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# NONLINEAR DYNAMICS OF AN ENERGY HARVESTING ONBOARD PENDULUM

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***Abstract.*** *This paper presents a computational method in the MATLAB environment for numerical simulation and analysis of the ordinary differential equations of the nonlinear dynamics of mechanical and electromechanical systems.*

*As a motivation for the study, the differential equations studied are mathematical models of energy harvesting devices of existing vibrations in the environment. In particular, we present a model of a on board pendulum (in a naval vehicle), which, for certain parameters and initial conditions, can present a permanent rotation regime that can be used for electric power generation.*

***Keywords:*** *Nonlinear dynamics, onboard pendulum, energy harvest*

INTRODUCTION

In this research, a computational method is developed, in the MATLAB program environment, for numerical simulation and analysis of the ordinary differential equations of the nonlinear dynamics of mechanical systems.

The area of multidisciplinary research of dynamic nonlinear systems, based on mathematics, can be applied to almost all areas of knowledge, such as engineering, physics, chemistry, economics, meteorology, astronomy etc. It has advanced exponentially in the last decades, thanks to the computational simulation of the behavior of the differential equations involved. With the current resources, it is possible to develop graphical analysis tools such as time histories, phase plans, Poincaré maps, bifurcation diagrams, attraction basins, Lyapunov coefficients etc.

As a motivation for the study, the differential equations being studied are mathematical models of devices for collecting the energy present in existing vibrations in the environment. In particular, we present a model of an onboard pendulum (in a naval vehicle) which for certain parameters and initial conditions, can present a permanent rotation regime that can be used for electric power generation. The energy potential represented by open ocean waves is immense, with wave energy being more consistent than wind power, which encourages the development of devices for the collection of such energy (Falnes, 2007). The most important research group worldwide in this subject is Prof. Marian Werciengovich at the University of Aberdeen, United Kingdom, with whom the supervisor of this work develops joint research.

# 1 INTRODUCTION

The programming and simulation of the nonlinear dynamics of some proposals of floating mechanisms to extract alternative energy from the motions of sea waves can improve the knowledge about the forms of exploitation of this form of energy. For this, it is possible to use analytical and mathematical models proposed in previous researches in a computational environment that allows simulating some solutions already found and the evolution of computational codes for analysis of nonlinear systems, besides of documenting these codes.

Computer programs for simulation of mathematical models represented by nonlinear differential equations require intensive numerical analysis. In addition, it is important that the user interface to be very flexible, allowing for easier edition of parts of the code by the user. However, many programs have their own way of specifying the system or are written in relatively low-level programming languages, making it difficult to extend them to more advanced calculations.

A powerful and widely used computing environment for scientific computing is the MATLAB. However, because it is an interpreted language, not compiled, in some cases the processing time of the computational simulations can be very high or inadequate for the solution of dynamic nonlinear problems.

# 2 ENVIRONMENTAL MOTIVATION

Every year the demand for energy on the planet increases, but most of the world's energy matrix is made of fossil fuels. This fact is of great concern to the whole population, including the discussions at the Paris Conference in 2016.

It is well known that fossil fuels have a very large impact on the Earth's ecosystem. The use of these fossil fuels occurs mainly through their burning, which generates large quantities of pollutants. In Brazil, in 2015, 56.9% of the domestic energy supply (DES) came from fossil fuels. Because fossil fuels are not renewable, the amount of these resources available is limited. As they are being used more and more, their reserves are running out.

This way, a movement to seek cleaner and more efficient sources of energy has appeared and has been improving. 41.2% of the DES is made of renewable sources but 61% of these renewable sources are exploited through combustion, which generates, among other damages, pollutants such as CO2. One strand of this movement is that which seeks the collection of energy from vibrations, such as winds, people and vehicle motions, and sea waves.

# 3 AN ONBOARD PENDULUM MODEL

## 3.1 Nomenclature

$l$ : stem length

$m$ : mass of the pendulum

$\theta$ : angle between rod and axis y

$v_s(t)$: vertical excitation

$g$ : acceleration of gravity

$c$ : damping coefficient

$\tau$ : Independent variable obtained by the dimensionless

$\omega_n$ : natural frequency of the pendulum

$p$ : amplitude of the dimensionless excitation related to the natural frequency



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$\omega$ : frequency of the dimensionless excitation related to the natural frequency

$\gamma$ : dimensionless damping related to natural frequency

$Y$ : real amplitude of excitation

$\Omega$ : actual frequency of excitation

$N_c$ : Linear damping

## 3.2 Deduction of the non-linear differential equation of motion

Initially we deduced the equation that is being used in MATLAB environment for simulations.

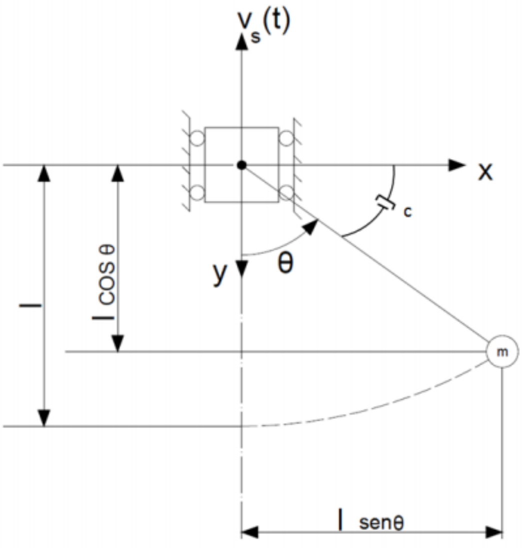



**Figure 1. Physical model**

By the configuration shown in the Fig. 1, it is possible to infer the position of mass $m$ .

$$x = lsen(\theta) \tag{1}$$

$$y = l\cos(\theta) + v_s \tag{2}$$

By deriving in time the two equations, we get the velocities:

$$\dot{x} = l\dot{\theta}\cos(\theta) \tag{3}$$

$$\dot{y} = -l\dot{\theta}sen(\theta) + \dot{v}_s \tag{4}$$

The kinetic energy of the system is obtained by equation

$$T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) \tag{5}$$

$$T = \frac{1}{2}m\left(\left(l\dot{\theta}\cos(\theta)\right)^2 + \left(-l\dot{\theta}sen(\theta) + \dot{v}_s\right)^2\right) \tag{6}$$

Deriving T in relation to θ:

$$\frac{\partial T}{\partial \theta} = -ml\dot{v}_s\dot{\theta}\cos(\theta) \tag{7}$$

Deriving T in relation to and after in relation to *t*:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta} - ml\dot{v}_s\dot{\theta}\cos(\theta) - l\ddot{v}_s msen(\theta) \tag{8}$$

As for the potential energy:

$$V = -mg\left(l(1-\cos(\theta)) + vs\right) \tag{9}$$

$$\frac{\partial V}{\partial \theta} = -mglsen(\theta) \tag{10}$$

Linear viscous damping is the only non-conservative force considered

$$N_c = -c\dot{\theta} \tag{11}$$

By the Euler-Lagrange equation, we can obtain the following equation of the motion of the onboard pendulum:

$$ml^2\ddot{\theta} - ml\ddot{v}_s sen(\theta) + mglsen(\theta) + c\dot{\theta} = 0 \tag{12}$$

Dividing by the moment of inertia of the mass:

$$\ddot{\theta} + \frac{1}{l}(-\ddot{v}_s + g)sen(\theta) + \frac{c\dot{\theta}}{ml^2} = 0 \tag{13}$$

The excitation caused by the sea wave is modeled as harmonic, where Y is the amplitude and Ω is the frequency of the wave:

$$v_s = Y\cos(\Omega t) \tag{14}$$

$$\dot{v}_s = -Y\Omega sen(\Omega t) \tag{15}$$

$$\ddot{v}_s = -Y\Omega^2\cos(\Omega t) \tag{16}$$

Then the equation of motion can be written in the form

$$\ddot{\theta} = \frac{-sen(\theta)}{l}\left(Y\Omega^2\cos(\Omega t) + g\right) - \frac{c}{ml^2}\dot{\theta} \tag{17}$$

The natural frequency of the pendulum is represented by $\omega_n$. In order that the generated equation can be analyzed parametrically it is convenient to render them non dimensional. For this to be possible, the following parameters are defined:

$$\tau = \omega_n t \tag{18}$$

$$p = \frac{Y\Omega^2}{l\omega_n^2} \tag{19}$$

$$\beta = \frac{\Omega}{\omega_n} \tag{20}$$

With the chain rule, we get:

$$\frac{d\theta}{dt} = \frac{d\theta}{d\tau}\frac{d\tau}{dt} \tag{21}$$

$$\frac{d^2\theta}{dt^2} = \frac{d\dot{\theta}}{d\tau}\frac{d\tau}{dt} \tag{22}$$

This way:

$$\dot{\theta} = \theta'\,\omega_n \tag{23}$$

$$\ddot{\theta} = \theta''\,\omega_n{}^2 \tag{24}$$

This way we have $\gamma = \dfrac{c}{ml^2\omega_n}$, the equation of the dimensionless motion is given by:

$$\theta'' = -\gamma\theta' - sen(\theta)(1 + p\cos(\beta\tau)) \tag{25}$$

In state space, this dimensionless motion equation can be written as:

$$y_1 = \theta \tag{26}$$

$$y_2 = \theta' \tag{27}$$

So:

$$y_1{}' = y_2 \tag{28}$$

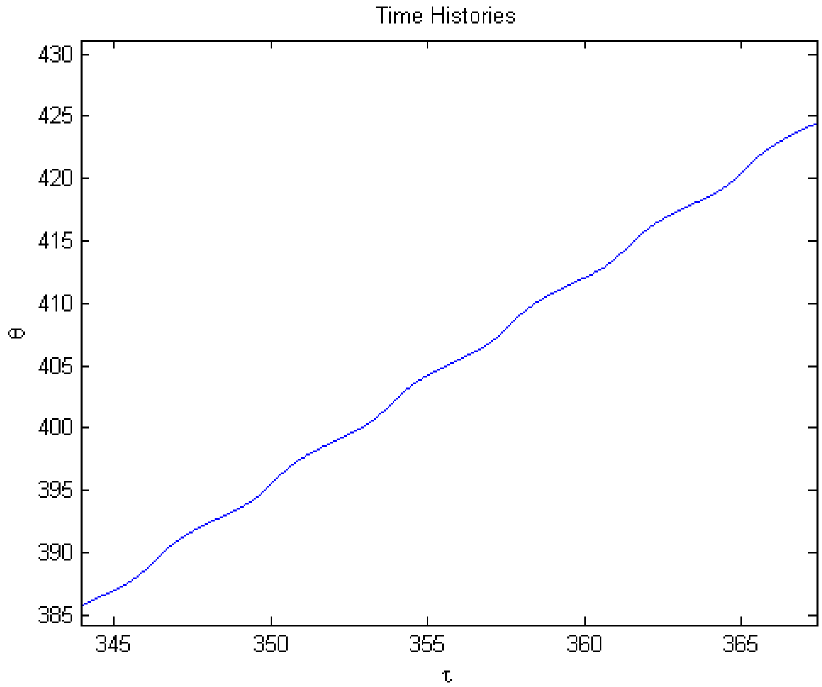
$$y_2' = -sen(y_1)(1 + p\cos(\beta\tau)) - \gamma y_2 \tag{29}$$

# 4 NUMERICAL INTEGRATION

With the dimensionless equation of the parametric pendulum developed in the previous item, numerical simulations were made based on some specific parameters. The fourth-order Runge-Kutta numerical integration method is being used.

Rotations are defined as motions that exceed 2π. For this pendulum, two types of rotation are possible: the rotations that always follow in the same sense, called pure rotations, and the rotations that change direction, that receive the name of rotations-oscillations.

Examples of pure rotation are:



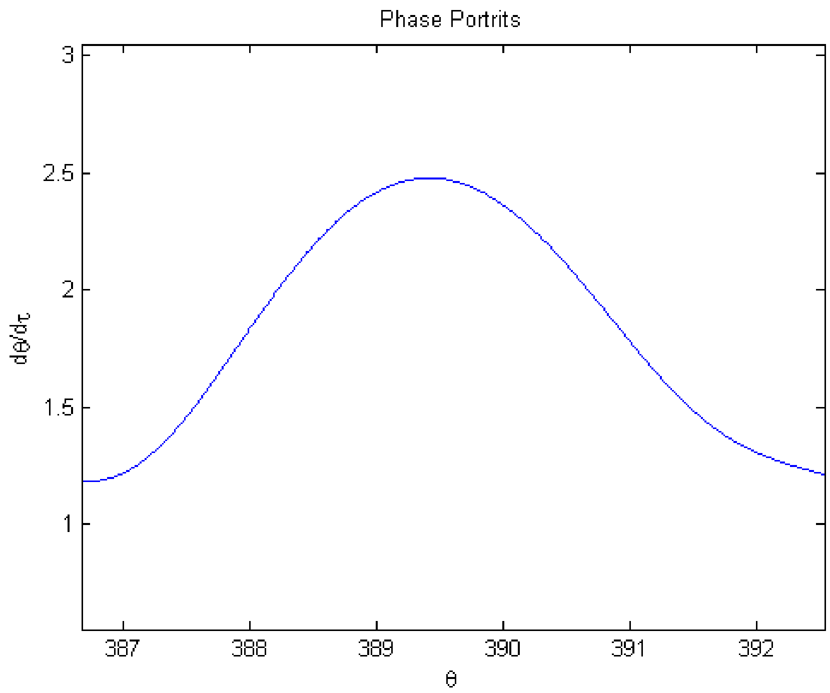
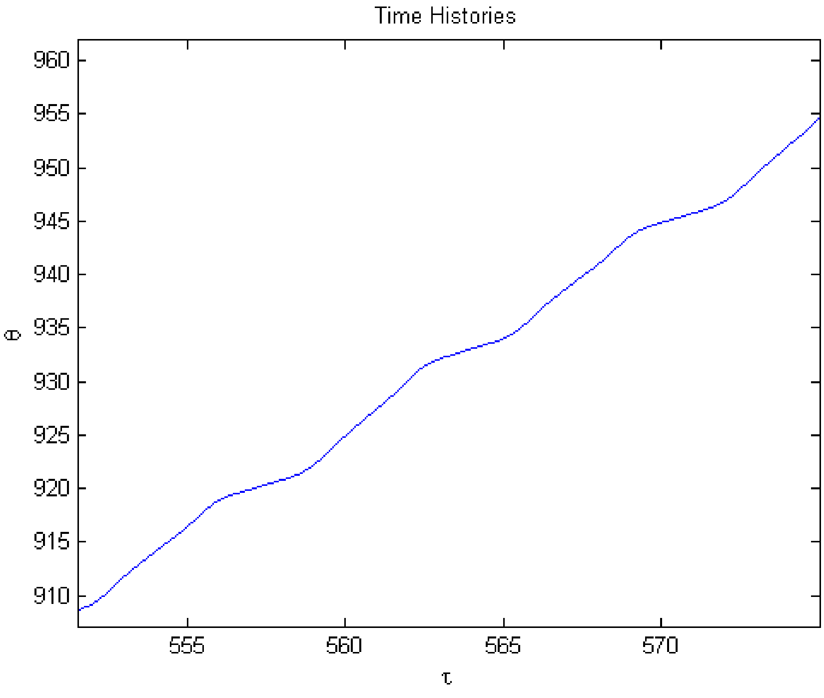



**Figure 2: History in time and phase plane for** $\gamma = 0.1, p = 0.8, \beta = 1.67$



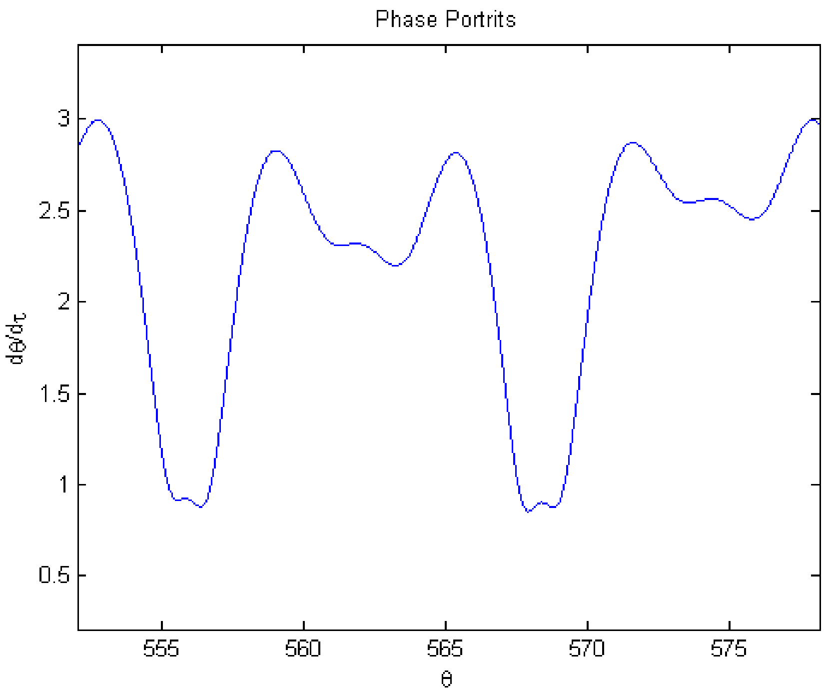
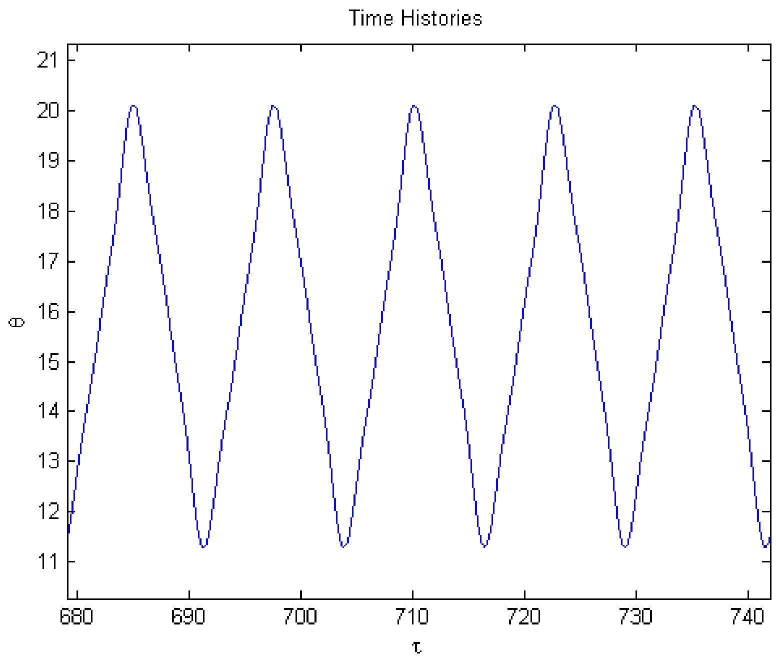



**Figure 3: Time history and phase plane for** $\gamma = 0.1$, $p = 1.5$, $\beta = 1.9$

Examples of rotation-oscillation are:



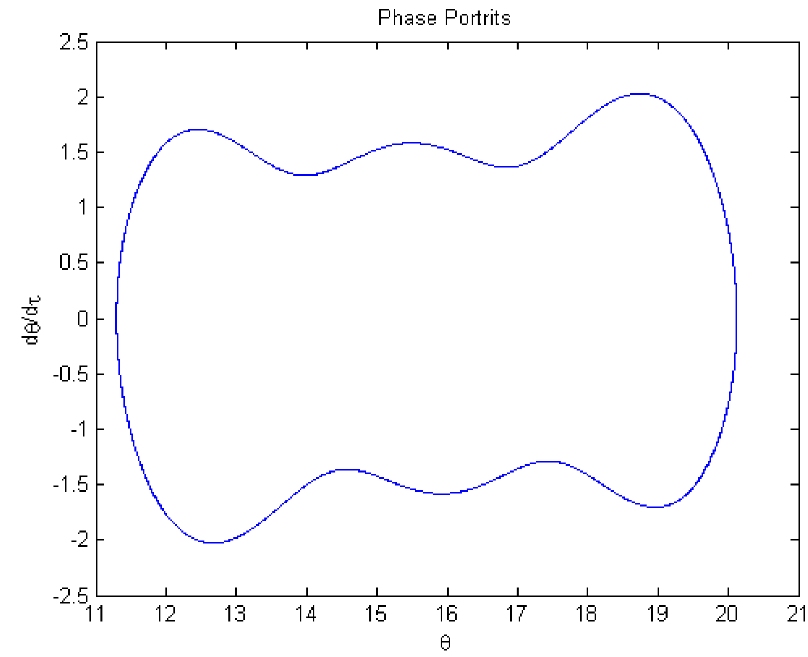
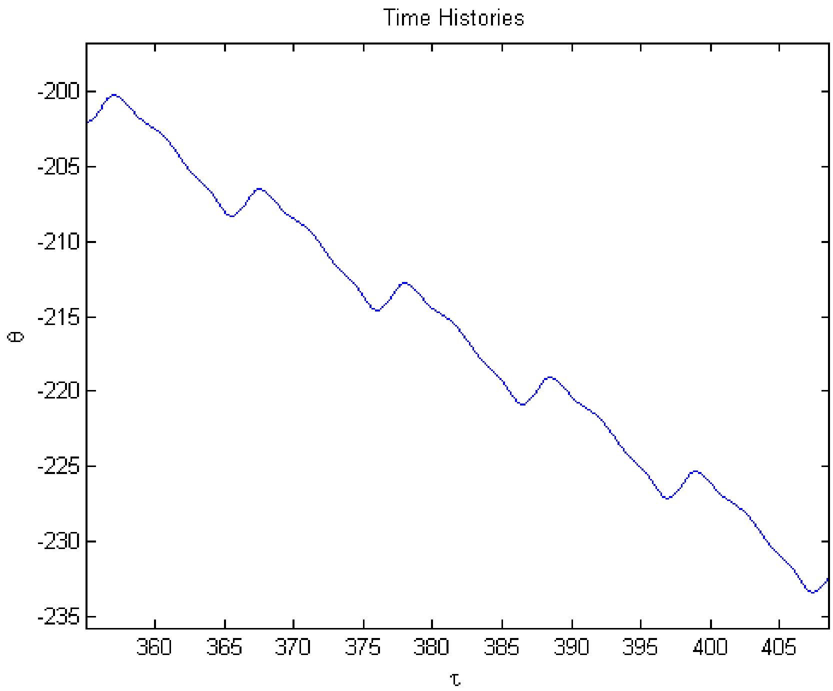



**Figure 4: Time history and phase plan for** $\gamma = 0.1$, $p = 1.8$, $\beta = 1.0$



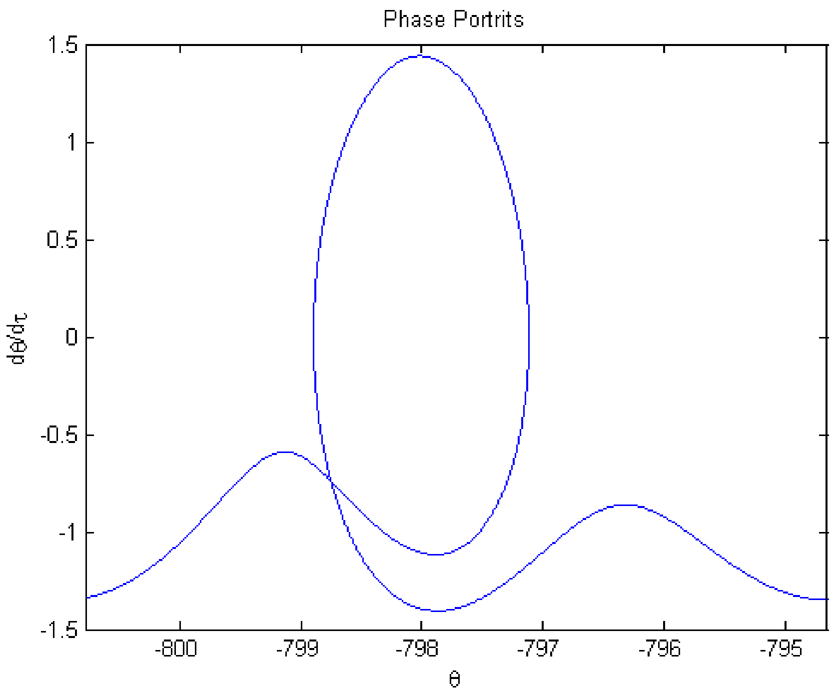
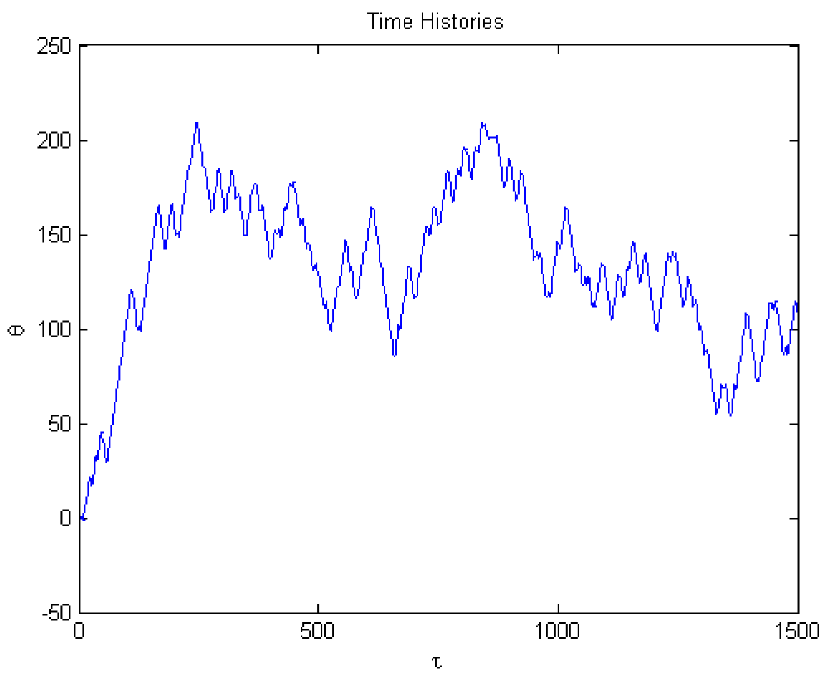



**Figure 5: Time history and phase plane for** $\gamma = 0.1, p = 1.9, \beta = 0.6$

An example of chaotic motions:



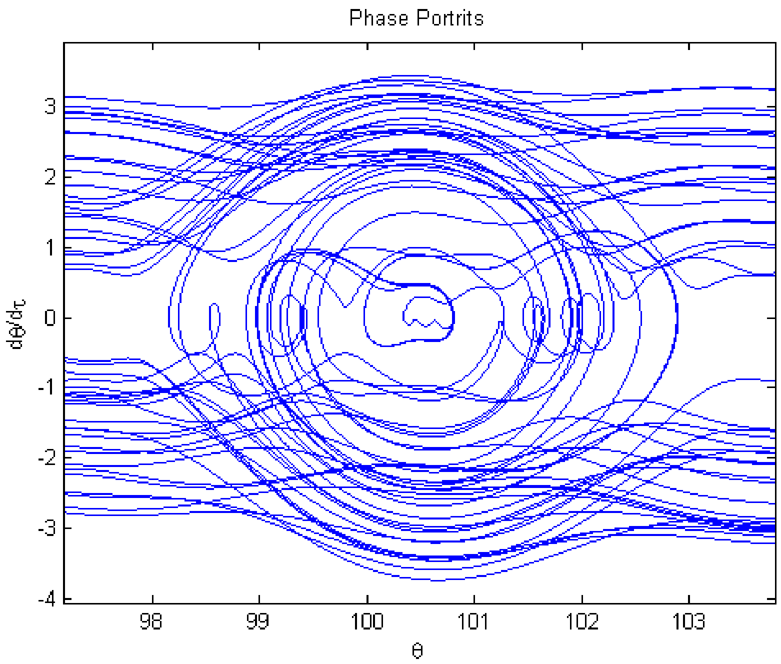



**Figure 6: Time history and phase plane for** $\gamma = 0.1, p = 2.04, \beta = 1.8$

Figure 2 and Fig. 3 represent pure rotation motions, Fig. 2 representing a one-period rotation and Fig. 3 period four rotation.

Figure 4 and Fig. 5 describe one oscillation and one rotation per period. The number of oscillations or rotations in one period maybe more than one.

Figure 6 represents chaotic motions. Chaos can be described as an irregular combination of rotations and oscillations, when the pendulum completes an apparently random number of clockwise (or anti-clockwise) rotations before changing direction. Such changes include many oscillations about the hanging position. They are very sensitive to initial conditions.

# 5 CONCLUSION

An initial numerical nonlinear dynamic study of motions of an onboard pendulum for several parameter settings and initial conditions was presented.

Future work will include Poincaré maps, bifurcation diagrams, Lyapunov exponents etc.

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## REFERENCES

Amaral, L. 2013. *Dinâmica não linear de dispositivos de coleta de energia alternativa em ambiente offshore,* MSc dissertation, Federal University of ABC.

Falnes, J., 2007. A review of wave-energy extraction. *Marine Structures,* v. 20, n. 4, pp. 185-201.

E.L. Allgower and K. Georg, 1990. *Numerical Continuation Methods: An introduction*, Springer-Verlag.

Beyn, W.-J., 1994. Numerical analysis of homoclinic orbits emanating from a Takens-Bogdanov point, *IMA J. Numerical Analysis*, n. 14, pp.381-410.

Xu, X., Wiercigroch, M., & Cartmell, M.P., 2005. Rotating orbits of a parametrically-excited pendulum, *Chaos, Solitons and Fractals,* n.23, pp. 1537-1548, Elsevier, 2005;

Xu, X., Wiercigroch, M., 2007. Approximate analytical solutions for oscillatory and rotational motion of a parametric pendulum, *Nonlinear Dynamics,* n. 47, pp. 311-320.

Nandakumar, K., Wiercigroch, M., & Chatterjee, A., 2012. Optimum energy extraction from rotational motion in a parametrically excited pendulum, *Mechanics Research Communications, n. 43, pp.7-14.*

*Brasil, R., Balthazar, J., & Góis, W., 2015. Métodos numéricos e computacionais em engenharia e ciências. São Paulo, Ed. Edgard Blucher.*

*MEIROVITCH, L., 2009. Methods of analytical mechanics. London, Dover.*

W. Szemplifiska-Stupnicka and E. Tyrkiel, 2002. Common features of the onset of

the persistent chaos in nonlinear oscillators: a phenomenological approach.

*Nonlinear Dynamics*, n. 27, pp. 271-293.