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HEALTH MONITORING OF ROTATING STRUCTURES: IDENTIFICATION OF CRACKS

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Abstract. *Health monitoring of rotating shafts in operation is performed generally by vibration measurement, that are taken in correspondence of the bearings of the machine. Different malfunctions generate typical vibration changes called symptoms. Symptoms of a propagating transverse crack are changes in 1xrev. and 2xrev. vibration components. In this paper it is shown how these symptoms are related to the position and the depth of the crack. Combining the crack propagation laws, as stated by fracture mechanics theory, with the model of crack induced vibrations in rotating shafts, the evolution of the vibrations of a cracked shaft can be calculated, and residual life can be estimated. The model of the shaft of an industrial turbo-generator unit is used as an example for simulating numerically the evolution of the vibrations. Different scenarios are presented, depending on the position the crack and on the operating conditions of the unit. The crack is assumed to develop in the high pressure steam turbine, where grooves and thermal stresses may activate the development and propagation of cracks. This example shows how with standard vibration measurements the health of a rotor affected by a crack can be monitored.*

Keywords: *Cracked shafts, propagation of cracks, identification of cracks, monitoring of cracked shafts.*

1 INTRODUCTION

Health monitoring of rotating shafts in operation of industrial machinery is performed generally by vibration measurements, that are taken in correspondence of the bearings of the machine, combined to a series of measurements, such as rotating speed, temperatures, pressures defining the operating condition of the machine. Different malfunctions generate changes in the vibrational behavior of the machine, that are typical and specific to each category of malfunction. These changes in vibrational behavior are called symptoms. One of the most dangerous malfunctions in rotating shaft is a propagating crack: it is difficult to identify the propagating crack at an early stage, the propagation speed is very slow at the beginning and becomes dramatically rapid just before failure.

Symptoms of a propagating transverse crack are changes in 1xrev. and 2xrev. vibration components. These symptoms are rather small at rated speed but become more evident and recognizable when during run down transients some critical speeds are crossed where the vibration amplitudes are amplified. It will be shown how these symptoms are related to the depth of the crack, by means of a very accurate model of the crack and of the cracked shaft dynamic behavior. Further combining the crack propagation laws, as stated by fracture mechanics theory, with the model of crack induced vibrations in rotating shafts, the evolution of the vibrations of a cracked shaft can be calculated, and residual life can be estimated.

It is known from field experience that vibration changes in cracked rotors develop very slowly as long as crack is not very deep, during its first stage of propagation. When the depth or extension of the propagating crack become large and reduces the resisting section to 50% or less, then the vibrations induced by the crack start to increase rapidly, with a trend which is almost exponential see e.g. Ziebarth et al.(1981) and Sanderson (1992). This has allowed in many cases to recognize the presence of the crack in rotating industrial machinery and to stop the machine just in time for avoiding catastrophic failures.

The fracture mechanics approach allows modelling the propagation velocities in radial and circumferential direction. Once a crack with a certain shape has developed in a shaft, the crack excited vibrations can be calculated using the model of the breathing mechanism, the model of the cracked beam element and the model of the cracked shaft dynamical behavior. These models developed in Bachschmid et al. (2010)[1] are recalled here briefly. Moreover with these models, the vibrations in correspondence of the bearings, where they are generally measured, can be calculated. This has been done in the presented case study, which has been discussed in Bachschmid et al. (2010)[2] and has been repeated for different time steps during the propagation of the crack, when also its depth and shape is changing, and the results allow forecasting the evolution of the vibrations versus time.

It should be pointed out the existence of new trends on crack detection. Cavalini Jr et al. (2014) proposed an electrical impedance based approach to experimentally detect incipient cracks in rotating shafts. More recently, a combinational frequency technique was tested to identify the location and depth of transversal cracks in rotating shafts was presented by Cavalini Jr et al. (2015).

2 DESCRIPTION OF MOST COMMON CRACK SHAPES

Cracks, which are propagating due to bending stresses in rotating shafts, have often an elliptical shape as long as the depth is small. Figure 1 and figure 2 show the shapes of cracks, which were obtained by applying a static bending moment to rotating shaft specimens, respectively in a very early propagation state and after the crack had propagated to a consistent depth. In both shafts crack initiation was obtained by means of a small cut (notch) visible in both figures.

Both specimens were produced by CETIM (Centre Technique des Industries Mécaniques) for R&D – AMA Department of EDF (Electricité de France) and were used for cracked shaft vibration analysis as described in Bachschmid et al. (2010)[1].

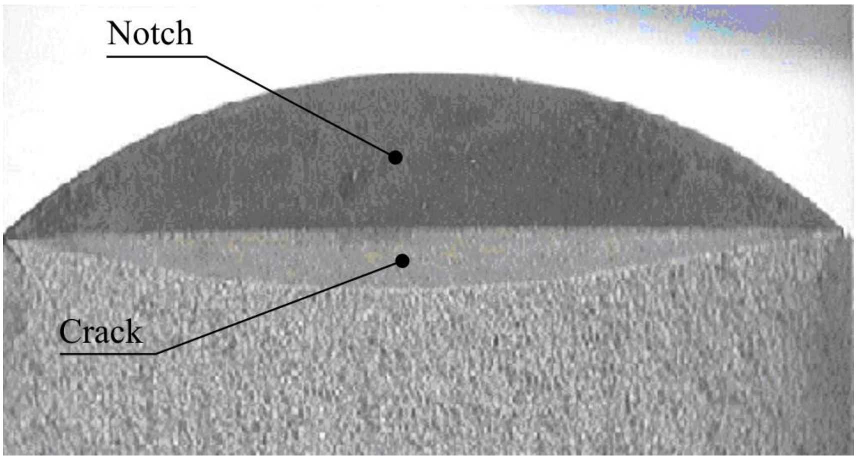

Figure 1. Crack in early propagation state.

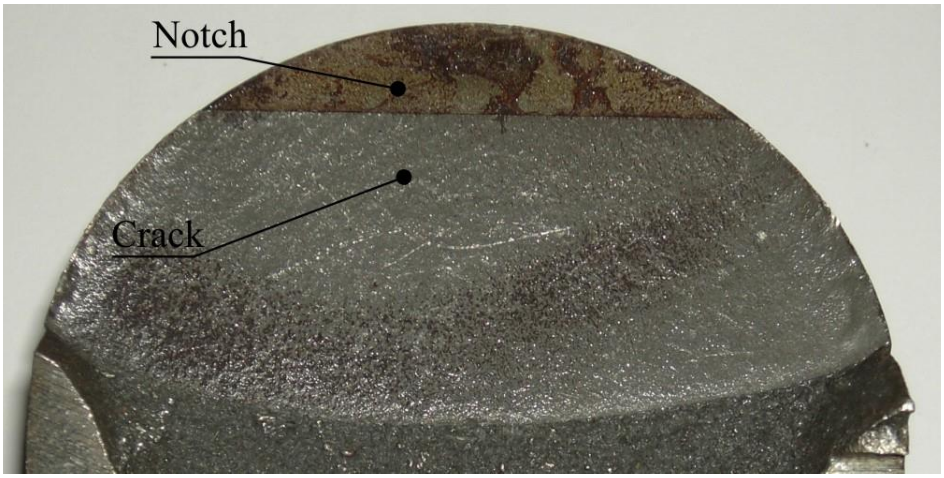

Figure 2. Deep propagated crack.

As can be seen in Fig.1 and Fig.2, the crack has preferably an elliptical shape when it is generated by static bending moment in a rotating shaft. The presence of torsion, which is always present in operating shafts of industrial machinery, can produce cracked surfaces which are not planar but slightly helical. Flexural vibrations are rather unaffected by the helical development of the crack. Planar cracks seem to be the most common consequence of crack propagation in rotating shafts. When the crack has propagated to a depth close to 50%

of the diameter, the propagation velocity in radial direction decreases and that in circumferential direction prevails. The crack tip becomes first roughly rectilinear and later elliptical but convex. This last situation is shown in Fig. 3. The related case history analysis is reported in Bachschmid et al. (2010)[1].

Figure 3. A very deep crack which has developed in a 320 MW generator.

These results about crack shapes as function of crack propagation depth (or length) can be deduced also from stress intensity factors (SIF) which have been calculated with 3D finite element models by several authors as reported in Murakami (1987). The crack propagation velocity, according to the Paris' formulation (Paris et al. 1960) is then function of the change of SIF during one load cycle. When the crack is breathing, which means opening and closing completely during one revolution, then the change of SIF is equal to the SIF with open crack, because one could assume that the SIF vanishes with closed crack. This might not be true because in closed crack configuration the distribution of axial stresses is generally not completely linear over the cracked section. Instead, when the crack does not close completely, due to the local bow that can develop during crack propagation, then it can be assumed that the SIF variation is twice the SIF with open crack.

Crack propagation involves radial propagation and circumferential propagation with velocities that depend respectively on the radial crack depth or on the circumferential crack length. Therefore, the crack tends to develop a shape in which the SIF at the center is equal to the SIF at the edge, so that the propagation velocity in radial direction at the center is equal to the velocity in circumferential direction at the crack edges.

An accurate analysis of the crack growth on a cracked shaft is reported by Shih et al. (1997).

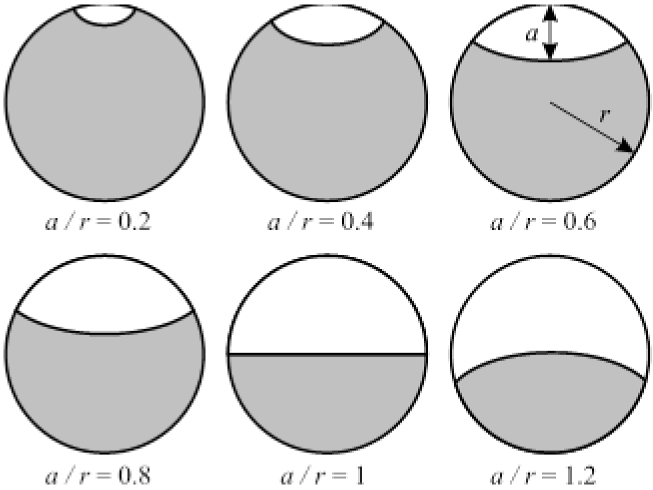

Figure 4. Crack shapes during propagation: from elliptical to straight and convex.

When the starting shape is a small ellipse with a ratio of minor *a* to major *b* axis (which is called aspect ratio) equal to 0.6 and with a relative depth *a/r* of 0.2, where *r* is the radius of the shaft, as shown in Fig. 4, then the SIF in radial direction is similar to the SIF in circumferential direction for all different relative depths between 0.2 and 0.8. This means that the propagation velocity in radial direction is similar to the propagation in circumferential direction: in other words the shape of the crack remains elliptical with the similar ratios , as it is shown in Fig. 4. For higher depths ($a/r \geq 0.8$), the SIF becomes higher in circumferential direction than in radial direction, the crack tip becomes rather straight and finally convex curvilinear.

The vibrations of a cracked shaft will be calculated with the procedure illustrated in following section, for each of the crack shapes and depths shown in Fig. 4. Also the actual propagation velocity can be derived from the static bending moment, from the corresponding nominal stresses, from the depth of the crack and from the corresponding SIF. This propagation velocity allows calculating the time interval required for crack propagation from step to step. The vibration amplitude evolution as a function of time can be calculated in this way. The complete procedure will be presented by means of a test case that involve the model of a real industrial machinery that has already been discussed in Bachschmid et al.(2010)[2].

3 TEST CASE: A TURBO-GENERATOR WITH A CRACK IN THE HP-IP STEAM TURBINE ROTOR

Figure 5 shows the finite element model of the 320 MW steam turbo-generator unit composed of a high-intermediate pressure (HP-IP) turbine, a low pressure (LP) turbine and a two-poles generator, rotating at rated speed at 3000 rpm. Oil-film bearings are modeled by means of linearized stiffness and damping coefficients, which are function of the rotating speed. Supporting structure is modeled by means of pedestals, i.e. equivalent single d.o.f.

mass, stiffness and damping systems, one for each bearing. The crack is located on the HP-IP turbine main body, roughly at mid-span close to the control stage, as shown in Fig.5. There is the experimental evidence of a crack developed due to a thermal shock that occurred during an operation accident in this position, in a similar steam turbine rotor (Passaleva, 1982).

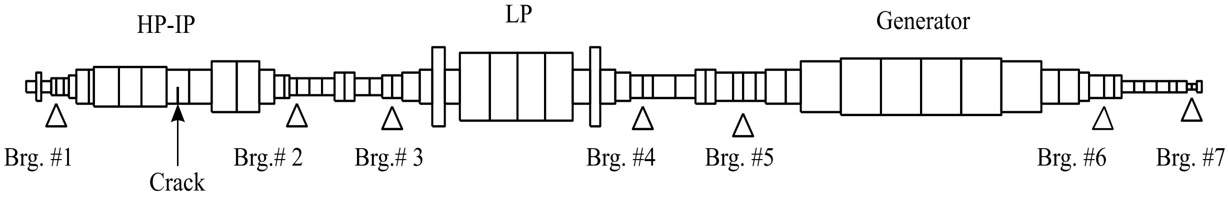

Figure 5. Model of the shaft-line, with a transverse crack at mid-span of the HP-IP rotor.

The bending moment due to the weight (and bearing alignment conditions) of the turbine in correspondence of the crack is M_f = 147 kNm, which generates a maximum of nominal axial alternate stress equal to σ_f = 4.97 MPa .

Also stress concentration due to grooves for blade attachment should be considered that might increase consistently the alternate stress level, which is then responsible for crack propagation. Anyway for the development of cracks, that is crack initiation, also the constant stresses, due to centrifugal effect, and even more to thermal stresses, during start up transients and during load variation at rated speed, give a relevant contribution.

Material is supposed to be 30CrMoV 412, which is a steel with 1% of Cr, 1.3% of Mo and 0.25% of V.

3.1 Modelling the Breathing Crack and the Cracked Shaft Dynamic Behavior

Breathing has to be determined accurately, in order to reproduce the transition from fully closed crack (full shaft stiffness) to the fully open crack (when the shaft has its weakest stiffness).

A simplified model, which assumes linear stress and strain distributions to calculate the breathing mechanism has been developed and is described in Bachschmid et al. (2010)[1], and will be recalled here. The determination of the breathing mechanism is a non-linear iterative procedure. The proposed model assumes:

- the initial position of the main inertia axes of the supposedly partially open crack surface, in a given angular position of the shaft
- a linear axial stress distribution due to the bending load, to which also thermal stresses are superposed if it is the case.
- then the compressive and tensile stresses due to the applied loads are defined: cracked surfaces where tensile stresses should appear are “open” areas; where instead compressive stresses appear these areas are considered “closed”.
- the second moments of area with respect to the fixed reference frame can be calculated from open and closed areas, as well as the new position of the inertia axes

• now previous steps have to be repeated until the angular position of the inertia axes converge

• this loop has to be repeated for each angular position of the shaft during one revolution.

Once the breathing mechanism and the second moments of area have been defined for the different angular positions, the stiffness matrix of an equivalent cracked beam element of suitable length l_c can be calculated, assuming a Timoshenko beam with constant section and second moments of area along l_c, as shown in Fig. 6, for each different angular position.

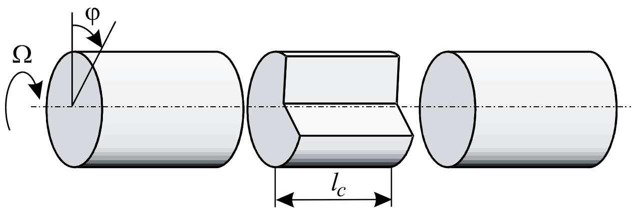

Figure 6. Equivalent cracked beam element: its configuration changes with its angular position, due to breathing.

The stiffness matrix of the equivalent cracked beam element, which is obviously periodical (period one revolution), is introduced in the stiffness matrix of the shaft-line of the turbo-generator unit. The angular position dependent stiffness matrix $[\mathbf{K}_c(\Omega t)]$ results obviously also periodical, has one constant term (the mean stiffness) $[\mathbf{K}_m]$ and several harmonic components $[\Delta\mathbf{K}_j]$ of which only the first three components (with j = 1, 2 or 3) are retained. The equation of motion of the cracked shaft-line is then:

$$[\mathbf{M}]\ddot{\mathbf{x}} + ([\mathbf{C}]+[\mathbf{Gyr}]\Omega)\dot{\mathbf{x}} + ([\mathbf{K}_m]+\textstyle\sum_j [\Delta\mathbf{K}_j]\, e^{ij\Omega t})\mathbf{x} = \textstyle\sum_j \mathbf{F}_{ej}\, e^{ij\Omega t} + \mathbf{W}, \quad j = 1, 2, 3, .. \qquad (1)$$

that is linear with variable parameters. $[\mathbf{M}]$, $[\mathbf{R}]$ and $[\mathbf{Gyr}]$ are respectively the mass, damping and gyroscopic matrices, $\mathbf{W}$ is the weight vector and $\mathbf{F}_{ej}$ are the external force vectors, which also can have 1X, 2X and 3X components. The rotor displacements $\mathbf{x}$ can be split in their static $\mathbf{x}_s$ and dynamic $\mathbf{x}_d$ components and eq. (1) can be rewritten in the following form:

$$[\mathbf{M}]\ddot{\mathbf{x}}_d + ([\mathbf{C}]+[\mathbf{Gyr}]\Omega)\dot{\mathbf{x}}_d + ([\mathbf{K}_m])\mathbf{x}_d = \textstyle\sum_j \mathbf{F}_{ej}\, e^{ij\Omega t} - \textstyle\sum_j [\Delta K_j] e^{ij\Omega t}(\mathbf{x}_s + \mathbf{x}_d), \quad j = 1, 2, 3,.. \qquad (2)$$

If only the steady state solution due to the exciting forces is of interest, being the exciting forces periodical, then it is possible to shift from time domain to the frequency domain.

The following equations are obtained:

$$\{-(j\Omega)^2[\mathbf{M}]+ij\Omega([\mathbf{C}]+[\mathbf{Gyr}]\Omega) + ([\mathbf{K}_m]\}\mathbf{x}_j = \mathbf{F}_{tj} \qquad j = 1, 2, 3, \qquad (3)$$

where $\mathbf{F}_{tj}$ includes the external forces and the equivalent crack forces, which depend on stiffness variation components and on static and dynamic vibration amplitudes of the nodes of the cracked element:

$$\mathbf{F}_j\, e^{ij\Omega t} = f\,(\Delta K_j, \mathbf{x}_s, \mathbf{x}_j) \tag{4}$$

Vibrations $\mathbf{x}_j$ that are present in Eq. (3) and in the exciting forces Eq. (4), have to be calculated by means of an iterative procedure. This iterative procedure has to be repeated for each different rotating speed.

In this way, the vibrations in all nodes of the finite element model and in particular in the bearing nodes, where they are measured on the real machine, can be calculated.

This has to be done for all different crack depths and shapes.

3.2 Modelling the crack propagation

Linear Elastic Fracture Mechanics (LEFM) states that the crack growth rate, which is the derivative of the crack depth *a* with respect to the number of fatigue cycles *N*, depends mainly on the variation of the SIF (Stress Intensity Factor) during one load cycle, which is generally indicated with ΔK or ΔK_I referring to mode *I* of crack opening. Figure 7 shows a typical graph of the crack growth rate (log *da/dN*), as function of the SIF variation (log ΔK).

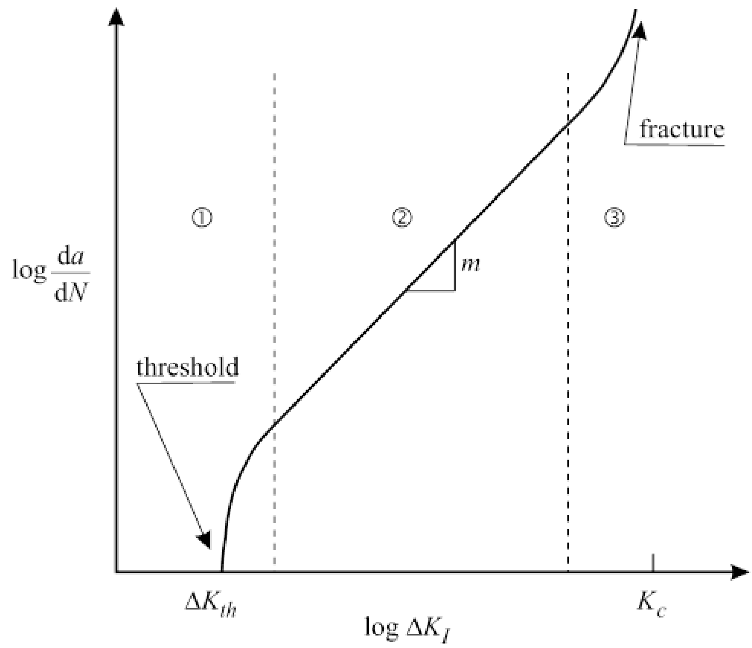

Figure 7. Propagation velocity as function of SIF variation during one cycle.

Once a threshold ΔK_{th} has passed, in zone ② of Fig.7, the logarithmic growth rate tends to a linear function of the logarithmic stress intensity variation with slope *m* . The growth rate increases at the end of zone ② and rapidly, in zone ③, the fracture will occur at the critical stress intensity. Paris' law (Paris, 1960) given by Eq. (5), holds in zone ②. The parameters C and m have to be assessed with fatigue tests and depend only on the material.

$$da/dN = C \, \Delta K^m \tag{5}$$

This law will be used for calculating the crack growth rate, assuming to have passed the threshold. The following values can be used for the shaft material of the steam turbine:

$$C = 0.65\ 10^{-18},\ m = 6.7$$

and the crack propagation threshold value can be assumed equal to ΔK_{th}=30 dN mm$^{-3/2}$.

The stress intensity factor K, due to the crack with a radial depth a is given by:

$$K = F\sigma_f\,(\pi a)^{1/2} \tag{6}$$

where σ_f is the nominal stress and F is a coefficient whose value depends only on the crack shape.

For the crack with aspect ratio a/b = 0.6, the assumed values of coefficient F are given in table 1 as a function of the crack depth.

Table 1 Values of coefficient F for different crack depths.

a/r	F
0.4	0.75
0.6	0.85
0.8	0.95
1.0	1.05

Different scenarios can then be analyzed:

i) the crack is breathing and with closed crack no SIF appears: in this case ΔK can be considered equal to K given by Eq. (6);

ii) the crack is not breathing and is always open due to a local bow, permanent or thermal: in this case ΔK can be considered equal to 2 K ;

iii) the crack has developed in a position where, due to the geometry of the shaft a high stress concentration occurs: nominal stresses are multiplied by a coefficient c, which value could range between 1 and 3. Also K of expression Eq. (6) will be multiplied by the same coefficient.

Combining Eq. (5) with Eq. (6) the following equation is obtained:

$$da/dN = C(F\sigma_f\,(\pi a)^{1/2})^m \qquad (7)$$

and the number of cycles N, required to propagate a crack from depth a_1 to depth a_2 can be calculated by the integral in following Eq.(8).

$$\int_0^N dN = \int_{a_1}^{a_2} \frac{a^{-m/2}}{C(F\sigma_f\sqrt{\pi})^m}\,da \qquad (1)$$

Therefore it results that:

$$N = \frac{1}{c(F\sigma_f\sqrt{\pi})^m}\,\frac{1}{p}\left|a^p\right|_{a_1}^{a_2} \text{ with } p = 1 - m/2 \qquad (2)$$

From the calculation of the number of cycles N, using Eq. (2), given the rotating speed, the time, required by the crack to propagate and deepen by a certain amount, can be determined.

3.3 Vibration evolution at rated speed

If scenario i) is considered the crack would never develop nor propagate, due to the low level of alternating stress. Even with a crack at 50% of the diameter (which could have been produced during abnormal severe operating conditions) propagation speed would be so slow that it would take several hundred years to propagate further in normal operating conditions. This result, which is certainly surprising, might be due to the very simple crack propagation model that has probably never been validated for huge shaft diameters and very deep cracks.

Similar results have been found if the stress concentration factor c is equal to 2.

Considering scenario ii) with a stress concentration factor of 3, then the crack would propagate once the threshold has over-passed; this situation is called case A. Propagation occurs only in normal operating conditions, when the crack has already reached a relative depth of 0.6. Figure 8 shows the evolution of the 1X vibration amplitude measured in bearing 1 of the turbine as function of time (days of operation), in a linear graph in the range between 2000 and 3000 days, considering the time origin (0) when the crack has a relative depth of 0.4. Assuming that threshold has passed during abnormal operating conditions, the crack would take 2455 days of operation to grow from 0.4 to 0.6 relative depth: the propagation velocity is vanishing small. At that depth of 0.6 the threshold for propagation in normal operating conditions is passed and propagation gets faster and faster. The last step takes only 17 days.

Figure 9 shows, in the same time scale, the evolution of the 2X component of the breathing crack (case A) and also that of the non-breathing, always open crack that excites only, but much more strongly, the 2X component. This condition is called case B and represents the combined scenarios ii) and iii). The time origin (0) of case B is when the crack starts propagating from relative depth of 0.2. At this depth the threshold is passed.

Propagation in case B (fully open crack) is much faster than in case A (breathing crack). The final step from 1.0 to 1.2 relative depth is covered in less than 4.5 hours, which is a dangerously short time, if compared to 17 days in which the same evolution occurs in case A.

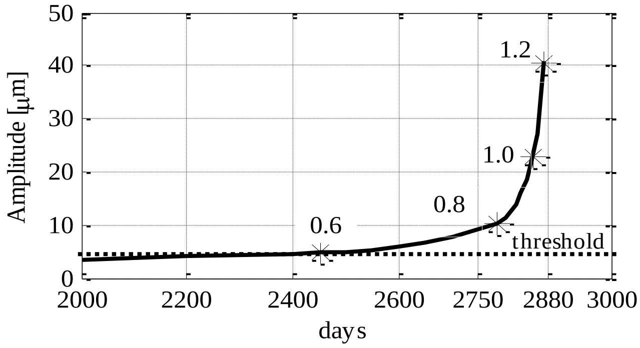

Figure 8. Evolution of 1X component (case A)

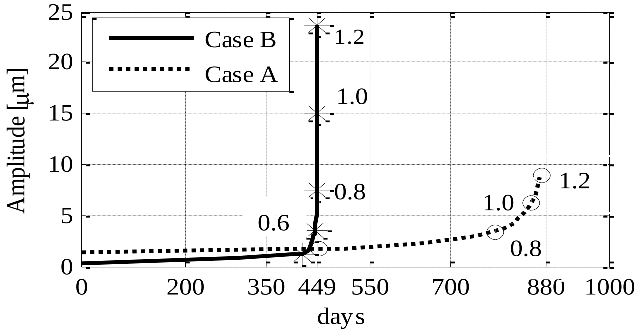

Figure 9. Evolution of 2X component (case A and B)

A logarithmic time scale is used in Fig. 10, where time is calculated as days of operation before reaching the dangerous relative depth of 1.2. The different symbols used define the corresponding depth: from star for 1.2 to the triangle for 0.2. The dramatic speed of propagation at the last steps of propagation of the always open crack is emphasized in the graph of figure . Considering the fully open crack (case B) the propagation from 0.6 relative depth to 0.8 takes some days of operation, but the propagation from 0.8 relative depth to 1.0 occurs in few hours!

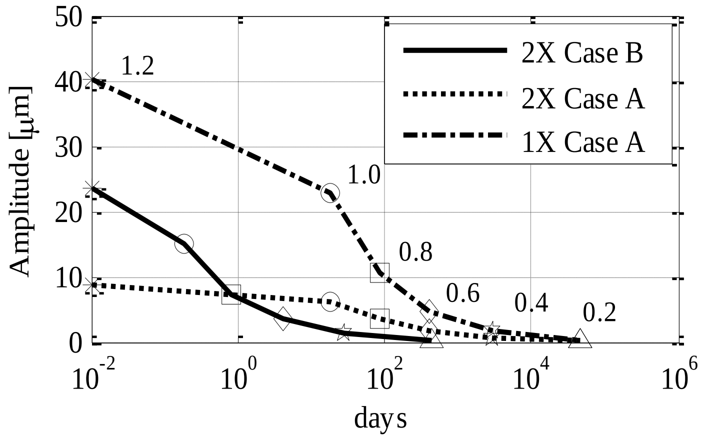

Figure 10. Days of operation before reaching a dangerous relative depth of 1.2

The 3X component is omitted because its amplitude is so small that it would be completely masked by noise. It should also be noted that the critical SIF at which failure occurs, which can be assumed equal to K_c = 200 dN mm$^{-3/2}$, is never reached even at the depth of 1.2. Anyway, failure can obviously occur when other stresses sum up to the dynamic stresses that are responsible for the propagation.

The simulated crack propagation laws show that it is difficult if not impossible to predict the residual life of a cracked rotor only from crack depth which can be determined in a model based approach from vibration measurements. Propagation from 0.8 to 1.0 can take 45 days in case A and only 3 days in case B. The time history of vibration evolution is needed for trying to evaluate residual life, provided that the model used in this analysis is accurate enough.

3.4 Run down transient –evolution of the Bode plots

Since the vibration levels excited by the crack are rather low at rated speed, it is interesting to observe the vibration amplitudes that develop in resonant conditions for the 1X and 2X components, during typical run down transients, which occur at each one of the periodical stops of the machine. Also in this case 3X component is omitted for the sake of brevity. Figure shows 1X amplitude Bode plots corresponding to the different depths for case A. Figure shows the close up of figure to highlight the amplitudes related to the first steps of propagation. It can be seen that in the initial stages of propagation (0.2 – 0.4) the 1X amplitudes are so small that they could hardly be discovered considering also that other 1X components due to unbalance and bow could completely mask the vibrations generated by cracks.

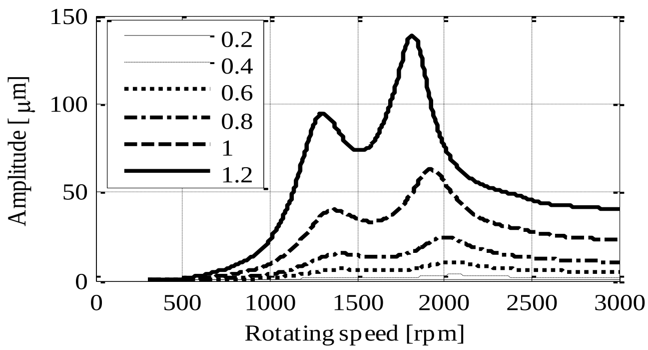

Figure 11. 1X Bode plot in bearing 1 (case A)

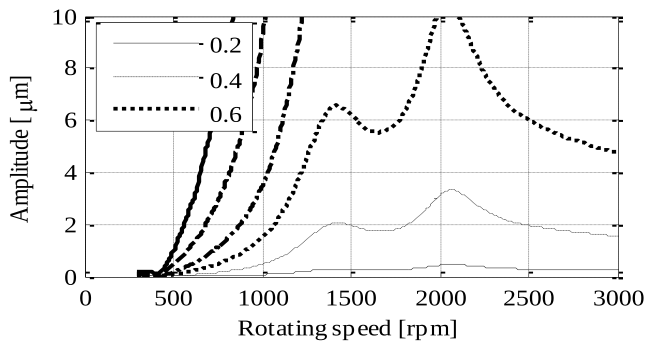

Figure 12. Close up of figure 11 (case A)

Figure shows the 2X component amplitudes of case A, and figure the close up. It is generally assumed that the 2X component is the most reliable symptom of the presence of a crack, but this symptom can be missed unless the crack has already propagated to a consistent depth, as shown also here in case A.

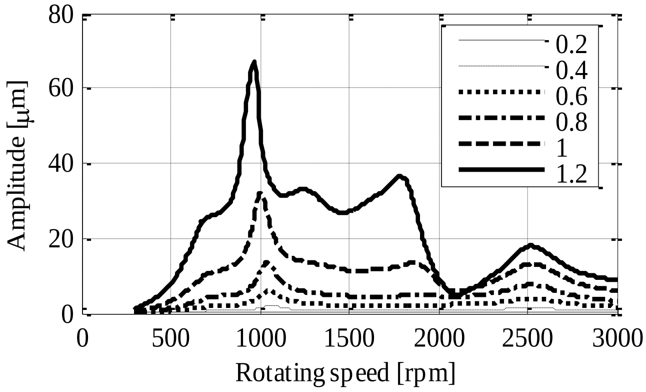

Figure 13. 2X Bode plot in bearing 1 (case A)

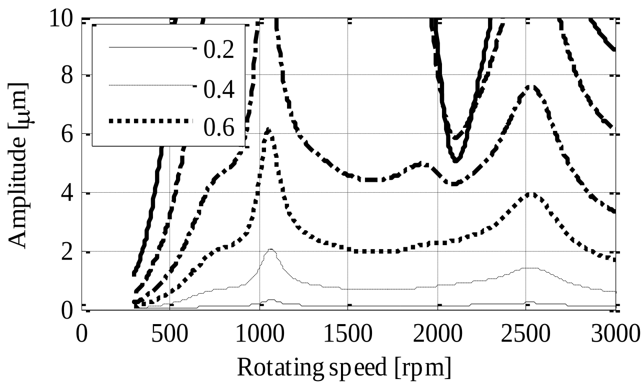

Figure 14. Close up of figure (case A)

Figure 15 shows the 2X amplitudes for case B and figure 16 the close up for the cracks with lower depths.

In this case crack detection would be easier, but only when the crack has already propagated to a relative depth of 0.4. It is further interesting to note that the shift in bending natural frequencies, or better the shift in rotating speed at which the resonance peak appears, is recognizable only for the higher depths (from 0.8 to 1.2). Also this result was predicted in Bachschmid et al.(2010)[1].

All these results, obtained with a rather refined model of the crack and with a simple frequency domain calculation procedure based on an harmonic balance approach, show that monitoring cracked rotors is not an easy task when cracks are in an early stage of propagation. The prediction of the residual life is a still more difficult task. It is nevertheless interesting to notice that when the crack is approaching a critical depth then the changes in vibrational behaviour become rather rapid (e.g significant changes during continuous operation occur from one day to the other). This behaviour has permitted in many cases to stop the machine in time for avoiding catastrophic failures.

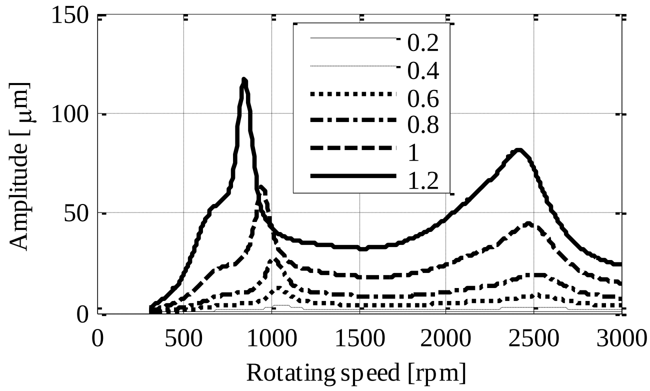

Figure 15. 2X Bode plot in bearing 1 (case B)

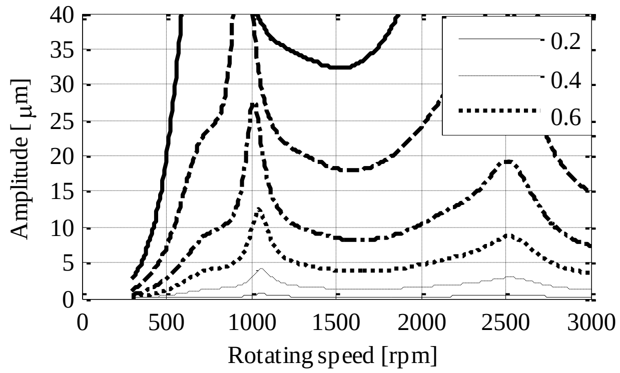

Figure 16. Close up of 15 (case B)

CONCLUSIONS

Health monitoring (HM) of rotors, and more in detail HM of cracked rotors during operation can be performed only by means of standard vibration measurements, when additional devices (e.g. sensors or exciters) are not available. Standard vibration measurements in industrial machinery are taken generally only in, or close to, the bearings of the machines.

The first step in HM is then modeling the relationship between vibrations and position and depth of the crack, that can be done accurately by suitable models as described in the paper.

The second step in HM is then to forecast the crack propagation velocity, or to calculate the residual life of the rotor. This is an important task because the management of the industrial plant needs to know how many months or days the machine can be operated, before the crack depth becomes critical, so that a spare rotor (if available) can be prepared or eventually ordered. But the plant management needs also to know which will be the evolution of the vibrations, in order to be able to monitor the crack propagation.

To accomplish this task, crack propagation laws have been combined to cracked rotor behavior for calculating the evolution of vibration amplitudes in different scenarios. Crack growth rate depends strongly on the initial stress concentration factors in correspondence of the initial crack position. It has been shown that high alternating stresses can be responsible for high, dramatic, crack propagation velocities, without generating remarkable vibration amplitudes, at least at rated speed. It resulted further that residual life cannot be evaluated unless the time history evolution of the vibrations is taken into account. Run down transients allow to highlight 1X and 2X amplitudes when passing critical speeds, that allow not only easier detection of cracks, but also to control the propagation of the crack, provided that they have developed in positions where the excitation of vibrations is significant.

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