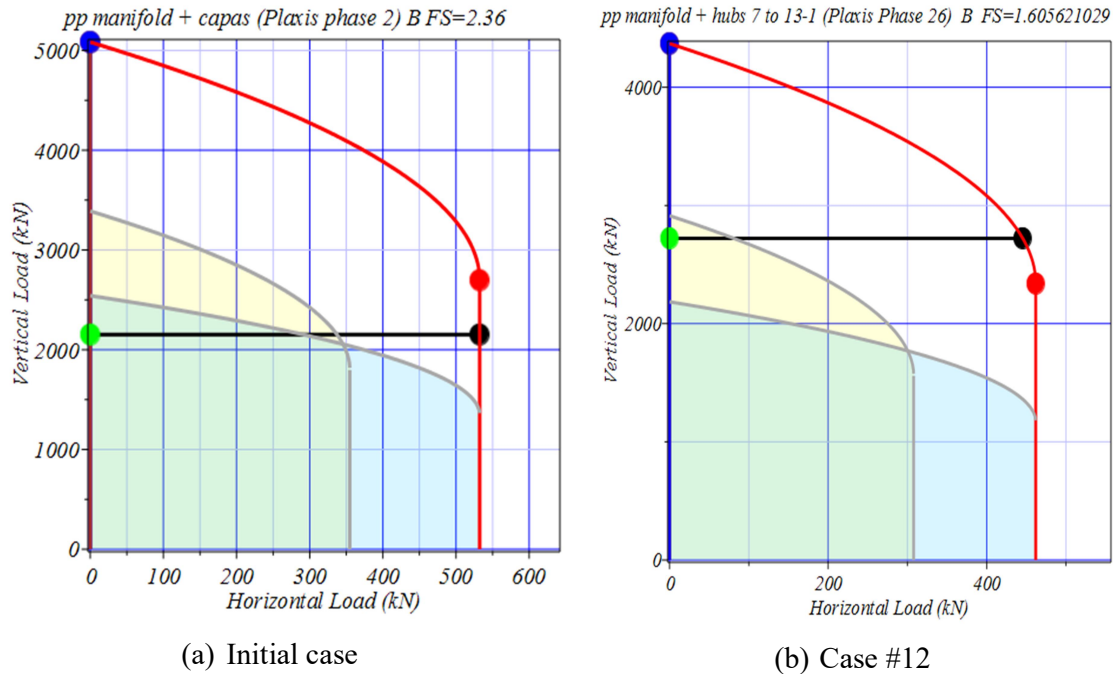


Figure 6. Envelope of bearing / sliding capacity

The long-term displacement result for drained loading condition, when pore water pressure is in equilibrium with the in situ conditions, shows a settlement of 0.31 m over an infinite time.

As can be seen in Table 5, the analytical calculation results obtained through the API RP 2GEO methodology alone suggest that the manifold foundation was designed to a safety factor of less than 2; however, this alone cannot be confirmed without the knowledge of the actual soil properties at the exact location of the manifold installation and with the execution of more refined analyses.

4 NUMERICAL SOLUTION

The mechanical behavior of the soil can be simulated with varying degrees of accuracy. Over the years, numerous soil constitutive models have been developed for analysis in EF (Lade, 2005); however, few of them have the capacity to reproduce the mechanical behavior of the material over any three-dimensional stress trajectory. Each model has different capacities and requires different input (experimental) data for its calibration. It is essential to model the behavior as realistically as possible, for reasons of economy, time, and safety. If the input parameters are sufficiently accurate, then the finite element model results are reliable. Three-dimensional models are computationally intensive. Because of this, it is important to select the simplest computational model that can accurately capture the behavior of the soil in question.

4.1 Finite Element Modeling

PLAXIS is a commercial finite element computer program developed by the University of Delft (Brinkgreve and Vermeer, 1998). It is used for simulation purposes in different geotechnical applications, mainly for stress-strain analysis and stability.

The numerical model created in PLAXIS 3D utilized specific constitutive models for each one of its parts (i.e. manifold structure, soil, and soil-structure interface) as described in Figure 7, below:

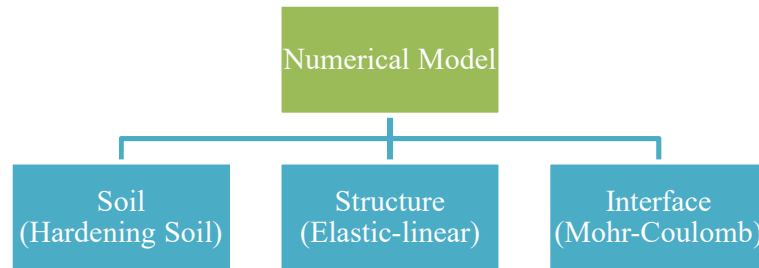


Figure 7. Numerical model structure

4.2 Soil Constitutive Model and Parameters

The PLAXIS 3D Hardening Soil (HS) is an advanced model that has been developed according to the Theory of Plasticity. The main feature of the Hardening Soil model is its ability to simulate hardening behavior (Schanz and Vermeer, 1999). The total strains are calculated by using a stress-dependent stiffness, which is unique for each loading and unloading/reloading. The hardening is assumed to be isotropic and dependent on the plastic shear and volumetric strains (Fig. 8).

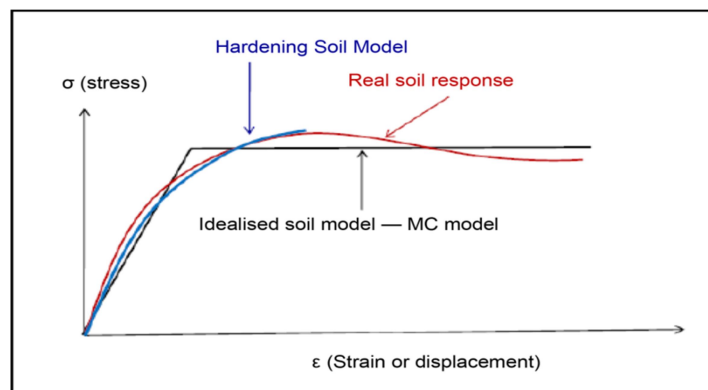


Figure 8. Behavior of the Hardening Soil model (Ehsan, 2013)

The soil parameters for the HS model were obtained from Fagundes et al. (2012) and are summarized in Table 6 below:

Table 6. HS Constitutive Model Parameters

Parameter	Nomenclature	Value
Specific weight	γ_{sat} (kN/m ³)	16.1
Failure ratio	R_f	0.9
Dilatancy	Ψ (degrees)	0
Secant stiffness module	E_{50}^{ref} (kPa)	1731 ^{b,c}
Oedometric stiffness module	$E_{\text{oed}}^{\text{ref}}$ (kPa)	1384 ^{b,c}
Unload-reload stiffness module	$E_{\text{ur}}^{\text{ref}}$ (kPa)	5834 ^{b,c}
Stiffness stress level dependency	m	1 ^a
Unload-reload Poisson coefficient	ν_{ur}	0.2
Undrained shear strength at	s_{uo} (kPa)	2
Rate of increase with depth of initial	k (kPa/m)	1.5
Coefficient of lateral pressure for	K_o^{nc}	1

Obs.: a - *default* PLAXIS 3D values

b – values inferred from parameters $C_c = 0.515$, $C_s = 0.11$ e $e_o = 2.1$

c – PLAXIS reference pressure 100 kPa (1 atm)

4.3 Geometry and Mesh Definition

The three-dimensional finite-element mesh is illustrated in figure 9a, below. Several sensitive analyses were carried out to define the mesh and model boundaries. This was necessary to prevent interference with the results as well as identify a mesh optimum density to reduce computational demand without compromising the model accuracy; therefore, the modeling soil was delimited between points $x_{\text{min}} = -18$ m, $x_{\text{max}} = 90$ m, $y_{\text{min}} = -15$ m, and $y_{\text{max}} = 75$ m, totaling an area of 90 m wide by 108 m long and 38 m deep.

The Average element size (I_e) parameter was set to 5.097. The relative element size factor (r_e) was 0.7. In order to improve the model accuracy, the mesh beneath and surrounding the manifold was further refined by creating a volume of 16 m x 13.5 m x 0.5 m, in which the refinement factor is 0.10, and other volume of 26 m x 24 m x 9 m, in which the refinement factor is 0.16. The horizontal displacements were prevented at the vertical limits of the model as well as all the vertical and horizontal displacements referring to the base of the model (Fig. 9a). The mesh is comprised of 137,161 10-node tetrahedral quadratic elements (Fig. 9b).

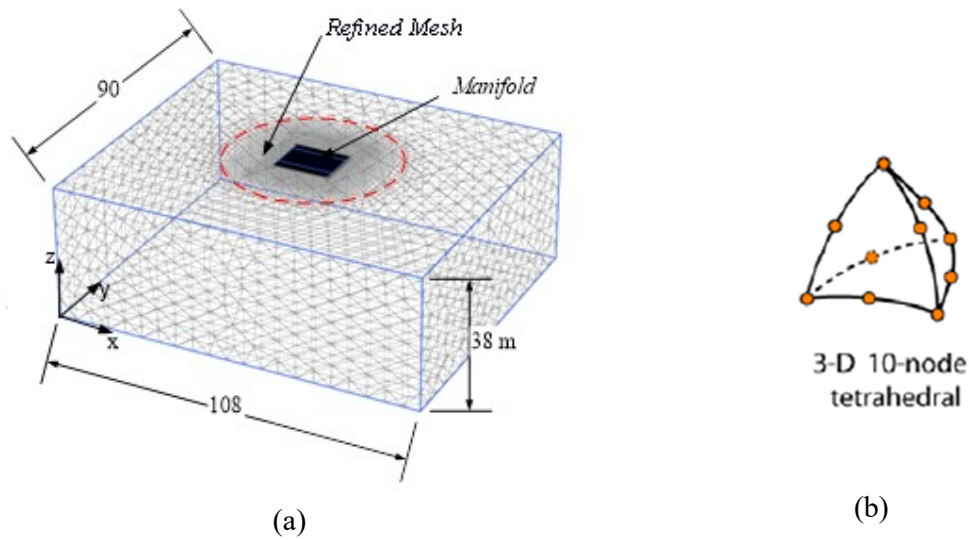


Figure 9. Finite element model boundaries and element type

The contact between the soil and the skirted mudmat is modeled with Mohr-Coulomb interface elements. The PLAXIS interface strength reduction factor (R_{inter}) was to 0.9 to define the adherence between soil-structure interface contact.

The soil behavior is considered undrained for all analysis cases. The HS option Undrained B was utilized. This option prevents volumetric changes and utilizes the soil undrained shear strength. The Poisson ratio adopted by default was 0.495 to prevent numeric problems during the stiffness matrix calculation (POTTZ and ZDRAVKOVIC, 1999).

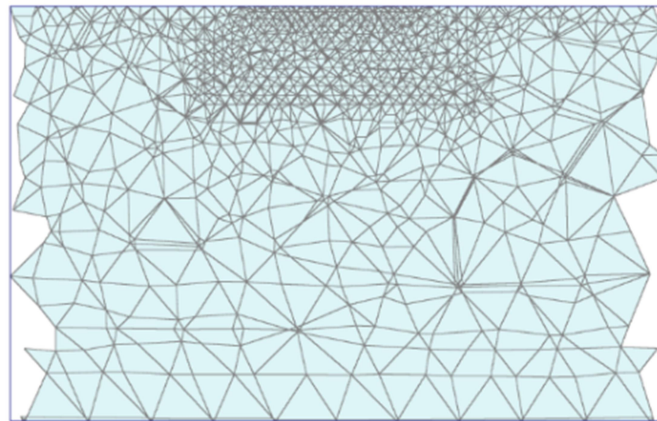


Figure 10. Model mesh refinement

4.4 Numerical Analysis Results

PLAXIS calculates the safety factors by using its ϕ -c reduction function. The program considers the shear strength of the soil and compares it with the minimum shear strength corresponding to the equilibrium limit (Brinkgreve et al., 2016). Table 7 below shows the calculated values:

Table 7. Numerical Results (PLAXIS 3D)

Case	SF	u_x (mm)	u_y (mm)	u_z (mm)	θ_r (rad)	θ_t (rad)
1	3.34	0	0	19.4	0	0
3	2.79	1	0	21.6	0	0
5	2.70	1	-1	23.7	0	0
7	2.51	1	-1	25.1	0	0
9	2.46	1	0	25.4	0	0
11	2.46	1	0	25.3	0	0
13	2.30	1	0	27.2	0	0
15	2.34	1.3	0	27.6	0	0
17	2.22	1.3	1	28.7	0	0
19	2.20	1	2	30.3	0	0
21	2.23	1	2	30.7	0	0
23	2.22	1	2	30.8	0	0
25	2.21	1.3	2	29.8	0	0
27	2.43	1.3	2	29.9	0	0
29	2.68	1.3	2	28.0	0	0

Obs.: the order of magnitude of the manifold rotation angles were so low (i.e. 10⁻⁴) that they were considered zero.

The short-term displacement values were obtained by calculating the arithmetic mean of the displacement values in 9 main points of the manifold (Fig. 11). The rotations around X and Y axis were obtained through the arithmetic mean of the vertical displacements along each side of the manifold.

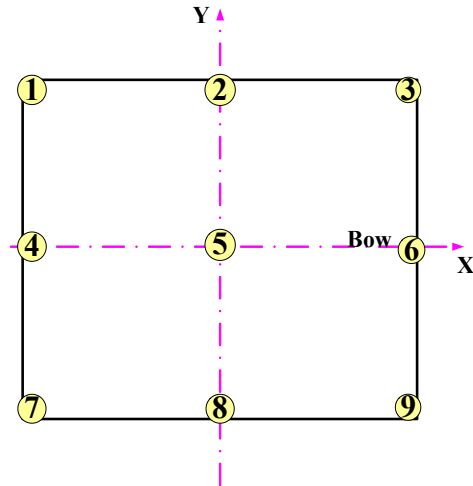


Figure 11. Short-term displacement measurement points

The figure below (Fig. 12) shows the displacement vectors, which indicate the soil short-term rupture type (general rupture).

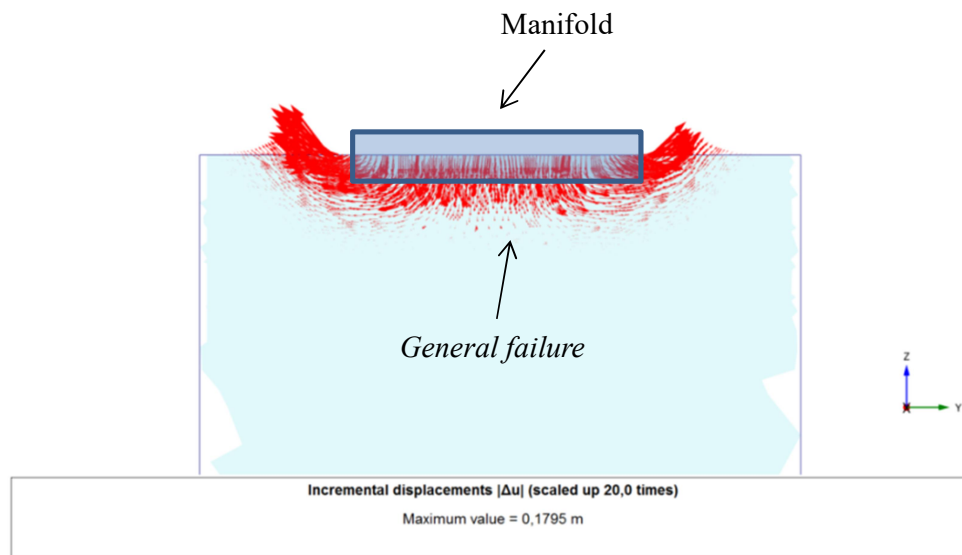


Figure 12. Displacement vectors during short-term settlement

Figures 13 and 14, below, show the foundation displacements when subjected to MCVs' loads:

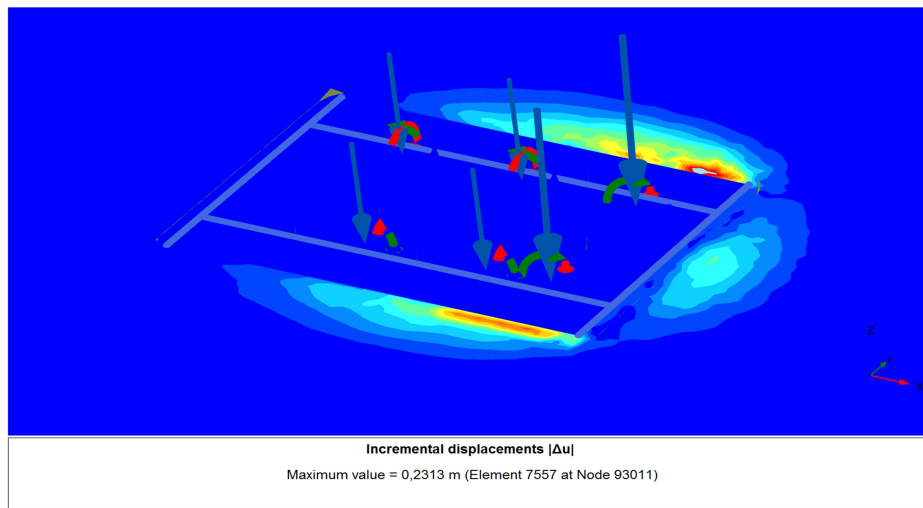


Figure 13. Short-term displacements due to MCV's and MTU's loads.

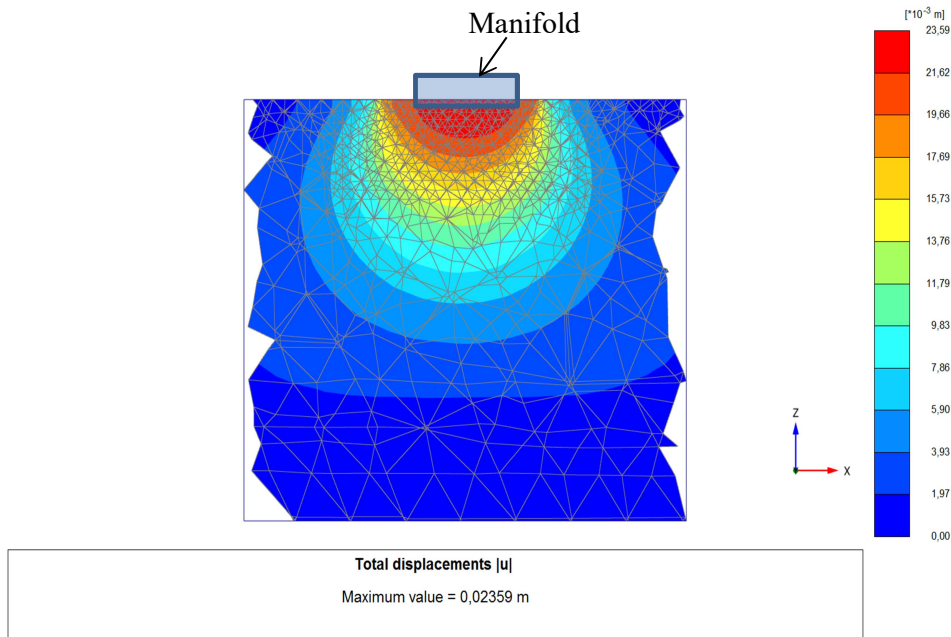


Figure 14. Short-term displacement

Figure 15 shows a graph of the long-term displacement (99.9% of the final settlement) calculated by PLAXIS 3D:

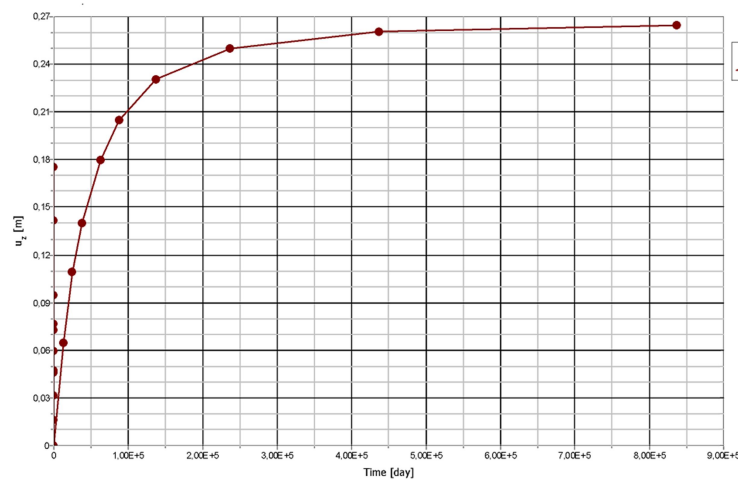


Figure 15. Long-term displacement (drained conditions)

5 CONCLUSION

The choice of the most adequate methodology to solve a foundation problem is of the paramount importance to achieve a safe and economical foundation design.

The comparison of the finite element case results, performed with the commercial finite element program, with the analytical methodology, recommended by the API RP 2GEO, has shown that the prediction of the safety factors and soil short- and long-term settlements are in good agreement and of the same order of magnitude; however, the former methodology provides less conservative results than the latter. The analytical method proposed by API RP 2GEO resulted in safety factors corresponding to 78% of those obtained from the PLAXIS 3D analyses. Similar results were obtained by Lai (2009) and Schmid (2009). The immediate displacements evaluated using formulae recommended by API RP 2GEO were almost the same as those obtained by PLAXIS 3D. The safety factor against general failure of a superficial foundation recommended by Standards for offshore applications is 2.0 (API RP 2GEO, 2014, p. 16). Thus, the design carried out by analytical methods will result in a heavier and larger footprint mudmat than one designed with numerical methods. This difference implies not only in additional material, project time, and labor costs, but will also reflect in higher installation cost. This fact suggests that the payback period of the investment in a finite element modeling software as well as in the corresponding user training is certainly short.

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