A HYBRID METHOD USING q-GRADIENT TO IDENTIFY STRUCTURAL DAMAGE

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Abstract. *The present investigation is focused on the solution of a dynamic inverse vibration problem for structural damage identification. This inverse problem has been formulated as an optimization problem, and it has already been solved by heuristic and metaheuristic methods. These techniques are successfully employed for low degrees-of-freedom (DOF) structures. In more complex structures, these techniques . A hybrid approach was used to overcome this difficulty, where a new optimization scheme based on q-gradient is employed to provide an initial guess for the Hooke-Jeeves Direct Search method. The damage estimation has been evaluated using simulated experimental data, and the reported results are presented to a three-bay truss structure.*

Keywords: *Structural damage, q-G method, Hooke-Jeeves, Inverse vibration problem.*

1 INTRODUCTION

Structural health monitoring is an important application on the field of system identification. This research branch combines the advances of sensing technology with theories and techniques for nondestructive damage detection. One relevant issue of this field is the monitoring while the structure is in operation. The capacity of early detection of possible damages is desirable in the proposed monitoring approaches, helping to repair or rehabilitate a structure before it has major damages.

For critical systems, as aerospace structures, it is essential to have the capability of detecting damages in an accurate and a safe way. In this case, if a damage is not detected and not repaired, a structural failure could happen, with catastrophic consequences, as human and material losses.

Dynamic processes modeling in mechanical vibration is characterized by knowing initial and boundary conditions, geometry of the structure, material properties, and forcing terms. The system output response is the measurement of displacement, accelerations, strains, stresses, natural frequencies, and modes shapes. In vibration systems, modal parameters are function of the physical properties of the structure (such as mass, damping, and stiffness). Therefore, some change in the physical properties (caused by cracks, loosening of connections or another possible damage) will cause detectable changes in the modal properties. The inverse problem of damage evaluation consists in detecting, localizing and evaluating the damage severity. In particular, the damage severity is obtained by estimating the stiffness values or from estimated from structural displacements and/or modal properties measurements. The forward and the inverse problems are described on Section 2.

In structures with a high number of degrees of freedom, the task of detecting and evaluating damages becomes computationally expensive. From this consideration, it is important to develop a method for solving the problem within a reasonable period of time, with low computational cost.

In this paper, the q-gradient (q-G) method, a novel optimization algorithm, is hybridized with the well known Hooke-Jeeves Pattern Search (HJ) algorithm, creating a method to solve the inverse problem of detecting damages on structures. The proposed approach combines the exploratory advantages of q-derivative with the intensification capability of the HJ approach.

The q-G method, explained in Section 3.1, has been proposed to solve unconstrained continuous global optimization problems with a competitive performance when compared to Evolutionary Algorithms (EAs), especially in multimodal problems with several minimum values (Soterroni et al. 2011, 2012, 2013). This new approach is a generalization of the classical *steepest descent* method where the search direction is defined by the q-gradient vector of the objective function. The q-gradient vector is a generalization of the standard gradient vector based on the concepts of q-derivative (or Jackson's derivative) from q-calculus, and its use provides an effective mechanism for escaping from local minima.

Hooke-Jeeve pattern search method is a simple and efficient local optimization method that consists in exploratory and pattern moves (Hooke et al. 1961). Such scheme uses a sequence of exploratory movements on a base point (candidate solution), considering a certain radius. If, during the exploration, a better solution is computed, the base point changes for this better point. Otherwise, the exploration radius is changed for a smaller value. The method is detailed on Section 3.3.

The methodology of solution of the inverse problem is evaluated over a three-bay truss structure, with twelve degrees of freedom. This case study is described on Section 4. Some statistical results are presented in Section 4.1, and a comparison is made with a method previously tested, the Multi-Particle Collision Algorithm, also hybridized with Hooke-Jeeve Pattern Search (MPCA-HJ) (Hernández Torres et al. 2015b).

2 STRUCTURAL DAMAGE DETECTION AS AN OPTIMIZATION PROBLEM

The dynamic response of motion of a D degrees of freedom structure is given by a second order, non-homogeneous ordinary differential equation:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t)\,; \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are the $D \times D$ mass, stiffness and damping matrices, respectively; $\mathbf{F}$ is the external forces vector; and $\mathbf{u}$, $\dot{\mathbf{u}}$, $\ddot{\mathbf{u}}$ represents the displacements, velocity and acceleration vectors, respectively. The initial conditions are given by:

$$\mathbf{u}(0) = \mathbf{u}_0\,, \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0\,. \tag{2}$$

The forward problem calculates the system displacement $\mathbf{u}$, knowing $\mathbf{M}$,$\mathbf{K}$, $\mathbf{C}$, and $\mathbf{F}$.

Since no analytical solution exist for any arbitrary functions of $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{F}$, a numerical solution for this forward problem is obtained using the Newmark method (Newmark 1959).

The inverse problem of localizing and quantifying damages on structures is solved as an optimization problem. It consists of minimizing the objective function:

$$J(\mathbf{K}) = \sum_{i=0}^{N} \left[\mathbf{u}_i(t) - \hat{\mathbf{u}}_i(t)\right]^2\,, \tag{3}$$

where $\mathbf{u}_i(t)$ and $\hat{\mathbf{u}}_i(t)$ are the computed and measured displacements at time t, respectively.

The inverse problem of detecting damages on the truss structure is solved using the hybrid approach of q-G-HJ and compared with results obtained with MPCA-HJ. The methods q-G, MPCA and HJ are described in the next section.

3 OPTIMIZATION ALGORITHMS

3.1 q-G Method

In the beginning of the last century, the English reverend Frank Hilton Jackson started to develop the q-calculus in a systematic way working on the q-analogues of functions, series, special numbers, and more importantly, the q-derivative concepts (Ernst 2003; Jackson 1908, 1910a,b; Kac et al. 2002). Key to the performance of the q-G method is the parameter q required to calculate the q-gradient vector. When $q \neq 1$, the search direction is likely to be either descent or not descent and the method performs a global search. For q equals 1, the q-gradient reduces to the classical gradient vector and the q-G method performs a local search. In other words, the

q-G method can be described as a q-analogue of the *steepest descent* method that reduces to its classical version whenever the parameter q equals 1. The method is implemented in a way that the search process gradually shifts from global in the beginning to almost local search in the end (Soterroni et al. 2012).

Let f be a differentiable function of n variables, the n first-order partial q-derivatives of f with respect to the variable s_i is given by (Soterroni et al. 2011, 2012, 2013)

$$D_{q_i,s_i}f(\mathbf{s}) = \begin{cases} \dfrac{f(s_1,\cdots,q_i s_i,\cdots,s_n) - f(s_1,\cdots,s_n)}{q_i s_i - s_i}, & \text{if } s_i \neq 0 \text{ and } q_i \neq 1, \\ \dfrac{\partial f(\mathbf{s})}{\partial s_i}, & \text{otherwise,} \end{cases} \tag{4}$$

where the parameter $\mathbf{q}$ is a vector $\mathbf{q} = (q_1,\cdots,q_i,\cdots,q_n) \in R^n$. Note that when $s_i = 0$ or $q_i = 1, \forall i$, the first-order partial q-derivative is the classical first-order partial derivative. Thus, similarly to the classical gradient vector, the q-gradient is the vector of the n first-order partial q-derivatives of f (Soterroni et al. 2011, 2012, 2013)

$$\nabla_{\mathbf{q}} f(\mathbf{s}) = [D_{q_1,s_1}f(\mathbf{s}) \cdots D_{q_i,s_i}f(\mathbf{s}) \cdots D_{q_n,s_n}f(\mathbf{s})]. \tag{5}$$

The q-G method uses an iterative procedure that, starting from an initial point $\mathbf{s}^0$, generates a sequence $\mathbf{s}^k$ given by

$$\mathbf{s}^{k+1} = \mathbf{s}^k + \alpha^k \mathbf{d}^k, \tag{6}$$

where k is the iteration number, $\mathbf{d}^k$ is the search direction and α^k is the step length or the distance moved along $\mathbf{d}^k$. Thus, the search direction $\mathbf{d}^k$ in the q-G method is the negative of the q-gradient of the objective function $-\nabla_{\mathbf{q}} f(\mathbf{s})$ (Eq. (5)).

An adopted strategy for generating the parameter q is to draw the values of q_i^k from a Gaussian probability distribution such that $q_i^k s_i^k$ has a standard deviation σ^k that decreases as the iterative search proceeds (Soterroni et al. 2012, 2013). Starting from σ^0, the standard deviation is decreased by $\sigma^{k+1} = \beta \cdot \sigma^k$, where $0 < \beta < 1$ is the reduction factor. As σ^k approaches 0, the values of q_i^k tend to unity and the q-G method shifts gradually from a global search in the beginning to a local search in the end behaving similarly to the *steepest descent* method. However, if σ^k decreases too quickly, the algorithm may be trapped in a local minimum. Then, the role of the standard deviation is reminiscent of the temperature in a Simulated Annealing (SA) algorithm, that is, to make the iterative procedure go from a random at beginning to a deterministic search in the end (Soterroni et al. 2013). Since the search directions given by the q-gradient are not necessarily descent directions, the use of a line search technique to determine the distance moved along direction generated by the q-G method may be not efficient. Here, as in (Soterroni et al. 2012, 2013), the step length α^k is computed by the geometric recursion $\alpha^{k+1} = \beta \cdot \alpha^k$, where, for the sake of simplicity, β is the same reduction factor used to compute the standard deviation σ^k. The algorithm for the q-G method is summarized as follows (Soterroni et al. 2013).

As can be seen in Algorithm 1, the q-G method has three free parameters: the initial standard deviation (σ^0), used to generate the values of the parameters q_i^k, the initial step length (α^0) and the reduction factor (β). Usually, the values of σ^0 and α^0 are normalized

Algorithm 1 q-G method for unconstrained continuous global optimization problems

Given $\mathbf{s}^0$ (initial point), $\sigma^0 > 0$, $\alpha^0 > 0$ and $0 < \beta < 1$
$k = 0$
$\mathbf{s}_{best} = \mathbf{s}^k$
while $NFE < NFE_{\mathrm{qG}}$ **do** ▷ Stopping criterion
    Generate $\mathbf{q}^k\mathbf{s}^k$ by a Gaussian distribution with σ^k, and $\mu^k = \mathbf{s}^k$
    Calculate the q-gradient $\nabla_q f(\mathbf{s}^k)$
    $\mathbf{d}^k = -\nabla_q f(\mathbf{s}^k)/\|\nabla_q f(\mathbf{s}^k)\|$
    $\mathbf{s}^{k+1} = \mathbf{s}^k + \alpha^k \cdot \mathbf{d}^k$
    if $f(\mathbf{s}^{k+1}) < f(\mathbf{s}_{best})$ **then**
        $\mathbf{s}_{best} = \mathbf{s}^{k+1}$
    $\sigma^{k+1} = \beta \cdot \sigma^k$
    $\alpha^{k+1} = \beta \cdot \alpha^k$
    $k = k + 1$
return $\mathbf{s}_{best}$ and $f(\mathbf{s}_{best})$

by $L = \sqrt{\sum_{i=1}^{D}(s_{max_i} - s_{min_i})^2}$, the largest distance within the search space. The stopping criterion can be a maximum number of the function evaluations (NFE_{qG}).

An appropriate specification of the free parameters allows that the search process gradually shifts from global exploration in the beginning to almost local search in the end. Although a bad choice may lead to some deterioration in its performance, the q-G method has shown to be sufficiently robust to still be capable of reaching the global minimum (Soterroni et al. 2012, 2013).

3.2 Multi-Particle Collision Algorithm

Multi-Particle Collision Algorithm (MPCA) is a metaheuristic algorithm based on the physics in the nuclear reactor, with particles (neutrons) traveling in the reactor (Luz et al. 2008). Two phenomena can be pointed out during the travel trajectory: absorption and scattering. This algorithm has been used successfully in the solution of many optimization problems such as fault diagnosis (Echevarría et al. 2012), automatic configuration of neural networks applied to different problems such as atmospheric temperature profile identification (Sambatti et al. 2012), data assimilation (Anochi et al. 2015), climate prediction (Anochi et al. 2014) and in the solution of an inverse radiative problem (Hernández Torres et al. 2015a).

MPCA consists in a set of particles (candidate solutions of dimension D) that travels inside a nuclear reactor. Initially, the set of particles is generated randomly. Three principal functions in the algorithm control all the process: perturbation, exploration and scattering. Particles are perturbed, and depending on their fitness, they are absorbed or scattered to other area of the space search. The MPCA pseudo-algorithm is shown in Algorithm 2.

The PERTURBATION function (see Algorithm 3) performs a random variation of a particle within a defined range, creating a new particle. If the new particle is better that the current particle, then the EXPLORATION function (see Algorithm 4) performs an intensification of the current particle, by means of small perturbations (see Algorithm 5). If the new particle is worse than the current particle, the SCATTERING function (see Algorithm 6) is activated. Scattering function uses a Metropolis scheme: with a defined probability, the current particle is replaced

Algorithm 2 Multi-Particle Collision Algorithm

for $i \leftarrow 1$ to $N_{\text{processors}}$ **do** ▷ Initial set of particles
  $NFE_i = 0, lastUpdate_i = 0$
  for $j \leftarrow 1$ to $N_{\text{particles}}$ **do**
    currentP$_{i,j}$ = RANDOMSOLUTION
    $NFE_i = NFE_i + 1$
for $i \leftarrow 1$ to $N_{\text{processors}}$ **do** ▷ Initial blackboard
  bestP$_i$ = UPDATEBLACKBOARD(currentP$_{i,-}$)
while $NFE < NFE_{\text{mpca}}$ **do** ▷ Stopping criterion
  for $i \leftarrow 1$ to $N_{\text{processors}}$ **do**
    for $j \leftarrow 1$ to $N_{\text{particles}}$ **do**
      newP$_{i,j}$ = PERTURBATION(currentP$_{i,j}$)
      if newP$_{i,j}$.Fitness $<$ currentP$_{i,j}$.Fitness **then**
        currentP$_{i,j}$ = EXPLORATION(newP$_{i,j}$)
      else
        currentP$_{i,j}$ = SCATTERING(currentP$_{i,j}$, newP$_{i,j}$, bestP$_i$)
      if currentP$_{i,j}$.Fitness $<$ bestP$_i$.Fitness **then**
        bestP$_i$ = currentP$_{i,j}$
  if NFE_i - $lastUpdate_i > NFE_{\text{blackboard}}$ **then**
    for $i \leftarrow 1$ to $N_{\text{processors}}$ **do** ▷ Blackboard
      bestP$_i$ = UPDATEBLACKBOARD(currentP$_{i,-}$)
      $lastUpdate_i = NFE_i$
for $i \leftarrow 1$ to $N_{\text{processors}}$ **do** ▷ Final blackboard
  bestP$_i$ = UPDATEBLACKBOARD(currentP$_{i,-}$)
return bestP$_1$

by a new random solution, or a series of small perturbations are performed over it (Luz et al. 2008; Sacco et al. 2005).

Particles in the whole population behave cooperatively, i.e., the best particle overall is over-copied for all other particles in the set, through a blackboard strategy. The UPDATEBLACKBOARD procedure is applied each a number of function evaluations ($NFE_{\text{blackboard}}$).

As stopping criterion, a maximum number of function evaluations (NFE_{mpca}) is defined.

Algorithm 3 Perturbation Function

function PERTURBATION(currentP)
  for $d \leftarrow 1$ to D **do**
    R $= rand(0, 1)$
    P.Solution$_{(d)}$ = currentP.Solution$_{(d)}$ + $\big(\big(\text{UB}_{(d)} - \text{currentP.Solution}_{(d)}\big) \cdot \text{R}\big) +$
      $-((\text{P.Solution}_{(d)} - \text{LB}_{(d)}) \cdot (1 - \text{R}))$
    if P.Solution$_{(d)} > \text{UB}_{(d)}$ **then**
      P.Solution$_{(d)} = \text{UB}_{(d)}$
    else if P.Solution$_{(d)} < \text{LB}_{(d)}$ **then**
      P.Solution$_{(d)} = \text{LB}_{(d)}$
  P.Fitness = f(P.Solution)
  $NFE = NFE + 1$
  return P

Algorithm 4 Exploration Function

function EXPLORATION(currentP)
  for $n \leftarrow 1$ to $NFE_{\text{internalMax}}$ **do**
    newP = SMALLPERTURBATION(currentP)
    $NFE = NFE + 1$
    if newP.Fitness $<$ currentP.Fitness **then**
      currentP = newP
  return currentP

Algorithm 5 Small Perturbation Function

function SMALLPERTURBATION(currentP)
  for $d \leftarrow 1$ to D **do**
    $u = \text{currentP.Solution}_{(d)} \cdot rand(1, \text{SL})$
    $l = \text{currentP.Solution}_{(d)} \cdot rand(\text{IL}, 1)$
    $\text{R} = rand(0, 1)$
    if $u > \text{UB}_{(d)}$ **then**
      $u = \text{UB}_{(d)}$
    if $l < \text{LB}_{(d)}$ **then**
      $l = \text{LB}_{(d)}$
    $\text{P.Solution}_{(d)} = \text{currentP.Solution}_{(d)} + ((u - \text{currentP.Solution}_{(d)}) \cdot \text{R}) +$
      $- \big((\text{currentP.Solution}_{(d)} - l) \cdot (1 - \text{R})\big)$
  P.Fitness = f(P.Solution)
  $NFE = NFE + 1$
  return P

Algorithm 6 Scattering Function

function SCATTERING(currentP, newP, bestP)
  $p_{\text{scattering}} = 1 -$(bestP.Fitness/newP.Fitness)
  if $p_{\text{scattering}} > rand(0, 1)$ **then**
    P = RANDOMSOLUTION
    $NFE = NFE + 1$
  else
    P = EXPLORATION(currentP)
  return P

3.3 Hooke-Jeeves Direct Search Method

The direct search method of Hooke-Jeeves (Hooke et al. 1961) consists of the repeatedly application of exploratory moves about a base point which, if successful, is followed by pattern moves. The HJ pseudo-algorithm is shown in Algorithm 7. In a D-dimensional problem, a candidate solution is denoted as a vector $\mathbf{s}$ of length D.

The exploratory move consists adding one column of the search directions matrix $\mathbf{V}$ (called v_d), scaled by a step size h, to the solution $\mathbf{s}$. This process is made over all the dimensions of the problem. A new solution is accepted if it is better than the previous $\mathbf{s}$. If the exploratory move was successful, it will return an improved solution $\mathbf{s}^n$.

The pattern move $\mathbf{s}^{n*}$ is computed adding a search direction $(\mathbf{s}^n - \mathbf{s}^c)$ to $\mathbf{s}^n$. If $\mathbf{s}^{n*}$ is better

than $\mathbf{s}^c$ then it will replace the latter, else $\mathbf{s}^n$ will become the new $\mathbf{s}^c$. If no improvement is found for $\mathbf{s}^c$, the step size h is reduced in ρ times. As stopping criteria, a minimum step size ($h_{\min}$) and a maximum number of function evaluations (NFE_{hj}) are defined.

Algorithm 7 Hooke-Jeeves

Choose $\mathbf{s}^c, \rho, h_{\min}$
while $NFE < NFE_{\text{hj}}$ **and** $h > h_{\min}$ **do** ▷ Stopping criteria
  $\mathbf{s}^n = \text{EXPLORATORY}(\mathbf{s}^c, h)$
  if $f(\mathbf{s}^n) < f(\mathbf{s}^c)$ **then**
    $\mathbf{s}^{n*} = \mathbf{s}^n + (\mathbf{s}^n - \mathbf{s}^c)$
    if $f(\mathbf{s}^{n*}) < f(\mathbf{s}^n)$ **then**
      $\mathbf{s}^c = \mathbf{s}^{n*}$
    else
      $\mathbf{s}^c = \mathbf{s}^n$
  else
    $h = h \times \rho$
return $\mathbf{s}^c$

function EXPLORATORY($\mathbf{s}^c, h$)
  $\mathbf{s} = \mathbf{s}^c$
  for $d \leftarrow 1$ to D **do**
    if $f(\mathbf{s} + hv_d) < f(\mathbf{s})$ **then**
      $\mathbf{s}^n = \mathbf{s} + hv_d$
    else if $f(\mathbf{s} - hv_d) < f(\mathbf{s})$ **then**
      $\mathbf{s}^n = \mathbf{s} - hv_d$
  return $\mathbf{s}^n$

3.4 Hybridization approaches

The hybridization of the q-G method with HJ, named q-G-HJ, works as a multi-stage structure (Ting et al. 2015), similar to the MPCA-HJ (Hernández Torres et al. 2015b). For the solution of the optimization problem, i.e. the inverse problem, the global optimization algorithm (q-G or MPCA) looks for "good" candidate solutions within the entire search space. When the global algorithm (q-G or MPCA) reaches its stopping criterion, the optimization procedure is interrupted. Then, the best solution found in the global exploration phase is used as starting point for the local optimization algorithm (HJ) to perform an exploitation (or local intensification) phase. Finally, the best solution found by the HJ method will be the final solution for the inverse problem.

4 CASE STUDY: THREE-BAY TRUSS STRUCTURE

A three-bay truss structure with 12-DOF is considered to test the methods, as shown in Figure 1. The structure is composed of 12 aluminum bars with a square cross section area $A = 9 \times 10^{-4}\ m^2$. The material has a Youngs modulus $E = 70\ GPa$, and a material density $\rho = 2700\ kg/m^3$. Each non-diagonal element is $l = 1.0\ m$ long. The damping matrix is assumed as proportional to the integral stiffness matrix ($\mathbf{C}_i = 10^{-5}\mathbf{K}_i$). Constant (over time) external forces $\mathbf{F}_i = 1000\ N$ are applied to the nodes A and B in the positive direction of the x-coordinate, as shown in Figure 1. Final time for the numerical simulations is $t_f = 5 \times 10^{-2}\ s$,

with a time step $\Delta t = 5 \times 10^{-4}\ s$. Initial conditions are adopted equal to zero ($\mathbf{u}(0) = 0\,, \dot{\mathbf{u}}(0) = 0$).

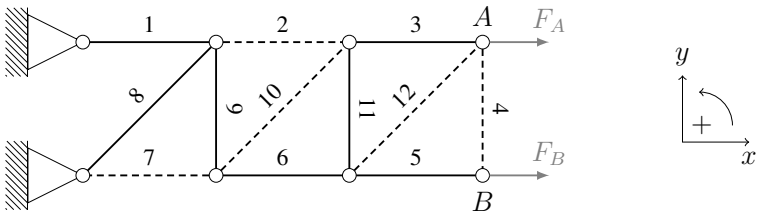

Figure 1: Three-bay truss structure (dashed lines represent damaged elements)

Experimental data were created *in silico*. Damages were assumed throughout the structure, by means of reducing values of stiffness of some elements, as shown in Table 1. A damage configuration of 15% over the 2^{nd} element, 5% over the 4^{th}, 30% over the 7^{th}, 10% over the 10^{th} and 20% over the 12^{th} element was considered. All the others elements have been assumed as undamaged. Damaged elements are represented on Figure 1 with dashed lines.

4.1 Damage Identification Results

The stiffness values for structure elements under investigation are shown in Table 1 – considering reference and damaged elements.

In the inverse problem of damage identification, small perturbations or noise on the measurements can cause large oscillations on the solution of the inverse problem. In order to simulate that situation and test the robustness of the methods, some experiments were performed with noisy data, adding a random error to the node displacements along the x and y coordinates, as shown in Eq. (7):

$$\hat{\mathbf{u}}_i(t) = \mathbf{u}_i(t) + \mathcal{N}(0, \sigma), \tag{7}$$

where $\hat{\mathbf{u}}_i(t)$ represents the noisy experimental data, $\mathcal{N}(0, \sigma)$ is a normal distribution with mean 0 and standard deviation σ. In the experiments, three cases are tested: $\sigma = 0.00$ (noiseless data), $\sigma = 0.02$ and $\sigma = 0.05$.

Table 2 shows the stopping criteria and the control parameters of the q-G, MPCA and HJ methods. The maximum number of function evaluations for each hybrid approach (q-G-HJ or MPCA-HJ) is 100,000 with 70,000 for the global optimization method (q-G or MPCA) and 30,000 for the local optimization method (HJ). For the HJ method the iterative procedure stops if either the minimum step ($h_{\min}$) or the maximum number of function evaluations (NFE_{hj}) is reached. In order to estimate the damaged stiffness values, the search domain used in the optimization procedures was defined as $[0.5, 1.05]\mathbf{K}_u$, where $\mathbf{K}_u$ represents the undamaged stiffness values.

Figure 2 shows the results for the estimated damage factor. Lighter gray shows the exact damage percentage. Darker gray bars represent estimates given by q-G-HJ and black bars, estimates given by MPCA-HJ. Negatives damages (stiffness values greater than integral value, which are not supported physically) are represented as zero. Table 3 presents the relative error between the real and the estimated values of the stiffness. For both methods (q-G-HJ and

Element	Undamaged Stiffness	Real Stiffness (damaged)	Damage percent
1	1,750,000	1,750,000	0 %
2	1,750,000	1,487,500	**15 %**
3	1,750,000	1,750,000	0 %
4	1,750,000	1,662,500	**5 %**
5	1,750,000	1,750,000	0 %
6	1,750,000	1,750,000	0 %
7	1,750,000	1,225,000	**30 %**
8	1,237,440	1,237,440	0 %
9	1,750,000	1,750,000	0 %
10	1,237,440	1,113,696	**10 %**
11	1,750,000	1,750,000	0 %
12	1,237,440	989,952	**20 %**

Table 1: Stiffness and damage percent values for each element

Algorithm	Parameter	Value
q-G	σ^0	$0.2 \cdot L$
	α^0	$0.1 \cdot L$
	β	0.999
	NFE_{qg}	70,000
MPCA	$N_{\text{particles}}$	10
	$NFE_{\text{blackboard}}$	$1,000$
	IL	0.7
	SL	1.1
	NFE_{mpca}	70,000
HJ	ρ	0.8
	$h_{\min}$	1×10^{-10}
	NFE_{hj}	$30,000$

Table 2: Control parameters and stopping criteria for q-G, MPCA and HJ methods

MPCA-HJ), final damage identification results are similar. Almost perfect damage estimation were obtained for noiseless data. Good results were obtained using 2% noisy experimental data: all damaged elements were well identified; in the sixth element (which is not damaged) a little false damage was identified; and eighth, ninth and eleventh elements presented negatives damages. Worse results were obtained using 5% noisy experimental data, but all damaged elements were identified. Again, a false damage appears on the sixth element and eighth, ninth and eleventh elements presented an increase of the stiffness.

Table 4 shows statistical results for the objective function values obtained from a set of

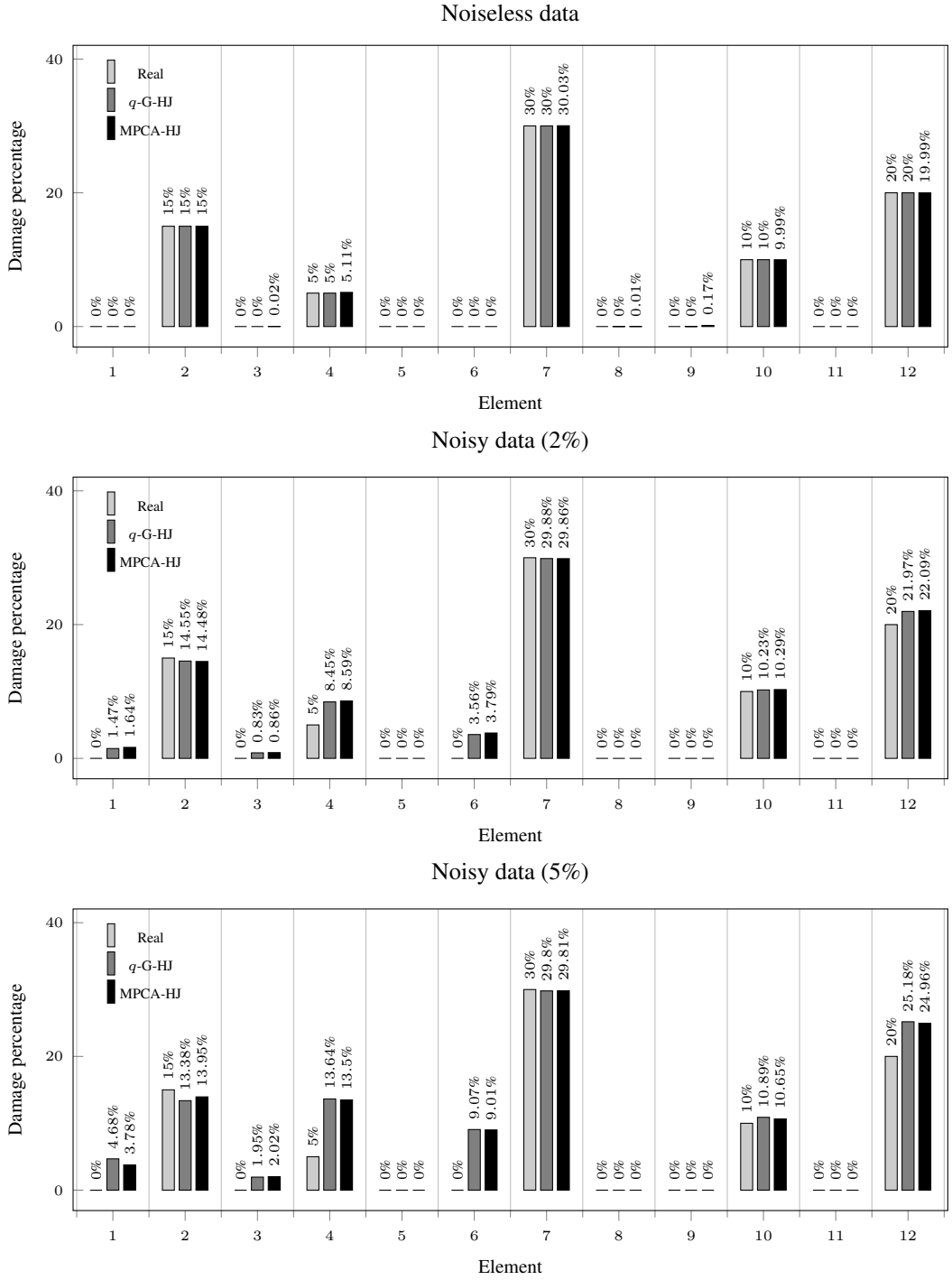

Figure 2: Damages identified by q-G-HJ and MPCA-HJ

50 runs of q-G and MPCA (with and without hybridization with HJ), in experiments with noiseless data. Overall, the q-G method outperforms MPCA. During the exploration phase, the q-G method shows objective function values up to three orders of magnitude better than the ones obtained by MPCA. The final results from the hybrid methods q-G-HJ and MPCA-HJ are similar for the minimum and median values of the objective function. However, for the maximum, mean and standard deviation, the q-G-HJ approach presents objective function

	Noiseless		Noisy 2%		Noisy 5%	
Element	q-G-HJ	MPCA-HJ	q-G-HJ	MPCA-HJ	q-G-HJ	MPCA-HJ
1	0.000	0.000	1.470	1.644	4.681	3.780
2	0.000	0.000	0.530	0.608	1.902	1.235
3	0.000	0.020	0.825	0.859	1.951	2.020
4	0.000	0.116	3.630	3.781	9.098	8.947
5	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	3.561	3.794	9.068	9.010
7	0.000	0.043	0.176	0.199	0.292	0.271
8	0.000	0.010	0.000	0.000	0.000	0.000
9	0.000	0.170	0.000	0.000	0.000	0.000
10	0.000	0.011	0.257	0.318	0.989	0.722
11	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.012	2.460	2.616	6.477	6.200

Table 3: Relative error between real and estimated stiffness

Statistic	qG	qG-HJ	MPCA	MPCA-HJ
Mean	4.57×10^{-6}	2.66×10^{-20}	1.14×10^{-3}	2.79×10^{-15}
Maximum	7.08×10^{-6}	5.84×10^{-19}	2.73×10^{-3}	5.58×10^{-14}
Minimum	8.83×10^{-7}	1.33×10^{-23}	3.92×10^{-5}	1.08×10^{-23}
Median	4.35×10^{-6}	1.04×10^{-21}	1.04×10^{-3}	2.60×10^{-21}
Standard Deviation	1.65×10^{-6}	1.14×10^{-19}	6.91×10^{-4}	9.76×10^{-15}

Table 4: Statistical results for the values of the objective function over 50 runs

values up to five orders of magnitude better than the ones obtained by MPCA-HJ.

In all the experiments, the number of function evaluations where the algorithm reaches a value of the objective function of 1×10^{-10} was saved. The q-G-HJ method reached the mentioned value for the objective function after 8342 function evaluations, while the MPCA-HJ reached it after 9353 function evaluations. These numbers of function evaluations are average values taken from a set of 50 runs of each optimization method.

5 FINAL REMARKS

In this work, the inverse problem of structural damage detection was solved by using the q-G method hybridized with the HJ method, named q-G-HJ. This novel approach was compared to the hybrid method MPCA-HJ for the same problem and under the same stopping criteria in order to ensure a fair comparison. Although q-G-HJ and MPCA-HJ obtained final damages estimates with a similar quality, the former reached the results faster. The statistical measures of the objective function values obtained from a set of runs indicated a quicker convergence of q-G-HJ.

In future works, the proposed hybrid method will be applied to more complex structures, such as International Space Station (Chiwiacowsky et al. 2008), and damage identification for experiments in a laboratory. Numerical experiments will be also performed using the forward problem expressed in frequency domain and incomplete experimental data.

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REFERENCES

Anochi, J. A. & Campos Velho, H. F. 2014. Optimization of feedforward neural network by Multiple Particle Collision Algorithm. *Foundations of Computational Intelligence (FOCI), 2014 IEEE Symposium on*. IEEE, pp. 128–134.

Anochi, J. A., Campos Velho, H. F., Furtado, H. C. M. & Luz, E. F. P. 2015. Self-configuring Two Types of Neural Networks by MPCA. *Journal of Mechanics Engineering and Automation* vol. 5, pp. 112–120.

Chiwiacowsky, Leonardo D., Gasbarri, Paolo & Campos Velho, Haroldo F. de. 2008. Damage assessment of large space structures through the variational approach. *Acta Astronautica* vol. 62, n. 1011, pp. 592 –604. ISSN: 0094-5765. DOI: http://dx.doi.org/10.1016/j.actaastro.2008.01.017.

Echevarría, L. C., Llanes Santiago, O. & Silva Neto, A. J. 2012. Aplicación de los algoritmos Evolución Diferencial y Colisión de Partículas al diagnóstico de fallos en sistemas industriales. *Revista Investigación Operacional* vol. 33, n. 2, pp. 160–172.

Ernst, T. 2003. A Method for q-Calculus. *Journal of Nonlinear Mathematical Physics* vol. 10, n. 4, pp. 487–525.

Hernández Torres, R., Luz, E. F. P. & Campos Velho, H. F. 2015a. Multi-Particle Collision Algorithm for Solving an Inverse Radiative Problem. *Integral Methods in Science and Engineering. Theoretical and Computational Advances*. Ed. by C. Constanda & A. Kirsch. Birkhäuser Basel. DOI: 10.1007/978-3-319-16727-5.

Hernández Torres, R., Campos Velho, H. F. & Chiwiacowsky, L. D. 2015b. Multi-Particle Collision Algorithm with Hooke-Jeeves for solving a Structural Damage Detection Problem. 10th International Conference on Composite Science and Technology (ICCST/10), Lisbon, Portugal.

Hooke, R. & Jeeves, T. A. 1961. "Direct Search" Solution of Numerical and Statistical Problems. *Journal of the ACM (JACM)* vol. 8, n. 2, pp. 212–229.

Jackson, F. H. 1908. On q-functions and a certain difference operator. *Trans. Roy Soc. Edin.* Vol. 46, pp. 253–281.

Jackson, F. H. 1910a. On q-definite integrals. *Quart. J. Pure and Appl. Math.* Vol. 41, pp. 193–203.

Jackson, F. H. 1910b. q-difference equations. *American Journal of Mathematics* vol. 32, pp. 307–314.

Kac, V. & Cheung, P. 2002. Quantum Calculus. New York: Springer-Verlag.

Luz, E. F. P., Becceneri, J. C. & Campos Velho, H. F. 2008. A new multi-particle collision algorithm for optimization in a high performance environment. *Journal of Computational Interdisciplinary Sciences* vol. 1, n. 1, pp. 3–10.

Newmark, N. M. 1959. A method of computation for structural dynamics. *Journal of the Engineering Mechanics Division* vol. 85, n. 3, pp. 67–94.

Sacco, W. F. & Oliveira, C. R. E. 2005. A new stochastic optimization algorithm based on a particle collision metaheuristic. *Proceedings of 6th WCSMO*.

Sambatti, S. B. M., Anochi, J. A., Luz, E. F. P., Carvalho, A. R., Shiguemori, E. H. & Campos Velho, H. F. 2012. Automatic configuration for neural network applied to atmospheric temperature profile identification. *3rd International Conference on International Conference on Engineering Optimization*, pp. 1–9.

Soterroni, A. C., Galski, R. L. & Ramos, F. M. 2011. The q-Gradient Vector for Unconstrained Continuous Optimization Problems. English. *Operations Research Proceedings 2010*. Ed. by B. Hu, K. Morasch, S. Pickl & M. Siegle. Operations Research Proceedings. Springer Berlin Heidelberg, pp. 365–370. ISBN: 978-3-642-20008-3. DOI: 10.1007/978-3-642-20009-0_58.

Soterroni, A. C., Galski, R. L. & Ramos, F. M. 2012. The q-gradient method for global optimization. *arXiv:1209.2084* vol. math.OC.

Soterroni, A. C., Galski, R. L. & Ramos, F. M. 2013. The q-gradient method for continuous global optimization. *American Institute of Physics Conference Series*. Ed. by T. Simos, G. Psihoyios & C. Tsitouras. Vol. 1558. American Institute of Physics Conference Series, pp. 2389–2393. DOI: 10.1063/1.4826022. arXiv: 1209.2084 [math.OC].

Ting, T.O., Yang, Xin-She, Cheng, Shi & Huang, Kaizhu. 2015. Hybrid Metaheuristic Algorithms: Past, Present, and Future. English. *Recent Advances in Swarm Intelligence and Evolutionary Computation*. Ed. by Xin-She Yang. Vol. 585. Studies in Computational Intelligence. Springer International Publishing, pp. 71–83. ISBN: 978-3-319-13825-1. DOI: 10.1007/978-3-319-13826-8_4.