CODING THE 'NEW IMPLICIT METHOD' IN MATLAB FOR PRELIMINARY TUNNEL DESIGN

Wallace C. Ferrão

Denise Bernaud Maghous

wallace.ferrao@ufrgs.br

denise.bernaud@ufrgs.br

Federal University of Rio Grande do Sul - UFRGS

99 Osvaldo Aranha Ave – Centro Histórico, 90035-190. Porto Alegre, Rio Grande do Sul, Brazil. Programa de Pós-Graduação em Engenharia Civil.

Abstract. *The tunnel design is a subject that demands a three-dimensional analysis dealing with two different players: the lining and the rockmass. In most of the cases, the traditional methods of getting the necessary parameters for design: convergences (closure) and stresses, do not consider a correct interaction between the two players. Concerning this, the New Implicit Method (NIM), that uses principles like those of the Convergence-Confinement Method, shows that the lining stiffness influences the convergence in the moment of lining installation of the tunnel. The NIM develops formulations for time-independent conditions (elasticity and plasticity) and for time-dependent conditions (viscoplasticity) of rockmasses. A code in MatLab of NIM for rockmasses is developed and validated through Finite Element analysis in elasticity, plasticity for Tresca and Coulomb failure criteria and viscoplasticity with the same previous criteria. The FEM numerical analysis are performed with the numerical code GEOMEC91. This code uses the deactivation/reactivation method for the elements to simulate the steps of excavation and the lining installation. The obtained results give an average accuracy of 12%. The code in MatLab is simple and its results agree very well with the numerical results, what represents a good approximation regarding geotechnical issues.*

Keywords: *Tunnel design, New Implicit Method, Convergence, Confinement*

1 INTRODUCTION

Designing a tunnel support has been for many years considered very complex, even for experienced engineers. Most of the projects remain, empirically, analyzed and designed. This practice is getting unsuitable for tunnel support design, since it is being proved that the interaction of the soil and lining matters in a way to change the stresses taken by the concrete as well as the deformation caused by the rockmass (BERNAUD et al., 1996).

The *Association Française des Tunnels et de L'Espace Souterrain* (AFTES, 2001) classifies the tunnel support design methods into four types: (i) purely empirical methods indicating the most appropriate type of support for a situation defined from various geotechnical classification systems; (ii) methods for determining the loads acting on the support, regardless of support type and deformation; (iii) support design methods which consider loads exerted by the ground as input data with some integration of support stiffness, deformation and the reactions of the surrounding ground; (iv) and more recent methods taking full account of the ground/support interaction.

This paper is focused on the fourth classification and aims to present a simplified method for this coupled interaction. The code derived from this method can be used on previous analysis of the lining structure defining the stress and deformation on the concrete surface by considering the characteristics of the rockmass and it is a new tool based on the New Implicit Method developed by BERNAUD and ROUSSET (1996).

The determination of the load transferred to the support requires an analysis of the interaction of the load-deformation characteristics of the elements comprising the system ground/support. Let us consider a circular deep tunnel (radius R_i), driven in a homogeneous and isotropic rockmass [Fig. 1], initially subjected to a geostatic stress field:

$$\sigma_0 = -P_\infty \underline{\underline{1}} \text{ with } P_\infty = \rho\ g\ z \tag{1}$$

Where:

ρ: specific mass; g: gravity; z: mean depth.

With the assumption that the lining is a ring of constant thickness e, made of a homogeneous and isotropic material; and, moreover, that the lining is set at a distance d_0 from the plane and vertical tunnel face, the problem becomes an axisymmetric problem dependent only on the radial distance r to the tunnel axis and the time t.

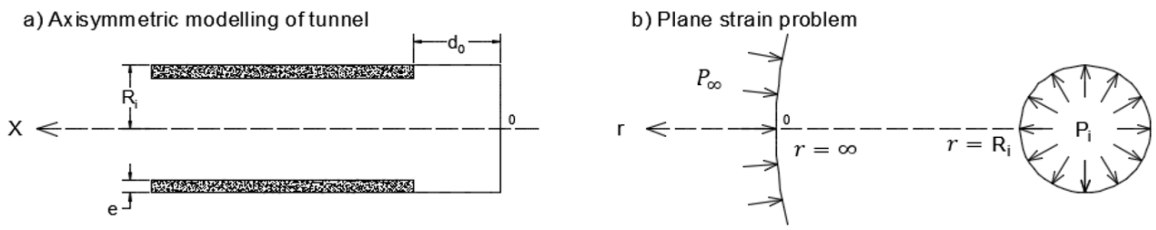

Figure 1. Simplification of the behavior of the problem

In the other hand, the deep section of the tunnel can be treated as a Plane Strain problem since the lining speed is constant (time dependent), so the parameters will depend only on the radial distance r as well.

Firstly, the most used method to predict the stresses upon the lining is the Convergence-Confinement method (AFTES, 2001) which is a procedure that allows the load imposed on a support installed behind the face of a tunnel to be estimated. Per CARRANZA and FAIRHURST (2000), when a section of support is installed in the immediate vicinity of the tunnel face, it does not carry the full load to which it will be subjected eventually; a part of the load is redistributed around the excavation face itself, however, as the tunnel and face advance, this ‘face effect’ decreases and the support must carry a greater proportion of the load that the face had carried earlier.

The method explained in the previous paragraph is defined by two curves [Fig. 2]: the convergence curve (or ground reaction curve) gives the internal pressure exerted at the tunnel wall by the rockmass. Depending on the behavior, it may be a straight line for an elastic situation, a curved line for a plastic case or both for an elastoplastic field. The CV dashed line in Fig. 2 illustrates an elastoplastic rockmass. The second curve is the confinement one (or support characteristic curve), it is the straight CF line in Fig. 2 and demonstrates an elastic lining. The curvature of it is defined as lining stiffness K_s.

The curves illustrate the relationship between the pressure P_i versus the closure of the lining ${U_i}^s$ and the meeting point is the equilibrium value for pressure exerted on the lining ring and its respective convergence.

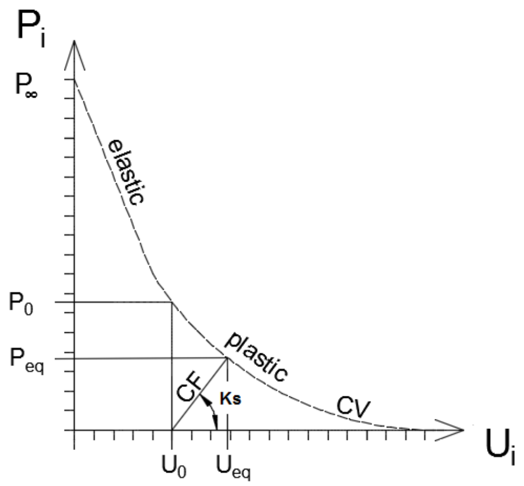

Figure 2. Tunnel equilibrium in a diagram Pi x Ui

The parameters in the equilibrium are:

U_o and P_0: the convergence (radial deformation) and the pressure at the distance d_0 (distance between the lining installation and tunnel face);

U_{eq} and P_{eq}: the convergence and the pressure at a section far from the tunnel face, where both pressure and convergence are stabilized;

R_p: the plastic radius is the limit between the elastic and plastic zone around the tunnel.

Eventually, the Convergence-Confinement is drawn and the safety factor can be obtained by comparing the concrete strength with the stress at a distance far from the tunnel face that is imposed by the rockmass upon the lining structure.

2 THE NEW IMPLICIT METHOD (NIM)

The New Implicit Method (NIM) was developed by BERNAUD and ROUSSET (1996) and consists on deducing the curve of convergence with a geometrical transformation from unsupported to supported tunnel. The transformation is a function $a^0(x)$ of the unsupported tunnel and it was approximated by:

$$a^0(x) = \frac{U_i^f(x) - U_i^f(0)}{U_i^f(\infty) - U_i^f(0)} = 1 - \left(\frac{0.84R_p}{x + 0.84R_p}\right)^2 \tag{2}$$

Where:

R_p: is the plastic radius.

Where $U_f = U_i^f(0)$ is the convergence at the tunnel face of the unsupported tunnel and $U_i^f(\infty) = U_\infty$ is the convergence of the unsupported tunnel far from the tunnel face. The NIM proved that $a^s(x)$ depends on the K_s (stiffness of the lining) that had not been predicted by the CV-CF method; so, a function for the supported tunnel is proposed:

$$a^s(x) = a^0(\alpha x), \text{with } \alpha = \alpha(K_s') \tag{3}$$

Then, for a given constitutive law of rockmass, the application of the NIM can be summarized by the following steps:

Choose the reduced lining function $\alpha^*(K_s')$.Then, for the supported tunnel, equation (3) becomes:

$$a^s(x) = 1 - \left(\frac{0.84R_p}{\alpha x + 0.84R_p}\right)^2 \tag{4}$$

$$a^s(x) = 1 - \left(\frac{0.84}{{\alpha x}/{R_p} + 0.84}\right)^2 \tag{5}$$

$$x' = \frac{x}{R_i};\ \alpha^* = \alpha\frac{R_i}{R_p};\ a^s(x) = 1 - \left(\frac{0.84}{\alpha^* \frac{x}{R_i} + 0.84}\right)^2 \tag{6}$$

$$a^s(x') = 1 - \left(\frac{0.84}{\alpha^* x' + 0.84}\right)^2 \tag{7}$$

The parameter N_s gives an idea of the plastification of the tunnel and, for Tresca, it is given in function of the cohesion (C) as follows:

$$N_s = \frac{P_\infty}{C} \tag{8}$$

The function $\alpha^s(x')$ is given from (7) depending on the lining function (α^*) that is illustrated in Figure 3:

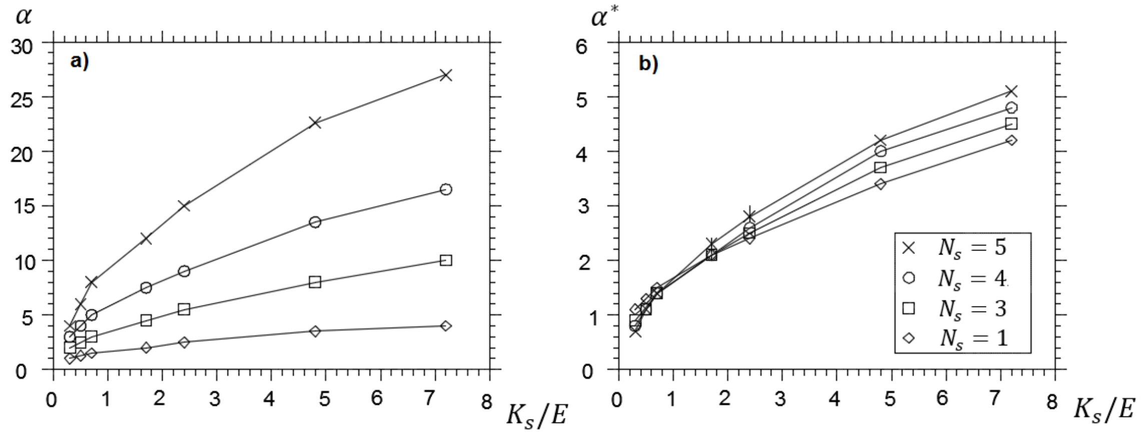

Figure 3. Lining function α (a) and non-dimensional lining function α^* (b) for many N_s.

Where:

K_s: is the lining stiffness; E: Young`s Modulus of the rockmass.

The approximate method by BERNAUD and ROUSSET (1996) proposed an average function for the reduced lining function, so $\alpha^* = \alpha^*_{average}$ which is illustrated in Figure 4:

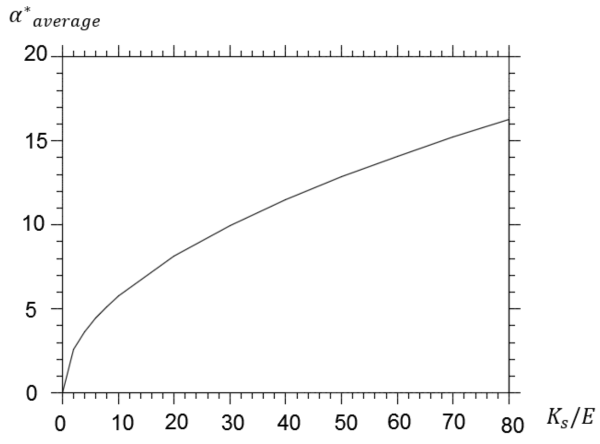

Figure 4. Lining function $\alpha^*_{average}$.

$$\alpha^* = \alpha^*_{average} = 1.82\sqrt{K_s'} \tag{9}$$

Choose the convergence of the tunnel face U_f.

The solution of the problem is then obtained by solving the following equations for the convergence CV and confinement CF curves:

$$P_{eq} = C_F(U_0, U_{eq}) \tag{10}$$

$$P_{eq} = C_V(U_{eq}) \tag{11}$$

In the system (10), the convergence U_0 is a function of U_{eq} (that explains the name of 'implicit method'). By (3) and (10) we can write:

$$\mathrm{U_0 = a^s(d_0)(U_{eq} - U_f) + U_f} \tag{12}$$

Where:

$\mathrm{d_0}$: is the distance between the lining and tunnel face.

Figure 5 sums up how the NIM method works integrating the two charts which correlate the convergence, the pressure and the location of the studied section by setting the lining supported function and getting the value for U_0 and its respective pressure.

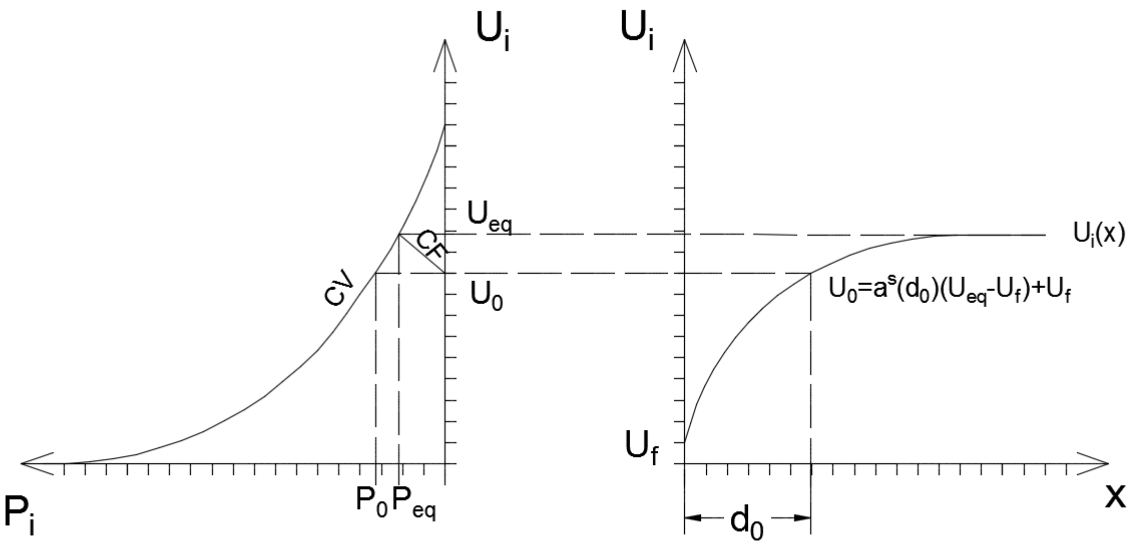

Figure 5. Calculation of a tunnel with the NIM.

Some dimensionless parameters are going to be introduced to turn the following equations valid for any incompressible rockmass.

Elastic Solution

$$P'_\infty = \frac{P_\infty}{E}; K'_s = \frac{K_s}{E}; d'_0 = \frac{d_0}{R_i} \tag{13}$$

Where:

P_∞: Initial geostatic stress;

E: Modulus of Young of the rockmass;

K_s: Stiffness of the lining.

Plastic Solution with a Tresca Criterion

$$P'_\infty = \frac{P_\infty}{E}; K'_s = \frac{K_s}{E}; d'_0 = \frac{d_0}{R_i};\ C' = \frac{C}{E} \tag{14}$$

Where:

C: cohesion of the rockmass for a Tresca Criterion.

The dimensionless parameters for a plastic criterion of Coulomb and for viscoplastic rockmasses are going to be shown as the text follows, and the main added parameter is going to be related to the friction angle. Figure 6 shows that when using the surface of Coulomb the friction angle ϕ is greater than zero.

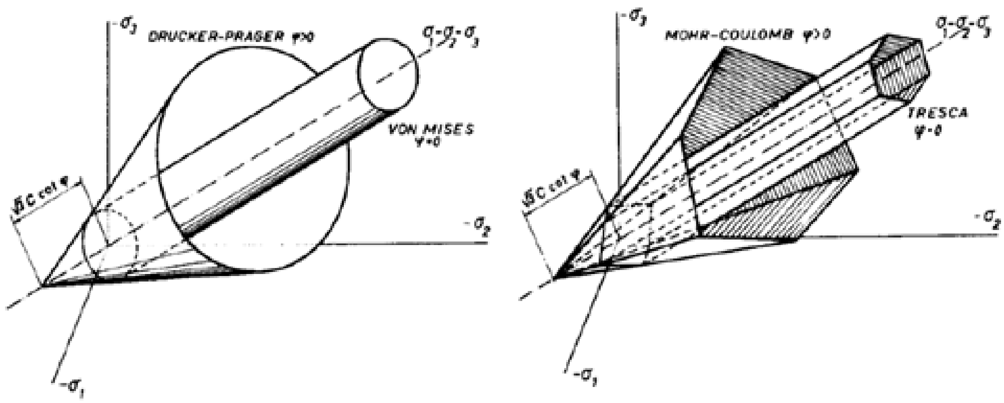

Figure 6. Plasticity surfaces criteria. (ZIENKIEWICZ & CORMEAU, 1974)

Figure 6 shows that for cohesive material, the surfaces of Tresca and Von-Mises are adequate, however for friction materials require either Drucker-Prager or Coulomb surface. So, the selection of the adopted criterion depends on the characteristics of the rockmass material.

The value of the convergence at equilibrium as well as the stress at equilibrium are two of the driving characteristics to build a tunnel, and as the text follows, the solution of the incognitos cited before are going to be given for different behavior of the rockmasses depending upon the geology, type and creep characteristic of the rock around the lining.

2.1 Elastic Rockmass

The steps to follow by the NIM to find the convergence and pressure at equilibrium are as follow:

Get the entry parameters: $P_\infty, E, K_s, d_0, \nu$ (Poisson coefficient of the rockmass) and R_i;

Calculate the convergence at P_∞ and at the tunnel face (U_f), as follow:

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$$U_\infty = \frac{1+\nu}{E} P_\infty;\ U_f = 0.27 U_\infty \tag{15}$$

So the function $a^s(x)$ from (7) is going to be changed by replacing the function α^* as it is given in terms of average solution from (9) where $R_p = R_i$:

$$a^s(x) = 1 - \left(\frac{0.84}{1.82\sqrt{K_s' \frac{x}{R_i}} + 0.84} \right)^2 \tag{16}$$

$$\mathrm{a^s}(\mathrm{d_0}) = 1 - \left(\frac{0.84}{1.82\sqrt{\mathrm{K_s'}} \frac{\mathrm{d_0}}{\mathrm{R_i}} + 0.84} \right)^2 \tag{17}$$

Calculate the convergence and stress at equilibrium:

$$\mathrm{U_{eq}} = \frac{\mathrm{P_\infty} + \mathrm{K_s U_f}\left(1 - \mathrm{a^s}(\mathrm{d_0})\right)}{\mathrm{K_s}\left(1 - \mathrm{a^s}(\mathrm{d_0})\right) + \frac{\mathrm{E}}{1+\nu}} \tag{18}$$

$$\mathrm{P_{eq}} = \mathrm{P_\infty} - \frac{\mathrm{E}}{1+\nu}\mathrm{U_{eq}} \tag{19}$$

The convergence at d_0 is given by (12).

2.2 Plastic Rockmass

The rockmasses that present a cohesion and a friction angle will be treated here with Tresca ($\phi = 0$), Coulomb ($\phi > 0$) and Von Mises ($\phi = 0$) criteria. However, some other criteria like Hoek-Brown is valid and presented in the original text of NIM (BERNAUD, D. *et al*, 1994).

Tresca criterion

This criterion is valid for rockmasses whose main geotechnical characteristic is given by the cohesion. The steps to follow by the NIM to find the convergence and pressure at equilibrium are as follow:

Get the entry parameters: $P_\infty, E, K_s, d_0, C, \nu = 0.5$ and R_i.

The reduced non-dimensional lining function for Tresca is the function given in (9).

Calculate the convergence U_∞ and at the tunnel face U_f, as follows:

$$U_\infty = \frac{1.5\mathrm{C}}{E} e^{(N_s - 1)}\ for\ incompressible\ material\ (\nu = 0.5) \tag{20}$$

$$\frac{U_f}{U_\infty} = 0.413 - 0.0627N_s \; for \; 1 < N_s \leq 5 \tag{21}$$

The $a^s(d_0)$ is the same as the shown one in (16).

The solution of this criterion valid for $P_{eq} < P_\infty - C$; if $P_{eq} > P_\infty - C$ the rockmass is elastic. So, solving the system (10), the convergence and the stress at equilibrium are as follow:

$$a \ln U_{eq} + bU_{eq} + c = 0 \tag{22}$$

$$\begin{cases} a = -\dfrac{C}{K_s}; b = a^s(d_0) - 1 \\ c = \left(1 - a^s(d_0)\right)U_f + \dfrac{P_\infty}{K_s} + \dfrac{C}{K_s}\left(\ln\left(\dfrac{1.5C}{E}\right) - 1\right) \end{cases} \tag{23}$$

$$P_{eq} = (U_{eq} - U_0)K_s \tag{24}$$

The convergence at d_0 is given by (12);

The plastic radius is:

$$R_p = R_i e^{-\left(\frac{P_{eq}-P_\infty}{2C}+1\right)} \tag{25}$$

For Von-Mises criterion, as it is shown in the Figure 6, its surface is completely curved, while Tresca is hexagonal. Mises does not present vertex when θ is thirty degrees and so the solution is given by the system (10) following the same steps listed for Tresca with C replaced by ${}^{2C}/_{\sqrt{3}}$.

Coulomb criterion

This criterion is valid for rockmasses whose main geotechnical characteristic is given by cohesion and friction angle when $P_{eq} < \frac{2}{K_p+1}\left\{P_\infty + \frac{H}{2}(1 - K_p)\right\}$. The steps to follow by the NIM to find the convergence and pressure at equilibrium are as follow:

Get the entry parameters: $P_\infty, E, K_s, d_0, \mathrm{C}, \phi, \nu = 0.5$ and R_i ;

For Coulomb, two new parameters depending on the friction angle ϕ are introduced:

$$R_c = \frac{2Ccos\phi}{1 - sin\phi} \tag{26}$$

$$N_s = \frac{2P_\infty}{R_c} \tag{27}$$

When the Friction Angle is zero, the solution is for Tresca criterion shown in (8).

For Coulomb, the reduced lining function (9) becomes:

$$\alpha^* = \alpha^*_{average} + 0.035\phi = 1.82\sqrt{K_s'} + 0.035\phi \qquad (\phi\ in\ degrees) \tag{28}$$

The equation (28) changes the equation $a^s(d_0)$ (6) as follows:

$$a^s(d_0) = 1 - \left(\frac{0.84}{(1.82\sqrt{K_s'} + 0.035\phi)\frac{d_0}{R_i} + 0.84} \right)^2 \tag{29}$$

Calculate the parameters K_p and H for Coulomb:

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} \tag{30}$$

$$H = C\frac{\cos\phi}{\sin\phi} \tag{31}$$

Calculate the convergence U_∞ and at the tunnel face U_f from (21), as follows:

$$U_\infty = \frac{1.5}{E}\frac{K_p - 1}{K_p + 1}(P_\infty + H)\left\{\frac{2}{K_p + 1}\frac{P_\infty + H}{H}\right\}^{2/(K_p-1)} \tag{32}$$

The solution of the system (10) gives the convergence and stress at equilibrium:

$$aU_{eq}^{n_1} + bU_{eq}^{n_2} + c = 0 \tag{33}$$

$$\begin{cases} a = \left\{\frac{(K_p + 1)E}{1.5(K_p - 1)(P_\infty + H)}\right\}^{\frac{K_p-1}{2}} \left(\frac{K_p + 1}{2}\right)\frac{K_s}{E}(1 - a^s(d_0)) \\ b = \left\{\frac{(K_p + 1)E}{1.5(K_p - 1)(P_\infty + H)}\right\}^{\frac{K_p-1}{2}} \left(\frac{K_p + 1}{2}\right)\left\{U_f\frac{K_s}{E}(a^s(d_0) - 1) + \frac{H}{E}\right\} \\ c = -\frac{(P_\infty + H)}{E}; n_1 = \frac{K_p + 1}{2}; n_2 = \frac{K_p - 1}{2} \end{cases} \tag{34}$$

2.3 Viscoplastic rockmass

The NIM for viscoplastic rockmasses was developed as the same way as the elastic and plastic rockmasses. The solution is given by getting a transformed equation from unsupported to supported tunnel to get how the convergence changes with the rockmass strength. The reduced velocity of excavation V^* is the new parameter for viscoplasticity and it is written as:

$$V^* = \frac{\eta V}{R_i C} \tag{35}$$

Where:

η: viscosity constant.

Figure 7 illustrates the influence of the reduced velocity on the convergence profile of the tunnel, showing that the faster it gets, the smaller is the convergence.

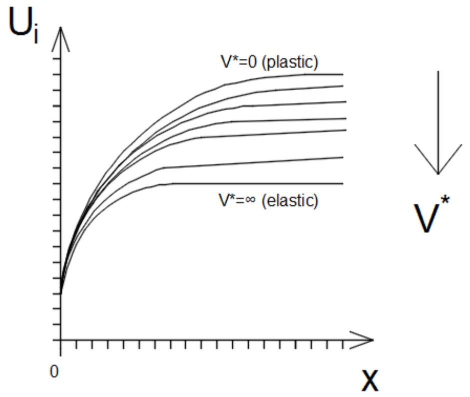


Figure 7. The influence of the reduced velocity on the convergence

The function of transformation obtained from the unsupported tunnel is:

$$a_{vp}^{0}(x) = \frac{U_i^f(x) - U_i^f(0)}{U_i^f(\infty) - U_i^f(0)} = 1 - \left(\frac{0.84}{\frac{x}{AR_p} + 0.84}\right)^{2B} \tag{36}$$

The parameters A and B are:

$$A(V^*) = 0.018 + 0.982e^{\left(-\left(\frac{\log(1+V^*)}{2.8}\right)^{6.4}\right)} \tag{37}$$

$$2B(N_s, V^*) = b_{02}(p_1 + (1 - p_1)e^{\left(-\left(\frac{\log(1+V^*)}{2.3}\right)^{3.3}\right)} \tag{38}$$

$$b_{02}(V^*) = 0.195 + 1.805e^{\left(-\left(\frac{\log(1+V^*)}{2.71}\right)^{4.84}\right)} \tag{39}$$

$$p_1(N_s) = e^{(1.756 - 0.878N_s)} \tag{40}$$

However, for the supported tunnel, as it was developed for the elastoplastic rockmasses, the relationship between the unsupported and supported equation is given as:

$$a_{vp}^{s}(x) = a_{vp}^{0}(\alpha x); a_{vp}^{s}(x) = 1 - \left(\frac{0.84}{\frac{\alpha x}{AR_p} + 0.84}\right)^{2B} \tag{41}$$

Calculate the convergence at the supported tunnel face (U_f), as follows:

$$\frac{U_f}{U_\infty} = p_2 + (0.413 - 0.0627N_s - p_2)e^{\left(-\left(\frac{\log(1+V^*)}{2.70}\right)^{2.9}\right)} \tag{42}$$

$$p_2(N_s) = \frac{0.27N_s}{e^{N_s-1}};\ 1 < N_s \leq 5 \tag{43}$$

Tresca Solution

The calculation of the lining function α^* is different from the ones presented before and it is as follows:

$$\alpha^* = \alpha\frac{R_i}{R_p} = K_s{'}^{C_1}e^{C_2} \tag{44}$$

$$C_1(N_s, V^*) = p_{c1} - (p_{c1} - 0.5)e^{\left(-\left(\frac{\log(1+V^*)}{3.54}\right)^{m_{c1}}\right)} \tag{45}$$

$$p_{c1}(N_s) = e^{(-0.118+0.817N_s)} \tag{46}$$

$$m_{c1}(N_s) = \ e^{(2.094+0.0467N_s)} \tag{47}$$

$$C_2(N_s, V^*) = p_{c2} - (p_{c2} - 0.6)e^{\left(-\left(\frac{\log(1+V^*)}{3.70}\right)^{m_{c2}}\right)} \tag{48}$$

$$p_{c2}(N_s) = e^{(-0.713+1.172N_s)} \tag{49}$$

$$m_{c2}(N_s) = \ -0.564 + 2.51N_s \tag{50}$$

So, the $a_{vp}^s(d_0)$ by joining (41) and (44) gets:

$$a_{vp}^s(d_0) = 1 - \left(\frac{0.84}{\frac{d_0}{A}\frac{\alpha^*}{R_i} + 0.84}\right)^{2B} = 1 - \left(\frac{0.84}{\frac{d_0}{A}\frac{K_s{'}^{C_1}e^{C_2}}{R_i} + 0.84}\right)^{2B} \tag{51}$$

The convergence U_∞ is given by (20).

The solution of the system (10) is:

$$a\ln U_{eq} + bU_{eq} + c = 0 \tag{52}$$

$$\begin{cases} a = -\dfrac{C}{K_s} \\ b = a_{vp}^s(d_0) - 1 \\ c = \left(1 - a_{vp}^s(d_0)\right) U_f + \dfrac{P_\infty}{K_s} + \dfrac{C}{K_s}\left(\ln\left(\dfrac{1.5C}{E}\right) - 1\right) \end{cases} \tag{53}$$

P_{eq} is given by equation (24).

The convergence at d_0 is given by:

$$U_0 = a_{vp}^s(d_0)\left(U_{eq} - U_f\right) + U_f \tag{54}$$

The plastic radius is given by (25).

Coulomb solution

The convergence U_∞ is given by (32);

Calculate the convergence at the supported tunnel face (U_f) with (42);

The friction angle in Coulomb Criterion changes the equation (44), however the parameters C_1 and C_2 remain the same as the calculated in (45) through (50). The function (51) of $a_{vp}^s(d_0)$ is changed and becomes:

$$a_{vp}^s(d_0) = 1 - \left(\frac{0.84}{\dfrac{d_0}{A}\dfrac{\left({K_s'}^{C_1} e^{C_2} + 0.035\phi\right)}{R_i} + 0.84}\right)^{2B} \qquad (\phi\ in\ degrees) \tag{55}$$

The solution of the system (10) is:

$$aU_{eq}^{n_1} + bU_{eq}^{n_2} + c = 0 \tag{56}$$

$$\begin{cases} a = \left\{\dfrac{\left(K_p + 1\right)E}{1.5\left(K_p - 1\right)\left(P_\infty + H\right)}\right\}^{\frac{K_p-1}{2}} \left(\dfrac{K_p + 1}{2}\right)\dfrac{K_s}{E}\left(1 - a_{vp}^s(d_0)\right) \\ b = \left\{\dfrac{\left(K_p + 1\right)E}{1.5\left(K_p - 1\right)\left(P_\infty + H\right)}\right\}^{\frac{K_p-1}{2}} \left(\dfrac{K_p + 1}{2}\right)\left\{U_f \dfrac{K_s}{E}\left(a_{vp}^s(d_0) - 1\right) + \dfrac{H}{E}\right\} \\ c = -\dfrac{\left(P_\infty + H\right)}{E} \\ n_1 = \dfrac{K_p + 1}{2} \\ n_2 = \dfrac{K_p - 1}{2} \end{cases} \tag{57}$$

P_{eq} is given by equation (24).

The convergence at d_0 is given by (54);

3 RESULTS AND VALIDATION

The code is developed in *MatLab* and gives the *.gui* executable. The code has been developed with the equations shown before, and afterwards some response charts and solutions are given to validate the method.

Another use of the simplified method is to determine the ratio λ_0 (or the convergence U_0) at the moment of lining installation and then use this value as a good approach to calculate the tunnel in a 2D plane strain condition with a more complex geometry and loading (anisotropic loading for instance). Figure 8 shows how the pressure P_i is obtained by using the ratio λ_0.

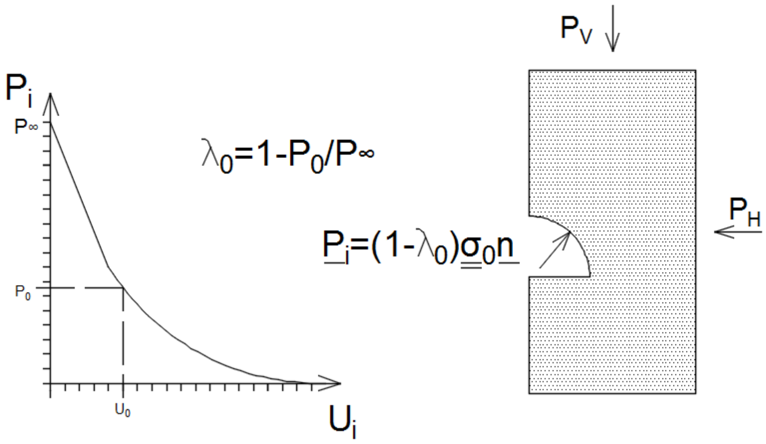


Figure 8. Application of the ratio λ₀ in a plane strain model.

3.1 Elastic solution

The first results are shown for elasticity, we adopted:

$E = 500\ MPa\ and\ \nu = 0.5$ (Matlab) and 0.498 (GEOMEC91); (58)

The solution obtained for the convergence through the Finite Element Analysis ($U_{eq}{}^{FE}$(%)) was taken from the GEOMEC91 solution introduced by (BERNAUD, D. and ROUSSET, G., 1996). The errors between the FEM and NIM presented good approximation as shown in Table 1.

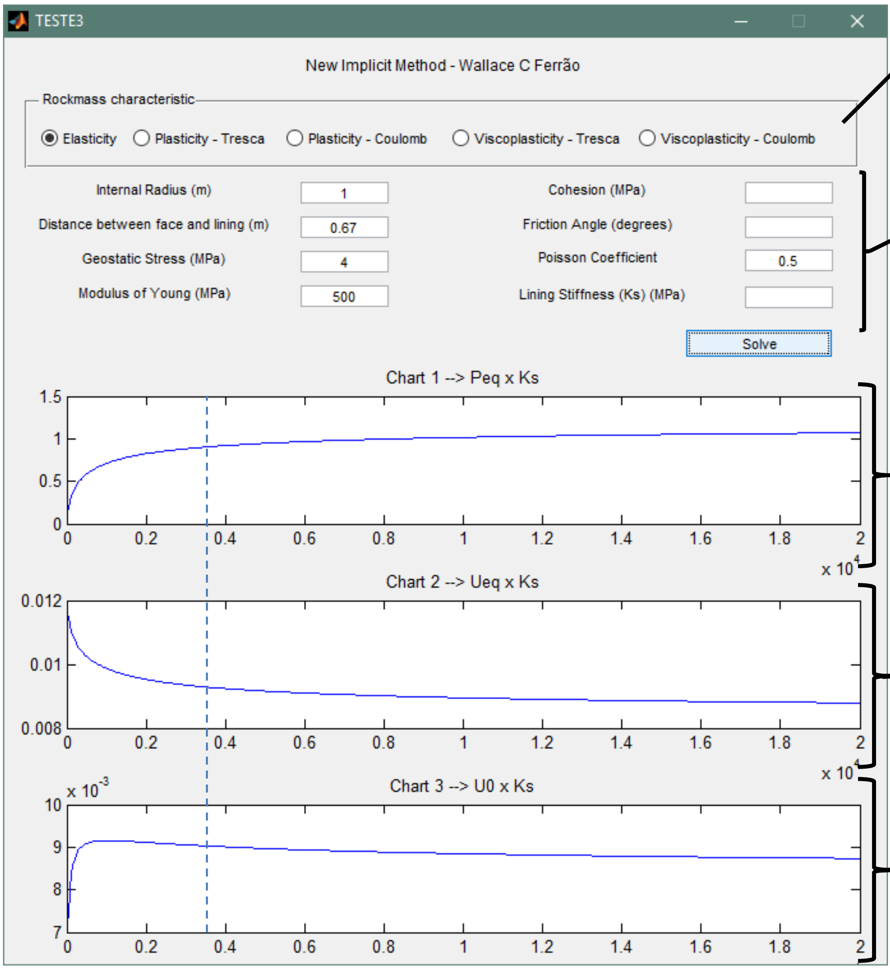
The *.gui* executable of Matlab shown in Figure 9 is in accordance with the results of Table 1, and besides the graphic output shown in the figure, the executable outputs an *.xlsx* document with the solutions for different lining stiffness K_s.

$U_{eq}{}^{FE,NIM}$ are the solutions for the convergence at a far distance from d_0 through Finite Element (GEOMEC91) and NIM method analysis respectively. Table 1 shows the solution for different situation according to the parameter defined in (14).

Table 1. Validity of the method for elastic rockmass

Constitutive Law	P_∞'	d_0'	K_s'	$U_{eq}{}^{FE}$ (%)	$U_{eq}{}^{NIM}$ (%)	$\frac{U_{eq}{}^{NIM} - U_{eq}{}^{FE}}{U_{eq}{}^{FE}}$
Elasticity	0.008	0	0.72	0.87	0.76	12.64%
	0.008	0.67	7.2	0.90	0.93	-3.33%
	0.008	1.67	72	1.02	1.11	-8.82%

Figure 9 shows the results obtained through the *.gui* executable for the third $d_0{}'$ shown in the Table 1 above. Three charts for the stress at equilibrium P_{eq}, convergence at equilibrium U_{eq} and convergence in the moment of the lining installation U_0 are developed automatically by the executable for stiffness varying from 0 to 20 000 MPa.



Define the behavior of the studied strain-zone of the rockmass.

Fill out the blanks, in exception of Cohesion (Elasticity), Friction Angle (Elasticity and Plasticity – Tresca).

The first chart is the solution of the stress at equilibrium (Peq) [MPa] versus the lining stiffness (Ks) [MPa].

The second chart is the solution of the convergence at equilibrium (Ueq) [m/m] versus the lining stiffness (Ks) [MPa].

The third chart is the solution of the convergence at a distance d0 (U0) [m/m] versus the lining stiffness (Ks) [MPa].

Figure 9. Solution for an elastic rockmass using .gui MatLab

The dashed line in the Figure 9 represents the solution for $P_\infty{}' = 0.008$, $d_0{}' = 0.67$, and $K_s' = 7.2$.

3.2 Plastic Solution

The results shown in Table 2 concern the plasticity with Tresca and Coulomb criteria as illustrated in Figure 6. For this analysis, we adopted:

$$E = 500\ MPa\ and\ \nu = 0.5 \text{ (Matlab) and } 0.498 \text{ (GEOMEC91)}; \qquad (59)$$

${P_{eq}}^{FE,NIM}$ are the solutions for the stress at a far distance (equilibrium) from d_0 through Finite Element (GEOMEC91) and NIM method analysis respectively.

Table 2. Validity of the method for plastic rockmass

Constitutive Law	P_∞'	d_0'	K_s'	${P_{eq}}^{FE}$	${P_{eq}}^{NIM}$	$\frac{{P_{eq}}^{NIM} - {P_{eq}}^{FE}}{{P_{eq}}^{FE}}$
Tresca						
C = 1 MPa	0.008	0.33	7.2	1.48	1.50	-1.35%
C = 1.33 MPa	0.008	1.00	72	0.90	0.85	5.56%
Coulomb						
C = 0.8 MPa ∅=4º	0.008	0.67	0.24	0.76	0.75	1.32%
C = 0.8 MPa ∅=15º	0.008	1.67	2.4	0.35	0.31	11.43%
C = 0.8 MPa ∅=30º	0.01	1.67	24	0.309	0.31	-0.32%

The results for Coulomb and Tresca criteria present some errors smaller than 12% which is acceptable if compared to the results of the Finite Element analysis. This indicates that the NIM is an adequate method for predicting the convergence and stress at equilibrium (the far distance from the tunnel face).

3.3 Viscoplastic Solution

Tresca Solution

The results shown in Table 3 uses the parameters N_s defined in (8), V^* defined in (35), ${d_0}'$ and ${K_s}'$ which have been defined in (14). This solution shown in Table 3 concerns a rockmass characterized by the cohesion only, since Tresca criterion does not include the influence of the friction angle.

The method works well for most recurrent rockmass condition. However, for some quite high N_s the solution starts to get too far from the gotten through the Finite Element analysis. This shows that the method is limited for $N_s < 5$ as it is shown in (50).

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Table 3. Validity of the method for viscoplastic rockmass with Tresca criterion

<table>
<tr><th>N_s</th><th>d_0'</th><th>V^*</th><th>K_s'</th><th>P_{eq}^{FE}</th><th>U_{eq}^{FE} (%)</th><th>P_{eq}^{NIM}</th><th>U_{eq}^{NIM} (%)</th><th>$\frac{P_{eq}^{NIM} - P_{eq}^{FE}}{P_{eq}^{FE}}$</th><th>$\frac{U_{eq}^{NIM} - U_{eq}^{FE}}{U_{eq}^{FE}}$</th></tr>
<tr><td rowspan="2">2</td><td>0.33</td><td rowspan="2">1000</td><td>0.72</td><td>0.86</td><td>1.06</td><td>0.77</td><td>1.11</td><td>-10.43%</td><td>4.72%</td></tr>
<tr><td>1</td><td>36</td><td>0.72</td><td>0.38</td><td>0.79</td><td>0.37</td><td>10.33%</td><td>-2.63%</td></tr>
<tr><td rowspan="4">3</td><td>0.33</td><td>2000</td><td>18</td><td>1.78</td><td>0.39</td><td>1.77</td><td>0.39</td><td>-0.68%</td><td>0.00%</td></tr>
<tr><td rowspan="3">1</td><td rowspan="2">150</td><td>0.24</td><td>0.92</td><td>2.8</td><td>0.92</td><td>2.79</td><td>0.51%</td><td>-0.36%</td></tr>
<tr><td>2.4</td><td>1.59</td><td>2.0</td><td>1.74</td><td>1.86</td><td>9.54%</td><td>-7.00%</td></tr>
<tr><td>500</td><td>0.72</td><td>1.49</td><td>2.1</td><td>1.44</td><td>2.15</td><td>-3.09%</td><td>2.38%</td></tr>
<tr><td>5</td><td>1</td><td>500</td><td>0.72</td><td>1.53</td><td>1.93</td><td>1.60</td><td>1.77</td><td>4.67%</td><td>-8.29%</td></tr>
</table>

Coulomb Solution

The results shown in Table 4 concern the viscoplasticity with a Coulomb criterion developed in (55) through (57). The parameter N_s is now defined in (27), the other parameters remain with the same definition as the one for Tresca solution. For this analysis, we adopted:

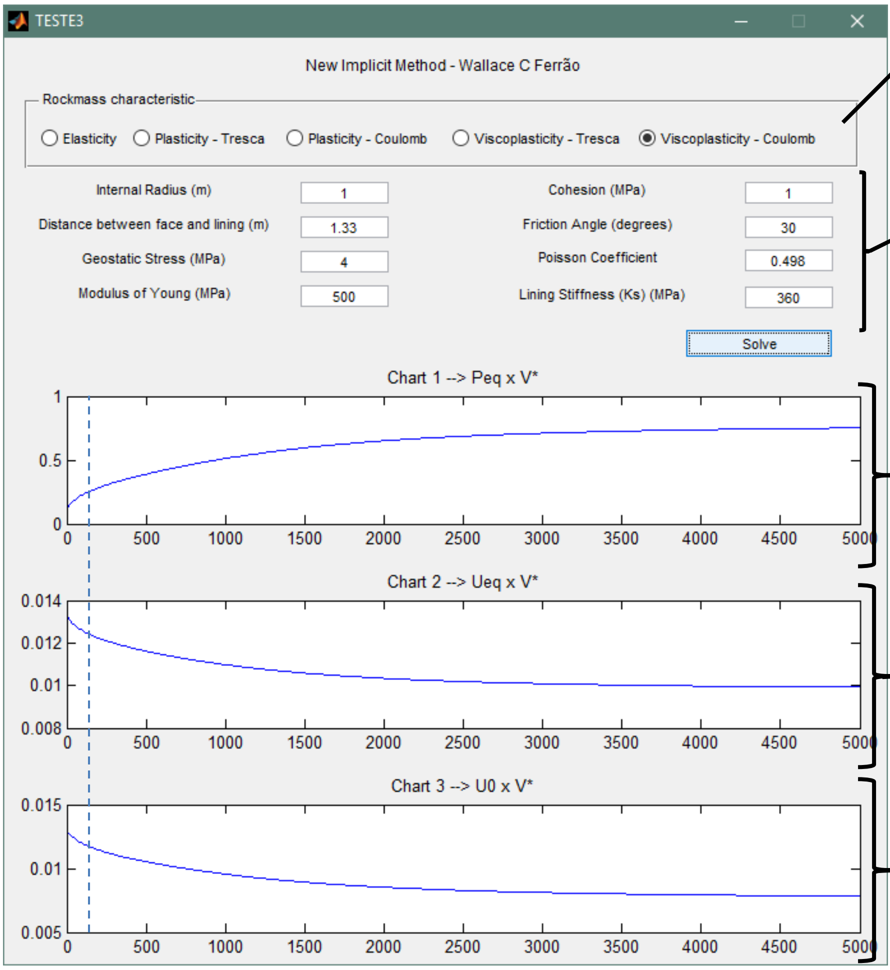
$$\phi = 30º \; and \; \eta = 115.74 \; MPa \times Day \tag{55}$$

The η is taken as 115.74 MPa x Day as an approximation to the $\frac{1}{\eta} = 0.0000001 \; sec^{-1}$ used by the validation Finite Element analysis used by the GEOMEC91.

Table 4. Validity of the method for viscoplastic rockmass with Coulomb criterion

<table>
<tr><th>N_s</th><th>d_0'</th><th>V^*</th><th>K_s'</th><th>U_{eq}^{FE}(%)</th><th>U_{eq}^{NIM}(%)</th><th>$\frac{U_{eq}^{NIM} - U_{eq}^{FE}}{U_{eq}^{FE}}$</th></tr>
<tr><td rowspan="3">2.31</td><td>0</td><td>1157.41</td><td>7.2</td><td>0.48</td><td>0.43</td><td>-10.42%</td></tr>
<tr><td>0.67</td><td>578.70</td><td>0.72</td><td>1.11</td><td>1.07</td><td>-3.98%</td></tr>
<tr><td>1.33</td><td>115.74</td><td>0.72</td><td>1.28</td><td>1.25</td><td>-2.73%</td></tr>
<tr><td rowspan="3">1.73</td><td>0</td><td>2314.81</td><td>1.00</td><td>0.61</td><td>0.55</td><td>-9.84%</td></tr>
<tr><td>0.67</td><td>1157.41</td><td>1.50</td><td>0.76</td><td>0.79</td><td>4.02%</td></tr>
<tr><td>1.33</td><td>578.70</td><td>1.00</td><td>0.79</td><td>0.89</td><td>11.14%</td></tr>
<tr><td>2.89</td><td>0.67</td><td>115.74</td><td>0.48</td><td>1.60</td><td>1.45</td><td>-10.45%</td></tr>
</table>

The dashed line in Figure 10 shows the graphic result shown by MatLab for the third line solution of Table 4. The $N_s = 2.31$ and $d_0' = 1.33$, Fig. 10 shows for this case a Pressure at Equilibrium (first chart) equal to 0.3MPa and an equilibrium convergence of 0.0125 what matches the Table 4.



Viscoplastic Solution for Coulomb Criterion

Fill all the blanks, including Ks, since the viscoplasticity solution is given for a defined lining stiffness in this executable.

The first chart is the solution of the stress at equilibrium (Peq) [MPa] versus the reduced velocity (V*).

The second chart is the solution of the convergence at equilibrium (Ueq) [m/m] versus the reduced velocity (V*).

The third chart is the solution of the convergence at a distance d0 (U0) [m/m] versus the reduced velocity (V*).

Figure 10. Solution for a viscoplastic rockmass with Coulomb Criterion using .gui MatLab

Figure 10 shows that the executable gives the charts in function of the reduced velocity instead of the lining stiffness as it was for the elastic and plastic solutions shown before.

4 CONCLUSION

A supported tunnel behaves as a three-dimensional structure for which strain and stress fields of the surrounding soil are strongly influenced. This paper develops an easy tool executable with MatLab to solve the problem of convergence and stress at equilibrium for elastic, elastoplastic and viscoplastic tunnels.

It is important to know beforehand some geotechnical characteristics of the rockmass and the loading pattern to select the stress-strain criteria correctly. It is known that if the rockmass does not present permanent strain after being stressed it is going to behave as an elastic model, however the existence of permanent and creep deformations implies a plastic or a viscoplastic behavior which give different solutions.

The problem was proved to be slightly different from the one solved by the Convergence-Confinement method, since the New Implicit Method highlights that the stiffness of the supporting is a great player on convergence U_0 (at a distance d_0 from the tunnel face).

A point to be added is to take the permanent deformation of the supporting lining, for instance the concrete one, since it may change the curvature of the confinement curve.

The accuracy of the new method is very satisfying for a geotechnical study and the average error stays under 12%, and, generally, the errors are for the safety, since they increase the value of the stress at equilibrium.

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