TOPOLOGY OPTIMIZATION OF STEEL PLATES USING THE CONCEPT OF PRINCIPAL STRESSES

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Abstract. *We intend in this work to present some research results on the topology optimization of steel plates considering the concept of principal stresses. An initial topology design of a constant thickness plate is considered and loads are applied on this. The structural system is modelled with the FEM (Finite Elements Method). In each element is computed the principal stresses. The element with the smallest principal stress is extracted from the system and its mass is redistributed between the others elements, improving their thickness, keeping them constant. The process is repeated until the system becomes unstable. The last stable design obtained in this process is kept. Then another continuous optimization problem is formulated, considering the thickness as the design variable and applying constraints on the stresses and displacements. The design obtained here is adopted as the optimum topology design. Some examples are designed and suggestions for future works are given.*

Keywords: *topology optimization, plates, steel structures, principal stresses*

1 INTRODUCTION

The theme of the present paper is treat the problem of topology optimization using the concept of principal stress. In this case, the topology optimization becomes a shape optimization, where the structure must have an optimal geometry for its purpose of receiving loads and transmitting them to the supports. Shape optimization is one the most important subject in structural design, besides of the minimization of mass and size, in cases where the structure must to travel to delivery or in case of aerospace structures the cost of the transportation is very important and must be taken into consideration.

A painting showing the problem analyzed here is shown is Fig. 1. In this figure we can see a square plate with a horizontal load applied on the top of the left side. The track of the stress is shown from the load until the supports. The part of the structure domain that really works is that one around of the tracks. The regions far from this can be extracted from domain and the structure be redesigned with its new domain to obtain an optimal shape.

Fig. 1 – Drawing of a plate under a point load

In the work of Harasaki and Arora (2001) was treated the problem of shape optimization using the transferred forces along the structure. These forces and transfer diagrams show relative importance of various parts of the structure that can be useful in redesign or optimum design of the structure. Inspired in this method the authors propose here something similar, but with the concept of principal stress. The plate is discretized in finite elements the elements with the smallest principal stress, in absolute value, are extracted from the domain and the structure is redesigned, until achieve the optimum shape design. After that, a continuous optimization problem is formulated, considering the thickness as the design variable and applying constraints on the stresses and displacements. The design obtained here is adopted as the optimum topology design.

To solve the problems discussed here are used the finite element method to accomplish the structural analysis and ABNT (2008) to do the verification of the allowable stress. In case of the discrete and topology optimization is used an algorithm developed by the authors. For the continuous optimization process, the augmented Lagrangian method, as described by Chahande and Arora (1994) and Arora (2012) is used. This method transforms a constrained optimization problem into an unconstrained optimization problem. The objective and constraints functions are combined using the Lagrange multipliers and penalty parameters to create an augmented Lagrangian functional. A sequence of functionals is created by properly altering the penalty parameters and Lagrange multipliers. The unconstrained minimum value of the functional in this sequence converges to the minimum of the constrained problem. Considering all the methods describe in this section we developed the work here presented.

2 DESCRIPTION OF THE PROBLEM

Consider the plate shown in Fig. 2. The reference system is x1, x2, x3. We can see the lateral dimensions as (l_x = 1000 mm) x (l_y = 1000 mm) and the initial thickness is h = 10 mm.

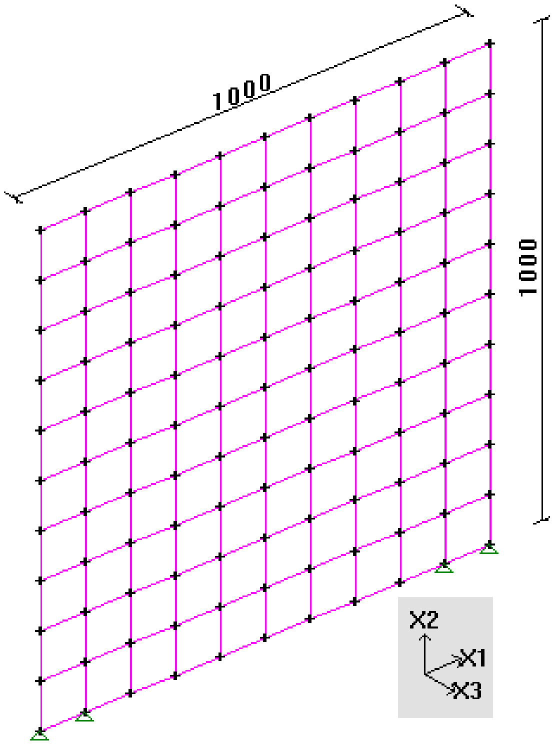


Fig. 2 – 3-D view of the plate structural model

The material of the plate is steel with density of ρ_{st} = 7850 kg/m^3 and the structure mass is m = 78.5 kg. The yield stress is fy = 250 MPa and the allowable stress is fyd = 225 MPa. The plate is supported in the inferior corners with two supports in each corner.

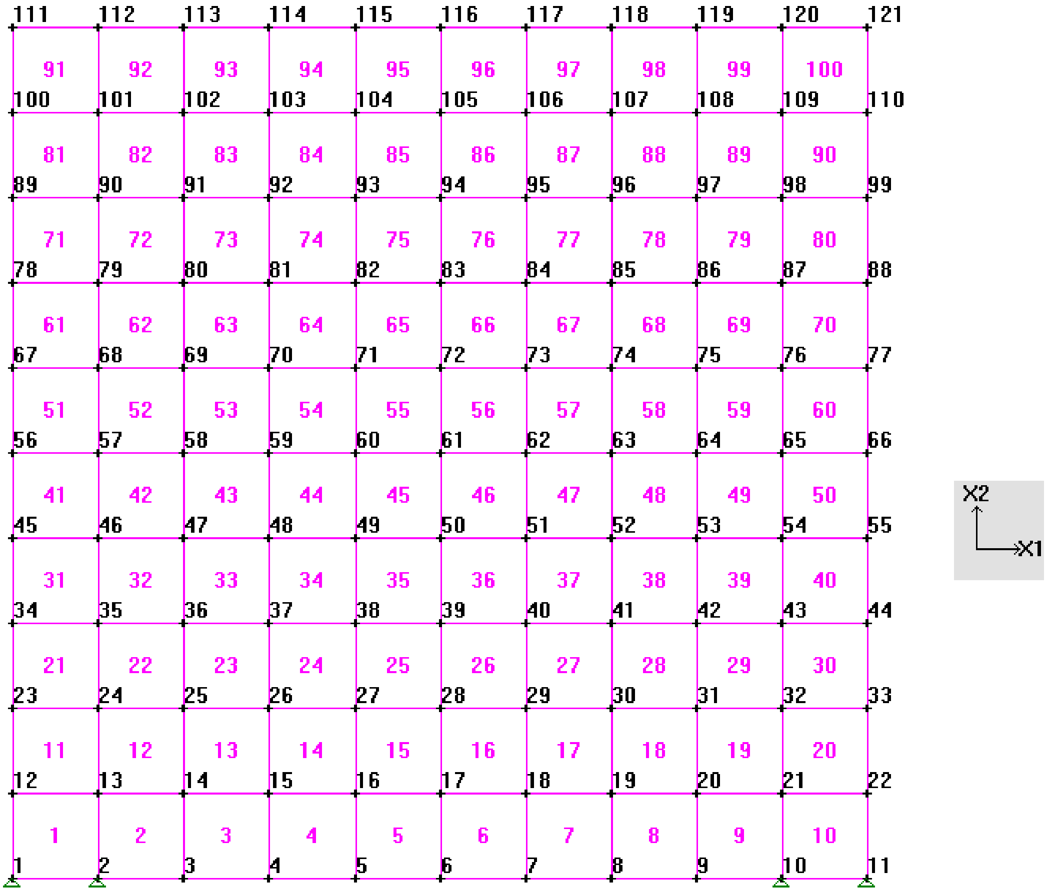


Fig. 3 – Plate discretization

The plate domain is discretized in ne = 100 elements and nn = 121 nodes, according to Fig. 3. In the FEM program used is accomplished a static linear elastic analysis where is considered 4 nodes per element and 2 degrees of freedom per node. As shown in Fig. 4, at the left superior corner of the plate is applied a unitary load F = 1 tf = 10 kN.

In structural analysis, the linear elastic problem can be defined by the system:

$$\boldsymbol{Ku} = \boldsymbol{f} \tag{1}$$

where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{u}$ and $\boldsymbol{f}$ are respectively the displacement and force vector. In function of the displacements $\boldsymbol{u}$ it is possible to compute the strains, stress, internal loads and support reactions.

First is proposed the following optimization problem to determine the maximum load resisted by the plate. Determine α that minimize the objective function

$$f_1 = - \alpha \ F \tag{2}$$

subjected to

$$|\sigma_{1i}| \leq fyd \qquad \text{for } i = 1, \ldots, ne \tag{3}$$

$$|\sigma_{2i}| \leq fyd \qquad \text{for } i = 1, \ldots, ne \tag{4}$$

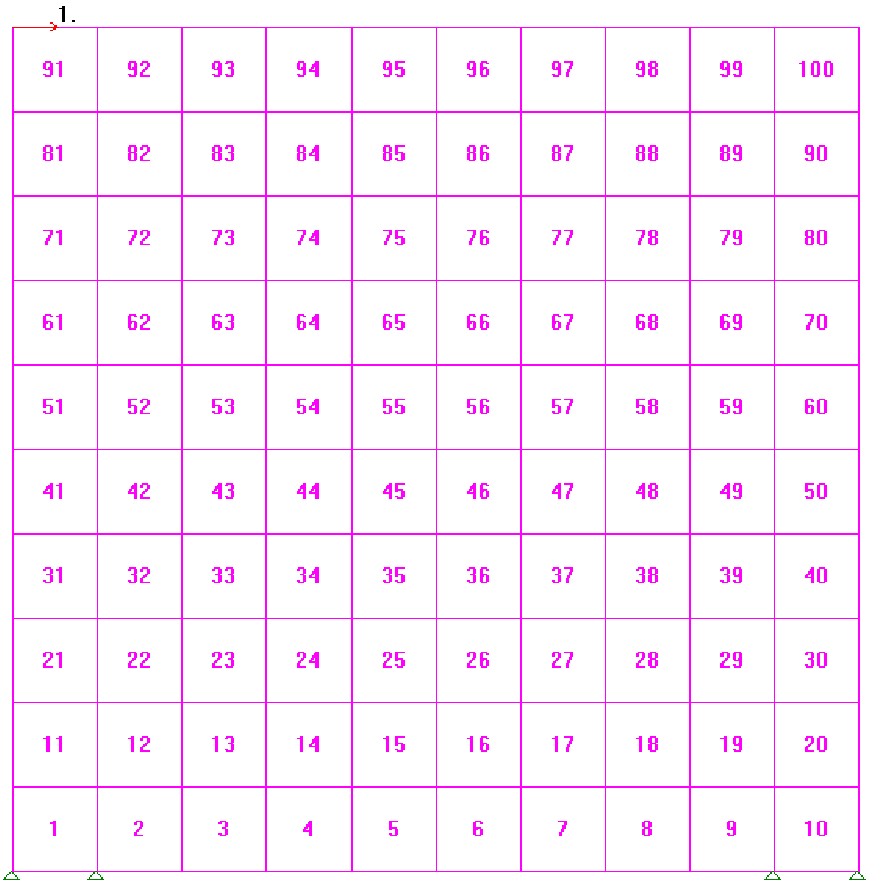


Fig. 4 – Point load applied in the superior left corner

In Eq. (3) and (4) σ_{1i} and σ_{2i} are respectively the maximum and minimum principal stresses of the i^{th} element. The initial value of α = 1.0 and the optimum value is α = 8.4 obtained in 5 steps of the optimization method. The problem defined by Eqs. (2) to (4) is known as limit analysis and it is an optimization problem.

With this we have

$$F_d = \alpha \; F = 84 \text{ kN}, \tag{5}$$

where F_d is taken as the design load.

Now the topology optimization problem is determine the optimal shape of the plate that resist to the load F_d.

3 TOPOLOGY OPTIMIZATION PROCESS

3.1 Topology optimization procedure

The procedure proposed here is solve the system $\boldsymbol{Ku} = \boldsymbol{f}$ and extract nr elements, which present the smallest values of principal stress. After that run the system again and repeat the process of extracting the elements until the system becomes unstable. We consider initially

$$nr = \text{int}(ne/10). \tag{6}$$

If, in a given step, the system becomes unstable with a number of elements extracted $j < nr$, we take $nr = j$ -1. We consider the system unstable when

$$d_j = \text{Det}(\boldsymbol{K}_j) = 0. \tag{7}$$

where $\boldsymbol{K}_j$ is the stiffness matrix in the step j with the current number of elements. The total mas of the plate at the initial design is m and the thickness is h. During a current design k the mass of the plate remains m and the thickness is $h(k)$. Note that $h(k)$ is computed such as m keeps always with the same value.

The topology optimization problem can be resumed with the procedure:

Step 0 – Do $k = 0$, $ne(k) = ne$, $nr(k) = ne/10$, $h(k) = h$;

Step 1 – Compute the principal stresses σ_{1i} and σ_{2i} and consider $\sigma_i = \max\{\,|\sigma_{1i}|; |\sigma_{2i}\,|\}$ for all $ne(k)$ elements;

Step 2 – Classify σ_i in increasing order; consider that the increasing order is given as $j = 1, \ldots, ne(k)$;

Step 4 – For $j = 1$ to $nr(k)$, compute $dj(k)$, if $dj(k) \neq 0$ extract the j^{th} element; else do not extract the element, do $nr(k) = j - 1$; compute $h(k)$ and go to Step 5;

Step 5 – If $nr(k) = 0$ stop the process, else do $k = k + 1$ and go to Step 1.

The final domain obtained here, with the number of elements $ne(k)$ and the thickness $h(k)$ is the shape of the plate optimized by the proposed method. Note that the initial value of nr is and adopted value given by the Eq. (6). Different rules can be adopted for this purpose. The authors have seen that this choice can change drastically the final design. In step 4, the computation of $d_j(k)$ is not needed, but the important is to check the singularity of the matrix $\boldsymbol{K}_j$.

3.2 Topology optimization of the plate

Consider the plate given by Figs. 2 to 4. For the initial design with h = 10 mm and F_d = 84 kN we have the principal stresses σ_1 and σ_2 as shown in Figs. 5 and 6.

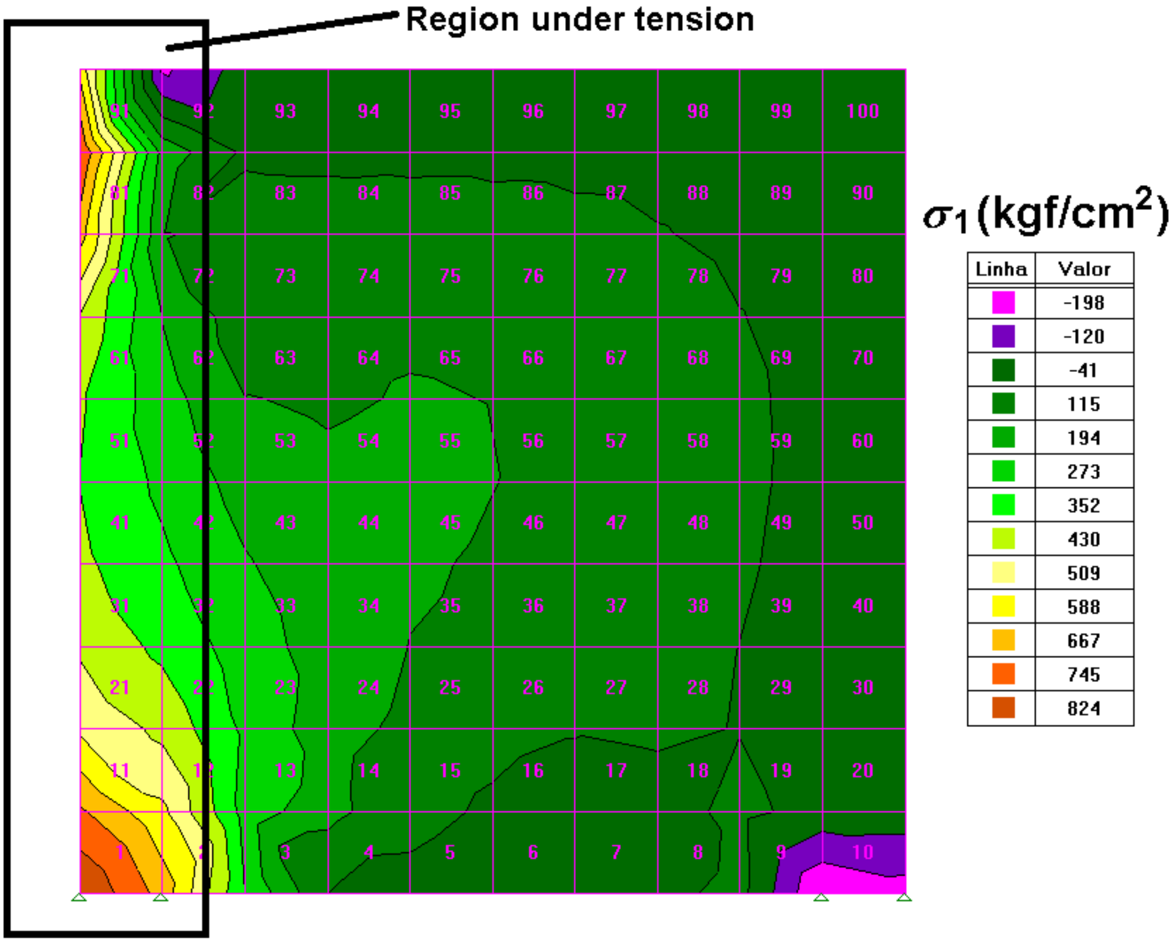


Fig. 5 – Principal stress σ_1 in the initial design

Note that the values are given in kgf/cm^2. We can see in Figs. 5 and 6 that the maximum principal stress in the plate is of compression (σ_2) equal to 2249 kgf/cm^2 = 224.9 MPa. The allowable stress is fyd = 250 MPa. So the limit analysis shown in Section 2 led to a design very close to the boundary value fyd.

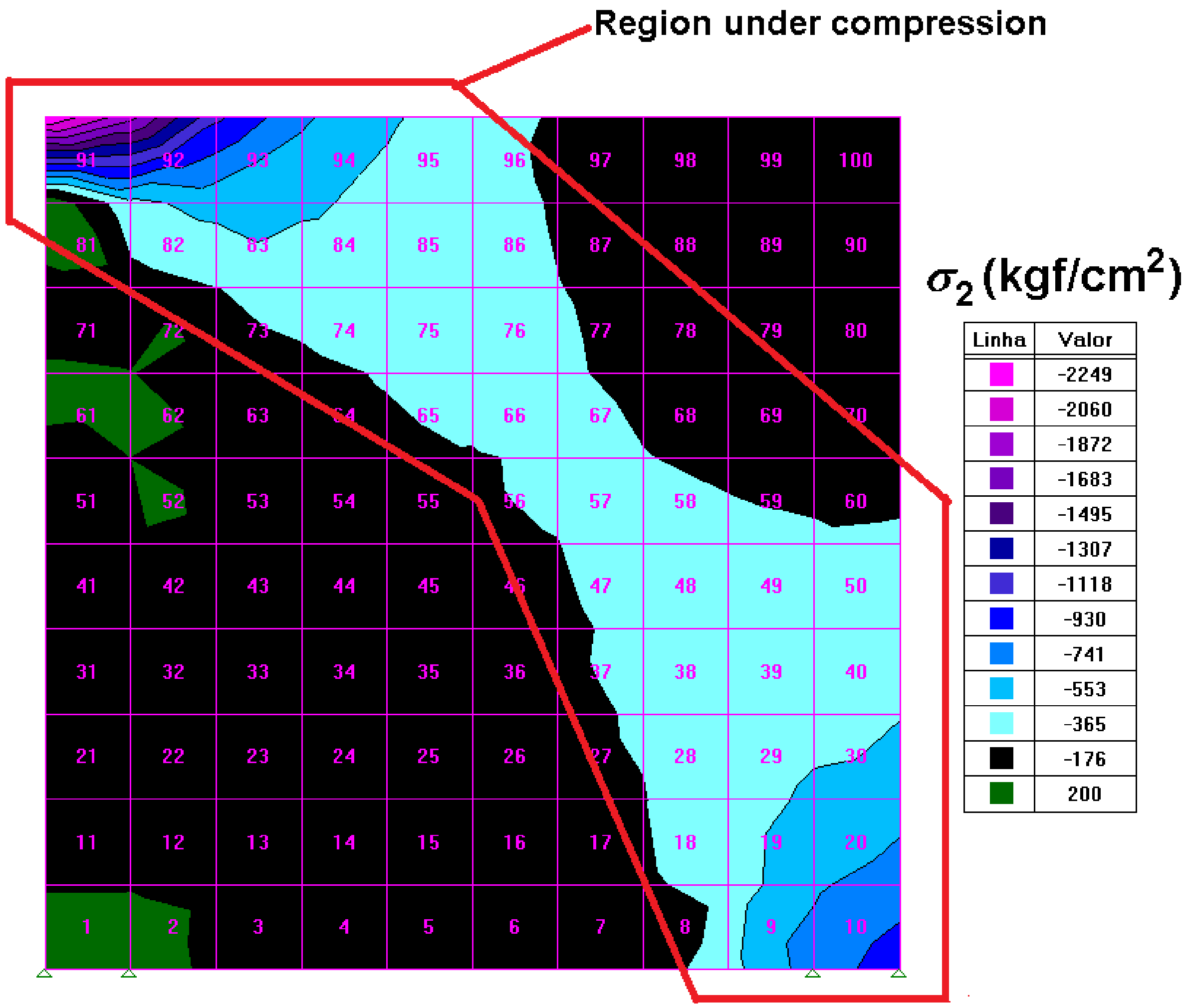


Fig. 6 – Principal stress σ_2 in the initial design

One can note in Figs. 5 and 6 that there are clearly two well-defined regions: one under tension (Fig. 5) and another under compression (Fig 6). The interpretation of this information can show us a possible geometrical way of optimize the shape, leading to a design similar to that shown in Fig. 1.

Let us apply the procedure described in Section 3.1. Taking the information of Figs. 5 and 6, we classify the values of principal stress from the smallest to the largest. Table 1 shows the classification of principal stress for the first iteration (k = 0). For question of space, we did not include in this table all the 100 elements. Considering $nr(0) = \text{int}(ne/10) = 100/10 = 10$ we extracted from the domain the first 10 elements shown in Table 1.

Table 1 – Classification of the principal stress for the initial step (k = 0)

Element	σ1 (MPa)	σ2 (MPa)	|σmax| (MPa)
100	-0.1	-0.3	0.3
6	1.0	-0.4	1.0
5	2.2	-0.2	2.2
90	0.1	-2.2	2.2
99	0.1	-2.2	2.2
7	0.0	-3.1	3.1
4	3.9	0.8	3.9
16	2.7	-4.9	4.9
15	5.1	-1.8	5.1
80	0.5	-5.5	5.5
98	0.5	-5.9	5.9
89	1.1	-6.0	6.0
72	7.0	3.5	7.0
25	8.8	-5.5	8.8
3	9.0	-0.8	9.0
26	6.3	-9.7	9.7
14	9.7	-0.6	9.7
63	9.7	-5.0	9.7

The domain remained for k = 1 is shown in Fig. 7. The structural system, now with $h(1)$ = 11 mm, is solved again and Table 2 shows the values of principal stress for the elements.

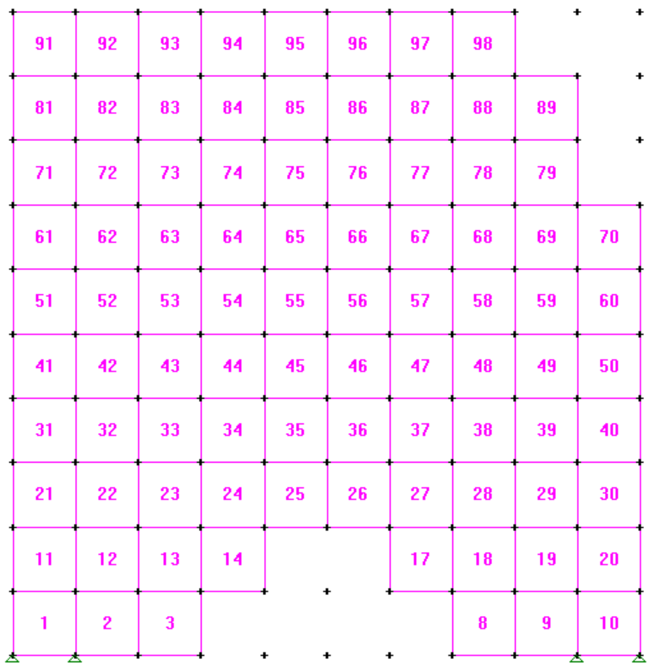


Fig. 7 – Structure domain for k = 1

Table 2 – Classification of the principal stress for the step 1 (k = 1)

Element	σ1 (MPa)	σ2 (MPa)	|σmax| (MPa)
98	0.1	-4.3	4.3
89	0.2	-4.8	4.8
3	6.3	-0.3	6.3
72	6.3	3.2	6.3
70	0.1	-8.1	8.1
63	8.8	-4.5	8.8
8	0.3	-9.3	9.3
97	1.1	-9.5	9.5
88	1.6	-10.4	10.4
14	10.9	0.1	10.9
54	11.4	-8.2	11.4
26	3.9	-11.5	11.5
53	11.5	-1.5	11.5
35	11.7	-8.4	11.7
45	11.9	-10.8	11.9
25	12.4	-1.2	12.4
44	12.7	-5.4	12.7
60	1.8	-12.7	12.7

Again, considering nr = 10 we extracted from the domain the first 10 elements shown in Table 2. The domain remained now with k = 2 is shown in Fig. 8.

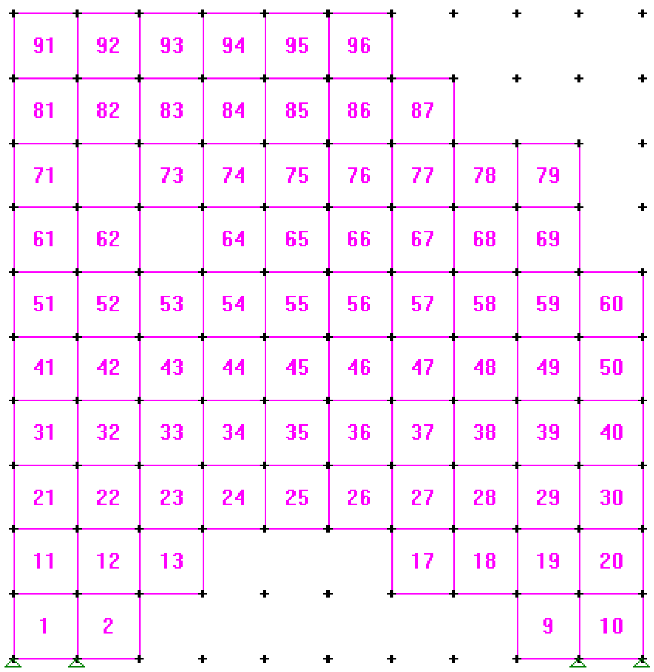
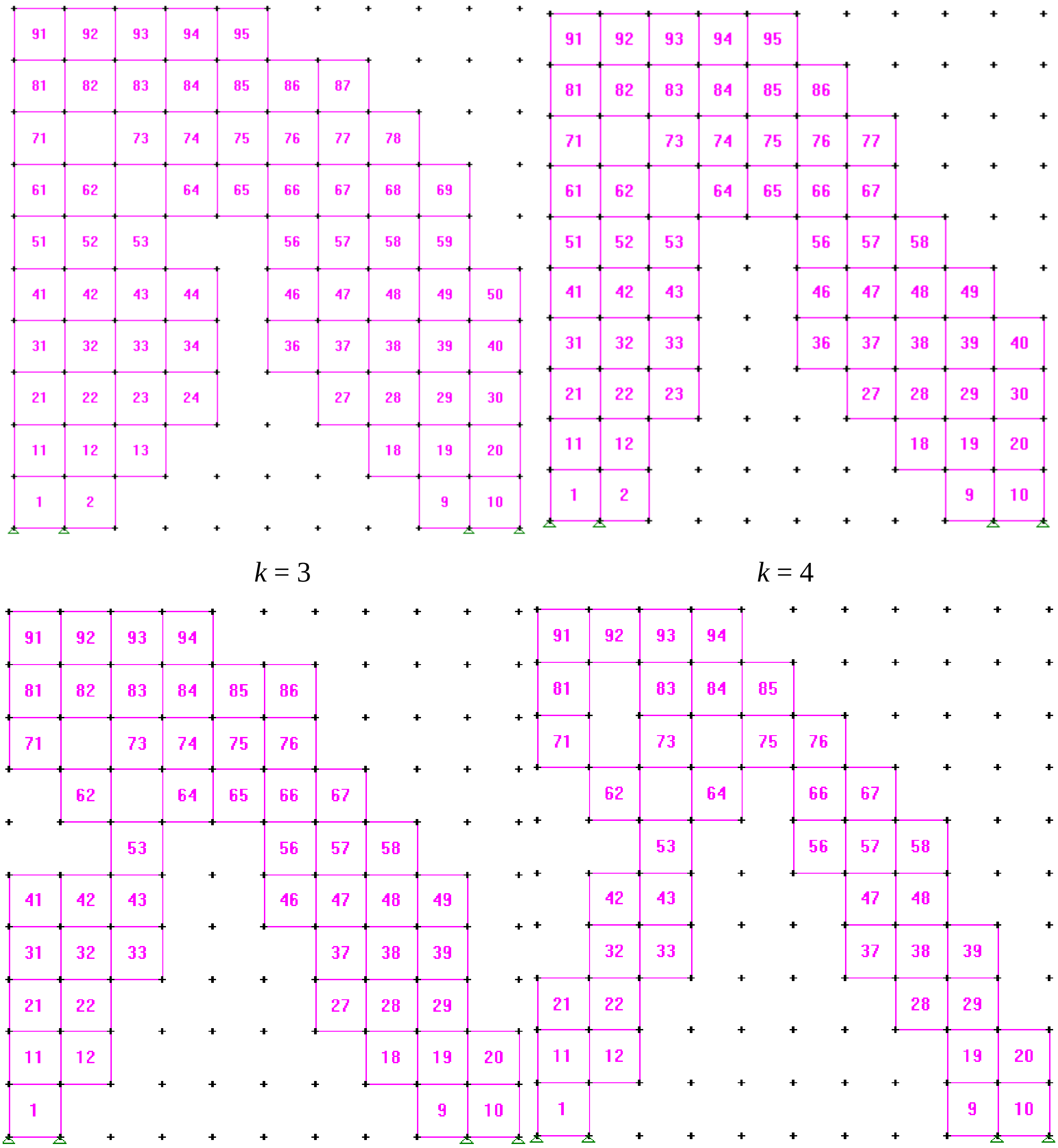


Fig. 8 – Structure domain for k = 2

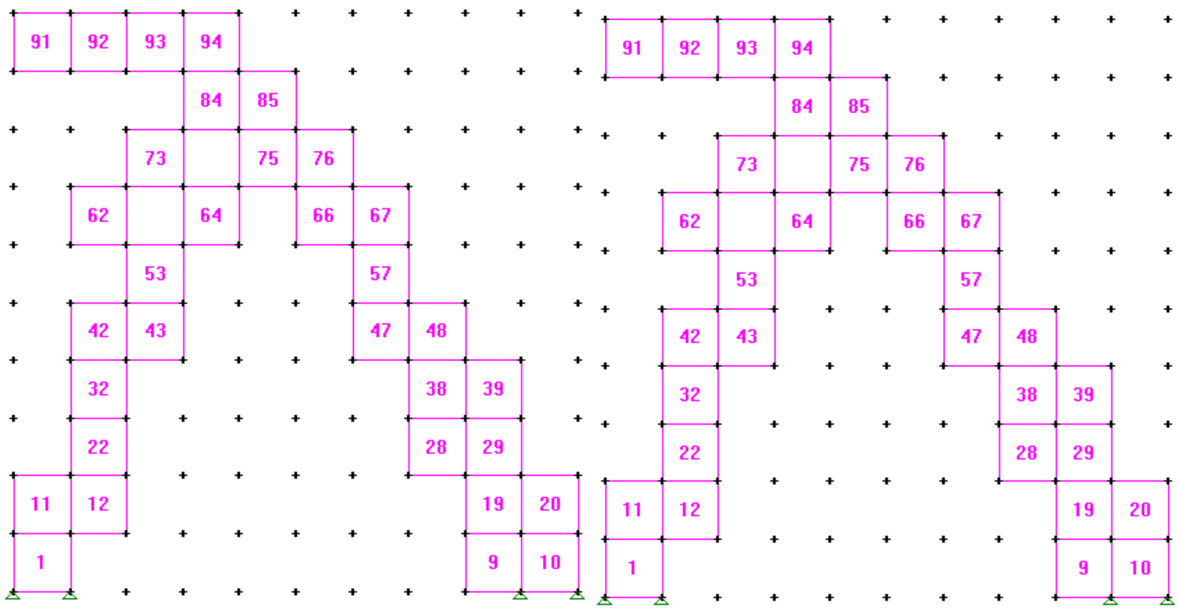
Continuing with the process we have the sequence of domains shown in Figs. 9 and 10.



$k = 3$ $k = 4$

$k = 5$ $k = 6$

Fig. 9 – Structure domains for steps $k = 3$ to 6



$k = 7$ $k = 8$

Fig. 10 – Structure domains for steps $k = 7$ and 8

One can note that until step $k = 6$ we had $nr = 10$, or 10 elements were extracted in each step. But in step $k = 7$ only 8 elements could be extracted. It means that $nr(7) = 8$. and in step $k = 8$ any element could be extract otherwise the system becomes unstable. In step 8 $nr = 0$ and the process was stopped. The thickness in the final design is $h(8) = 31$ mm.

Table 2 – Classification of the principal stress for the steps $k = 7$ and $k = 8$

k = 7				k = 8			
Element	σ1 (MPa)	σ2 (MPa)	|σmax| (MPa)	Element	σ1 (MPa)	σ2 (MPa)	|σmax| (MPa)
71	8,9	-1,5	8,9	62	11,1	0,0	11,1
81	9,5	-0,6	9,5	39	0,7	-11,2	11,2
33	9,8	-0,2	9,8	9	0,6	-11,4	11,4
37	0,0	-9,9	9,9	42	14,1	2,3	14,1
58	0,1	-12,0	12,0	47	-2,4	-14,1	14,1
83	13,0	-12,5	13,0	20	0,5	-14,3	14,3
56	-0,1	-13,6	13,6	28	0,4	-14,8	14,8
21	13,9	-0,4	13,9	75	15,5	3,5	15,5
73	14,0	-2,3	14,0	12	15,9	2,1	15,9
9	0,9	-14,2	14,2	66	1,5	-17,8	17,8

Figs. 11 and 12 show the distribution of stresses inside the optimized plate domain, for $h(8) = 31$ mm. Observe in these figures that it converged to a very slim structure. Other important question is the rule to compute nr defined in Eq. (6). Note that we considered $ne/10$. Smaller values of nr lead to better designs, but with larger computational time. One important field of research in this method is to define rules to compute nr.

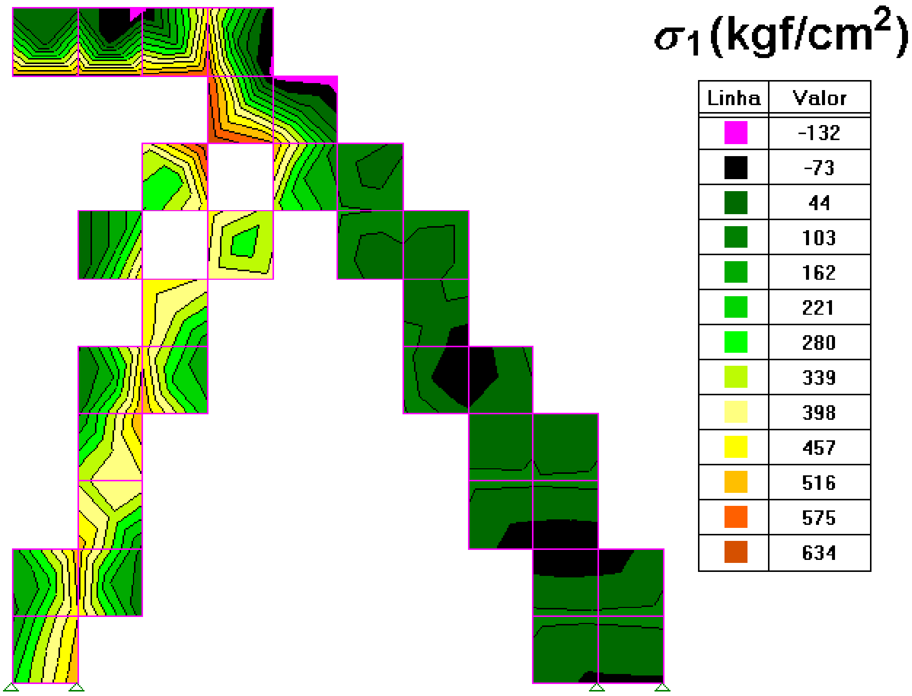


Fig. 11 – Principal stress σ_1 in the final design $h(8)$ = 31 mm

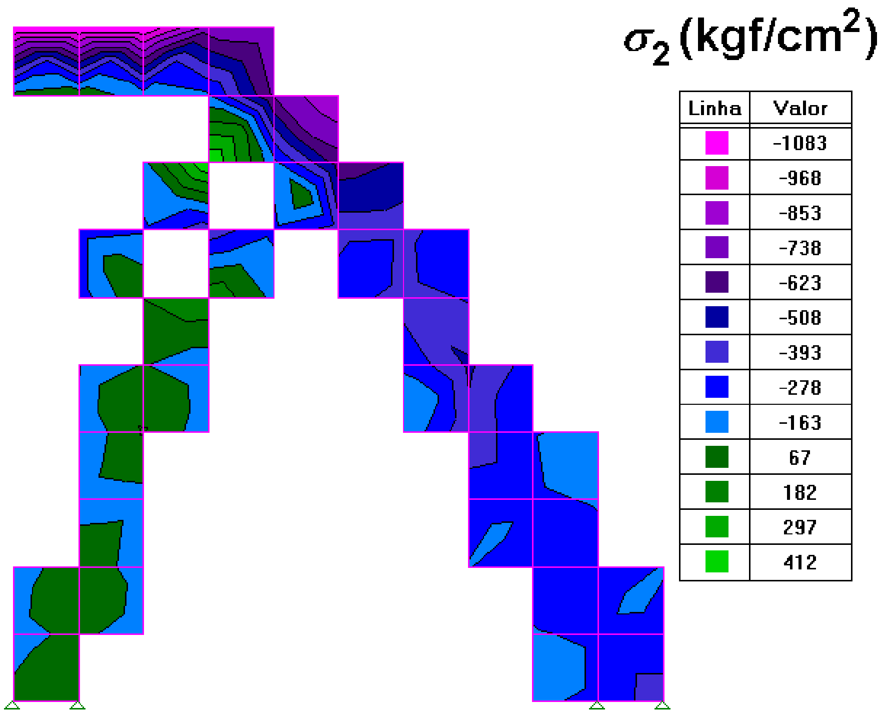


Fig. 12 – Principal stress σ_2 in the final design $h(8)$ = 31 mm

With the design obtained here we proceed a continuous optimization as describe in next Section 3.3.

3.3 Continuous optimization problem

Note in Fig. 12 that the maximum principal stress is σ_2 = 1083 kgf/cm^2 = 108.3 MPa, while the allowable stress is fyd = 225 MPa. There are a great gap between the current design (σ_2) and the possible design (fyd). At this stage we have h = 31 mm and the structure mass m = 78.5 kg. The objective is minimize the mass. As the plate presents constant thickness, minimize the thickness is the same as minimize the mass. So considering the thickness as the design variable, applying constraints on the stresses σ_1 and σ_2 we formulate the following optimization problem.

Determine h that minimize the objective function

$$f_2 = h \tag{8}$$

subjected to

$$|\sigma_{1i}| \leq fyd \qquad \text{for } i = 1, \ldots, ne \tag{9}$$

$$|\sigma_{2i}| \leq fyd \qquad \text{for } i = 1, \ldots, ne \tag{10}$$

In six iteration the augmented Lagrangian method converged to the optimum design with h = 15 mm and mass m = 37.68 kg. As we had at the beginning of process (Section 3.1) a thickness of 10 mm and a mass of 78.5 kg, we can observe a reduction of 52% in the mass of the structural system with the method proposed here.

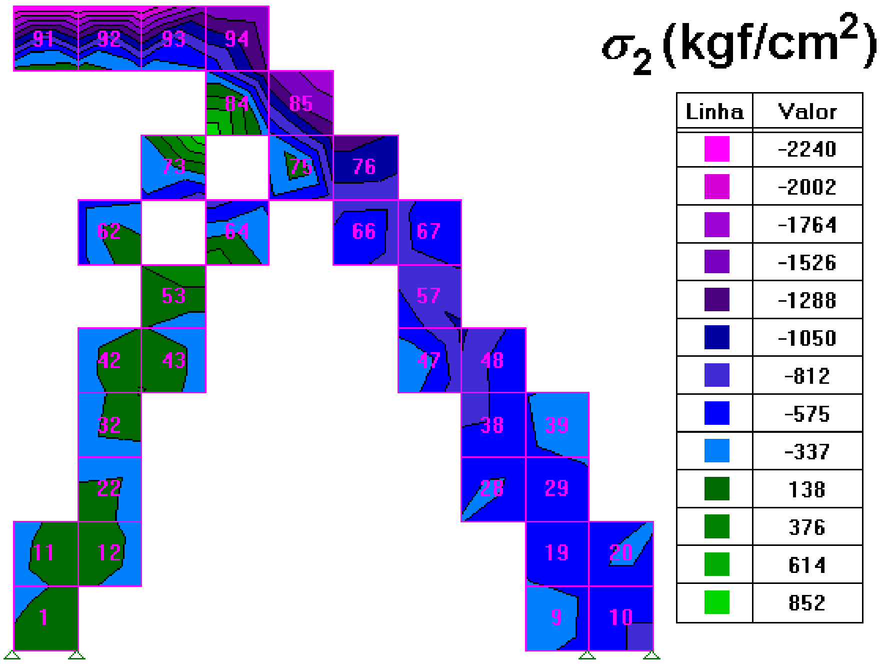


Fig. 13 – Principal stress σ_2 in the final design h = 15 mm

We can see in Fig. 13 that the maximum stress for the optimum design is σ_2 = 2240 kgf/cm^2 = 224 MPa while the allowable is *fyd* = 225 MPa. The method is very easy to be implemented and very efficient to accomplish the topology optimization as well as mass minimization.

It was not imposed constraints on the displacements of the structure because they were very small and were not important to the design of the shape, but this feature must be analyzed carefully in each case and if it is necessary impose these constraints.

To compare different formulations, the authors analyzed other procedure for shape optimization. This is called Procedure 2. We took the values classified in Table 1 and extracted the elements until the structure becomes unstable. But in this case we jumped and kept the element that became the structure unstable and continued the process with this same table until being impossible to extract any element without the structure becomes unstable. We accomplished the shape optimization in only one step extracting in the sequence of Table 1 all the elements that was possible. After that we proceed the optimization process described by Eqs. (8) to (10). The optimum design and σ_2 for this case is shown in Fig. 14.

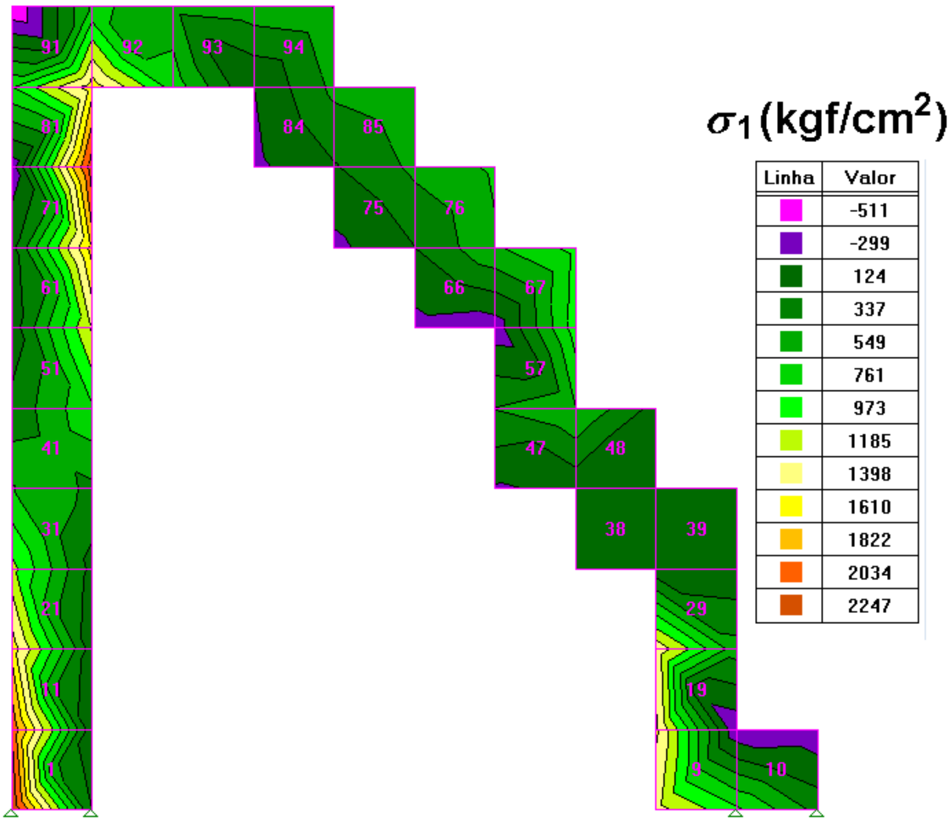


Fig. 14 – Principal stress σ_1 in the final design h = 18.9 mm

In this case the ε-active constraint is related to σ_1, while in the first formulation the ε-active constraint was related to σ_2. Note that the mass here is 41.5 kg, larger than the value obtained in the first formulation (37.68 kg) showed above. So the first formulation is more efficient than this one.

Again, the success of the method here proposed to optimize the shape strongly depends on the choice of the rule to compute *nr*. It is a very important feature to be researched and it suggestion for further works. Another important aspect is the computational time involved in the optimization problems, specially when the degrees of freedom are larger and the problem more complex.

4 CONCLUSIONS

We presented a method to accomplish the topology optimization of a steel plate with constant thickness. The structure was discretized and analyzed using the FEM. The shape optimization method is based on the evaluation of the principal stresses of the elements and the extraction of elements that present the lower value of principal stress. In each step a certain number of elements are extracted from the structural model until the structure becomes unstable. The method is very sensitive to the number of elements that is extract in each step. At the point that is not possible extract more elements we use a continuous optimization method to minimize the structural mass. The procedure proposed here reduced the plate mass in 52% in the example analyzed. Suggestions for futures works were given.

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