A COMPARATIVE STUDY ON FINITE ELEMENT METHODS FOR CRACK PROPAGATION IN CONCRETE

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Abstract. *A research about numerical modeling for crack propagation in concrete is explored using finite element software Abaqus. Three methods are considered: the extended finite element (XFEM), element elimination technique and interelement crack method. The XFEM is a method for arbitrary crack propagation that introduces fracture implicitly in the model without remeshing. The implementation of the XFEM formulation used in this work is based on the Phantom Node Method in which the discontinuity is introduced through new degrees of freedom of overlapping elements. This method is used in combination with a cohesive fracture law. In the element elimination technique, elements that meet a fracture criterion are deleted. In order to account for the energy dissipation, concepts from smeared crack models are applied. In the interelement crack methods, the crack is limited to element edges; the separation of these edges is governed by a cohesive law. Two problems are considered: mode I and mixed-mode fracture of a notched concrete beam. Comparison of the numerical model results with experimental data from the literature reveals the ability and disadvantages on the methods.*

Keywords: *extended finite element method (XFEM), element elimination technique, interelement crack method, concrete fracture.*

1 INTRODUCTION

The prediction of the behavior of concrete structures is of a major importance in engineering. In the past two decades, a huge effort was made to develop novel, efficient, and accurate computational methods, and an enormous progress was made. The focus of this paper will be on computational methods for discrete cracks. The performance of finite element methods for crack propagation, in quasi-brittle material like concrete, is studied. This research was carried out with the finite element software Abaqus and three methods are considered for crack propagation:

1. the extended finite element method,
2. the interelement crack method, and
3. the element elimination technique.

The Extended Finite Element Method (XFEM) is a numerical method that enables a local enrichment of approximation spaces. The method is useful for the approximation of solutions with pronounced non-smooth characteristics in small parts of the computational domain. The presence of fracture is ensured by the special enriched functions in conjunction with additional degrees of freedom with greater accuracy and computational efficiency. Furthermore, it is important to note that this method does not require the mesh to match the geometry of the fracture. It is a very attractive and effective way to simulate initiation and propagation of a crack along an arbitrary, solution-dependent path without the requirement of remeshing.

In the interelement crack method, the crack is modeled by separation along the element edges. In the approaches by Xu and Needlemen (1994), cohesive surfaces are introduced from the beginning in the simulation model. In contrast, Camacho and Ortiz (1996) adaptively introduced cohesive surfaces at element edges when a certain cracking criteria is met.

The Element Elimination Technique (EET) is the easiest way to deal with discrete fractures. There is no need to explicitly represent strong discontinuities in displacement fields since fractured elements are generally reflected by a set of elements in which the stress is set to zero. In order to account for the correct energy dissipation in the post-localization domain, concepts from smeared crack models are applied.

In this work, examples including mode I as well as mixed-mode fracture problems are analyzed. Comparison of the numerical model results thus obtained with experimental data from the literature reveals the ability and disadvantages of the methods for the structural failure modes.

2 REVIEW OF XFEM

The XFEM is implemented within Abaqus using the so called “phantom node method” (Song et al., 2006; Abaqus, 2011). The key idea in the formulation of this method is that the displacement field incorporates the discontinuity through additional terms in the conventional displacement approximation. A crack is introduced by addition of an extra element on top of an existing element, as shown in Figure 1. The jump in the displacement field is realized by simply integrating only over the area from the side of the real nodes up to the crack, i.e. domain Ω_A and Ω_B. Thus, the approximation of the displacement field within an element is then given (Song et al., 2006):

$$u(X,t)=\sum_{I\in S_A}\underbrace{u_I^A(t)N_I(X)}_{u^A(X,t)}H(-f(X))+\sum_{J\in S_2 B}\underbrace{u_J^B(t)N_J(X)}_{u^B(X,t)}H(f(X))\,. \tag{1}$$

where S_A and S_B are the index sets of the nodes of superposed element A and B, respectively, and $H(X)$ is the Heaviside function.

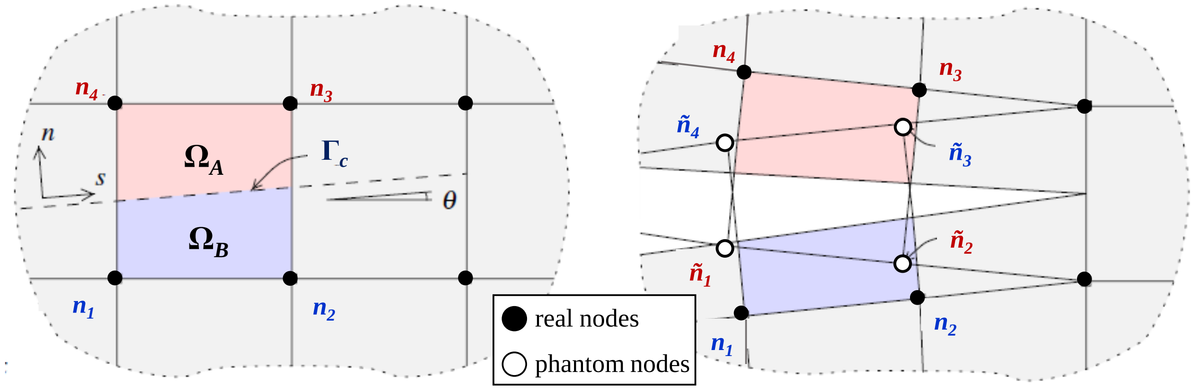


Figure 1. Connectivity and active parts of two overlapping elements in phantom node method.

The phantom node method is only applicable to cohesive crack modeling where singularity in the stress field is removed due to the presence of a cohesive traction. The discontinuity grows elementwise and the tip is always located at an element boundary.

3 REVIEW OF THE INTERELEMENT CRACK METHOD

The crack propagation is modeled in Abaqus through zero-thickness interface elements. The interface elements are defined a priori and placed between the faces of solids elements. This is the approach by Xu and Needlemen (1994), also called intrinsic cohesive zone model (CZM). In the Camacho and Ortiz (1996) approach, elements are allowed to separate along faces only when a criterion is met. This approach is called extrinsic CZM, and normally a remeshing algorithm is used. Figure 2 shows a scheme of the two approaches.

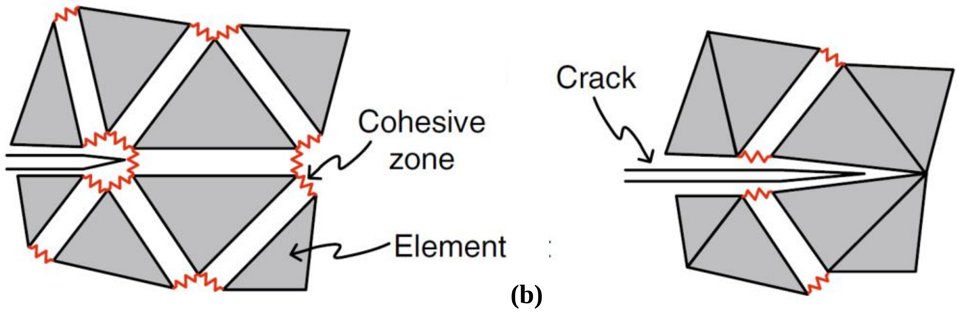


Figure 2. Schematic showing: (a) Xu and Needlemen (1994) and (b) Camacho and Ortiz (1996).

The interface elements are based on a traction-separation description for the modelling of cohesive cracks. The relation between the stress and crack opening for a 2D configuration are given by Eq. (2) (Abaqus 2011):

$$\begin{bmatrix}\sigma\\ \tau\end{bmatrix}=\begin{bmatrix}K_{nn} & 0\\ 0 & K_{tt}\end{bmatrix}\begin{bmatrix}\delta_n\\ \delta_t\end{bmatrix} \tag{2}$$

where σ and τ are the normal and shear stresses acting along the interface, respectively, K is the constitutive matrix, and δ_n and δ_t are the crack opening in the normal and tangential direction, respectively.

4 COHESIVE ZONE MODEL

This model treats fracture as a gradual process in which separation between incipient material surfaces is resisted by cohesive traction. In the cohesive zone model (CZM), there is no stress singularity near the crack tip and the traction on the crack surface is a function of the crack-opening displacement. Crack initiation and propagation are natural outcomes of a CZM analysis.

A cohesive law with linear softening is used for modeling of crack surface traction. For the XFEM procedure, a schematic is shown in Figure 3-a, where the area under the curve is the fracture energy, G_f, of the material, σ_{max} is the cohesive strength and δ_{max} is the maximum crack opening displacement. For the interface cohesive element procedure, it is also required to define the material behavior prior to damage initiation, which is assumed to be linear with initial stiffness K_n, as shown in Figure 3-b.

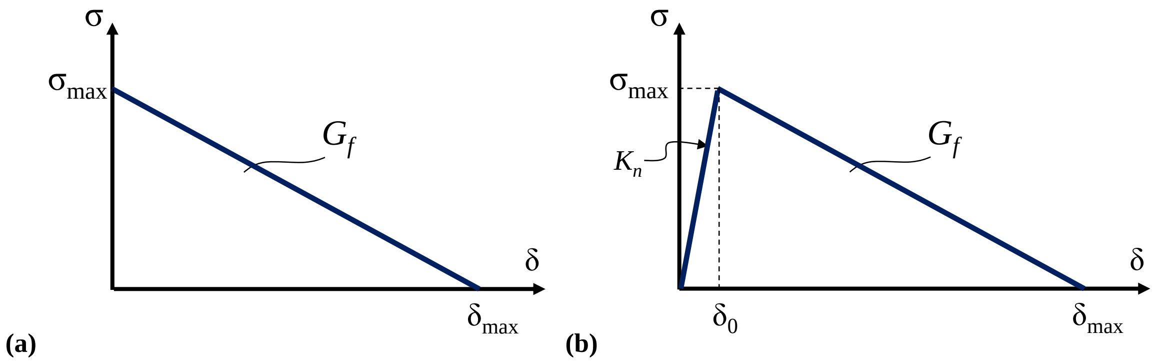


Figure 3. Law cohesive for (a) XFEM and (b) interface cohesive elements procedure.

The onset of damage refers to the beginning of the degradation of the response of a material point. A maximum principal stress criterion to predict damage initiation is adopted in the current research. The damage evolution describes the rate at which the stiffness of the material is degraded once the corresponding initiation criterion is reached. The damage evolution for mixed mode failure is defined based on the power law criterion, which is established in terms of an interaction between the energy release rates (Camacho and Davila, 2002):

$$\left(\frac{G_I}{G_{IC}}\right)^{\eta} + \left(\frac{G_{II}}{G_{IIC}}\right)^{\eta} = 1 \tag{3}$$

where G_I and G_{II} refer to the work done by the traction and its conjugate relative displacement in the normal and shear direction, respectively. The parameters G_{IC} and G_{IIC} refer to the critical fracture energies required to cause failure in the normal and the shear direction, respectively, and η is a material parameter.

5 REVIEW OF THE ELEMENT DELETION METHOD

The Element Elimination Technique (EET) is one of the simplest methods for fracture simulation within the framework of the conventional FEM. There is no need to explicitly represent strong discontinuities in displacement fields since fracture is modeled by a set of deleted elements. The cracked element is removed from the calculations after an amount of crack opening displacement is attained at which it can no longer sustain stresses. To eliminate an element, all components of stress tensors in this element are set to zero. As a result, all forces in this element become zero as well and therefore, this element stops to transmit load to neighboring non-eliminated elements. In order to avoid numerical problems related to strong local loss of equilibrium, the stress is set to zero in several relaxation steps. The modulus of elasticity in the eliminated elements is set to zero in the last relaxation step (Saharan and Mitri, 2008). A removed element represents a macro crack or fracture in the context of the present development. Figure 4 provides information about logical steps used by the EET using the brittle failure material model.

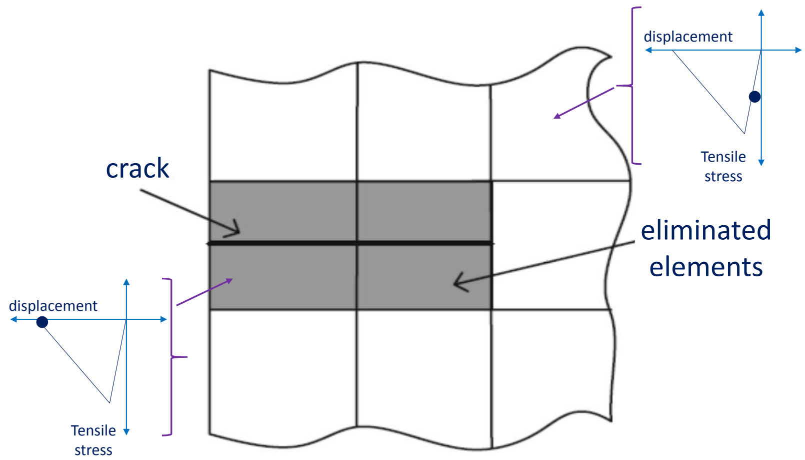


Figure 4. Schematic representation of the EET approach using brittle failure model.

A simple Rankine criterion is used to detect crack initiation. This states that a crack forms when the maximum principal tensile stress exceeds the tensile strength of the material.

The brittle-crack model is characterized by a stress-displacement response rather than a stress-strain response. Many researchers have addressed this concern, and the consensus is that Hillerborg's (1976) approach, based on brittle fracture concepts, is adequate to deal with this issue for practical purposes. A length scale, typically in the form of a “characteristic” length, is introduced to “regularize” the smeared continuum models and attenuate the sensitivity of the results to mesh density. Thus, crack-opening displacement is selected as a criterion for element elimination from the model. The post-failure stress-displacement curve is show in Figure 5-a.

Although crack detection in the present development is based purely on Mode I fracture considerations due to the adoption of the Rankine failure criterion, ensuring cracking behavior includes both Mode I (tension softening) and Mode II (shear softening/retention) behavior. In this model, the dependence is defined by expressing the post-cracking shear modulus G_c as a fraction of the uncracked shear modulus.

$$G_c = \rho(e_{nn}^{ck})G \tag{4}$$

where G is the shear modulus of the uncracked material and the shear retention factor, $\rho(e_{nn}^{ck})$, depends on the crack opening strain e_{nn}^{ck}. The shear retention is defined in the power law form (Abaqus, 2011):

$$\rho(e_{nn}^{ck}) = \left(1 - \frac{e_{nn}^{ck}}{e_{\max}^{ck}}\right)^p \tag{5}$$

where p and $e_{\max}^{ck}$ are material parameters. This curve is shown in Figure 5-b.

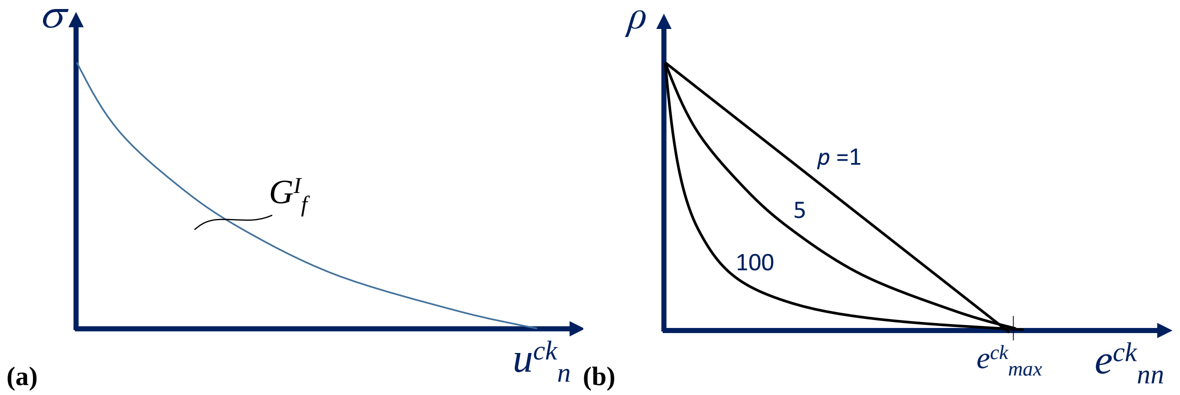


Figure 5. (a) Post-failure stress-facture energy-curve and (b) shear retention model.

6 BENCHMARK PROBLEMS

6.1 Mode I fracture of a notched concrete beam

From among the great variety of mode I fracture experiments a notched concrete beam tested by Peterson (1981) was selected for analysis various numerical methods. The experiments were conducted on a 2000 x 200 x 50 concrete beam with an initial notch midway from the bottom boundary. A schematic showing the experimental setup is shown in Figure 6. The specimen is loaded by prescribing the vertical displacement at the center of the beam until it reaches a value of 1 mm.

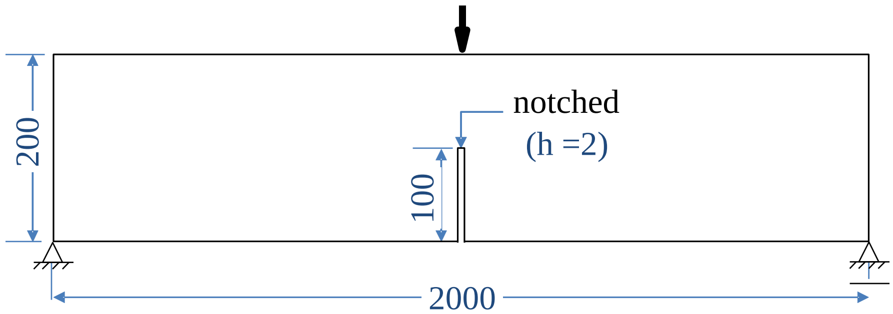


Figure 6. Experimental set up for notched beam in mode I fracture (dimensions in mm).

The material properties of the concrete are ρ = 2400 kg/m^3, E = 30000 MPa, ν = 0.2, mean tensile strength σ_{max} = 3.33 MPa, and fracture energy for mode I G_{IC} = 124 N/m.

For the XFEM simulations, the bulk material is linear elastic and when the maximum principal stress at an integration point of the element reaches the maximum tensile strength, 3.33 MPa, the formulation adds a strong discontinuity that is governed by a linear cohesive law. The critical crack opening which corresponds to the fracture energy G_{IC} = 124 N/m is $\delta_{max} = 7.44 \cdot 10^{-5}$ m.

For the interelement crack method, the intrinsic cohesive zone model is used with equivalent parameters: σ_{max} = 3.33 MPa, $\Delta\delta = 1.49 \cdot 10^{-7}$ m and $K_n = 2.24 \cdot 10^7$ MPa/m.

By the element elimination technique, the tensile strength and fracture energy do not completely define the evolution of the postcracking stress. The exponent in the definition of the shear retention factor by Eq. 5 is *p*=200. The width of the crack band was assumed *h* = 2 mm. Moreover, in this case a dynamic analysis is carried out (Abaqus/Explicit). Hence, care must be taken that the beam is loaded slowly enough to eliminate significant inertia effects. A vertical velocity at the center of the beam increased linearly up to 0.01mm/s to eliminate elastic waves. Thus, a final vertical displacement of 1 mm in the center of the specimen is obtained after 10 s.

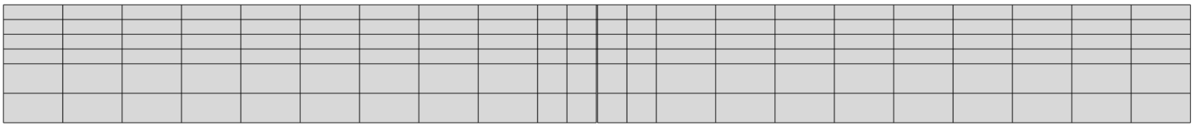
Four finite element meshes are used to show the influence of mesh refinement on the load-displacement response of the concrete beam. Figure 7 shows the meshes used for this purpose: a coarse mesh of 136 elements, a medium mesh of 536 elements, a fine mesh of 2128 elements and a very fine mesh of 8480 elements. The numerical calculations were made with structured quadrilateral meshes. The meshes consist of four-node plane stress elements, numerically integrated by the four-point Gaussian scheme. The element elimination technique in Abaqus/Explicit only supports reduced integrated elements. In the XFEM and interface cohesive elements, the Riks method is used since the solution is quite unstable when cracking progresses.

The numerical load-deflection curves obtained with the four meshes are shown in Figure 8. Furthermore, Figure 8 compares experimental and numerical load-deflection curves for XFEM, interface elements and the element elimination technique. These figures show that the coarse mesh gives a bad prediction of the post-failure behavior by all three methods. By the first two methods, some mesh sensitivity remains comparing the medium and fine meshes. However, the fine and very fine meshes give similar results.

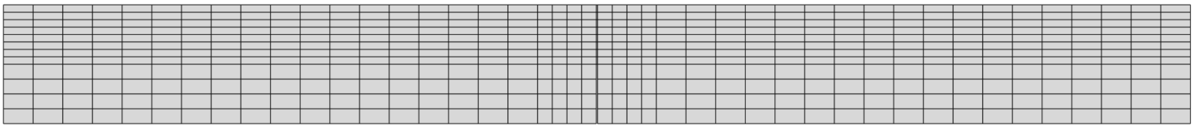
The numerical solution for the element elimination technique (EET) is found to be extremely sensitive to the size of the mesh for such type of fracture problems. Only the very fine mesh gave an approximate result. These results with the element deletion method were achieved by adopting a very low shear retention factor. In order to avoid energy consumption in crack shear a very low value should be adopted, because for mode I experiments no crack shear is expected.

In all cases, the linear softening diagram leads to a solution that clearly lies outside the experimental data from Peterson (1981). However the solution procedures were able to follow a post-peak path. The maximum load and the pre-peak response of the beam can be satisfactorily simulated for all cases.

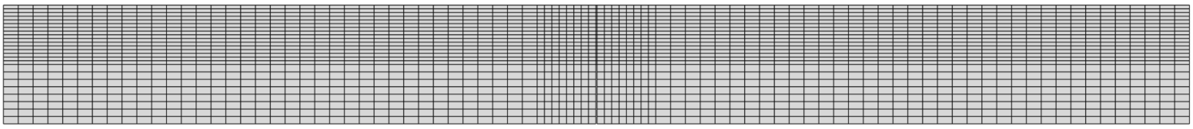
Based on these observations, all subsequent studies are done using the fine mesh, except for the element deletion method. The displaced shapes obtained for the three methods considered are shown in Figure 9 using the fine mesh.



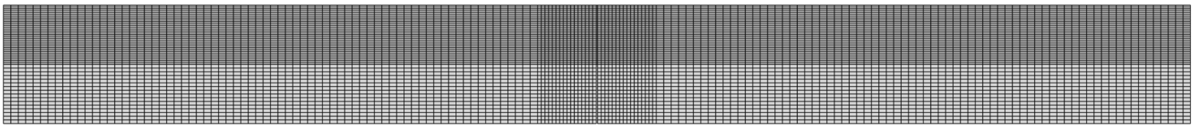
Coarse mesh



Fine mesh

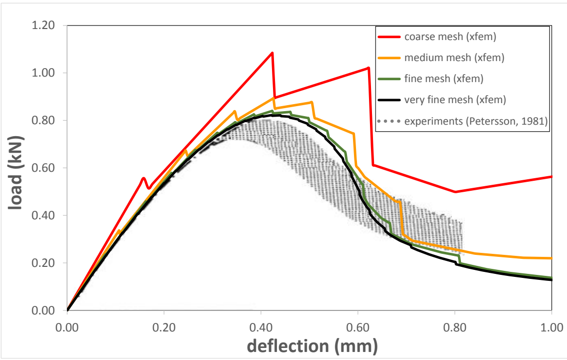


Fine mesh

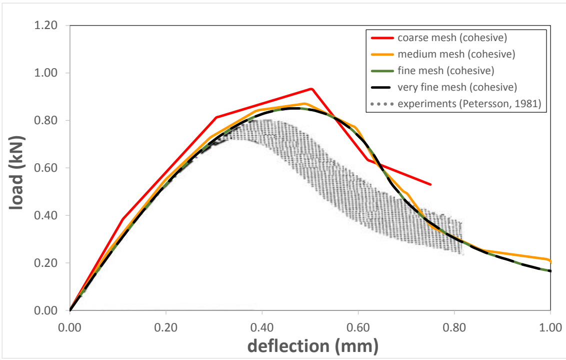


Very fine mesh

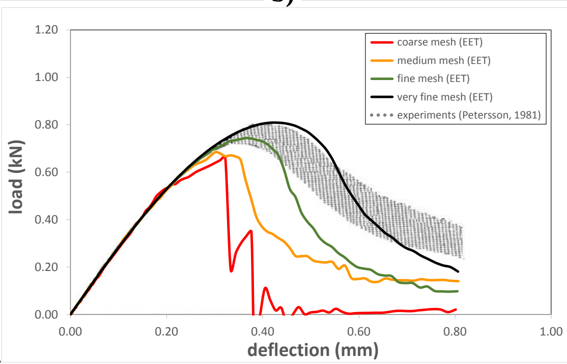
Figure 7. Finite element meshes of half of the notched beam.



a)



b)



c) .

Figure 8. Mesh refinement study for the notched beam in mode I fracture:
a) XFEM, b) Cohesive and c) EET

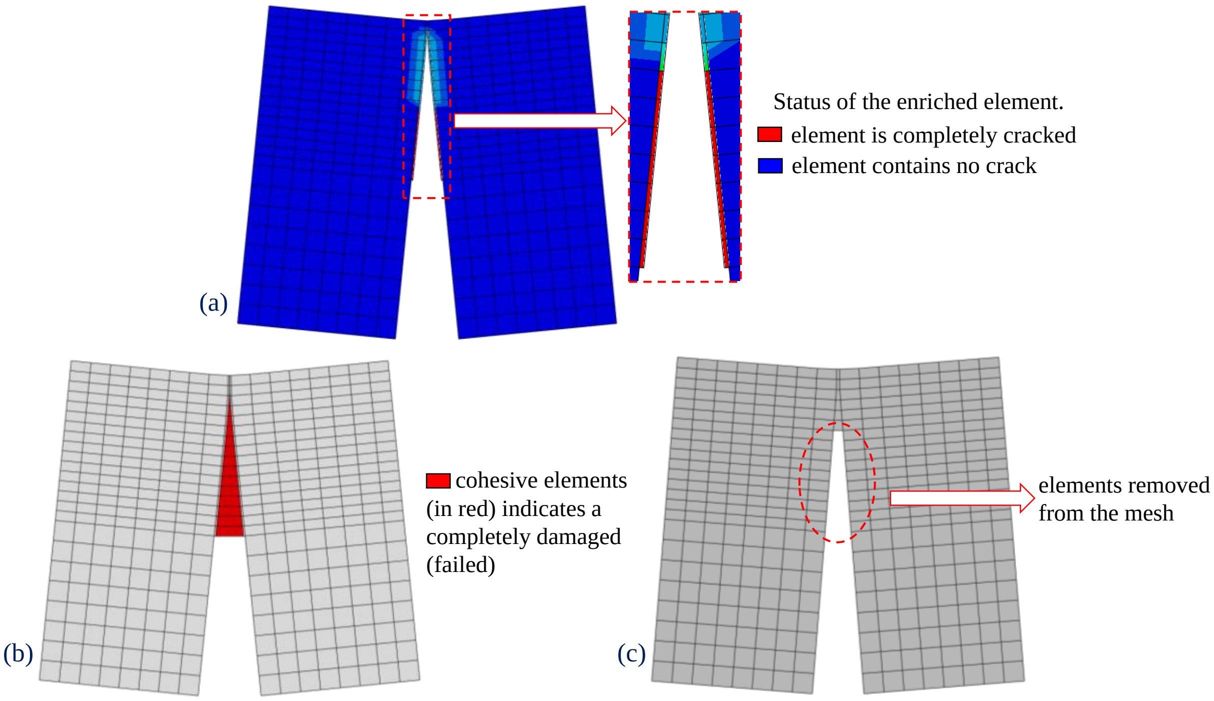


Figure 9. Displaced shapes obtained in the fine mesh (x 100): a) XFEM, b) cohesive and c) EET.

The results described above are obtained using linear tension softening. Such a choice leads to a response that is stiff compared with the experimental observations of Petersson (1981). The bilinear tension softening for the interelement crack method and element elimination technique was used to clarify the influence of the softening model. These functions are shown in Figure 10. The area under the softening curve is the same in all cases so that the Mode I fracture energy of the material is preserved.

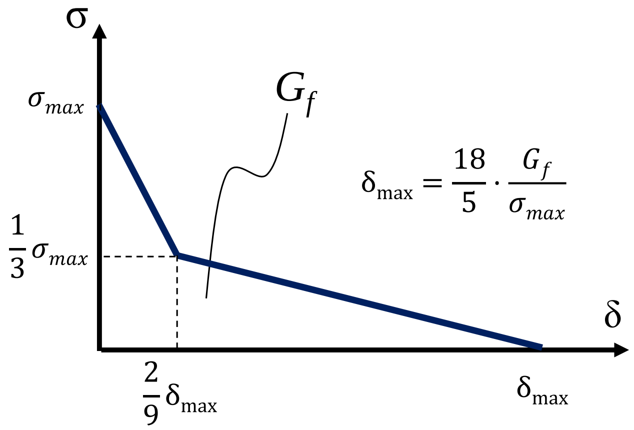


Figure 10. Bilinear softening diagram (Rots et al., 1985).

The load-deflection responses obtained with the interelement crack method and element elimination technique for the linear and bilinear softening diagram are shown in Figure 11 for the finest mesh. The interface cohesive elements agree well with the experiment result when the bilinear softening diagram is applied. It is clear that the stress drop after initial cracking lead to less stiff responses.

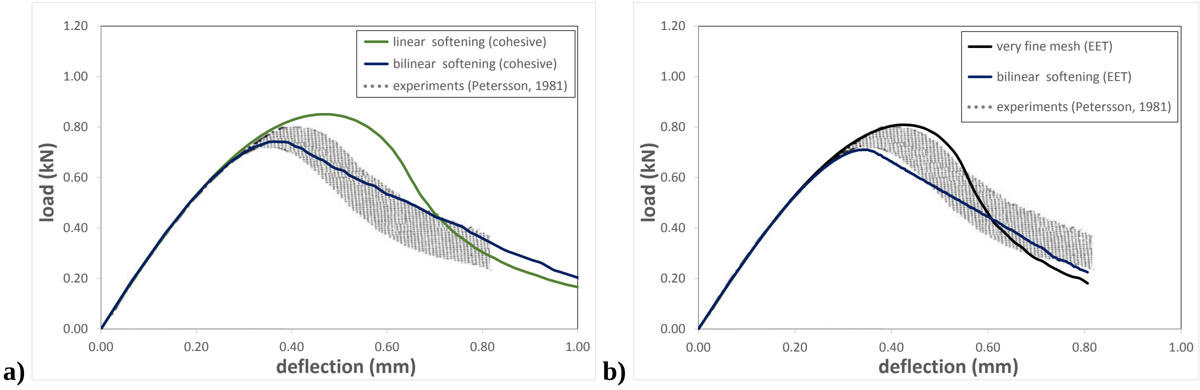


Figure 11. Tension softening study for the notched beam in mode I fracture: a) Cohesive (fine mesh) and b) EET (very fine mesh).

6.2 Mixed-mode fracture of a notched concrete beam

The preceding example was exceptional in that the crack path was purely straight and directed parallel to the lines of the mesh. In structural practice, cracks are seldom straight. Their opening path will be a mix between mode I (crack opening displacements) and mode II (crack sliding displacements).

From among the scarce experiments on mixed-mode concrete fracture a notched concrete beam, tested by Arrea and Ingraffea (1982), was selected for assessing various numerical methods. A schematic of the experiment is show in Figure 12. The supports and the loading conditions are non-symmetric with respect to the notch, as seen in Figure 12.

Note that the load was applied via a steel beam that distributed it to the concrete beam. The specimen is loaded by vertical velocity control at point C and transmitted to the notched beam through the rigid beam AB. The beam itself is not modeled since its kinematic motion can be introduced using constraint options. The applied velocity is linearly and slowly increased from 0 to 0.02 mm/s in 9 s to eliminate elastic waves. Thus, a final vertical displacement of 0.09 mm in the center of the specimen is obtained. The velocity chosen ensures that a quasi-static solution is obtained.

The material properties of the concrete are ρ = 2400 kg/m^3, E = 24800 MPa, ν = 0.18, mean tensile strength σ_{max} = 2.8 MPa, and fracture energy for mode I G_{IC} = 55 N/m.

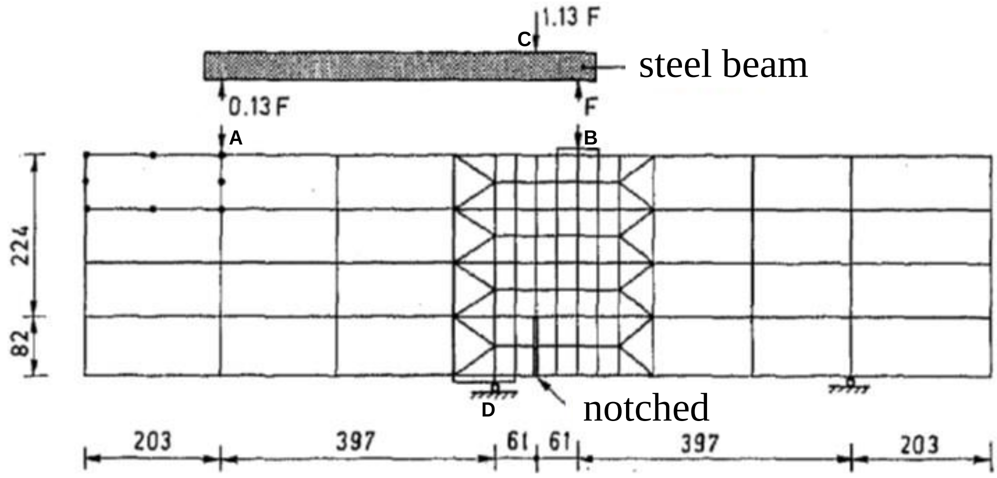


Figure 12. Finite element idealization for analysis of a notched beam in mixed-mode fracture (dimension in mm).

The displaced shapes and crack patterns obtained at the end of the analysis are shown in Figure 13 for the XFEM method and for the element elimination technique.

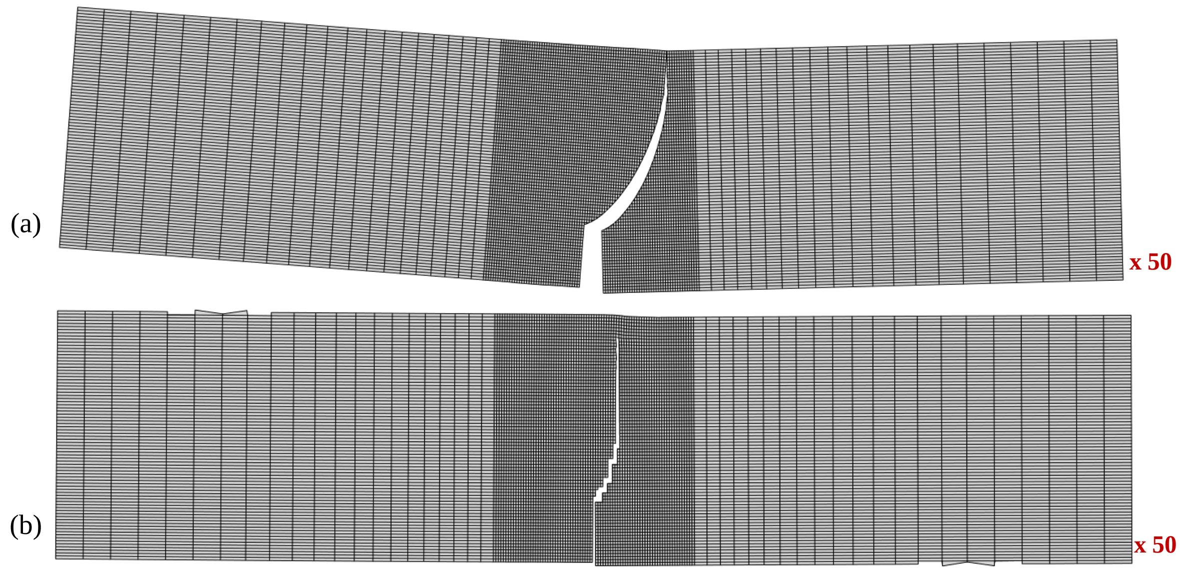


Figure 13. Displaced shapes obtained for (a) XFEM and (b) EET.

Figure 14 presents the nonlinear response of the beam in terms of load-CMSD curves. The CMSD is the Crack Mouth Sliding Displacement, which is indicated. Although the maximum load is captured quite well, the element elimination technique predicts a post-peak response that is far too ductile compared with the XFEM method.

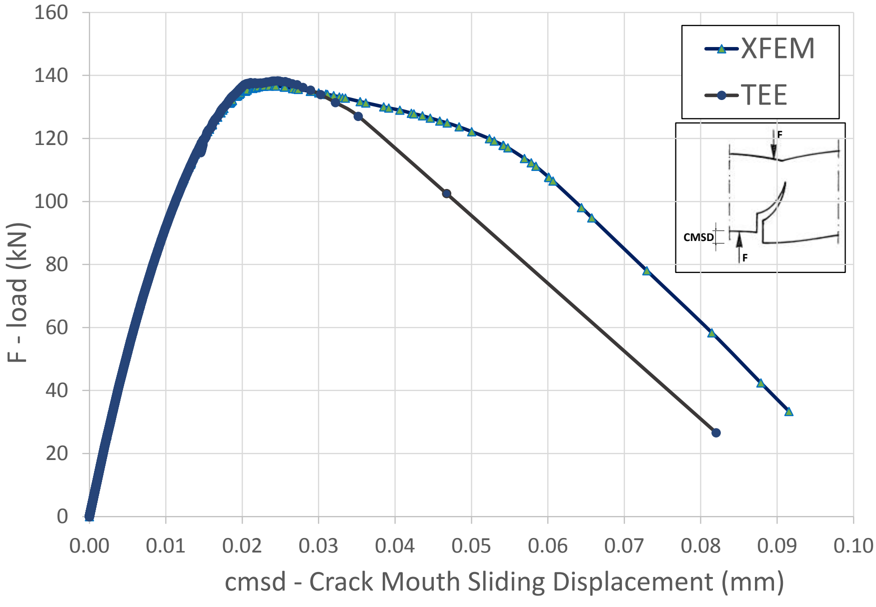


Figure 14. Load vs CMSD response for mixed-mode fracture problem.

7 CONCLUSIONS

Two examples were used to assess the applicability and accuracy of the three methods: extended finite element (XFEM), interelement method and element elimination technique. In the first example, the simulation of the notched beam under 3-point bending, represents mode I cracking, providing good verification of this aspect of the cohesive zone model. The sensitivity of the numerical results to the finite element discretization and the choice of the softening model were investigated. A mixed-mode fracture in a notched beam was also simulated, and the numerical predictions were compared with available experimental data.

All these methods have proven to exhibit mesh dependence. The XFEM and interelement method performed reasonably well when the mesh is sufficiently refined. One could improve the predictions by using a very fine mesh, but with a very high computational cost.

The element elimination technique performed especially poorly by the coarse mesh, and it is found to be extremely sensitive to the input of the shear retention factor. This technique can be used to predict the gross features of certain tests. So, perhaps in the context of million element simulations this method will give better results.

A shortcoming of XFEM as implemented in ABAQUS is the requirement for the crack to propagate across an entire element in a time step. It is apparent that element-by element cracking requires finer meshes to match the accuracy of methods with partial element cracking. Other methods that can insert an arbitrary crack, such as proposed by Remmers et al. (2003), may be more suitable for arbitrary crack propagation. Furthermore, the numerical integration in the cracked elements, the number of Gauss points and the type of sub-elements used in the implementation in Abaqus are unknown to the authors. For the XFEM it was not possible to simulate the bilinear softening diagram in Abaqus.

The interelement method appears to be more accurate when the crack path is known a priori. However, if the crack path is unknown, a remeshing at every time step should be used.

ACKNOWLEDGEMENTS

This research was carried out in the Tecgraf Institute/PUC-Rio and was funded by Petrobras.

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