A COMPARISON BETWEEN NUMERICAL AND ANALYTICAL METHODS FOR PREDICTION OF TEMPORARY GROUNDWATER DRAWDOWN

Daniel S. Vélez

Celso Romanel

daniels.velez@yahoo.com

romanel@puc-rio.br

Pontifical Catholic University of Rio de Janeiro (PUC-Rio)

Department of Civil Engineering

Marquês de São Vicente 225 Gávea, 22451-900, Rio de Janeiro, Brazil

Abstract. *When deep excavations reach the phreatic level in soil deposits, either for construction execution or excavation stability, a temporary groundwater drawdown is generally required. For initial design of the pumping system the engineer frequently makes use of analytical formulation for confined or unconfined aquifers, developed on basis of homogeneous and isotropic half-spaces with water in steady state flow occurring only in the horizontal direction (Dupuit assumption). The analytical method approximates the dewatering system as an equivalent circular well or a trench of great length, taking into account their distances to a recharge aquifer source. In this paper comparisons are made between analytical and numerical solutions obtained by the finite element method through 3D simulations of groundwater drawdown, considering several excavation geometries and recharge source distances.*

Keywords: *Temporary groundwater drawdown, finite element model, analytical drawdown prediction.*

1 INTRODUCTION

Dewatering systems are frequently implemented in construction to achieve an adequate groundwater level to execute foundations extending below the phreatic level. These systems generally are formed with an array of shallow or deep pumping wells providing dry conditions to improve slope stability during excavations. The design project of a temporary dewatering system (deep pumping wells) involves several parameters such as number, capacity, spacing, elevation, and the rate of water to be extracted from the aquifer. By this way analytical simplified functions (Mansur & Kaufman, 1962) based on Darcy assumptions (Darcy, 1856) were established in order to quantify the pumping rate required to result in a water lowering value.

In this paper is investigated the range of validity of analytical equations by a comparison of the flow rates obtained from these functions and from finite elements simulations for variations on the geometry of excavations and aquifer recharge source distance. The numerical modelling was carried out using 3D flow analysis in the geotechnical finite elements software Plaxis (Plaxis v.2013 3D).

2 FINITE ELEMENT MODEL

The numerical modelling was developed in 3D steady state flow conditions with the geotechnical finite element software Plaxis (Plaxis v.2013 3D). The model contains boundary conditions in terms of a very low permeability top layer (k = 1x10^{-9} cm/s) for simulation of confining strata, a hydraulic head (H = 38 m) was keep constant along all lateral boundaries and flow restriction in the bottom, for this confined aquifer was assumed a hydraulic conductivity (k = 5 x 10^{-5} m/s). The pumping system containing full penetrating wells was simulated as a rectangular array with dimensions length a, width b and distance to the recharge source represented by L_o. The finite element mesh generated in the software was composed by tetrahedral finite elements with 15 nodes, varying the number of the elements as the flow cases analyzed and with a refined mesh zone inside the excavation area. Figure 1 present the model with its characteristics.

In this form, as mentioned above, the flow rate values resulting from analytical equations Q and the numerical modelling Q_{ef} was compared for three main flow conditions: plane flow, radial flow and combined plane and radial flow.

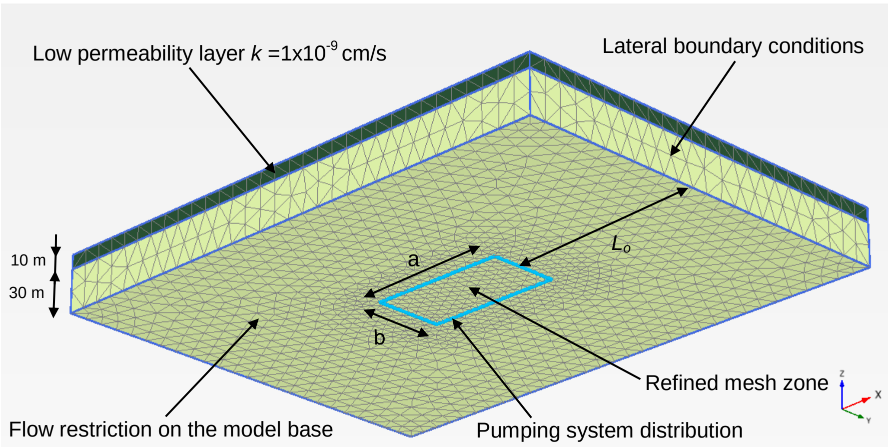


Figure 1. 3D finite element model mesh with excavation array (*a* x *b*) and recharge distance L_o

2.1 Plane flow case

For a rectangular dewatering system with geometric dimensions $a > b$ and relatively close to the recharge source, the flow could be considered essentially plane with a small contribution due to the radial flow incoming from the excavation edges (Fig. 2a). For this case, the following analytical equation gives a reasonably approximation of total flow rate.

$$Q = 2kD(H - h_w)\left[\frac{a}{L_o} + \frac{\pi}{\ln\left(\frac{2L_o}{b}\right)}\right] \quad (1)$$

Where D refers to the confined aquifer thickness, (H-h_w) represents the drawdown, L_o is the distance from the recharge source to the excavation and k the soil hydraulic conductivity.

If the excavation length a is quite different in relation with its width b ($a>>b$), Eq. (1) could be simplified taking into account only the plane flow incoming Eq. (2).

$$Q = 2kD(H - h_w)\left(\frac{a}{L_o}\right) \quad (2)$$

For this simulation and subsequent flow analysis was assumed a confined aquifer thickness D = 30 m as well as soil hydraulic conductivity k = 5 x 10^{-5} m/s. Otherwise the excavation relationship length/width was varied on values a/b = 10, 20, 50 while keeping constant the recharge boundary distance L_o = 50 m and drawdown value (H - h_w) = 5 m.

The flow rate values calculated from the finite element model Q_{ef} with those obtained from analytical equations Q are presented in Fig. 3 in the form of flow ratio (Q/Q_{ef}) and compared with the ratio of the recharge distance/excavation length (L_o/a).

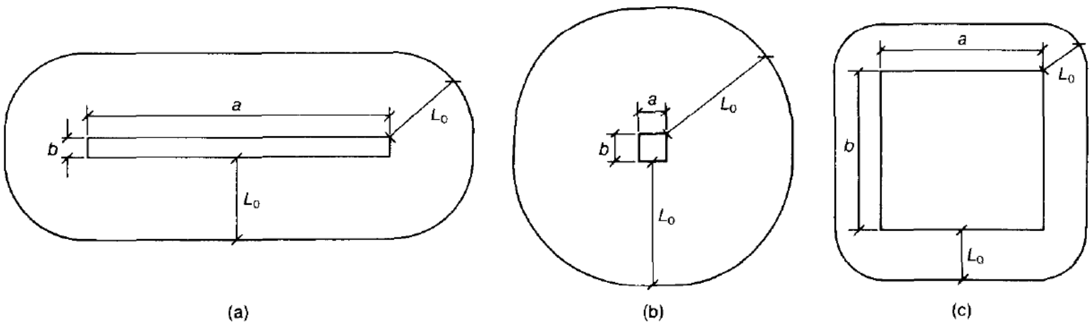


Figure 2. Idealized flow configurations: (a) mainly plane flow; (b) radial flow with distant recharge boundary; (c) plane and radial flow with a close recharge boundary (Powrie & Preene, 1992).

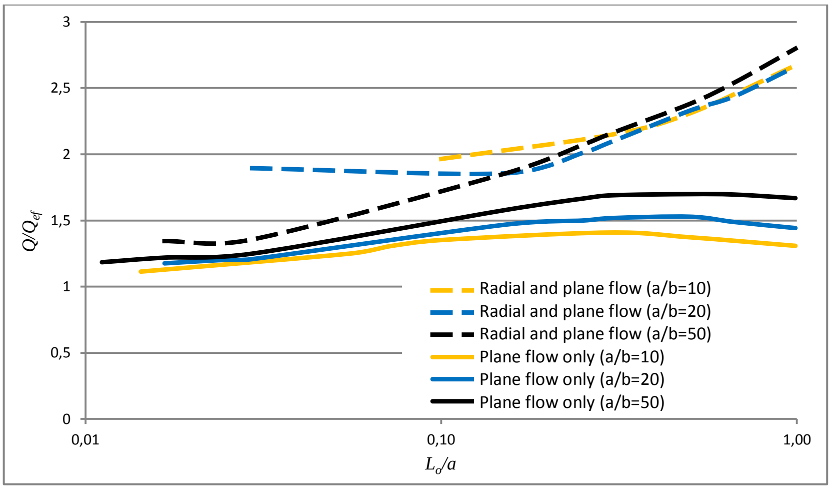


Figure 3. Flow rate variation with normalized recharge distance (L_o/a) obtained with the 3D finite element model and analytical equations.

As can be seen in Fig. 3, the consideration of an additional flow rate incoming from the excavation edges Eq. (1) tends overestimate the flow with respect to the finite element results, mainly for recharge source distances values $L_o/a > 0.10$, with approximately equal behavior for the three excavation dimensions ratio analyzed (a/b = 10, 20, 50). When applying the hypothesis of plane flow only Eq. (2) the analytical formulation overestimates the flow rate moderately, ranging from approximately 20% for $L_o/a = 0.01$ to 50% for $L_o/a = 0.10$ and then remaining constant up to $L_o/a = 1$. It may be concluded that as far the recharge source is located, the greater the influence of the excavation geometry (a/b) on the plane flow conditions.

2.2 Radial flow case

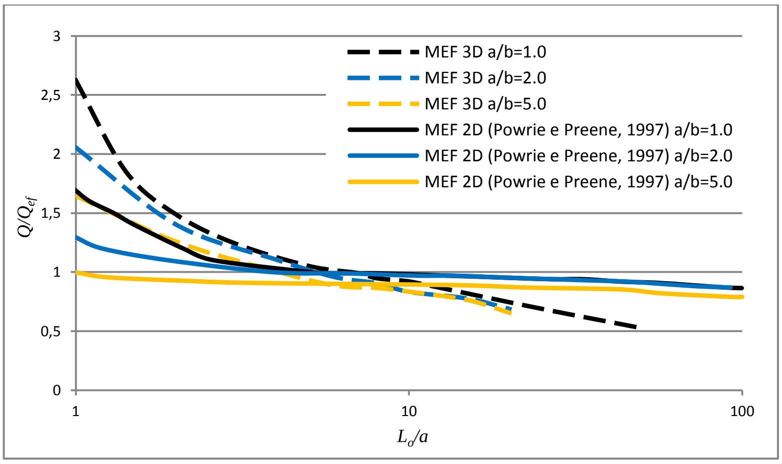
For an excavation with similar dimensions ($a \approx b$) and recharge source relatively distant, the flow turns essentially radial and the problem could be analyzed as a circular well with an equivalent radius. In this case the flow rate can be calculated by Eq. (3) (Thiem, 1906).

$$Q = \frac{2\pi k D (H - h_w)}{ln\left(\frac{L_o}{r_e}\right)} \tag{3}$$

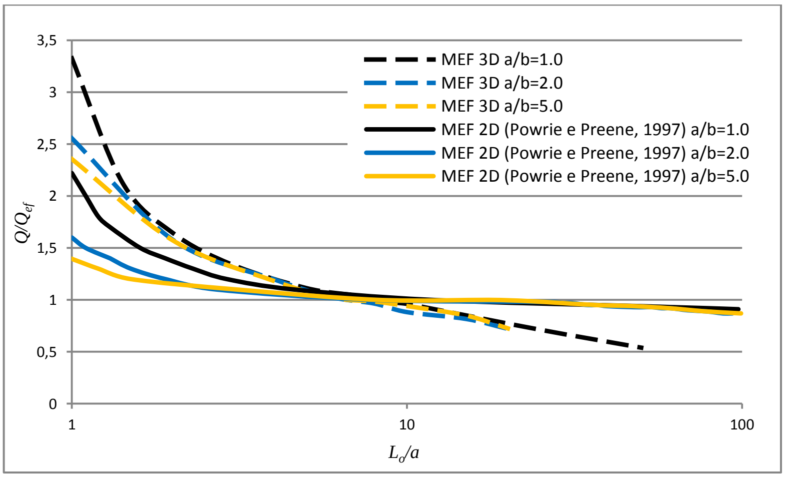
In this analysis the drawdown value ($H - h_w$ = 5 m) was kept constant, considering three excavation geometries (a/b = 1, 2, 5) while varying the distance to the recharge source $a \leq L_o \leq 20a$. The equivalent well radius (r_e) in Eq. (3) is determined in two different forms: (a) equivalent well radius with same excavation area or (b) equivalent well radius with same excavation perimeter. Figure 4 presents a comparison of the flow rates calculated with analytical formulation Eq. (3) and finite element simulation (Q_{ef}). The numerical results were determined with a previous 2D analysis by Powrie and Preene (1992) and 3D analysis (this research) with finite element method (MEF) in both cases.

From Fig. 4 can be seen that the analytical solution approaches the numerical results (2D and 3D cases) for recharge source distances between $7a < L_o < 10a$. Otherwise, flow rate is underestimated for $L_o > 10a$ and overestimated for $L_o < 7a$. The way to determine the equivalent well radius (r_e) shows irrelevant effects on the results, and a minor influence of excavation geometry *a/b* was detected for values $L_o < 5a$, i.e. only when the recharge boundary L_o is close enough to the excavation.

Analytical equations are made in the base of plane flow or axisymmetric flow only assumptions, for the present model conditions could be seen an approximately match in values achieved with this formulation and 2D finite elements model (Powrie and Preene, 1992). On the other hand a tridimensional model is capable to simulate the flow incoming on perpendicular direction that with a 2D approach is impossible to establish as well as the real effects of flow contribution from the edges of the excavation geometry that has an important influence in dependence of the recharge boundary distance.



a)



b)

Figure 4. Comparison of flow determined from numerical and analytical analyses considering an equivalent well radius as: a) $r_e = \sqrt{ab/\pi}$; **b)** $r_e = (a+b)/\pi$.

2.3 Combined plane and radial flow case

For a rectangular excavation located close enough to the recharge source, the flow generated is mainly plane with a radial flow contribution incoming to the corners as represented in Fig. 2c. For this situation, Cedergren (1989) suggested the following modifications for Eq. (1) and Eq. (2).

$$Q = 2kD(H - h_w)\left[\frac{a+b}{L_o} + \pi\right] \tag{4}$$

$$Q = 2kD(H - h_w)\left[\frac{a+b}{L_o}\right] \tag{5}$$

The first expression above consider the radial flow effects for the total flow rate estimative, the second one does not take into account those effects.

Figure 5 presents the ratio of total flow (Q) calculated with Eq. (4) and Eq. (5), with respect to the numerical modelling flow rate (Q_{ef}) for rectangular excavations with geometries a/b = 1, 2, 5 situated at distances from the recharge boundary in range: 0,1a < L_0 < 10a.

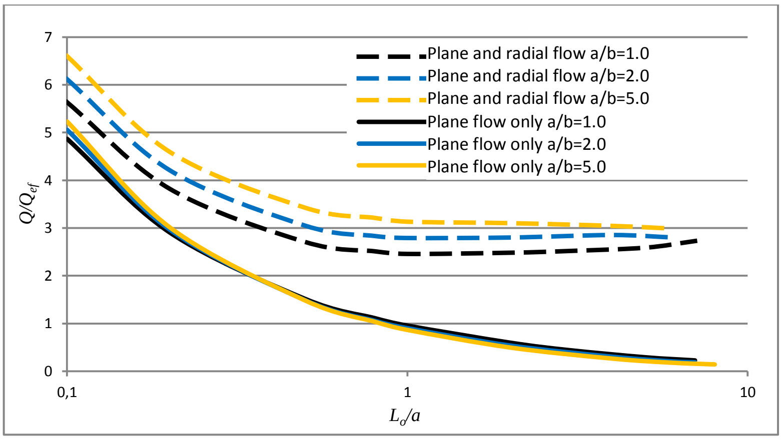


Figure 5. Comparative flow rates calculated with Eq. (4) and Eq. (5) and 3D finite elements analyses.

The following conclusions can be drawn:

a) For L_o/a < 0.2 there are significant differences between analytical and numerical results, since the flow occurs in the condition of plane flow essentially;
b) As the distance L_o/a becomes larger, the flow calculated from Eq. (5) results smaller, indicating that the incoming flow to the excavation area is turning gradually to radial;
c) The excavation geometry (a/b) has a greater influence on results obtained with the plane and radial flow hypothesis (Eq. 4) than those derived from plane flow only (Eq. 5);
d) The application of either Eq. (4) or Eq. (5) overestimates the flow rate (Q) with respect to finite elements calculated flow when L_o/a < 1;
e) When L_o/a >1 the combined radial and plane flow hypothesis Eq. (4) gives an overestimation greater than 150%.

3 CONCLUSIONS

In this paper results obtained by analytical equations based on assumptions of homogeneity, infinite lateral extension and isotropic confined aquifer were compared with approximate flow solutions provided by finite element method with the commercial software Plaxis 3D v.2013.

The following table presents a summary of the results. As mentioned before the conclusions in terms of overestimation and underestimation refers to flow calculated with analytical formulation as a comparison with the finite elements method for 3D analysis.

Table 1. Overview of results

Flow type	Model assumptions	L_o/a	Conclusions
Plane flow	with flow contribution	> 0.10	strong overstimated
	from the corners Eq. (1)	< 0.10	overstimated
	without flow contribution	> 0.10	slightly overestimate
	from the corners Eq. (2)	< 0.10	almost same values
Radial flow	equivalent radius Eq. (3)	> 10	slightly understimated
		< 10; > 7	good aproximation
		< 7	strong overstimated
Plane and radial flow	with radial flow contribution Eq. (4)	< 1	strongly overstimation
		> 1	overstimation (tending to a constant value)
	without radial flow	< 1	overstimated
	contribution Eq. (5)	> 1	understimation

With the flow cases analyzed can be pointed out that the flow assumption in a preliminary dewatering project (plane flow or radial flow) has not only exclusive dependence on the dewatering system geometrical distribution, an important factor influencing is the distance between excavation area and recharge boundary, as far is located this recharge, as mainly the flow becomes radial.

REFERENCES

Cedergren, H. R., 1989. Seepage, Drainage and Flow Nets. California: John Wiley & Sons.

Darcy, H., 1856. Lês Fontaines Publiques de la Ville de Dijon. Dalmont, Paris.

Dupuit, J., 1863. Études théoriques et pratiques sur le movement des eaux dans les canaux découverts et à travers les terrains perméables. Paris.

Mansur, C.I., Kaufman, R. I., 1962. Dewatering. In Foundation Engineering. Leonards, G. A., ed. Chapter 3, pp. 265-275. New York: McGraw – Hill.

Plaxis Finite Element Code for Soil and Rock Analysis. Scientific Manual. version 2013.1.

Powrie, W., Preene, M., 1992. Equivalent well analysis of construction dewatering systems. *Géotechnique*, vol. 42, pp. 635-639.

Thiem, G., 1906. Hydrologische methoden. J.M. Gebhardt: Leipzig, 1906.