XXXVI Ibero-Latin American Congress on Computational Methods in Engineering
CILAMCE
Rio de Janeiro, 22-25 Nov 2015

STRESS-BASED TOPOLOGY OPTIMIZATION CONSIDERING ERROR ESTIMATES

Guilherme Varella

Jun Sérgio Ono Fonseca

gvarella@gmail.com

jun@ufrgs.br

UFRGS - Federal University of Rio Grande do Sul

Rua Sarmento Leite, 425, 90050-170, Porto Alegre, Rio Grande do Sul, Brazil

Abstract. *This work presents a methodology for stress-constrained topology optimization, focusing on the employment of methodologies to improve the computational processing performance. Structural analysis is accomplished by the finite element method, and stress is computed at the elemental Gaussian integration points, and then smoothed over the mesh. In order to avoid the stress singularity phenomenon a constitutive tensor penalization is employed. A normalized version of the p-norm is used as a global stress measure instead of local stress constraint. A finite element error estimator is considered in the stress constraint calculation. In order to solve the optimization process, Sequential Linear Programming is employed, with all derivatives being calculated analytically. A evolutionary criterion is proposed to remove low density elements, contributing for well-defined structures and reducing significantly the computational time. Checkerboard instability is circumvented with a linear density filter. To reduce the computational time and enhance the performance of the code, a surrogate model is used in inner iterations of the Sequential Linear Programming. The results of the present algorithm is compared with the ones obtained by other authors, presenting good results and thus verifying the validity of the procedure.*

Keywords*: topology optimization, stress constraint, surrogate model, error estimator.*

1 INTRODUCTION

Topology optimization (Bendsoe and Sigmund, 2004) is an important tool to help the design of structures with better performance and lower material consumption, with a direct impact in the production cost of components. This paper presents some aspects of an algorithm for stress-based topology optimization. This problem aims to determine the existence or not of material in a structure subjected to certain loads and boundary conditions, and considering the stress as constraint.

There is a significant difficulty in solving the optimization problem considering stress constraint. This difficulty is related to the fact that stress is a local variable, thus each point of the structure has to be below the imposed limit. If each local stress is treated as a separated constraint during the optimization, there is a high computational cost. In order to reduce the computational cost, related to the number of constraints a global stress measure can be used, as the p-norm proposed by Yang and Chang (Yang and Chang, 1996).

However, even considering a global stress measure, it is still necessary to calculate the stress derivatives for each node of the structure in relation to all elements. Even if the stress derivatives are calculated analytically, this part of the methodology is responsible for the greatest part of the computational time of the algorithm, increasing exponentially with mesh refinement. With a high computational cost, the optimization methodology might become unfeasible for practical applications. Therefore a good optimization algorithm ideally needs to be robust, to achieve good results with minimal intervention, and has a small processing and configuration time. To improve the performance, related to the derivative calculations, a surrogate model (Queipo et al., 2005) is proposed.

During the optimization process it is usually seen that some regions of the structure converge to void during the initial iterations. To reduce the number of nodes and consequently the number of derivatives to calculate at each iteration, a criterion to remove low density elements is proposed (Xie and Steven, 1997) .

2 METHODOLOGY

The finite element method is employed to model the structural behavior, considering linear infinitesimal strain theory. Taylor-Beresford-Wilson non-conforming element is considered for better modelling of the shear behavior. The stress is calculated at the integration points of each element and then extrapolated to the nodes, using the element shape functions.

A stress smoothing scheme is applied to reduce the discontinuities among neighbor elements. The stress value in shared nodes is averaged to represent the nodal stress:

$$\sigma_P = \sum_{i=1}^{npg} \frac{\sigma_n}{npg} \tag{1}$$

where σ_P is the smoothed nodal stress, σ_n the nodal stress obtained from each neighbor element and *npg* the number of elements that share the same node.

An *a posteriori* error estimator (Zienkiewicz and Zhu, 1987) is used. The nodal stress extrapolated from each Gaussian point of the element is compared with the smoothed stress to estimate the error:

$$e_{ZZ_n} = \max\left(\sqrt{\int_{\Omega_e} \frac{(\sigma-\sigma_P)^T \mathbf{C}(\sigma-\sigma_P)}{\det(J_e)} \partial\Omega_e}\right) \tag{2}$$

where σ is the nodal stress extrapolated from each Gaussian point, $\mathbf{C}$ the constitutive matrix, J_e the Jacobian of the element and Ω_e the element domain. The estimated error is then added to the smoothed nodal stress value.

To avoid the checkerboard instability (Sigmund and Petersson, 1998) during the optimization process, a density filter is employed (Bourdin, 2001).

2.1 Stress singularity

The issue of stress singularity appears, in this work, when the global solution is not part of a continuous domain, but of a degenerated subset. The methodology employed to handle the stress singularity was proposed by Le (Le *et al.*, 2010). Initially the discrete optimization problem is relaxed to a continuum optimization process. To implement this strategy it is necessary to define ways to interpolate the stiffness, volume and stress between its minimum and maximum values.

To penalize the stiffness tensor, volume and stress tensor the following term are defined:

$$\mathbf{C} = n_c(\rho)\mathbf{C}_0 \tag{3}$$

$$\mathrm{V} = \int_{\Omega} n_v(\rho) dv_0 \tag{4}$$

$$\sigma_r = n_T(\rho)\mathbf{C}_0\varepsilon \tag{5}$$

where the index *0* refers to the initial value (no penalization) of the variables. These equations must obey the following restrictions:

- n_c, n_v e n_t are monotonic functions
- $0 < n_c(\rho) \le \rho$, $0 < n_v(\rho) \le \rho$ e $0 < n_T(\rho) \le \rho$ for $0 < \rho \le 1$;
- $n_c(1) = 1$, $n_v(1) = 1$ and $n_T(1) = 1$
- $n_c(0) = 0$, $n_v(0) = 0$ and $n_T(0) = 0$

2.2 Global stress measure

Usually when stress based topology optimization is employed together with the finite element method, one stress constraint for each node of the mesh is considered, which implies the number of constraints would be as large as the mesh node count.

There is a great difficulty in solving a stress based topology optimization problem. This difficulty is due, mainly, to the fact that stress is a local parameter, so each position of the structure under investigation needs to have its stress obeying the constraint. However, if each stress is treated as a separated constraint, an elevated computation cost is implied. An efficient way to reduce the computational time related to this issue, is a global stress measure. Although a global stress measure reduces the computational time, it also makes more difficult to control stress peaks and stress concentrations (Xia, 2011).

To define a global stress measure, initially the original problem, with stress constraints for each node is considered:

$$\sigma_n = \sigma_Y \text{, } n\text{=1,2 ... } ntn \tag{6}$$

where σ_n is the stress measure for the node, *n* the number of nodes in the structure and σ_Y the stress limit. These *ntn* constraints could be rewritten in terms of a single one:

$$\max_{n=1..ntn} (\sigma_n(\rho)) \leq \sigma_Y \tag{7}$$

which means: if the element with the higher stress is below the stress limit, consequently all other elements are also respecting the constraint. The problem is that *maximum* is not a differentiable function, so other method should be employed to approximate *maximum* function, as the p-norm described by Yang (Yang, 1996):

$$\sigma_{NP} = \left(\sum_{e=1}^{n} \sigma_n{}^P \right)^{1/P} \leq \sigma_Y \tag{8}$$

where *P* is the value of the norm and σ_{NP} the global stress measure. When $P \to \infty$, the stress σ_{NP} tends to the maximum stress calculated at the structure, $\sigma_{\max}$, and there is no approximation. On the other hand, when $P \to 1$ there is an approximation, and the stress σ_{NP} is farther from the maximum stress of the structure.

It is inadequate to impose a maximum stress constraint based on the p-norm, as there is no mathematical expression to relate the $\sigma_{\max}$ and σ_{NP}. To solve this issue Le (Le *et al.*, 2010) proposed a normalized stress measure, to better approximate the maximun stress.

$$\sigma_{\max} \approx c\sigma_{NP} \leq \sigma_Y \tag{9}$$

where *c* is calculated in each iteration $i \geq 1$ and is given by:

$$c^i = \alpha^i \frac{\sigma^i_{\max}}{\sigma^i_{NP}} + \left(1 - \psi^i\right) c^{i-1} \tag{10}$$

The parameter $\psi^i \in (0,1]$ controls the variation of c^i between the iterations. It is evident that c^i is a parameter to correct the difference between the maximum stress in the structure and the value obtained by the p-norm.

It can be noticed that $c\sigma_{NP} \leq \bar{\sigma}$ is not differentiable, as *c* varies in a discontinuous manner, resulting in a slightly different problem at each iteration. However, as the optimization process starts to converge, the difference between the successive problems is reduced, and *c* also converges, reducing the effect of the non-differentiability (Le *et al.*, 2010)

2.3 Criterion to remove low density elements

During the optimization process the density of the elements could tend to zero, increasing substantially the local stress. This goes against that assumption that density near zero would represent void regions, and so is not desirable. If low density elements are employed to simulate void regions, but for the definition of the global stress measure only high density elements are considered, the result for the optimization problem could be trivial. By the other hand, if the global stress measure is defined based on the low and high density elements, it could be necessary to manually adjust the stress in the low density elements so they are lower than in the high density elements (Xia et al., 2011)

To avoid this matter, a evolutionary criterion to remove low density elements of the structure is implemented. The element removal strategy is employed starting from the first iteration, and is given by the following equation:

$$\rho_e \leq mean(\rho) - \left(mean(\rho) - \min(\rho)\right)\eta_{BD} \tag{11}$$

ρ_e is the density of the element under consideration, $mean(\rho)$ the mean density of all elements of the structure, $\min(\rho)$ the minimum density in the structure and η_{BD} a parameter to calibrate the elements removal.

Considering equation (11), only elements with density below the structure mean density are candidates for removal. Additionally, a restriction is implemented to verify, based on the stress derivatives, which would be the maximum stress of the structure if a given element is removed, so elements that would increase significantly the stress of the structure are not removed:

$$\max\left(\frac{\sigma_P - \dfrac{\partial \sigma_P}{\partial \rho}\rho}{\sigma_Y}\right) \leq 1{,}2 \tag{12}$$

Another advantage of removing the low density elements from the structure is the reduction of computational time. As with removal of the unused elements, some nodes not linked to the structure are removed, and consequently the number of degrees of freedom is also reduced.

The disadvantage of considering this method is that the removed elements does not have the possibility of being re-integrated, thus the function could converge to a local minimum instead of a global minimum.

2.4 Surrogate model

The goal of using a surrogate model, is to accelerate the optimization process through the employment of an approximated model for the objective function and restrictions, resulting in a lower computational time.

The surrogate model is based in data generated by high fidelity models, being capable of quickly provide approximations of the objective functions and restrictions, reducing the computational effort to calculate the derivatives (Queipo *et al.*, 2005). Multiple surrogate models could provide good estimative for the data being analyzed. For some problems there might be preferences in considering one than another, but usually it is not known *a priori* which is the best method (Forrester *et al.,* 2009).

Surrogate models can be divided in two main groups:

- Parametric – apply a model to estimate the behavior of a global function through the relationship of the project variables and answers obtained in known positions;

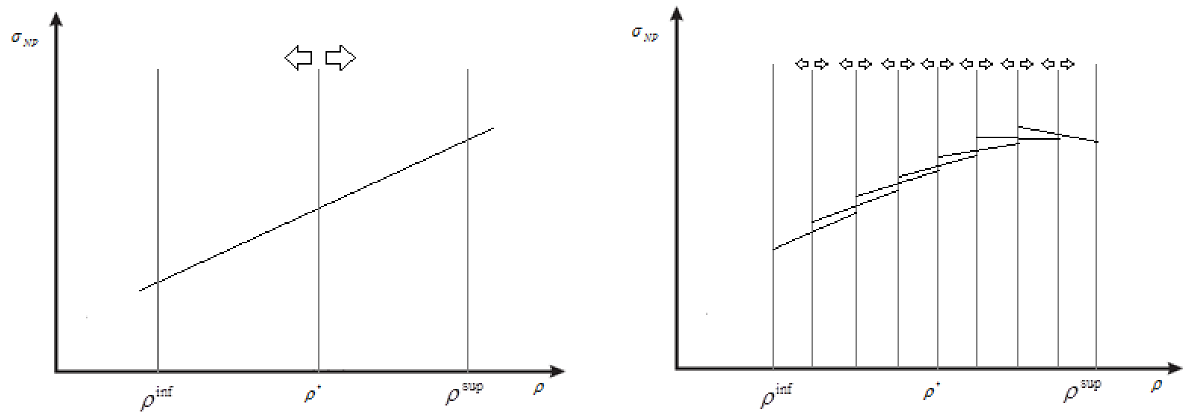
- Non-parametric – apply simpler local models to estimate the behavior of the functions in regions close of the known points.

In the present work a non-parametric surrogate model is employed. There are two non-parametric models more disseminated, which are more likely to converge to a local optima: Approximation Model Management Framework (AMMF), which explores the use of trust regions near the positions where the results are known; and the Surrogate Management Framework (SMF), which employs a pattern search methodology.

The method employed here is the AMMF, for being easily implemented to the algorithm and for the guarantee of convergence for a local optima. This method is generally associated with the use of algorithms based on gradients, with the assumption that the surrogate model is good enough to determine the direction which will improve the initial condition.

The AMMF method has the same convergence properties from the classical methods based on trust regions. Considering first order approximations, the method is convergent for local optima when used together with continuum functions and with the boundarie conditions correctly applied. There is a guarantee of the algorithm convergence, but this comes with the cost that it is needed to calculate the derivatives in every iteration (Queipo et al., 2005).

In the way it is usually employed, the SLP considers trust regions, allowing the design variable to either change to the upper or to the lower limits. This might be an ideal case when there is one design variable. However when there are multiple design variables, in this case the density of each element, it might be that the best solution having at each iteration the projects variables being set to the upper or lower limits, but to values between them. In cases like this, the surrogate model allows the sub-division of the trust region, having as consequence that the project variable could end the iteration in an intermediate position. Figure 1 illustrates the behavior of the design variable considering the usual SLP and the surrogate model.



SLP **Surrogate model**

Figure 1. Behavior of the design variable

It is proposed that at each iteration the stress, derivate of stress and sensitivities of the objective function and restrictions are calculated, and n_{it} iterations are done considering this values.

The trust region for the density variation is divided in n_{it} parts:

$$\rho^{it} = \left(\frac{\rho^{\sup} - \rho^{\inf}}{n_{it}} \right) \tag{13}$$

where $\rho^{\sup}$ and $\rho^{\inf}$ are the upper and lower limits of the trust region respectively. In each iteration the surrogate model will approximate n_{it} sub-iteration inside the same trust region. The density variation in each sub-iteration is given by:

$$\left(\rho_{it-1}{}^{*} - \rho^{it}\right) \le \rho_{it}{}^{*} \le \left(\rho_{it-1}{}^{*} + \rho^{it}\right) \text{ for } it=1,2..n_{it}$$
$$\text{and for } it=1,\ \rho_{i-1}{}^{*} = \rho^{*} \tag{14}$$

The stress in each element is updated to consider the change in the element density $\rho_{it}{}^{*}$ for each sub-iteration, considering a linear approximation and the derivatives already calculated. It is assumed that in the interval $[\rho^{\inf}, \rho^{\sup}]$, the stress derivative with respect to element density is constant.

$$\sigma_{it} = \sigma^{*} + \left(\rho_{it}^{*} - \rho^{*}\right) \frac{\partial \sigma_{vm}\left(\rho^{*}\right)}{\partial \rho} \tag{15}$$

Considering the equations (14) and (15), the sensitivity of the constraint is given by:

$$\frac{\partial \sigma_{it\,NP}}{\partial \rho_j} = \frac{\sigma_{it\max}}{\sigma_{it\,NP}} \left(\sum_{n=1}^{n_{nó}} \sigma_{it\,n}{}^{P} \right)^{1/P-1} \left(\sum_{n=1}^{n_{nó}} \sigma_{it\,n}{}^{P-1} \frac{\partial \sigma_{vm_i}}{\partial \rho_j} \right) \tag{16}$$

It is important to notice that in each sub-iteration based on the surrogate model, the only parameter which remains constant is the derivative of the stress in relation to the elements density, $\frac{\partial \sigma_{vm_i}}{\partial \rho_j}$.

Sequential linear program is iterated n_{it} times during each iteration. This approach allows the design variable to finish the iteration with intermediate values inside the trust region interval, and not only its extremes.

3 RESULTS

The present methodology is evaluated in a well-known (Bendsoe and Sigmund, 2004) L-shaped domain test case for stress concentrator algorithms. Figure 2 presents the geometry and boundary conditions of the problem, considering $F = 3N$, $E = 1MPa$, $\upsilon = 0{,}3$ and $\sigma_Y = 6MPa$. The remaining parameters, related to the configuration of the algorithm, are presented in Table 1. The mesh has 4,100 elements and 8,514 nodes.

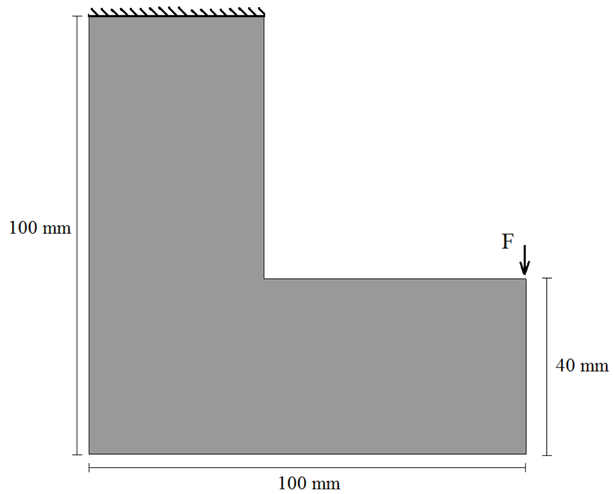


Figure 2. Geometry and boundary conditions

Table 1. Parameters considered

Description	**Parameter**	**Value**
Density filter	R_{max}	1
Removal of low density elements	η_{BD}	0,95
Constitutive tensor penalization	n_c	3
Volume penalization	n_v	1
Stress tensor penalization	n_t	0,5
Global stress measure (p-norm)	P	100

The results for the convergence of the objective function is presented in Figure 3, the optimal density distribution is presented in Figure 5, and the maximum von Mises nodal stress for each element is presented in Figure 4.

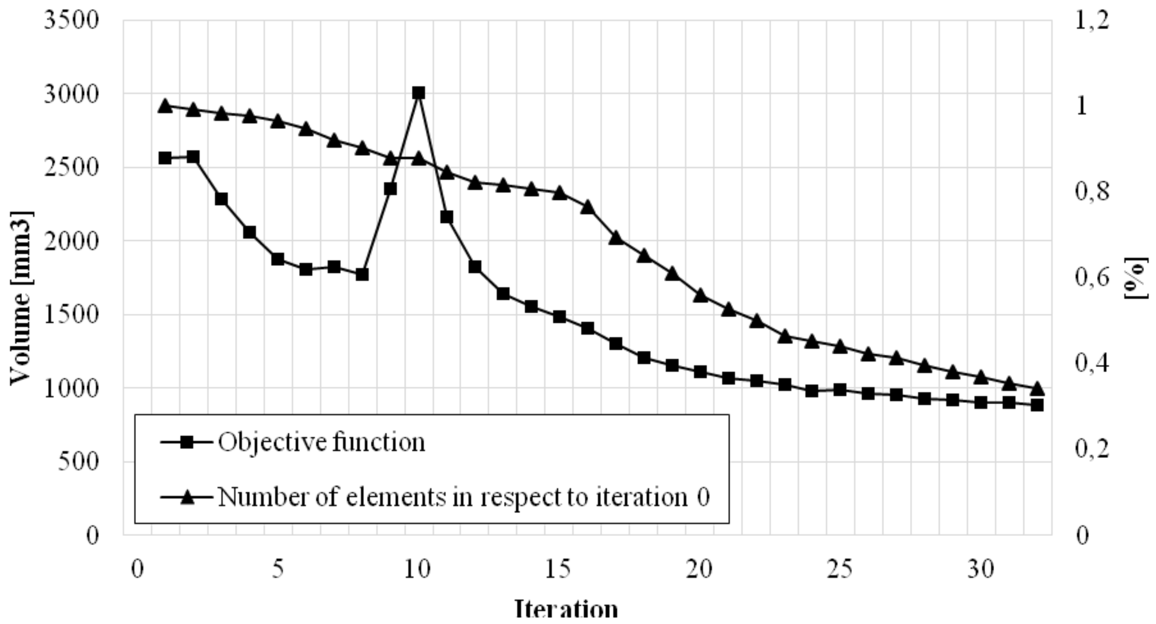


Figure 3. Convergence of the objective function

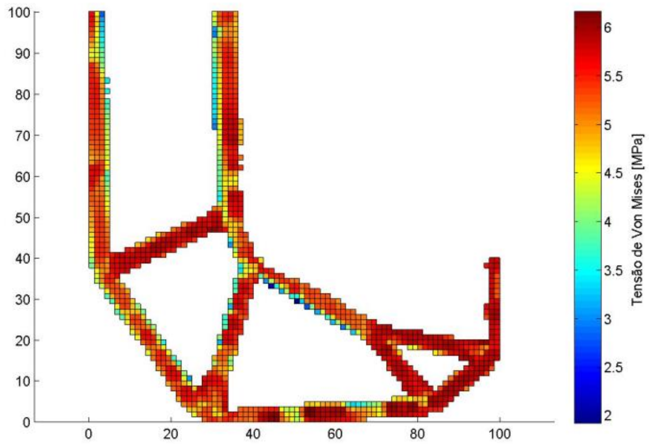


Figure 4. Maximum element stress

Evaluating Figure 3 and Figure 5, it can be noticed that the optimization process created an additional stress concentrator during the element removal process, this concentrator was removed in iteration 11.

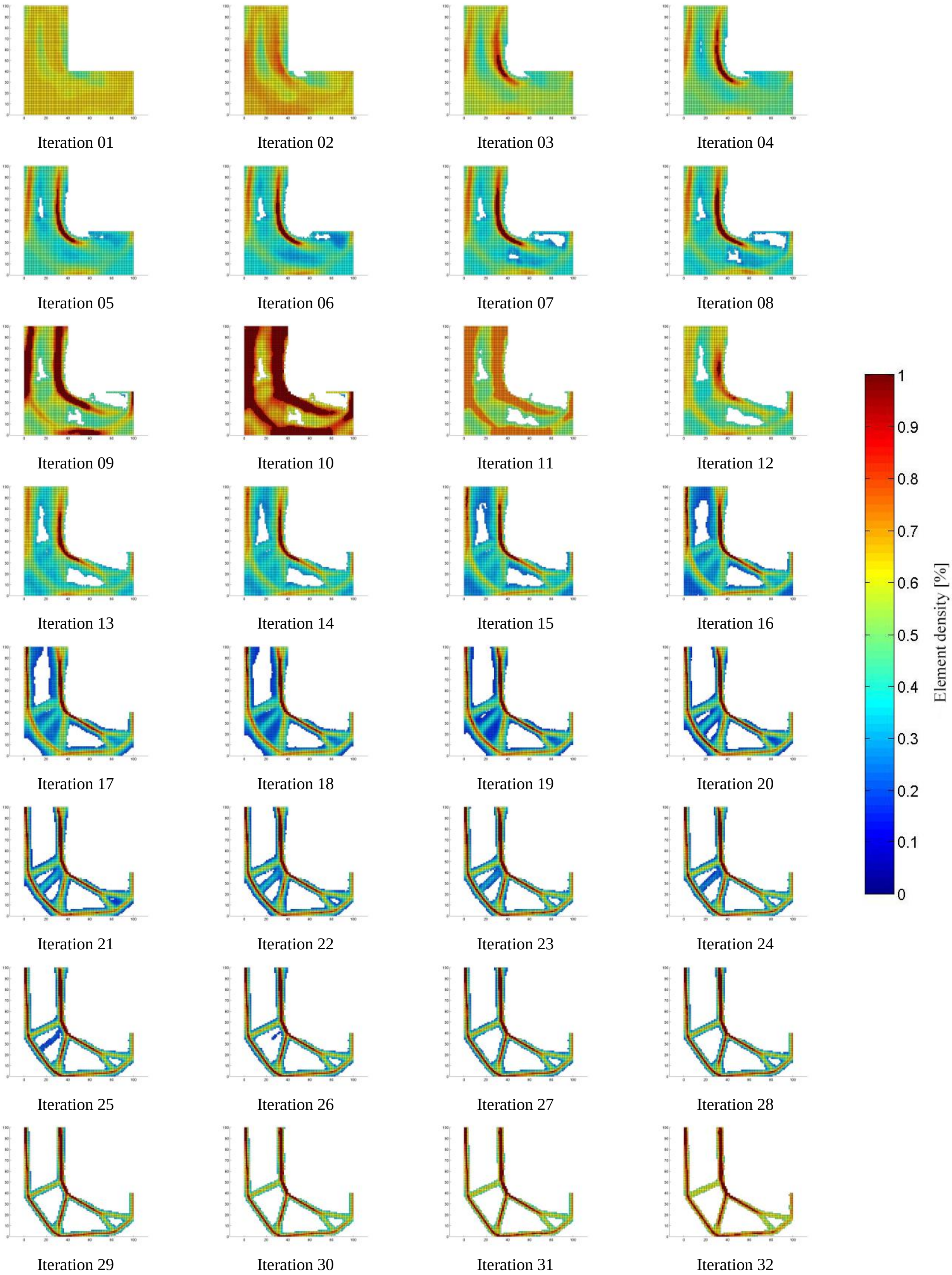


Figure 5. Topology optimization evolution

Interpretation of the results of the last image from Figure 5, the final smoothed structure is presented in Figure 6.

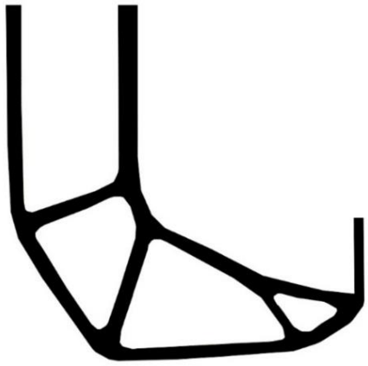


Figure 6. Final structure

The test-case with the L-shaped structure has been studied by many authors. The main challenge in optimizing a structure like this is the stress concentrator. Figure 7 presents results obtained by other authors to the same problem, it is important to notice that the same structure is considered, however in some cases the force is applied either at the middle of the tip of the extremity of the "L".

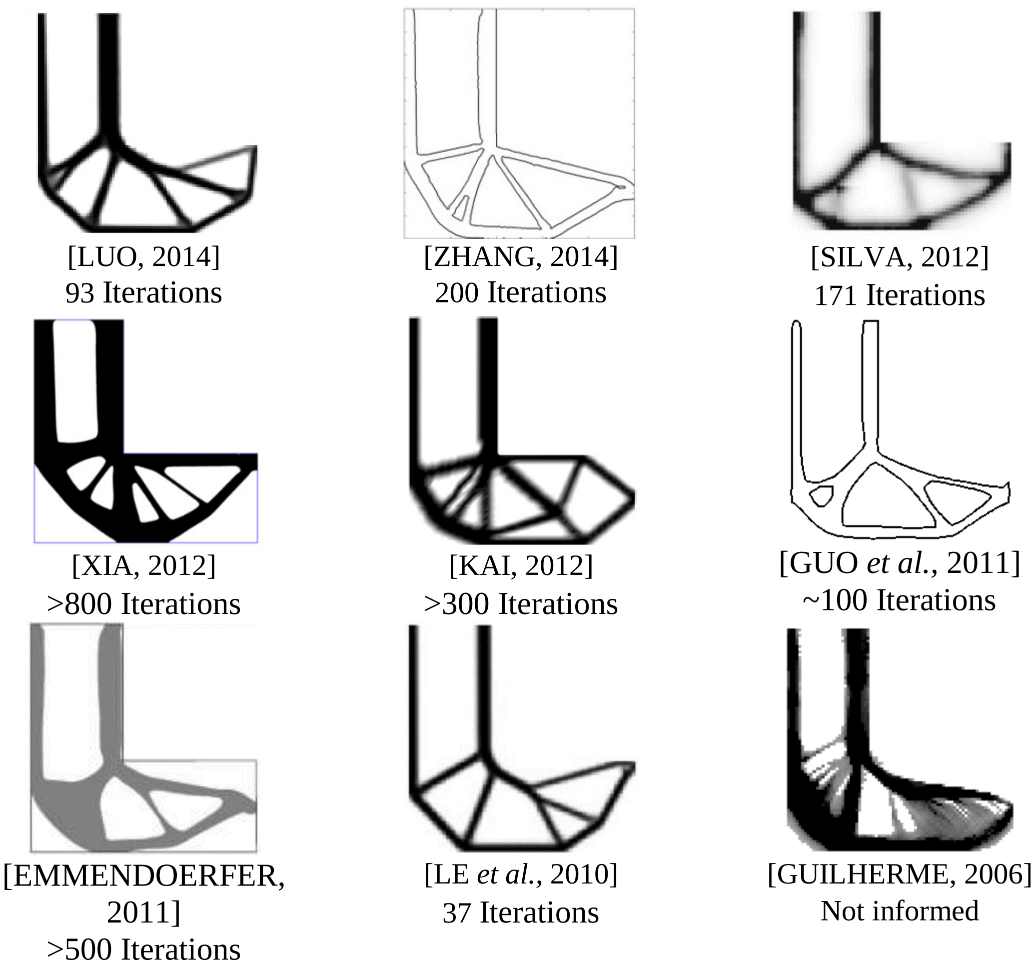


Figure 7. Results obtained by other authors

Many authors employed optimization algorithms that were able to remove the stress concentrator. However most of this optimization processes took many iterations to finish. In the methodology presented in this paper, the lack of material is not simulated with a low density element, but the elements are removed from the structure, resulting in faster calculation for the subsequent iterations.

Figure 8 shows the reduction of computational time with the iterations. The first iteration tooks about 220 minutes, and the last iteration about 15 minutes, a reduction of 93% in the processing time. If the low density removal is not active, and assuming the convergence would take the same number of iterations, the optimization would have taken 120 hours.

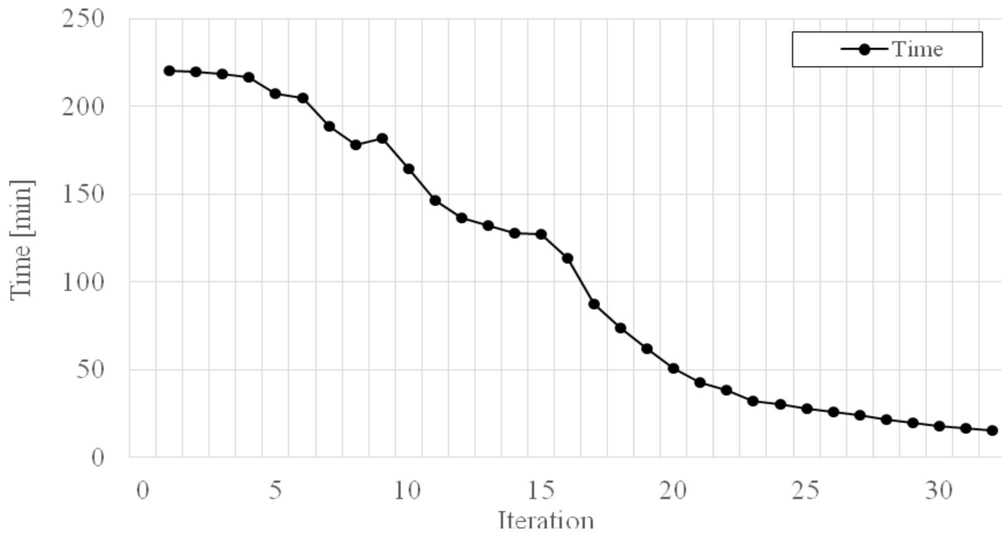


Figure 8. Computational time

4 CONCLUSION

The proposed surrogate model showed significant reduction of the computational time, as it makes possible to run many sub-iterations without the need to recalculate the derivatives.

The use of a global stress measure together with the proposed surrogate model is very beneficial, as in this type of problem many project variables have impact over only one constraint. The fact that the surrogate model makes possible to do small variations in elements density in each sub-iteration, without increasing the computation burden, reflects in the reduction of the number of iteration needed for convergence.

The criterion to remove low density elements presented good results reducing the total computational time. However it is important to notice that using this method it is not possible to re-add an element that was previously removed, and as consequence the function might converge to a local minimum instead of the global.

Adding the estimated error to the stress values for the calculation of the stress constraint showed beneficial results, having the advantage of obtaining safer structures.

REFERENCES

Bendsoe, M.P., Sigmund, O., *Topology Optimization - Theory, Methods and Applications,* Springer, 2004.

Bourdin, B.; *Filters in topology optimization,* **International Journal for Numerical Methods in Engineering,** v. 50, p. 2143-2158, 2001.

Emmendoerfer, H.J. *Problema de otimização estrutural com restrição de tensão local usando o método level set,* Dissertação de mestrado, UFSC, 2011.

Guilherme, C.E.M. *Otimização topológica e cálculo do gradiente de forma para estruturas submetidas à restrição de fadiga,* Dissertação de Mestrado, UFRGS, 2006.

Guo, X.; Zhang, W.S.; Wang, M.Y.; Wei, P. *Stress-related topology optimization via level set approach,* **Computational Methods Applied to Mechanical Engineering**, v. 200, p. 3439-3452, 2011.

Kai, J.A.; Lee, E.; Martins, J.R.R.A. *Stress-based topology optimization using an isoparametric level set method.* **Finite Elements in Analysis and Design,** v.58, p.20-30, 2012.

Le, C.; Noratto, J.; Bruns, T.; Ha, C.,; Torrorelli, D. *Stress-based topology – optimization for continua***. Structural Multidisciplinary Analysis**, v. 41, p. 605-620, 2010.

Luo, Y.; Zhou, M.; Wang, M.Y.; Deng, Z. *Reliability based topology optimization for continuum structures with local failure constraints,* **Computer & Structures,** v.143, p.73-84, 2014.

Forrester, A.I.J., Keane, A.J. *Recent advances in surrogate-based optimization,* **Progress in Aerospace Sciences**, v.45 p. 50-79, 2009.

Queipo, N.V., Haftka, W.S., Shyy, W., Goel, T., Vaidyanathan, R., Tucker, P.K. *Surrogate-based Analysis and Optimization,* **Progress in Aerospace Sciences,** v.41, p.1-28, 2005.

Silva, E. *Uma formulação de tensão topológica com restrição de tensão suavizada*, Dissertação de mestrado, UFRGS, 2012.

Sigmund O., Petersson, J. *Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima,* **Structural Optimization,** v. 16, p. 68-75, 1998.

Xia, Q.; Shi, T.; Liu, S.; Wang, M.Y. A level set solution to the stress-based structural shape and topology optimization, **Computer & Structures**, v.90-91, p. 55-64, 2011.

Xie, Y.M., Steven, G.P., *Evolutionary Structural Optimization,* Springer-Verlag, 1997.

Yang, R.J.; Chen, C.J. *Stress-based topology optimization,* **Structural Optimization**, v. 12, p. 98-105, 1996.

Zhang, S.C.; Zhu, J.; Gao, T. Stress constrained shape and topology optimization with fixed mesh: A B-spline cell method combined with level set function, **Computed Methods in Applied Mechanics and Engineering,** v.278, p.361-387, 2014.

Zienkiewicz, O.C.; Zhu, J.Z. *A simple estimator and adaptative procedure for practical engineering analysis*, **International Journal for Numerical Methods in Engineering,** v.24, p.337-357, 1987.