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VALIDATION TEST OF A NUMERICAL SCHEME MODELING USING THE LATTICE-BOLTZMANN AND THE COHESIVE FRACTURE MODELS FOR HYDRO-FRACKING SIMULATION

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Abstract. *This research considers the hydraulic fracturing or hydro-fracking process in an impermeable material. The PPR potential-based cohesive model (Park et. al., 2009) (Park & Paulino, 2012) was implemented in the finite element method (FEM) to simulate the crack propagation. In this implementation, interface cohesive elements are adaptively inserted into the mesh to capture the softening fracture process (extrinsic implementation). To model the fluid injected, the lattice-Boltzmann model (LBM) is used through an iterative procedure (Mejia et. al., 2014). In the FEM, boundaries of cracks are represented by the faces of the cohesive elements. The position and velocity of these faces are transferred to the LBM as a boundary condition, where the forces applied on these boundaries, caused by the fluid injected, can be calculated. These forces are transferred to the FEM as external force applied on the cohesive element's faces, where the new position and velocity of the crack's boundary is computed. These values are transferred to the LBM, initializing a new calculation cycle. The purpose of this paper is to present results concerning the validation of the proposed procedure against available analytical solutions.*

Keywords: *Hydro-fracking, Cohesive fracture model, Lattice-Boltzmann model, PPR model*

1 INTRODUCTION

In hydraulic fracturing process, a discontinuity is created inside a solid through the application of hydraulic pressure. To create a crack, the fluid pressure injected has to be higher than the sum between the tensile strength and the minor confined stress. The crack propagation happens when the fluid injected flows into the crack. Several applications of this process may be observed in different areas (civil, mining and oil industries) having their most relevant application in oil industry. To improve the oil production, hydraulic fracturing process is used to create cracks inside the rock reservoir, increasing the permeability. Several authors have studied the hydraulic fracturing process from analytical (Detournay, 2004)(Burger et. al., 2005)(Adachi and Detournay, 2008) and numerical (Lecampion, 2009)(Hunsweek and Shen, 2012)(Carrier and Granet, 2012) perspective, generally based in continuum mechanics or using interface elements through a known propagation path. The hydraulic fracturing process is a coupled phenomenon that involves a moving boundary and fluid flow into the fractures, where the fluid flow depends of the crack aperture. When the material has heterogeneities or discontinuities, a fracture could be propagated in an irregular path, showing branching. Conventional finite element method (FEM) does not easily allows simulating irregular crack propagation. The use of enrichment elements can become a difficult task to be implemented when the fracture has an irregular path or show branching. Otherwise, interface elements used to represent the non-linear fracture process zone ahead of a crack tip, allow simulating these irregular paths and their implementation may be easier than the enrichment elements. The interface elements can be inserted into the mesh at the beginning of the calculation process (intrinsic) or inserted into the mesh to capture the softening process fracture (extrinsic). The softening fracture process is modeled by the interface elements and the crack is considered opened when their aperture reach a critical value.

A fluid flow into an interface element (crack) may be considered as a flow between parallels plates, where its motion can be described by the Navier Stokes equations. An alternative method to solve these equations is the lattice Boltzmann method (LBM) (McNamara and Zanetti, 1988)(Lallemand and Luo, 2003)(Noble and Torcynsky, 1998). This method allows simulating the fluid flow through complex geometries as a fracture. The proposal of this research is to simulate the hydraulic fracturing process using the FEM coupled with the LBM.

2 PPR POTENTIAL-BASED COHESIVE MODEL IMPLEMENTATION TO SIMULATE THE CRACK PROPAGATION

Finite element method (FEM) is used to simulate the mechanical process of the hydraulic fracturing. The crack propagation is modeled using the PPR potential-based cohesive model (Park et. al., 2009)(Park and Paulino, 2012) by means of an extrinsic implementation. In this case, the cohesive elements are adaptively inserted in the mesh to capture the softening process (extrinsic implementation). The consequence of this insertion is the duplication of some nodes during the calculation process. To manage the mesh information and the changes in its topology we use the Tops code (Paulino et. al., 2008)(Espinha et. al., 2009). The Tops code is a topological data framework developed in C++ that supports these modifications in the mesh and allows optimum information management. To insert the cohesive elements, the tensile traction is checked at all facets of the mesh and is inserted when this tensile traction supersedes the normal or tangential cohesive strengths. The cohesive element allows simulating the softening process through the cohesive traction-separation relationship given by:

$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) + \Big[\Gamma_n\Big(1 - \frac{\Delta_n}{\delta_n}\Big)^{\alpha} + \langle \phi_n - \phi_t \rangle\Big]\Big[\Gamma_t\Big(1 - \frac{|\Delta_t|}{\delta_t}\Big)^{\beta} + \langle \phi_t - \phi_n \rangle\Big] \quad (1)$$

Where Ψ is the potential function for cohesive element, ϕ_n, ϕ_t are the normal and tangential cohesive strengths, Γ_n, Γ_t are the energy constant in the PPR model, Δ_n, Δ_t are the separation field in local coordinates, δ_n, δ_t are the normal and tangential final crack opening widths and α, β are the shape parameters in PPR model. The gradient of the PPR potential leads to the normal and tangential cohesive tractions along the fracture surface by:

$$T_n(\Delta_n, \Delta_t) = -\alpha\frac{\Gamma_n}{\delta_n}\Big(1 - \frac{\Delta_n}{\delta_t}\Big)^{\alpha-1}\Big[\Gamma_t\Big(1 - \frac{|\Delta_t|}{\delta_t}\Big)^{\beta} + \langle \phi_t - \phi_n \rangle\Big] \quad (2)$$

$$T_t(\Delta_n, \Delta_t) = -\beta\frac{\Gamma_t}{\delta_t}\Big(1 - \frac{|\Delta_t|}{\delta_t}\Big)^{\beta-1}\Big[\Gamma_n\Big(1 - \frac{\Delta_n}{\delta_n}\Big)^{\alpha} + \langle \phi_n - \phi_t \rangle\Big]\frac{\Delta_t}{|\Delta_t|} \quad (3)$$

The finite element code implemented uses linear triangular elements (T3) and the cohesive element is composed of 4 nodes.

3 FLUID FLOW SIMULATION USING THE LATTICE-BOLTZMANN MODEL

Fluid modeling may be accomplished by using the LBM. In this model, space is discretized in cells where the probable number of particles that comprise them, (on time t and with velocity **v**) is represented by the distribution function $f(x, v, t)$. From this function, fluid macroscopic variables may be calculated. Usually, the solution of LBM is accomplished via two-step process of collision and propagation. The collision phase redistributes the particles on the node (particles collide with each other) due to the collision effect and the propagation phase propagates the particles to neighboring nodes. For the case of the incompressible LBM, the evolution equation is given by:

$$p_\alpha(x + v_\alpha \Delta t, t + \Delta t) = p_\alpha(x, t) - \frac{1}{\tau^*}(1 - B)(p_\alpha(x, t) - p_\alpha^{eq}(x, t)) + B\Omega_\alpha^s \quad (4)$$

$$p_\alpha \equiv c_s^2 f_\alpha \quad (5)$$

$$p_\alpha^{eq}(x, t) = w_\alpha\Big\{p + p_0\Big[3\frac{(v_\alpha \cdot u)}{c^2} + \frac{9}{2}\frac{(v_\alpha \cdot u)^2}{c^4} - \frac{3}{2}\frac{u^2}{c^2}\Big]\Big\} \quad (6)$$

Where the $\tau^* = \tau/\Delta t$, τ is the time relaxation, p_α^{eq} is the pressure distribution function in the equilibrium, x is the position of the node and t is the time. For the two-dimensional case, it is usual to choose a D2Q9 cell that allows nine velocity directions, four on the vertex of the cell, four at the center of each cell side and one in the center of the cell itself. The solids can be included in the mesh. Each cell is identified with a number: one for solids and zero for fluids. For the interface between solid and liquid, the cells partially saturated (Noble and Torczynsky, 1998) were implemented. The B parameter, shown in the Eq. 4 represents a weight function dependent of the solid fraction ϵ in the cell. The value of the B parameter is given by:

$$B(x, t) = \frac{\epsilon(x, t)(\tau^* - 0.5)}{1 - \epsilon(x, t) + (\tau^* - 0.5)} \quad (7)$$

and Ω^s_α is an adicional term to modify the pressure function distribution of the solid-liquid interface (Noble and Torczynsky, 1998). Its equation introduces the term v_p that represents the wall velocity.

$$\Omega^s_\alpha(x,t) = p_{-\alpha}(x,t) - p_\alpha(x,t) + p^{eq}_{-\alpha}(p, v_p) - p^{eq}_{-\alpha}(x,t) \tag{8}$$

4 COUPLING IMPLEMENTATION

Two meshes are used in the coupling process between FEM and LBM. The fracture boundaries are obtained from the finite element mesh. Their position, which is computed using FEM, are transferred to the LBM. The force applied on the boundaries, caused by the fluid pressure, can be computed using LBM. After this calculation, these forces are transferred to the FEM and applied as external forces on the faces of the crack. The new position of the crack boundaries is calculated using FEM and then is transferred to the LBM to update the boundary conditions. Similar to the coupling process between the discrete element method (DEM) and LBM (Velloso, 2010)(Han et. al., 2007), the coupling process between FEM and LBM has two time steps. Normally, the time step for the FEM is smaller than the time step for the LBM. For this feedback-loop for fluid-structure interaction, it is necessary to define a subcycle number. The subcycle number can be described by:

$$n(subcycle) \approx \Delta t/dt + 1 \tag{9}$$

where *n(subcycle)* is an integer number, Δt is the LBM time step and *dt* is the FEM time step. Thus, for one step in lattice Boltzmann, *n* steps of FEM are needed.

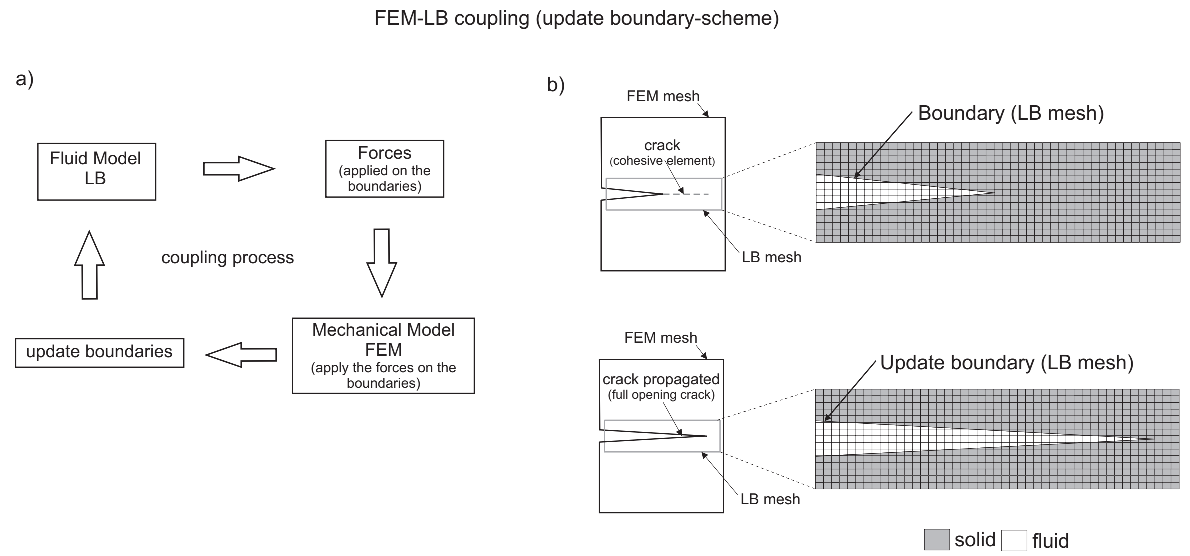

Figure 1: a) The coupling process between FEM and LBM for each subcycle; b) detail of the two meshes used on the numerical simulation. The LB mesh is created in a zone where the fracture propagation happens.

5 VALIDATION OF THE IMPLEMENTATION MODEL

To validate de model, the results obtained from the numerical implementation were compared with the analytical solution (Bunger el at, 2005)(Garagash and Detournay, 2005)(Garagash, 2006). The assumptions considered in these analytical solutions are:

- The material is homogeneous and isotropic.
- The material strain is based in the elastic theory.
- The fluid is incompressible and Newtonian.
- The fluid flow into the fracture has a laminar regime, given by the lubrification theory.
- The geometry of the fracture is simple, with a rectangular or radial form.
- The crack propagation is caused by the fluid injected and its propagations happen in the Mode I of rupture. The process is based on the linear elastic fracture mechanic theory.

For the numerical implementation, the considered assumptions are different of the analytical solution in some points. These are:

- The material is homogeneous and isotropic.
- The material strain is based in the elastic theory.
- The lattice-Boltzmann method for incompressible fluid is used.
- For this case, the geometry of the fracture is rectangular, generated by the insertion of the cohesive elements.
- The crack propagation is caused by the fluid injected and its propagation happens in the Modo I of rupture. The softening process of the fracture is simulated by the cohesive element adaptively inserted in the mesh. Its behavior is based in a potential function, corresponding to the nonlinear fracture mechanics.

Tab. 1 shows the properties of the materials used in the numerical simulation. In the analytical solution, two cases are treated: viscosity and toughness dominated regimes. The viscosity-dominated regime considers that the energy dissipated in extending the fracture in the material is negligible compared to energy losses in the viscous fluid flow, while the toughness-dominated regime considers the opposite (Garagash and Detournay, 2005). To simulate both conditions, the fracture energy was modified. A lower fracture energy (1N/m) was used in the cohesive element to simulate the viscosity regime condition and it was incremented (20N/m) to simulate the toughness regime condition.

Table 1: Properties of the material, fracture and fluid

E	ν	$\sigma_n = \sigma_t$	$\alpha = \beta$	$\phi_n = \phi_t$	Viscosity	Fluid density	Q_0
5 GPa	0.2	1 MPa	2	(1 or 20) N/m	1.5e-6 m/s	1000 Kg/m^3	7.6e-5 m^3/s

Fig. 2 and Fig. 3 show the results of the implementation model compared with the analytical solution.The first observation is about the pressurization process. In the numerical implementation, it is necessary to reach the strength of the cohesive element to initiate the fracture process. In fact, the typical curve of the hydraulic fracturing can be simulated, showing the pressure fluid break, the pressure fluid propagation and it is possible to simulate the shut-in process. This fluid injection process is very important to compare the results with the analytical solution. The second observation is concerned to the values of the results for the aperture and length of the crack. The Fig. 2e,f and Fig. 3e,f show that the aperture and length curves for both regimes have a similar behavior to the analytical solution. The differences between these values may be associated to the assumptions considered in the numerical implementation and to another mechanisms acting during the hydraulic process. The cohesive fracture model, used to capture the softening fracture process, has a nonlinear behavior. During this softening process, two types of forces act on the fracture's faces: the cohesive force, trying to close the fracture, and the fluid force (related to the fluid pressure) trying to open the fracture. Fig. 2b and Fig. 3b show the dependence of the pressure break and the fluid propagation pressure with the fracture energy and with the time that fluid force is activated in the softening process Fig. 3b.

The curves of the fluid pressure show a different behavior. This happens because the injection fluid process (Fig. 2c and Fig. 3c) is not considered in the analytical solution. In the analytical solution, the fluid pressure is uniform within the fracture, and has an asymptotic behavior near to the tip. The lattice-Boltzmann model shows another behavior. When the fracture is opening, a depressurization effect appears inside the fracture caused by the increment of its volume. This important effect, showed in the Fig. 2d and Fig. 3d, is not predicted by the analytical solution.

6 CONCLUSIONS

The methodology proposed to simulate the hydraulic fracturing process allows the reproduction of the fluid injection process, showing the initial pressurization, the break pressure and the propagation pressure. The comparison with the analytical solution showed a similar behavior for the aperture and the length of the fracture, but the fluid pressure had a different behavior due to the modeling process. The fluid flow behavior modeled by lattice-Boltzmann model shows a depressurization effect within the fracture caused by the increment of the cohesive element volume, an effect not taken into account by the analytical solution. This effect is important because it has influence in the aperture and in the length of the fracture.

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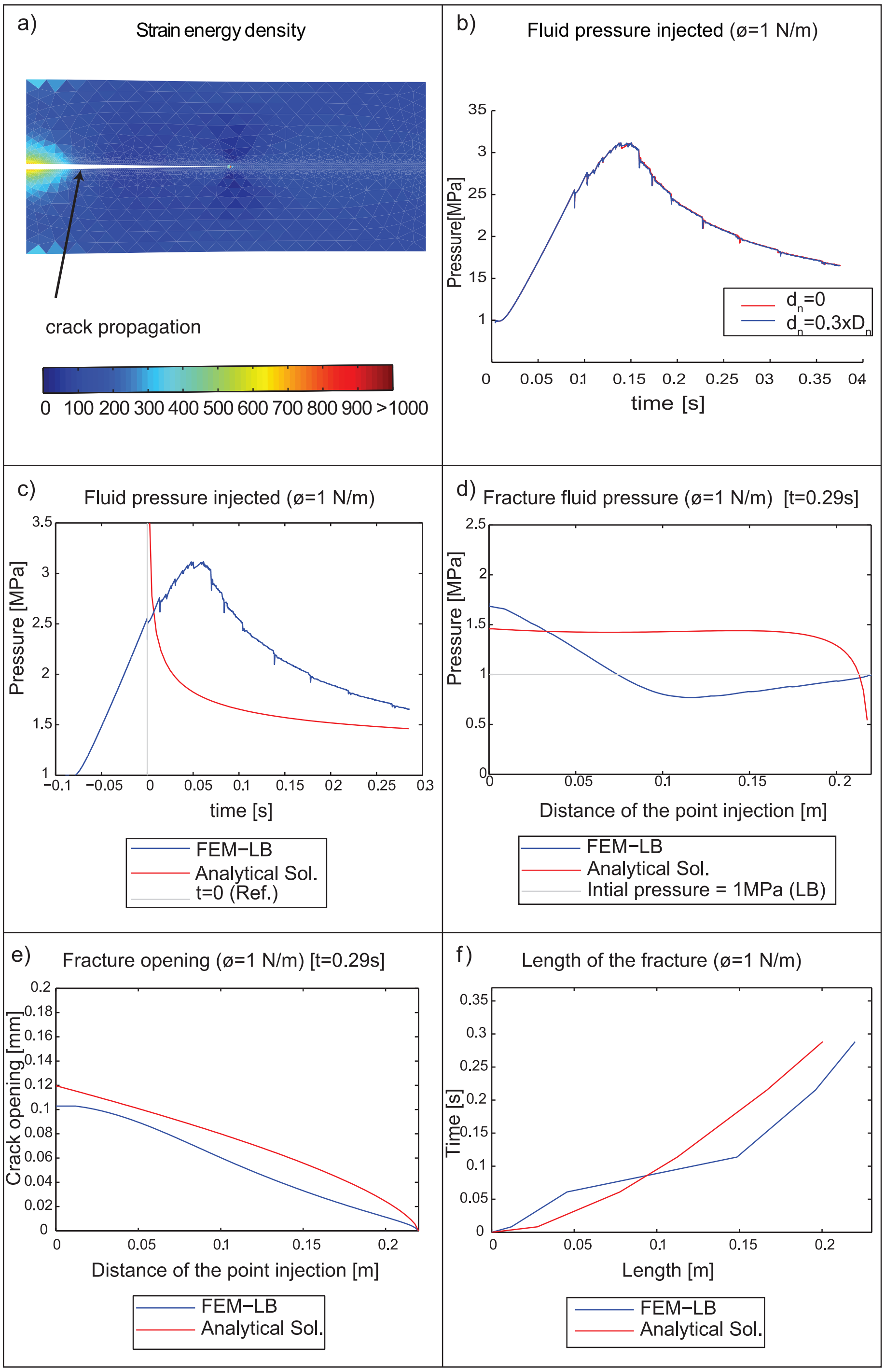

Figure 2: Hydraulic fracturing model using the numerical model proposed in the viscosity-dominated regime.

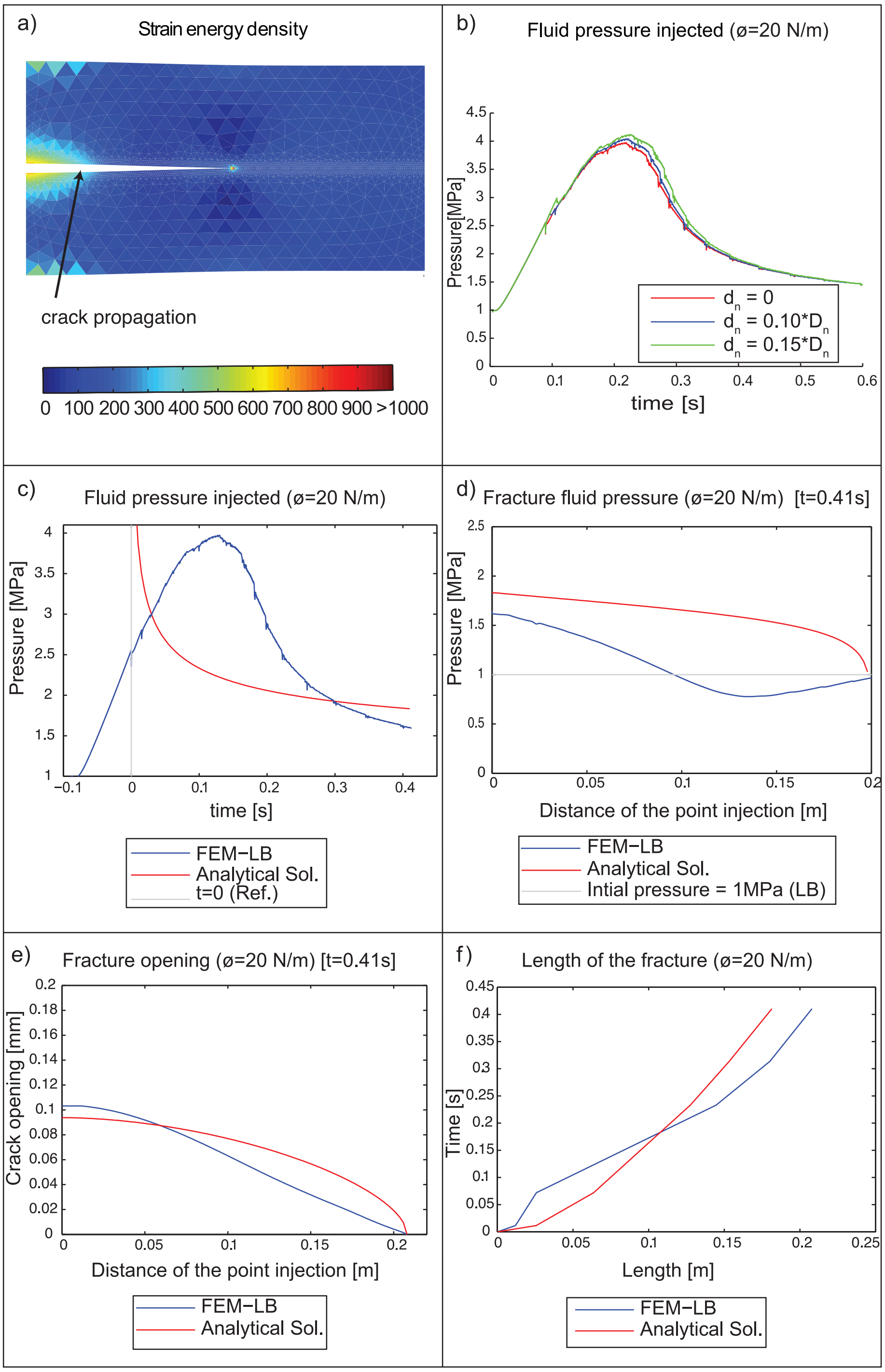

Figure 3: Hydraulic fracturing model using the numerical model proposed in the toughness-dominated regime.

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