NUMERICAL PLATES ANALYSIS BY FINITE DIFFERENCES IN AEROSPACE STRUCTURES

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Abstract. *The solution of the most varied types of structural problems, in general, in the aerospace engineering field, requires the use of adequate numerical methods, as well as the knowledge of their main characteristics and limitations. The present work presents a study on the numerical analysis of rectangular plates by means of the finite differences method, using Matlab® software for simulation. We obtain approximate solutions for the field of transversal displacements, as well as the corresponding bending moments. We consider problems involving a series of boundary conditions, especially aerospace engineering applications. Also, in this context, we perform a comparative analysis of the approximations obtained by means of the finite difference method with those with the finite element method, using commercial software Abaqus™, for same meshes. In both cases, comparisons are made with results found in the literature derived from trigonometric series. The results allow us to infer that, for certain problems, the finite difference method can be used with better efficiency, precision and simplicity - from the computational point of view - when compared with the finite element method.*

Keywords: *Finite Differences, Lagrange, Plates, Transversal displacements, Bending moments*

1 INTRODUCTION

In structural analysis in general, most of the problems require an appropriate numerical method. In this context, the present work approaches the plates behavior by the Finite Differences Method. As a first step, we present the Sophie-Germain and Lagrange differential equation for plates and corresponding boundary conditions that we must consider in each type of problems. As a second step, we present a methodology for the Finite Differences Method application, looking for an approximated solution to the problems here discussed. That way, it was possible to obtain the displacement fields and the corresponding bending moments in the x and y directions as well. For this, the Matlab® software was utilized for the Finite Differences modeling and numerical simulation and, in parallel, the AbaqusTM software was employed on the Finite Elements Method simulations for the same kind of problems.

Plates have large application in the industries in general, whether automobilist, naval or aerospace areas. At the aerospace sector, particularly in aeronautics, plates are very usual, because it's a fundamental element in aircraft structures. Depending on the type of aircraft, it is possible to find different structural component arrangements, as it can be seen in Fig. 1, where we have the case of the De Havilland Canada Twin Otter aircraft, with wings that consists of a *spars*, *ribs*, *stringers* and *skins* arrangement, which are – basically – mounted from a combination of several thin plates. All these components cited before, together, have as their main function resist and transmit the loads applied due the operation, providing aerodynamic shapes, security for the passengers, for the payload and for all the integrated systems as well.

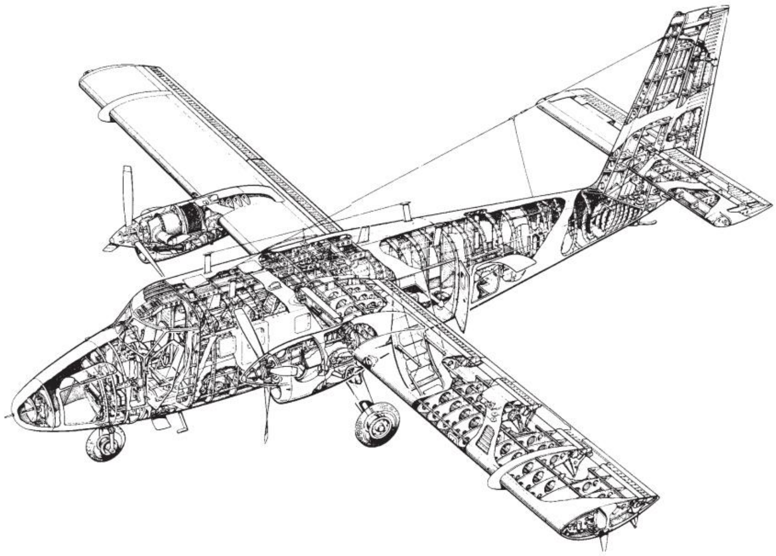

Figure 1: De Havilland Canada Twin Otter aircraft (Megson et al., 2007)

The primary function of a wing *skin* is to form an impermeable surface to support the aerodynamic pressure distribution due lift forces at the wings. These aerodynamic forces are transmitted to the *ribs* and *stringers* by the *skin*, via the forces action on the plates. The resistance to the shear and the torsion loads is provided by shear forces due the interaction of skin and spar webs, while the axial and torsion loads are resisted by the combination of the *skin* and *stringers* action. Although the *skin* is efficient to resist shear and torsion, it can be

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

easily bent, even under no significant compressible loads. A possible solution could be increase the plate thickness, which can lead to overweight problems.

2 SOPHIE-GERMAIN LAGRANGE EQUATION

The purpose of the discretization methods is reducing a continue problem in a system with a finite degrees of freedom number. In general, there is a physic phenomenon governed by a differential equation or a system of them, involving one or more functions. The solution of these equations, for general boundary conditions, by finding a closed form function is usually not possible. But a good approximation can be obtained by a limited number of parameters, if the analyst has a good sensitivity. These approximations, basically, involve exchanging a continue domain by a mash, with discrete points inside the domain. Instead of taking the continue function to these domain, we provide approximations to the value which this function assumes in each isolated point. In plates, the differential equation that govern the phenomenon is the Sophie-Germain and Lagrange equation:

$$\nabla^4 w = \frac{p}{D} \tag{1}$$

where

$$D = \frac{Et^3}{12(1-\nu^2)}$$

The sought for function, $w=w(x,y)$, is the transversal displacement of the surface points, and the *fourth order degree nabla operator* can be written as:

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{2}$$

In addition, the boundary conditions should be imposed at the plate faces. For example, if there is a simple support, the bending moments are equal zero and, therefore, the second derivative is also zero on this extremity. But, when there is a totally clamped face, rotation there does not exist, that is, the first derivative is zero.

3 FINITE DIFFERENCES METHOD

In the Finite Differences Method, all the derivates present in the differential equations inside the domain and the boundary conditions, are approximated by difference expressions, obtained by Taylor series, in general. The procedure was applied, for the first time, in two-dimensional domains, as in this case, by C. Runge, in 1908. The necessary expressions, for the purpose of this work are shown in what follows, for a point pertaining to a j line and a k column, from a rectangular mash with h_x spacing at the x direction and h_y at the y direction.

$$\left(\frac{\partial w}{\partial x}\right)_{j,k} = \frac{w_{j,k+1} - w_{j,k-1}}{2h_x} \tag{3}$$

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$$\left(\frac{\partial w}{\partial y}\right)_{j,k} = \frac{w_{j+1,k} - w_{j-1,k}}{2h_y} \tag{4}$$

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_{j,k} = \frac{w_{j,k+1} - 2w_{j,k} + w_{j,k-1}}{{h_x}^2} \tag{5}$$

$$\left(\frac{\partial^2 w}{\partial y^2}\right)_{j,k} = \frac{w_{j+1,k} - 2w_{j,k} + w_{j-1,k}}{{h_y}^2} \tag{6}$$

$$\left(\frac{\partial^4 w}{\partial x^4}\right)_{j,k} = \frac{w_{j,k+2} - 4w_{j,k+1} + 6w_{j,k} - 4w_{j,k-1} + w_{j,k-2}}{{h_x}^4} \tag{7}$$

$$\left(\frac{\partial^4 w}{\partial y^4}\right)_{j,k} = \frac{w_{j+2,k} - 4w_{j+1,k} + 6w_{j,k} - 4w_{j-1,k} + w_{j-2,k}}{{h_y}^4} \tag{8}$$

$$\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_{j,k} = \frac{w_{j+1,k-1} - 2w_{j+1,k} + w_{j+1,k+1} - 2w_{j,k-1} + 4w_{j,k} - 2w_{j,k+1} + w_{j-1,k-1} - 2w_{j-1,k} + w_{j-1,k+1}}{{h_x}^2{h_y}^2} \tag{9}$$

3.1 SQUARE PLATE EXAMPLE

Consider a square plate simply supported in its four borders, with *2x2 m* sides, $t = 2$ *cm* thickness, transversally loaded - from top to bottom - by uniformly distributed load $p = 2$ *KPa*, modulus of elasticity $E = 200$ *GPa* and Poisson ratio $\nu = 0.3$, so that it is possible to obtain the flexural stiffness $D = 146.52$. For comparison, the transversal displacement will be calculated for two mashes, one with $h_x = h_y = 1$ *m* and another one with $h_x = h_y = 0.5$ *m*. For the coarser mesh, Fig. 2, there is just one point at the interior domain and is necessary to consider external points, outside the domain, for applying the differential equation in terms of finite differences.

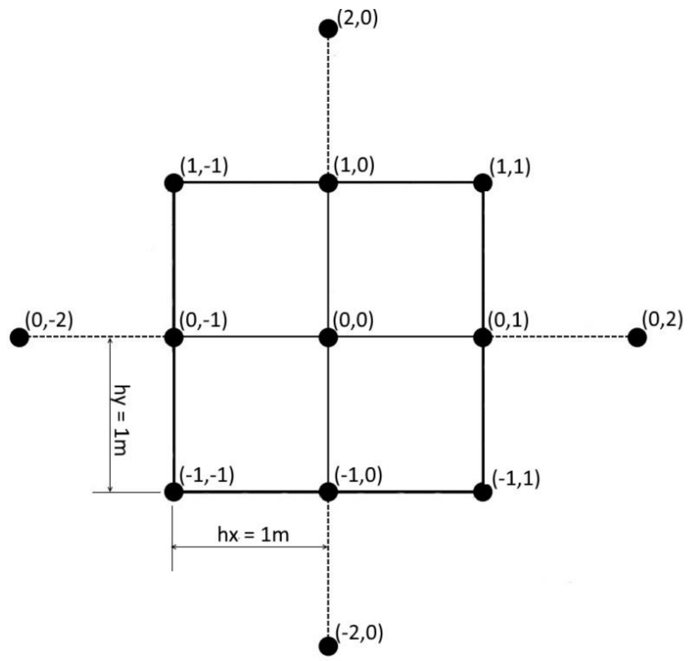


Figure 2: Simple supported square plate with two subdivisions mash

At the simple supported border lines, the transversal displacements are zero and, the bending moment (second derivative), is also zero. It means that the external point displacements are equal to minus the internal point displacement. That way is obtained:

$$16w_{0,0} = 2/146.52 = 0.01365 \qquad w_{0,0} = 0.8531 \text{ mm}$$

For the refined mesh, Fig. 3, there's nine points at the interior domain. Considering symmetry, is sufficient applying the differential equation based on the finite differences in just three internal points.

Point 0,0

$$w_{0,2} - 8w_{0,1} + 20w_{0,0} - 8w_{0,-1} + w_{0,-2} + w_{2,0} - 8w_{1,0} - 8w_{-1,0} + w_{-2,0}$$

$$+ 2\left(w_{1,1} + w_{1,-1} + w_{-1,1} + w_{-1,-1}\right) = 0.5^4 \frac{p}{D}$$

Point 0,1

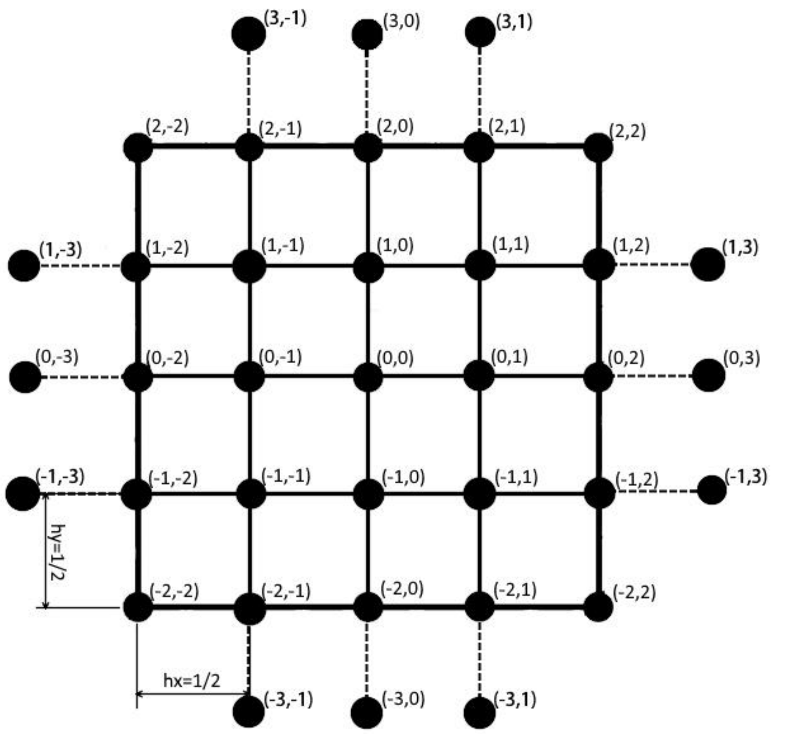
$$w_{0,3} - 8w_{0,2} + 20w_{0,1} - 8w_{0,0} + w_{0,-1} + w_{2,1} - 8w_{1,1} - 8w_{-1,1} + w_{-2,1}$$

$$+ 2\left(w_{1,2} + w_{1,0} + w_{-1,2} + w_{-1,0}\right) = 0.5^4 \frac{p}{D}$$

Point 1,1

$$w_{1,3} - 8w_{1,2} + 20w_{1,1} - 8w_{1,0} + w_{1,-1} + w_{3,1} - 8w_{2,1} - 8w_{0,1} + w_{-1,1}$$

$$+ 2\left(w_{2,2} + w_{2,0} + w_{0,2} + w_{0,0}\right) = 0.5^4 \frac{p}{D}$$



Considering the boundary conditions and symmetry, is possible to obtain the linear algebraic system with 3 equations and 3 unknowns.

$$\begin{bmatrix} 20 & -32 & 8 \\ -8 & 24 & -16 \\ 2 & -16 & 20 \end{bmatrix} \begin{Bmatrix} w_{0,0} \\ w_{0,1} \\ w_{1,1} \end{Bmatrix} = 0.5^4 \text{x} 0.01365 \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$w_{0,0} = 0.8798 \text{ mm}$$

For comparison, the result obtained by Timoshenko (1956) using trigonometric series was $w_{0,0} = 0.8867$ mm.

3.2 RECTANGULAR EXAMPLE

Consider the rectangular plate, displayed in Fig. 4, simply supported at the *x* parallel borders and clamped on the other 2 sides, with *8x4 m* dimensions, t = 5cm thickness, transversally loaded - from top to bottom - by uniformly distributed load *p = 0.1 kPa,* modulus of elasticity *E = 3 GPa* and Poisson ratio ν *= 0.25.*

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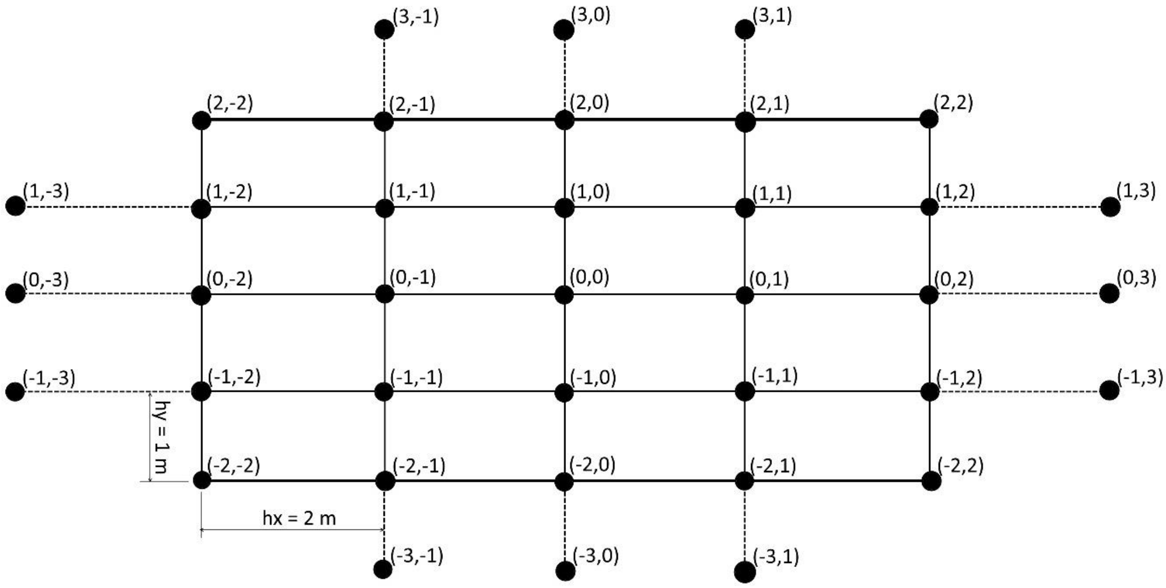

Figure 3: Rectangular plate with four subdivisions mesh

Through the symmetries in Fig. 4, it is enough to apply the differential equation in four points, named (0,0), (0,1), (1,0) and (1,1).

Point 0,0

$$\frac{1}{16}\left(w_{0,2}-4w_{0,1}+6w_{0,0}-4w_{0,-1}+w_{0,-2}\right)+\frac{1}{1}\left(w_{2,0}-4w_{1,0}+6w_{0,0}-4w_{-1,0}+w_{-2,0}\right)+$$

$$+\frac{2}{4}\left(w_{1,1}-2w_{1,0}+w_{1,-1}-2w_{0,1}+4w_{0.0}-2w_{0,-1}+w_{-1,1}-2w_{-1,0}+w_{-1,-1}\right)=\frac{p}{D}=0.003$$

Point 0,1

$$\frac{1}{16}\left(w_{0,3}-4w_{0,2}+6w_{0,1}-4w_{0,0}+w_{0,-1}\right)+\frac{1}{1}\left(w_{2,1}-4w_{1,1}+6w_{0,1}-4w_{-1,1}+w_{-2,1}\right)+$$

$$+\frac{2}{4}\left(w_{1,2}-2w_{1,1}+w_{1,0}-2w_{0,2}+4w_{0.1}-2w_{0,0}+w_{-1,2}-2w_{-1,1}+w_{-1,0}\right)=\frac{p}{D}=0.003$$

Point 1,0

$$\frac{1}{16}\left(w_{1,2}-4w_{1,1}+6w_{1,0}-4w_{1,-1}+w_{1,-2}\right)+\frac{1}{1}\left(w_{3,0}-4w_{2,0}+6w_{1,0}-4w_{0,0}+w_{-1,0}\right)+$$

$$+\frac{2}{4}\left(w_{2,1}-2w_{2,0}+w_{2,-1}-2w_{1,1}+4w_{1.0}-2w_{1,-1}+w_{0,1}-2w_{0,0}+w_{0,-1}\right)=\frac{p}{D}=0.003$$

Point 1,1

$$\frac{1}{16}\left(w_{1,3}-4w_{1,2}+6w_{1,1}-4w_{1,0}+w_{1,-1}\right)+\frac{1}{1}\left(w_{3,1}-4w_{2,1}+6w_{1,1}-4w_{0,1}+w_{-1,1}\right)+$$

$$+\frac{2}{4}\left(w_{2,2}-2w_{2,1}+w_{2,0}-2w_{1,2}+4w_{1.1}-2w_{1,0}+w_{0,2}-2w_{0,1}+w_{0,0}\right)=\frac{p}{D}=0.003$$

Considering the boundary conditions and symmetry, is possible to obtain a linear algebraic system with 4 equations and four unknowns.

$$\begin{bmatrix} 8.375 & -2.5 & -10 & 2 \\ -1.25 & 8.5 & 1 & -10 \\ -5 & 1 & 8.375 & -2.5 \\ 0.5 & -5 & -1.25 & 8.5 \end{bmatrix} \begin{Bmatrix} w_{0,0} \\ w_{0,1} \\ w_{1,0} \\ w_{1,1} \end{Bmatrix} = 0.003 \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

The solution is

$$\begin{Bmatrix} w_{0,0} \\ w_{0,1} \\ w_{1,0} \\ w_{1,1} \end{Bmatrix} = 0.001 \begin{Bmatrix} 7.09 \\ 5.20 \\ 5.09 \\ 3.74 \end{Bmatrix} m$$

It means that, for the central node, the transversal displacement is $w_{0,0} = 0.00709\ m$. Now, from the displacement field, it is possible to obtain the bending moments field in the *x* and *y* directions – respectively – in terms of finite differences.

$$M_x = -D\left(\nu \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right) \tag{10}$$

$$M_y = -D\left(\nu \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{11}$$

Equations 10 and 11 can be applied for any points at the interior plate domain to get the bending moments field. Applying these equations, in terms of finite differences, is possible to obtain:

$$M_{x_{0,0}} = 0.0649752\ \frac{kNm}{m}$$

$$M_{y_{0,0}} = 0.1415714\ \frac{kNm}{m}$$

4 NUMERICAL RESULTS

As proposed for the present work, after presentation of the theoretical foundation, the analyses become a quantitative object of study, in the computational point of view. That way, is possible to compare the results obtained by finite differences method and the finite elements method, and with the exact solutions obtained by Timoshenko (1956) using trigonometric series. Thus, an algorithm has been developed based on the finite differences method, using the Matlab® software. The inputs are the plate dimensions, material properties and the boundary conditions. The outputs are the displacement and the bending moments fields, as well as 3D deformed surface plot.

For the same problems and boundary conditions, the *Abaqus*^{TM} software has been used to obtain the finite elements method approximations, enabling a quantitative and qualitative analyses between the results, and comparing these with the values found in the literature, obtained by Timoshenko.

4.1 GEOMETRY ADOPTED FOR SIMULATIONS

For the simulations, a plate with the same dimensions of the plate shown in Fig. 4 has been adopted, with *8x4 m* sides, $t = 5\ cm$ thickness, transversally loaded - from top to bottom - by uniformly distributed load $p = 0{,}1\ kPa$, modulus of elasticity $E = 3\ GPa$ and Poisson ratio $\nu = 0.3$.

4.2 BOUNDARY CONDITIONS ADOPTED

The models, as well as the boundary conditions adopted for simulations were the same examples used and tabulated by Timoshenko, for thin plates under uniform load. We used rectangular plate models with the edges simply supported, all edges clamped, one edge clamped with the rest of them simply supported and, finally, a rectangular plate with two edges clamped with another two simply supported edges. The results for each boundary conditions, tabulated by Timoshenko, and used in this work, can be found in the table presented on the Table 1.

5 RESULTS

After the reference values were calculated, for the boundary conditions used by Timoshenko, we carried out the computational simulations for the same models and performed a comparative analysis between the outputs in each case. For the analysis, four decimal places and up to six significant digits were adopted, with truncation rules for the last digit.

5.1 PLATES WITH EDGES SIMPLY SUPPORTED

For the plate with the edges simply supported and mesh with four subdivisions, it was possible to obtain with our algorithm approximations as shown by the Fig. 5. Obviously, that is a “coarse” approximation, and has been obtained the following displacement and bending moments values:

$$w_{max} = 0.007512\ m$$

$$M_{x_{max}} = 0.073271\ \frac{kNm}{m}$$

$$M_{y_{max}} = 0.155022\ \frac{kNm}{m}$$

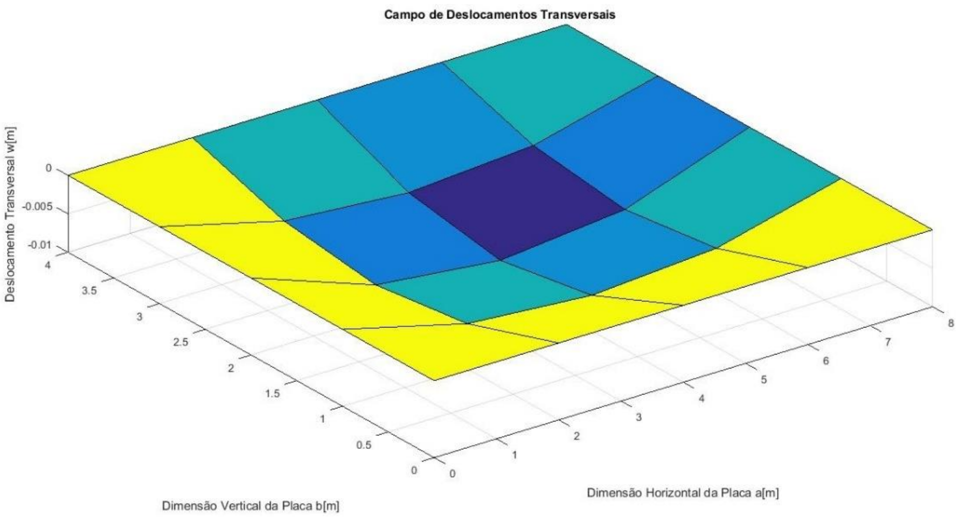

Figure 4: The transversal displacement field and mesh with four subdivisions

Using a mesh with more subdivisions, is possible to obtain a better approximation for the displacement fields and the bending moments, as shown in Fig. 6.

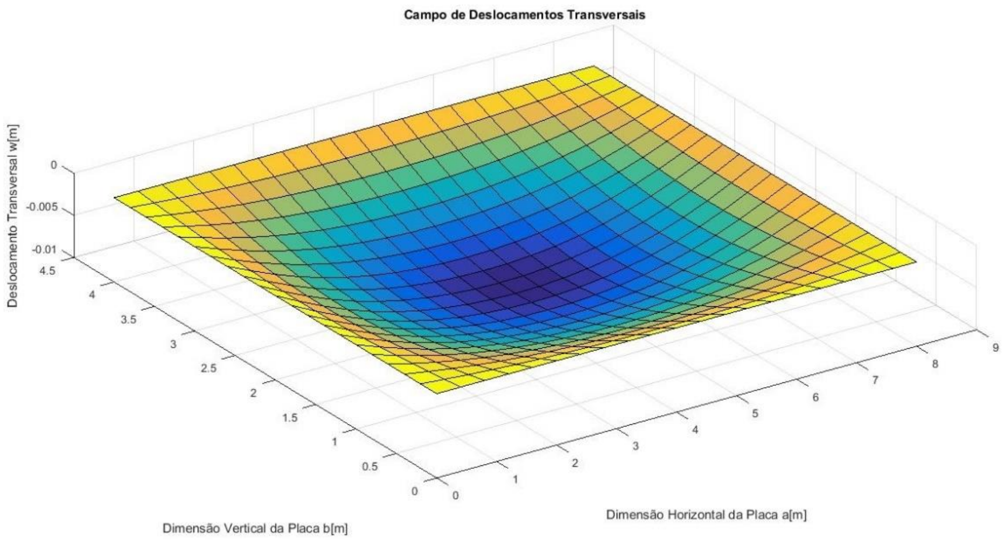

Figure 5: The transversal displacement field and mesh with twenty subdivisions

For the mesh with the twenty subdivisions, we obtained the following approximations by finite differences:

$$w_{max} = 0.007549\ m$$

$$M_{x_{max}} = 0.07412\ \frac{kNm}{m}$$

$$M_{y_{max}} = 0.162359\ \frac{kNm}{m}$$

For the simulations executed by finite elements and mesh with four subdivisions, we obtained de following approximations:

$$w_{max} = 0.007760\ m$$

$$M_{x_{max}} = 0.070770\ \frac{kNm}{m}$$

$$M_{y_{max}} = 0.143400\ \frac{kNm}{m}$$

The graphical representations for the displacement fields and the bending moments can be seen in the Figs. 7, 8 and 9 – respectively.

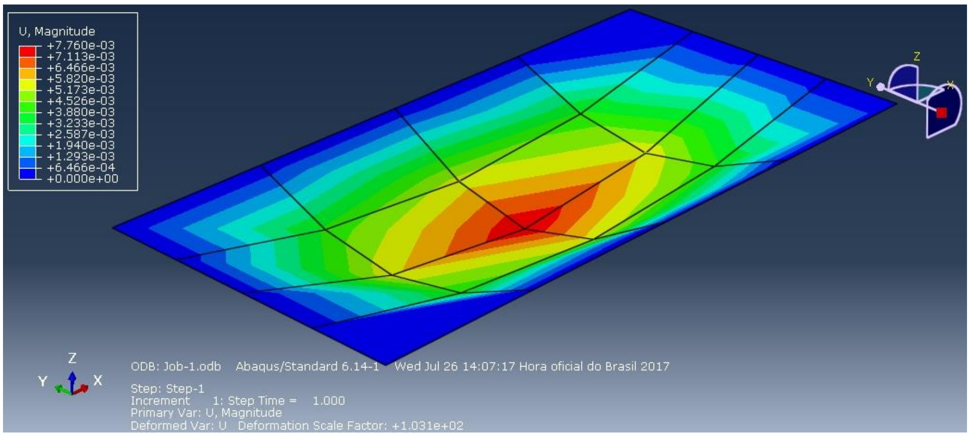

Figure 6: The transversal displacement field and mesh with four subdivisions

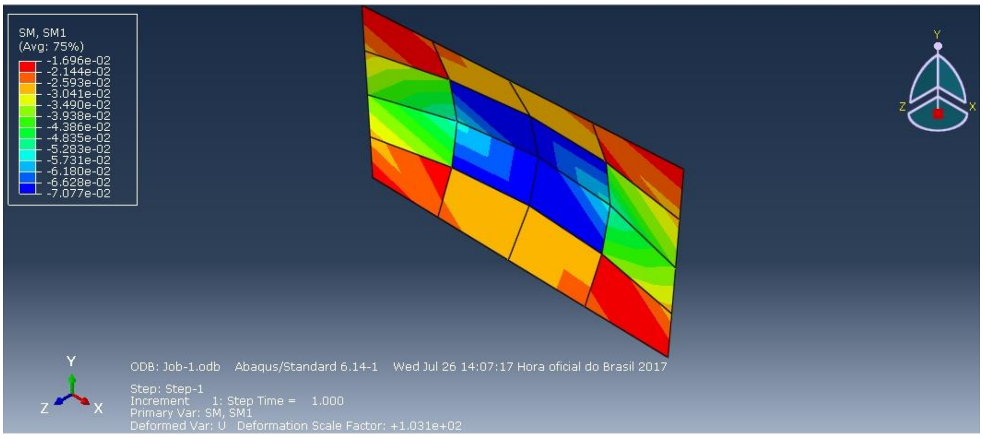

Figure 7: The bending moment field at the *x* direction and mesh with four subdivisions

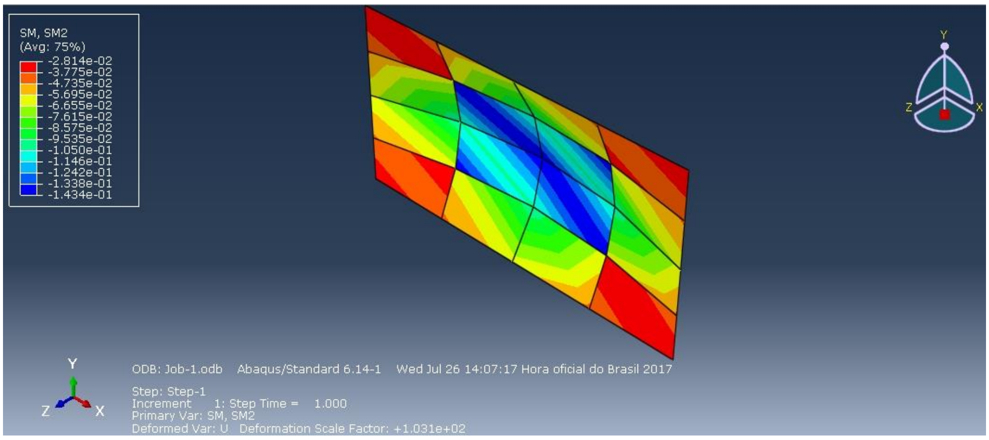

Figure 8: The bending moment field at the *y* direction and mesh with four subdivisions

For the simulations executed by finite elements and mesh with twenty subdivisions, we obtained de following approximations:

$$w_{max} = 0.007570\ m$$

$$M_{x_{max}} = 0.074320\ \frac{kNm}{m}$$

$$M_{y_{max}} = 0.162200\ \frac{kNm}{m}$$

The graphical representations for the displacement fields and the bending moments can be seen in the Figs. 10, 11 and 12 – respectively.

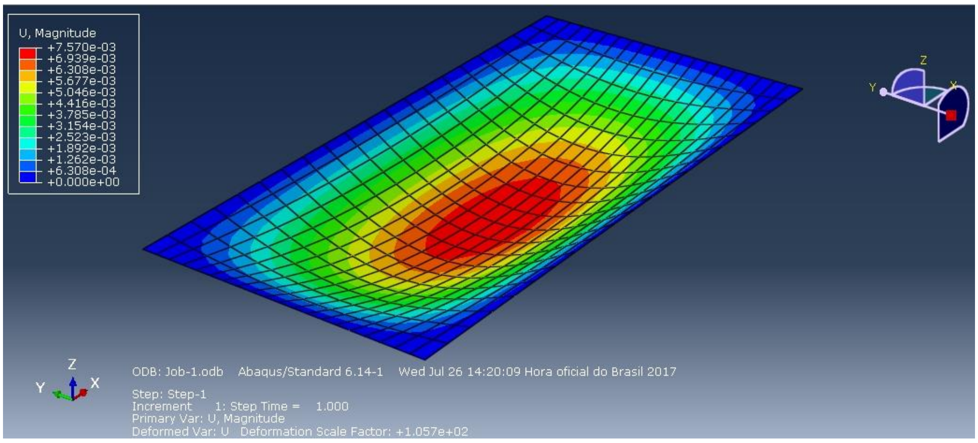

Figure 9: The transversal displacement field and mesh with twenty subdivisions

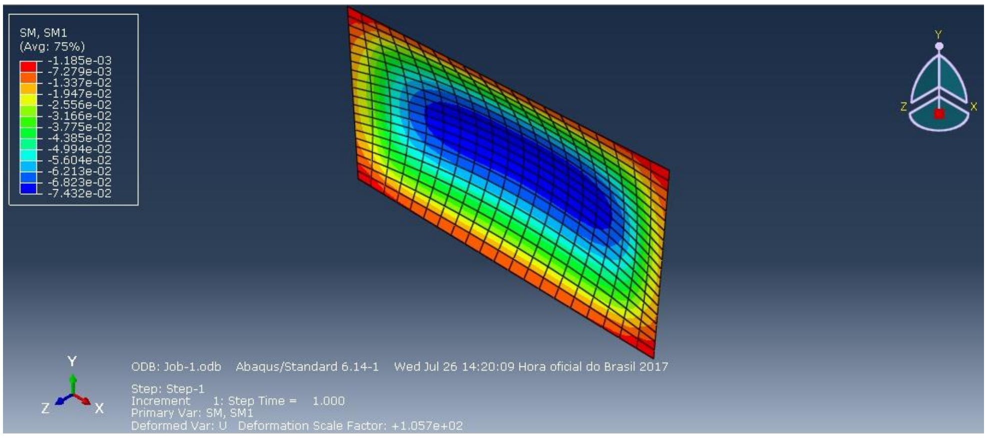

Figure 10: The bending moment field at the *x* direction and mesh with twenty subdivisions

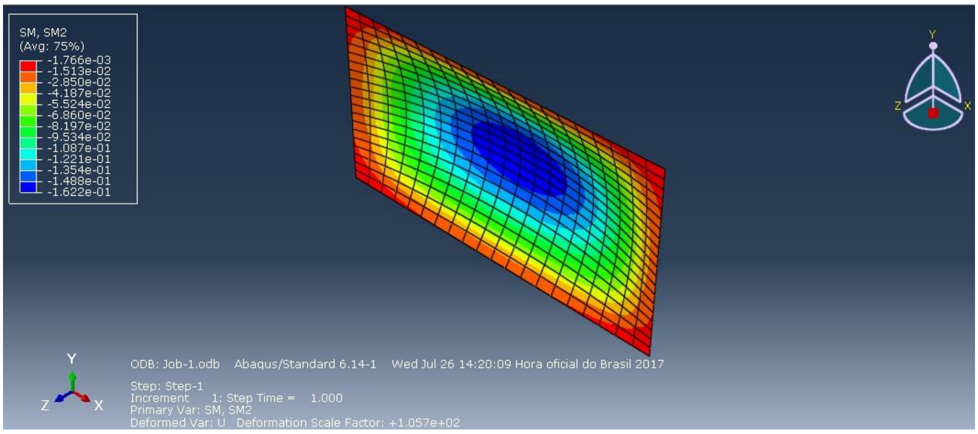

Figure 11: The bending moment field at the *y* direction and mesh with twenty subdivisions

5.2 OTHER BOUNDARY CONDITIONS

In addition to the case where the plate was in four simply supported edges, simulations for the cases where the plate was with all edges clamped, one edge clamped with the rest of them simply supported and, finally, a rectangular plate with two edges clamped with another two simply supported edges, were also made. The results for these simulations can be seen in Table 1.

Table 1. Results

SIMPLY SUPPORTED EDGES			
	FINITE ELEMENTS	FINITE DIFERENCES	TIMOSHENKO

MESH SUBDIVISIONS	4	20	4	20	
W [m]	0.007760	0.007570	0.007512	0.007549	0.007516
Mx [kNm/m]	0.070770	0.074320	0.073271	0.074124	0.074240
My [kNm/m]	0.143400	0.162200	0.155022	0.162359	0.162472
ALL EDGES CLAMPED					
	FINITE ELEMENTS		FINITE DIFERENCES		TIMOSHENKO
MESH SUBDIVISIONS	4	20	4	20	
W [m]	0.002000	0.001880	0.002704	0.001924	0.001893
Mx [kNm/m]	0.028280	0.028580	0.029729	0.025360	0.025280
My [kNm/m]	0.055240	0.065350	0.071153	0.066080	0.065920
ONE EDGE CLAMPED WITH THE REST OF THEM SIMPLY SUPPORTED					
	FINITE ELEMENTS		FINITE DIFERENCES		TIMOSHENKO
MESH SUBDIVISIONS	4	20	4	20	
W [m]	0.003533	0.003757	0.004315	0.003667	0.003653
Mx [kNm/m]	0.035270	0.037950	0.043214	0.037823	0.036800
My [kNm/m]	0.081660	0.082950	0.099547	0.096381	0.09600
TWO EDGES CLAMPED WITH ANOTHER TWO SIMPLY SUPPORTED					
	FINITE ELEMENTS		FINITE DIFERENCES		TIMOSHENKO
MESH SUBDIVISIONS	4	20	4	20	
W [m]	0.001741	0.001939	0.002775	0.001982	0.001938
Mx [kNm/m]	0.020300	0.026110	0.029134	0.022934	0.022720
My [kNm/m]	0.055450	0.066790	0.072816	0.067548	0.067200

Table 2. Percentage Errors

SIMPLY SUPPORTED EDGES				
%	FINITE ELEMENTS		FINITE DIFERENCES	
MESH SUBDIVISIONS	4	20	4	20
W [m]	3.25	0.72	0.05	0.44
Mx [kNm/m]	4.67	0.11	1.31	0.16
My [kNm/m]	11.74	0.17	4.59	0.07
ALL EDGES CLAMPED				
%	FINITE ELEMENTS		FINITE DIFERENCES	
MESH SUBDIVISIONS	4	20	4	20
W [m]	5.65	0.69	42.84	1.64
Mx [kNm/m]	11.87	13.05	17.60	0.32
My [kNm/m]	16.20	0.86	7.94	0.24
ONE EDGE CLAMPED WITH THE REST OF THEM SIMPLY SUPPORTED				
%	FINITE ELEMENTS		FINITE DIFERENCES	
MESH SUBDIVISIONS	4	20	4	20
W [m]	3.28	2.85	18.12	0.38
Mx [kNm/m]	4.16	3.13	17.43	2.78
My [kNm/m]	14.94	13.59	3.69	0.40
TWO EDGES CLAMPED WITH ANOTHER TWO SIMPLY SUPPORTED				
%	FINITE ELEMENTS		FINITE DIFERENCES	
MESH SUBDIVISIONS	4	20	4	20

W [m]	10.17	0.05	43.19	2.27
Mx [kNm/m]	10.65	14.92	28.23	0.94
My [kNm/m]	17.49	0.61	8.36	0.52

6 DISCUSSION

According to the present work, it is possible to conclude that the finite differences method presented better approximations for the displacement fields *w* and bending moments M_x and M_y, when compared with the approximations obtained by finite elements method, for the same mesh and boundary for *simply supported edges*. These results are shown in Table 2, where it can be seen the percentage errors obtained in the simulations. In Table 2, it is also possible to observe that for the mesh with the lower resolution – four subdivisions, the finite differences method shows better convergence when compared to the finite elements method with 0.05%, 1.31% e 4.59% in percentage errors for the approximations to the displacement fields and bending moments – respectively. At the same time, the finite elements method – for the same mesh – presented 3.25%, 4.67% e 11.74% in percentage errors for the displacement fields and bending moments – respectively.

For the same problem, the subdivisions number were increased, resulting in a mesh with twenty subdivisions which provided a better convergence for the approximations in both methods. However, the finite difference method presented lower percentage errors (0.44%, 0.16%, 0.07%), when compared to the finite elements method (0.72%, 0.11%, 0.17%). Is also possible to observe that when the subdivisions mesh increases, the maximum displacement field approximation obtained by finite differences increased slightly, compared to the mash with just four subdivisions, also by finite differences.

The situation also changes as the boundary conditions is changed, especially when some edge is clamped or even all of them. An example of this case is when the plate is with all of the edges clamped and, as it is possible to observe in the Table 2, the approximations by finite differences are considerably worse, with 42.84%, 17.60% e 7.94% percentage errors compared to the finite elements method, with 5.65%, 11.87% e 16.20% percentage errors. Is also possible to observe that the percentage errors for both methods were reduced as the subdivisions mesh number were increased. In this case, it can be noticed that the M_y approximation presented highest percentage error to the twenty subdivisions mesh compared to the four subdivisions mesh. As the same way, the M_x approximation presented lower percentage error by finite differences to the twenty subdivisions mesh while the M_y approximation for the both mashes have presented lower percentage errors by finite differences.

In the case where just the lower edges were clamped, all the approximations were better in terms of the convergence as the subdivisions mesh number were increased. For the four subdivisions mesh, the $w_{center,}$ and $M_{xcenter}$ approximations for the central mesh node were better in terms of convergence to the finite elements method, however, the M_y approximation presented lower percentage error by finite differences. In this context, when the subdivisions

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

mesh number were increased, the finite differences method presented better approximations (0.38%, 2.78% e 0.40%) compared to the finite elements method (2.85% 3.13% e 13.59%), to the w_{center}, $M_{xcenter}$ e $M_{ycenter}$ – respectively.

Finally, for the case in which the plate is with higher and lower edges clamped and the other two edges simple supported, for the mesh with four subdivisions, the finite element method presented better approximations to the transversal displacement w_{max} and the $M_{xcenter}$ bending moment, presenting percentage errors around 10.17% e 10.65%, respectively. However, the finite differences method presented a better approximation to the M_{ymax} bending moment, with percent error around 8.36%, compared to the 17.49% by the finite elements method. For the mesh with twenty subdivisions, the $M_{xcenter}$ e M_{ymax} approximations were better by the finite differences method, with percentage errors around 0.94% e 0.52%, however, the w_{max} approximation were better by the finite element method, with 0.05% percentage error.

ACKNOWLEDGEMENTS

The authors acknowledge support by FAPESP, CAPES and CNPq, all Brazilian research funding agencies.

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