IDENTIFICATION OF FATIGUE CRACKS IN THIN-WALLED BEAMS SUBJECTED TO INITIAL LOADS BY MEANS OF THE NON-LINEAR DYNAMIC ANALYSIS

Víctor H. Cortínez[1,2,3]

Patricia N. Dominguez[1,3]

vcortine@frbb.utn.edu.ar

pdoming@uns.edu.ar

1. Centro de Investigaciones en Mecánica Teórica y Aplicada, UTN-FRBB
   11 de abril 461, (8000) Bahía Blanca, Argentina.
2. Consejo de Investigaciones Científicas y Técnicas CONICET
3. Departamento de Ingeniería, Universidad Nacional del Sur
   Av. Alem 1253, (8000) Bahía Blanca, Argentina.

Abstract

Many structural systems are designed as thin-walled beams. Sometimes, these structures may suffer fatigue damage generated by dynamic loadings. It is important the early crack identification in order to avoid a catastrophic global failure. In this paper a methodology for identifying fatigue cracks in thin-walled beams, by measuring dynamic displacements, is developed. This approach is based on a theoretical model for the forced non-linear vibration of this kind of structures, taking into account the breathing effect of the cracks and the existence of initial stresses.

The identification problem consists in obtaining the depth and location of a fatigue crack in order to minimize the relative difference between theoretical and experimental displacement values.

Keywords: *fatigue damaged thin-walled beams, vibration based identification, breathing crack.*

INTRODUCTION

Early crack detection in engineering structures is a fundamental task in order to avoid the possibility of catastrophic failures.

In the last years, vibration-based techniques for damage identification have emerged as an attractive alternative to local inspection. They are based on the fact that damage induces a change in the structural stiffness distribution, thus modifying the dynamic response. Accordingly, by measuring vibration of the damaged structure under known loadings, it is possible to estimate the location and magnitude of the damage with the help of mathematical models as interpretive tools.

These techniques were extensively studied in relation to slender structures. For these ones, a beam model captures the most significant features of the structural dynamics.

Many studies were directed to the identification of cracks by measuring changes in natural frequencies or forced vibration amplitudes, assuming that the crack keeps open during the motion (Loya et al., 2006; Lee, J., 2009). However, a fatigue crack may present a breathing phenomenon when the structure is vibrating. In fact, the crack opens and closes in different moments of the vibration. Accordingly, the structural stiffness is modified in a non-linear form depending on the motion itself (Andreaus et al., 2007).

On the other hand, the existence of an initial stresses state may complicate the structural dynamics because it affects the moments of opening or closing of the cracks (Cortínez & Dominguez, 2014).

Some studies were directed to analyze the breathing crack effects in solid section beam modeled according to the Bernoulli-Euler or Timoshenko theories (Tsyfansky and Beresnich, 2000, Giannini et al., 2013). The case of thin-walled section beams has not been addressed in depth, perhaps due to their more complex dynamics presenting bending, twisting and axial vibrations in a coupled form (Cortínez & Piovan, 2002, Cortínez & Dotti, 2010).

Recently, the non-linear response of a thin-walled beam with a breathing crack was analyzed although initial stresses were not considered and the identification problem was not studied (Dotti et al., 2015).

This work deals with the identification of fatigue cracks in thin-walled beams considering the possibility of dynamic closing of the crack and also taking into account the existence of an initial stresses state.

The present approach is based on a theoretical model for describing the non-linear vibrations of the analyzed structures which constitutes a generalization of those formulations presented in Cortínez & Dotti, (2010) and Cortínez & Dominguez (2014).

The problem consists in obtaining the locations and depths of the fatigue cracks in order to minimize an objective function given by the quadratic relative difference between theoretical and experimental displacement values. The optimization is approached by means of the Finite Element Method (MEF) to evaluate the objective function and the Simulated Annealing technique (SA) to direct the search of the optimal parameters.

Some examples are given in order to show the effects of fatigue cracks and initial stresses on the forced vibration of thin-walled beams. It is shown that the proposed methodology allows identifying the crack parameters in a correct form.

1 STRUCTURAL MODEL

In this section, the equations governing the dynamics of the structural system shown in "Fig. 1" are presented. It is considered a non-homogeneous cross-section with an arbitrary shape (it is possible to analyze thin- or thick-walled sections) and composed of several materials. Considering the fact that the damaged zone is characterized by a different shape with respect to the intact cross-section, it is not possible to employ a global system of reference with principal axes. Accordingly, the theoretical model is developed using arbitrary reference axes.

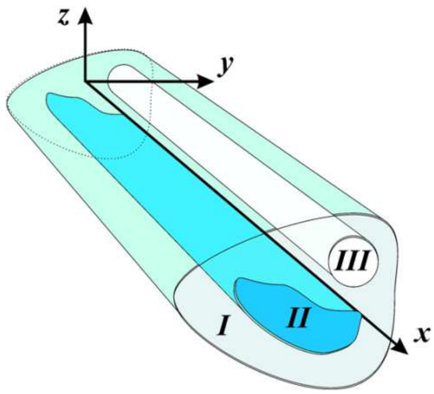

Figure 1.Analyzed Structural Element

1.1 Equations of motion

This theoretical model is based on the following assumption for the incremental displacements field:

$$
\begin{aligned}
u_x &= u(x,t) - \theta_z(x,t)y - \theta_y(x,t)z + \theta(x,t)\varphi(y,z) \\
u_y &= v(x,t) - z\phi(x,t) \qquad (1) \\
u_z &= w(x,t) + y\phi(x,t)
\end{aligned}
$$

where u_x represents the axial displacement in x- longitudinal direction, u_y and u_z transverse displacements, in y- and z-directions respectively, u, v and w constitute translational displacements of the cross-section, ϕ corresponds to twisting rotation, θ_y and θ_z are bending rotations, θ is a measure of the intensity of warping along the beam, and φ is the Saint-Venant torsional warping function defined as the solution of the following problem:

$$
\begin{aligned}
&\frac{\partial}{\partial y}\left(G\frac{\partial\varphi}{\partial y}\right) + \frac{\partial}{\partial z}\left(G\frac{\partial\varphi}{\partial z}\right) = 0 \qquad y,z \in \Omega \\
&\frac{\partial\varphi}{\partial n} = zl - ym \qquad y,z \in \Gamma
\end{aligned} \qquad (2)
$$

where Ω is the domain corresponding to the considered cross-section, Γ is the boundary of this domain, $l = \cos(n,y)$, $m = \cos(n,z)$, with n denoting the unit vector normal to Γ, and G is the transverse elasticity modulus.

The kinematic hypothesis (1) constitutes a generalization of that proposed by Cortínez & Piovan (2002) and is applicable to arbitrary cross-sectional shapes (including both cases, thin-walled or thick-walled sections).

From the displacement expressions (1), the following non-zero components of the Green-Lagrange strain tensor may be obtained:

$$\begin{aligned}
\varepsilon_x &= u' - \theta_z' y - \theta_y' z + \theta' \varphi \\
\gamma_{xy} &= \left(v' - \theta_z\right) - z\phi' + \theta\varphi_y \\
\gamma_{xz} &= \left(w' - \theta_y\right) + y\phi' + \theta\varphi_z \\
\eta_x &= \frac{1}{2}\left(v'^2 + z^2\phi'^2 - 2v' z\phi' + w'^2 + y^2\phi'^2 + 2w' y\phi'\right)
\end{aligned} \tag{3}$$

with the following notation $(\cdot)' = \partial(\cdot)/\partial x$. The fourth expression constitutes the non-linear longitudinal component.

Assuming an elastic material, the incremental stresses are obtained by using the Hooke’s Law:

$$\sigma_x = E\varepsilon_x \qquad \tau_{xy} = G\gamma_{xy} \qquad \tau_{xz} = G\gamma_{xz} \tag{4}$$

The Young modulus *E*, the transverse elasticity modulus *G* and the density ρ are supposed to vary with respect to *y* and *z*, making it possible to consider non-homogeneous cross-sections.

The dynamics of the analyzed structural system may be formulated by means of the Virtual Work Principle, including the contribution of initial stresses and inertial forces:

$$\int_V \left(\sigma_x \delta\varepsilon_x + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz}\right) dV + \int_V \sigma_0 \delta\eta_x dV - \sum_{i=x,y,z}\left(\int_V (f_i - \rho\ddot{u}_i)\delta u_i dV + \int_S p_i \delta u_i dS\right) = 0 \tag{5}$$

In (5) a dot over the variables denotes derivatives with respect to time, f_i and p_i are volume and surface forces, respectively, and σ_0 is the longitudinal initial stress. This last one is assumed to be in the following form:

$$\sigma_0 = \alpha + \beta y + \chi z \tag{6}$$

$$\begin{Bmatrix} \alpha \\ \beta \\ \chi \end{Bmatrix} = \begin{bmatrix} 1 & S_z & S_y \\ S_y & I_{yz} & I_y \\ S_z & I_z & I_{yz} \end{bmatrix}^{-1} \begin{Bmatrix} N_0 \\ M_{y0} \\ M_{z0} \end{Bmatrix} \tag{7}$$

where N_0 is the initial axial force, M_{y0} and M_{z0} are initial moments with respect to the y and z axes respectively.

Substituting expressions (1), (3), (4) and (6) into (5) and applying variational calculus, it is possible to arrive to the following equations of motion in terms of the beam stress resultants:

$$
\begin{aligned}
&-\frac{\partial N}{\partial x}+\rho_0\left(\ddot{u}A-\ddot{\theta}_z S_z-\ddot{\theta}_y S_y+\ddot{\theta}S_\varphi\right)=q_x\\
&\frac{\partial M_z}{\partial x}-Q_y+\rho_0\left(-\ddot{u}S_z+\ddot{\theta}_z I_z+\ddot{\theta}_y I_{yz}-\ddot{\theta}I_{\varphi y}\right)=-m_z\\
&\frac{\partial M_y}{\partial x}-Q_z+\rho_0\left(-\ddot{u}S_y+\ddot{\theta}_z I_{yz}+\ddot{\theta}_y I_y-\ddot{\theta}I_{\varphi z}\right)=-m_y\\
&-\frac{\partial\left(Q_y+P_v\right)}{\partial x}+\rho_0\left(\ddot{v}A-\ddot{\phi}S_y\right)=q_y\\
&-\frac{\partial\left(Q_z+P_w\right)}{\partial x}-+\rho_0\left(\ddot{w}A+\ddot{\phi}S_z\right)=q_z\\
&-\frac{\partial B}{\partial x}-T_W+\rho_0\left(\ddot{u}S_\varphi-\ddot{\theta}_z I_{\varphi y}-\ddot{\theta}_y I_{\varphi z}+\ddot{\theta}C_W\right)=b\\
&-\frac{\partial(T_{SV}+T_W+P_\phi)}{\partial x}+\rho_0\left(-\ddot{v}S_y+\ddot{\phi}I_y+\ddot{w}S_z+\ddot{\phi}I_z\right)=m_x
\end{aligned}
\tag{8}
$$

Along with the following end conditions:

$$
\begin{aligned}
&M_z=\tilde{M}_z && o && \delta\theta_z=0\\
&M_y=\tilde{M}_y && o && \delta\theta_y=0\\
&Q_z+P_w=\tilde{Q}_z && o && \delta w=0\\
&Q_y+P_v=\tilde{Q}_y && o && \delta v=0\\
&B=\tilde{B} && o && \delta\theta=0\\
&T_{SV}+T_W+P_\phi=\tilde{M}_T && o && \delta\phi=0
\end{aligned}
\tag{9}
$$

where:

$$
\begin{aligned}
P_v&=\alpha\left(v'\int_A dA-\phi'\int_A zdA\right)+\beta\left(v'\int_A ydA-\phi'\int_A yzdA\right)+\chi\left(v'\int_A zdA-\phi'\int_A z^2dA\right)\\
P_w&=\alpha\left(w'\int_A dA-\phi'\int_A ydA\right)+\beta\left(w'\int_A ydA-\phi'\int_A y^2dA\right)+\chi\left(w'\int_A zdA+\phi'\int_A zydA\right)
\end{aligned}
\tag{10}
$$

$$P_\phi = \alpha\left(-v'\int_A zdA + w'\int_A ydA + \phi'\int_A \left(y^2+z^2\right)dA\right) + \beta\left(w'\int_A y^2 dA + \phi'\int_A y\left(y^2+z^2\right)dA\right) +$$
$$+\chi\left(\phi'\int_A z\left(y^2+z^2\right)dA - v'\int_A z^2 dA + w'\int_A zydA\right)$$

In the above expressions, $q_x(x,t),\ q_y(x,t),\ q_z(x,t)$ are distributed incremental forces per unit length, $m_x(x,t),\ m_y(x,t),\ m_z(x,t)$ are incremental external moments per unit length and b is the incremental external bimoment per unit length. These forces include viscous damping. ρ_0 corresponds to the density of a reference material considering the fact that the section may be non-homogeneous. $\tilde{N}, \tilde{M}_y, \tilde{M}_z, \tilde{Q}_y, \tilde{Q}_z, \tilde{B}$ and $\tilde{M}_T$ are incremental external sectional stress resultants applied at the ends of the beam.

The incremental beam stress resultants are expressed in the following form:

$$\begin{aligned}
N &= \overline{EA}u' - \overline{ES_z}\theta_z' - \overline{ES_y}\theta_y' + \overline{ES_\varphi}\theta' \\
M_y &= \overline{ES_y}u' - \overline{EI_{yz}}\theta_z' - \overline{EI_y}\theta_y' + \overline{EI_{\varphi z}}\theta' \\
M_z &= \overline{ES_z}u' - \overline{EI_z}\theta_z' - \overline{EI_{yz}}\theta_y' + \overline{EI_{\varphi y}}\theta' \\
B &= \overline{ES_\varphi}u' - \overline{EI_{\varphi y}}\theta_z' - \overline{EI_{\varphi z}}\theta_y' + \overline{EC_W}\theta' \\
Q_y &= \overline{GA}\left(v' - \theta_z\right) - \overline{GS_y}\left(\phi' - \theta\right) \\
Q_z &= \overline{GA}\left(w' - \theta_y\right) + \overline{GS_z}\left(\phi' - \theta\right) \\
M_T &= T_{SV} + T_W \\
T_{SV} &= \overline{GJ}\phi' \\
T_W &= \overline{GS_z}\left(w' - \theta_y\right) - \overline{GS_y}\left(v' - \theta_z\right) + \overline{GD}\left(\phi' - \theta\right)
\end{aligned} \tag{11}$$

The sectional stiffness coefficients are determined as:

$$A = \int_A \frac{\rho}{\rho_0} dA,\quad S_z = \int_A \frac{\rho}{\rho_0} ydA,\quad S_y = \int_A \frac{\rho}{\rho_0} zdA,\quad S_\varphi = \int_A \frac{\rho}{\rho_0} \varphi dA,\quad C_W = \int_A \frac{\rho}{\rho_0} \varphi^2 dA$$
$$I_z = \int_A \frac{\rho}{\rho_0} y^2 dA,\quad I_y = \int_A \frac{\rho}{\rho_0} z^2 dA,\quad I_{yz} = \int_A \frac{\rho}{\rho_0} yzdA,\quad I_{\varphi z} = \int_A \frac{\rho}{\rho_0} \varphi zdA,\quad I_{\varphi y} = \int_A \frac{\rho}{\rho_0} \varphi ydA$$
$$D = \int_A \frac{\rho}{\rho_0}\left(\varphi_y^2 + \varphi_z^2\right)dA,\qquad J = \int_A \frac{\rho}{\rho_0}\left(y^2+z^2\right)dA - D \tag{12}$$

$$\overline{EA}=\int_A EdA,\quad \overline{ES_z}=\int_A EydA,\quad \overline{ES_y}=\int_A EzdA,\quad \overline{ES_\varphi}=\int_A E\varphi dA,\quad \overline{EC_W}=\int_A E\varphi^2 dA,$$

$$\overline{EI_z}=\int_A Ey^2dA,\quad \overline{EI_y}=\int_A Ez^2dA,\quad \overline{EI_{yz}}=\int_A EyzdA,\quad \overline{EI_{\varphi y}}=\int_A E\varphi ydA,\quad \overline{EI_{\varphi z}}=\int_A E\varphi zdA,$$

$$\overline{GA}=\int_A GdA,\ \overline{GS_y}=\int_A GzdA,\ \overline{GS_z}=\int_A GydA,\ \overline{GD}=\int_A G\left(\varphi_y^2+\varphi_z^2\right)dA,\ \overline{GJ}=\int_A G\left(y^2+z^2\right)dA-D$$

Expression (11) can be written in matrix form as:

$$\mathbf{F}=\mathbf{R}\,\mathbf{d} \tag{13}$$

where **F** corresponds to the vector of stress resultants, **d** is the vector of generalized strains and **R** is the matrix of sectional stiffness of the beam.

It should be observed that the present model takes into account shear deformation effects and rotary and warping inertias, and may be considered as an extension of the Timoshenko´s theory for three dimensional motion of beams (Cortínez & Piovan, 2002).

1.2 Fatigue model

It is considered a fatigue crack in the form shown in "Fig." 2 where x_f and a_f denote the longitudinal location and the depth, respectively. In order to formulate a simplified crack model, it is assumed that it can be approximately represented by means of a notch of depth a_f^* having a small length with respect to the total length of the beam. The equivalent depth a_f^* may be determined as a function of the real depth a_f comparing the natural frequencies given by the present model (with depth a_f^*) against a 3D model having a fatigue crack of depth a_f . In this way a_f^* is selected as the value that minimizes the difference among both results. This kind of calculation should be performed for several values of a_f. By applying this numerical procedure it was found that for small values of a_f^* with respect to the cross section dimensions, the equivalent depth a_f^* takes approximately the same value than the real crack depth a_f . Accordingly, in this work, it is assumed, as a simplification, that a_f^* and a_f coincide. An alternative methodology to find a_f^* is given in references Cortínez & Dotti (2013) and Dotti et al. (2015).

In this way, the structural flexibility due to the open crack may be appropriately modeled as a equivalent notch. However, the fatigue crack may suffer breathing during the motion. In order to consider this fact, the total normal stress at the end of the crack may be calculated for the intact section. If this value indicates compression, the crack is closed and then the cross-sectional properties correspond to that of the intact section, but if the total stress indicates traction, the crack is open and then the cross-sectional properties correspond to the damaged section shown in "Fig. 2".

Accordingly, in the damaged element (notch) the constitutive relationships for the beam stress resultants are given by equation (13) with the matrix of the cross-sectional stiffnesses expressed in the following form:

$$R=\begin{cases} R_U & si\ \ \left(\bar{\sigma}_x+\bar{\sigma}_0\right)\le 0 \\ R_D & si\ \ \left(\bar{\sigma}_x+\bar{\sigma}_0\right)>0 \end{cases} \tag{14}$$

where

$$\bar{\sigma}_x = E\left(\left.\frac{\partial u}{\partial x}\right|_{x=xf} - \left(b-b_1\right)\left.\frac{\partial \theta_z}{\partial x}\right|_{x=xf} - \left(h-h_1-\frac{e}{2}\right)\left.\frac{\partial \theta_y}{\partial x}\right|_{x=xf} + \varphi\left(b-b_1, h-h_1-\frac{e}{2}\right)\left.\frac{\partial \theta}{\partial x}\right|_{x=xf} \right)$$
$$\bar{\sigma}_0 = \alpha + \beta\left(b-b_1\right) + \chi\left(h-h_1-\frac{e}{2}\right) \quad (15)$$

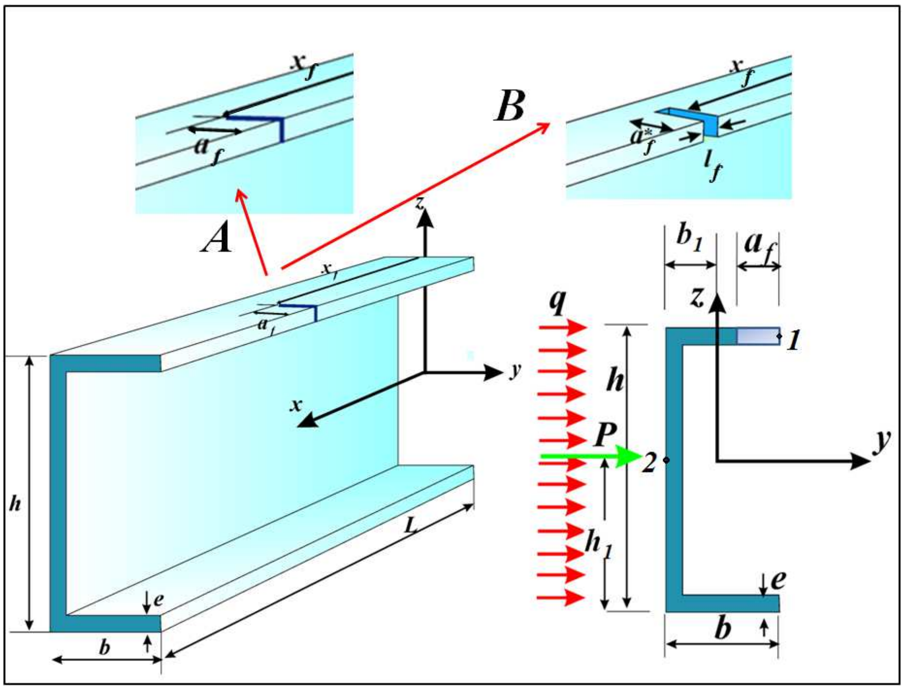

Figure 2. Damaged beam under study

2 DAMAGE IDENTIFICATION PROBLEM

The identification based on the modification in the dynamic response may be carried out by minimizing the difference, using an appropriate norm, between measured response magnitudes and those theoretically calculated with the present model. Such an estimation is performed for different feasible values of x_f and a_f .

In this work, the measurement of forced displacements under a known harmonic applied load is considered. Accordingly, the identification problem is formulated in the following form:

$$FO = \sum_{m=1}^{M}\left(\frac{\tilde{v}_m - v_m}{\tilde{v}_m}\right)^2 \quad (16)$$

s.t.

$$0 \le x_f \le x_f^{max}$$
$$a_f^{min} \le a_f \le a_f^{max} \qquad (17)$$

where $\tilde{v}_m$ are the maximum and minimum measured displacements, v_m are the calculated ones and M are the number of control points.

3 NUMERICAL MODEL

In order to solve the formulated problem, it is necessary to analyze different damage configurations given by the values adopted for a_f and x_f. In order to solve the forced vibration problem, equations (8-15) are employed. It should be noted that this problem is non-linear due to the breathing effect described by (14). The problem is solved by means of the Finite element method employing the FlexPDE software (2014).

The calculation has three stages. First, a sectional analysis has to be performed for the intact and the damaged cross-section, involving the solution of equation (2) and then the determination of the inertia and stiffness sectional coefficients given by expressions (12). As a second step, the initial stresses should be determined from the static version of equations (8-11)

These values (sectional properties and initial stresses) are used for solving the non-linear one-dimensional problem governed by the system (8-11) along with (14-15) (third stage).

A numerical study carried out by the authors has demonstrated that the virtual work of the initial stress gives practically the same value for both the intact and the damaged beam. Accordingly, the corresponding contribution is determined taking the intact cross section. This means that the crack only affects the linear stiffness.

Each crack configuration is evaluated according to the corresponding objective function (16). The search of its minimum value is performed by using an optimization technique known as Simulated Annealing (Cortínez & Dominguez, 2014).

The numerical model is implemented in the software Matlab (2010). From this last one FlexPDE (2014) is invoked.

4 NUMERICAL RESULTS

4.1 Validation of the proposed model

In order to validate the equations developed in section 1 and the corresponding numerical solution, the first five natural frequencies of vibration of a clamped-free beam were determined whose cross-sectional shape is shown in "Fig.2". The calculations were performed for both, the intact beam and the open cracked beam (without considering breathing effect). The features of the considered beam are the following: L=1m, h=0.1m, b=0.04m, e=0.006m, h_1=0.05m, b_1=0.011095m (centroid of the intact section), density $\rho = 2650$ kg/m^3, elasticity modulus $E = 4.5\text{x}10^{10}$ Pa and Poisson coefficient $\upsilon = 0.25$. For the damaged beam, the crack location and depth are x_f = 0.1m and a_f = 0.02m, respectively. The length of the 1D damaged element was assumed to be l_f=0.02m.

The results determined by means of the explained procedure are compared with a 3D finite element model developed in the program COMSOL (2013). Table “1” shows the corresponding natural frequencies. As it can be seen the maximum error is of the order of 1% for both, the intact and the cracked beam. This means that the 1D model captures appropriately the additional flexibility due to the crack.

Table 1: Natural frequencies of a Clamped-Free thin-walled beam with and without crack

	Intact			Damaged x_f = 0.1m a_f = 0.02m		
Frequency	3D FEM	Present approach	% error	3D FEM	Present approach	% error
f_1	26.95	26.98	-0.10	24.53	24.79	-1.06
f_2	59.49	59.50	-0.02	59.39	59.45	-0.10
f_3	104.71	105.05	-0.33	98.66	99.21	-0.55
f_4	165.62	167.94	-1.40	161.71	164.11	-1.49
f_5	226.25	228.01	-0.78	225.37	227.28	-0.85

4.2 Non-linear forced vibrations of thin-walled beam with and without crack

Forced displacements produced by a dynamic loading on a clamped-clamped beam, with the cross section shown in “Fig. 2”, were analyzed. The considered initial static load is q_y=1000N/m and the incremental dynamic load corresponds to a concentrated force at the middle point of the length P=1000*cos(ωt) N. The excitation frequency is ω=0.7ω_1 being ω_1 the fundamental frequency of the intact beam.

Figure “3” shows the forced y-displacement v corresponding to the point 2 indicated in “Fig.2”. Values are given for different crack locations and for the intact beam. A crack depth a_f =0.034m was adopted. It is possible to note the important differences in the displacement values with respect to the intact beam. This discrepancy is even higher when the crack is located in the fixed end, as expected.

In “Fig. 4” y-displacements along the beam are shown for different locations and depths of cracks. It is possible to observe that the different crack parameters cause appreciable differences in the dynamic response. This means that the identification of damage using measurements of displacements may be plausible for identifying crack parameters.

Figure “5 shows the time history of the y-displacement v corresponding to the cross section located at 0.2m from the clamped end (point 2 shown in “Fig. 2”). The crack parameters are a_f = 0.02m and x_f = 0.1m. The results corresponding to the cracked beam are compared with the corresponding displacements of the intact beam and also with values determined by neglecting the breathing effect, in fact, keeping the crack open. One may observe that the displacements corresponding to the cracked beam are appreciably higher than those corresponding to the intact beam. On the other hand, the displacements obtained without considering the breathing crack are erroneously higher. This means that the neglect of the breathing effect may cause a mistake in the identification process.

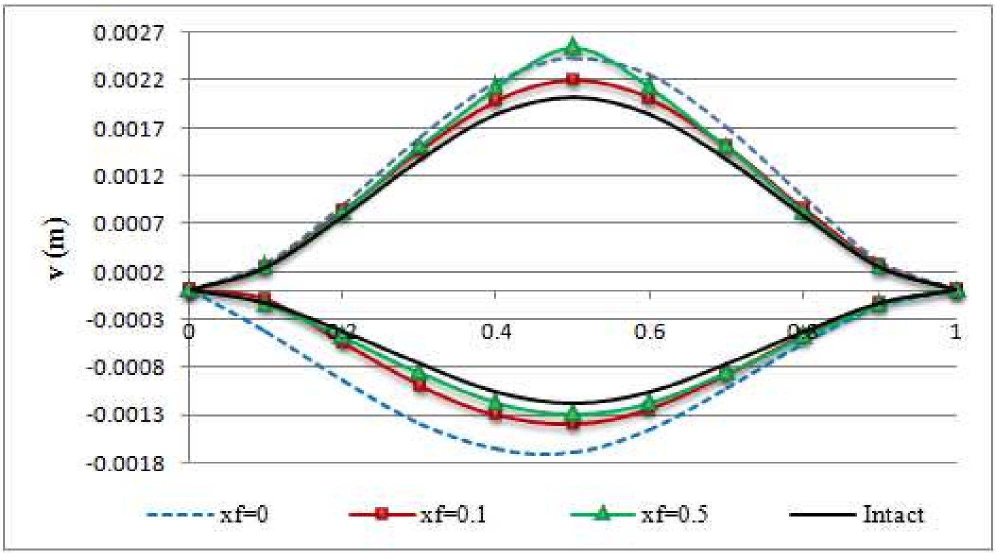

Figure 3. y-displacements corresponding to the intact and damaged beam for different locations of crack and depth a_f=0.034m

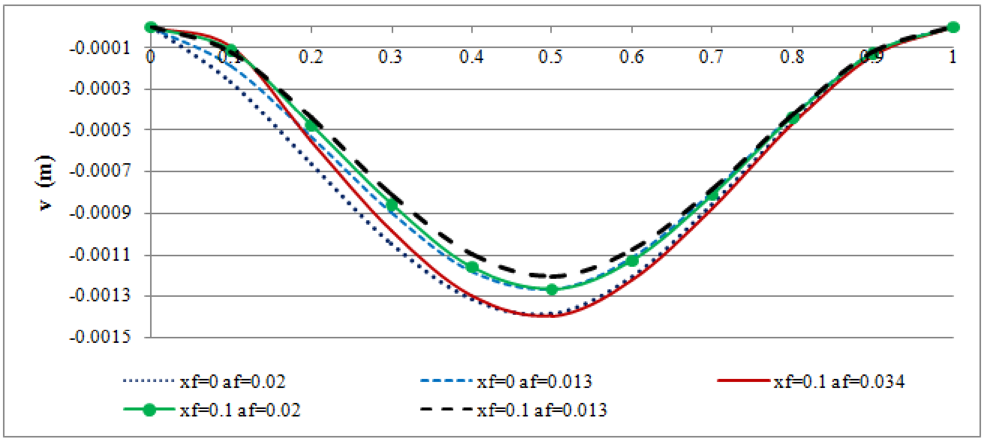

Figure 4. y-displacements corresponding to different locations and depths of cracks

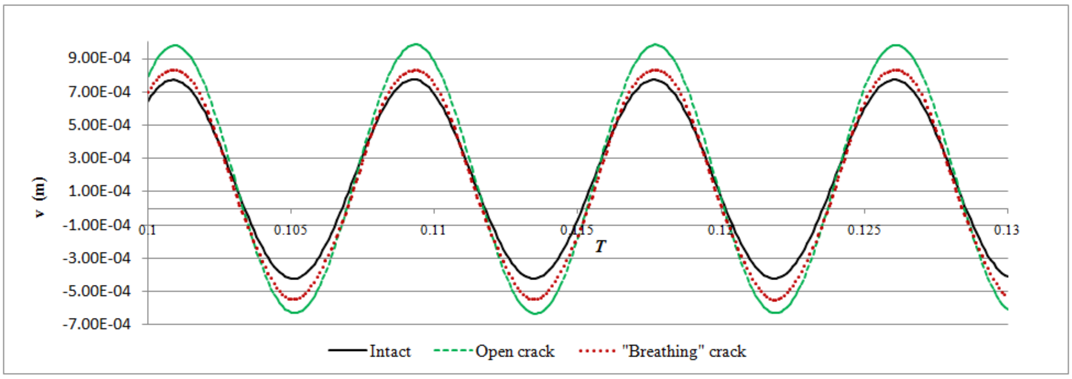

Figure 5. Time history for y-displacements

In "Fig. 6" the maximum displacements along the beam are shown for different initial loadings. The considered pre-loadings are q_y = 10^4 N/m and q_y = -10^4 N/m, and the dynamic load is P_y = 1000*cos(ωt). The crack parameters are x_f = 0.5m and a_f = 0.02m. As it can be appreciated, when the initial loading is in the positive y-direction the differences between damaged and intact beam are notable, but when the initial loading is in the negative y-direction, there is no difference between displacements corresponding to both the intact and the damaged beam. This last case occurs because the initial stress does not allow the opening of the crack. This fact should be taken into account in the design stage of the identification procedure. In some cases, it will be necessary to apply initial static loadings in order to open the investigated crack.

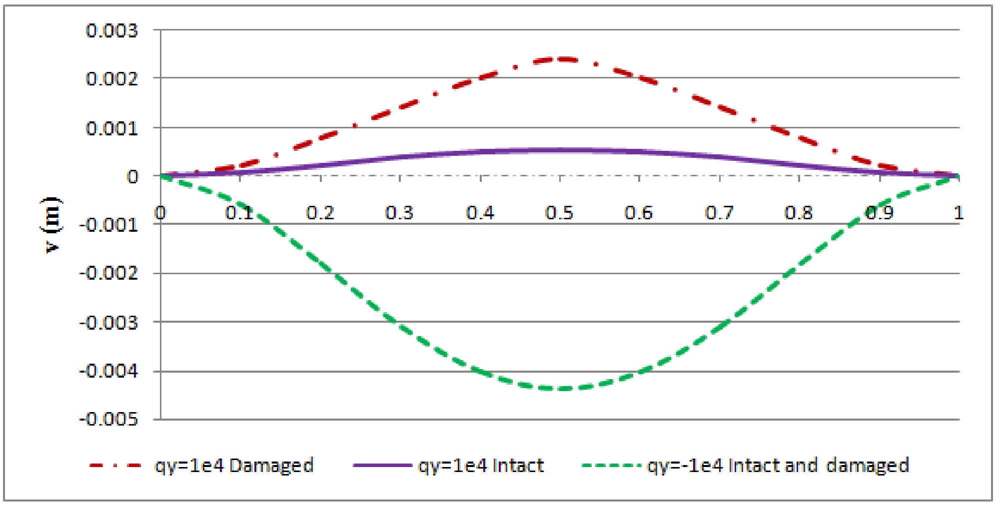

Figure 6. Maximum displacements along the beam for different initial loadings

4.3 Identification using forced displacements

In order to analyze the identification problem, the following damage parameters are selected: a_f = 0.02m and x_f = 0.1m. Nine points of measure for the forced displacements have been used. The experimental values were simulated by means of the theoretical model and adding stochastic errors of ±5%. Applying the explained procedure, the following damage parameters were obtained: $\bar{a}_f$ = 0.018m and $\bar{x}_f$ = 0.1m. These values present errors ($Error = 100\left|\left(a_f - \tilde{a}_f\right)/b\right|$) of the 5% with respect to the width of the flange, for the estimated depth, while the location is determined without error in this case.

CONCLUSIONS

A numerical procedure for identifying fatigue cracks in thin-walled beams by measuring forced displacements was developed. The objective was to determine the location and depth of the crack by minimizing the differences between calculated and measured values of forced displacements. The theoretical model takes into account the breathing effect of the crack and the existence of the initial stresses.

It was shown that the numerical solution of the model compare favorably with 3D FEM results.

The numerical studies demonstrate that the considered effects are very important for describing the dynamics of the beam. In fact, if the calculations do not consider the dynamic closure effect of the crack, the values of the corresponding displacements are inaccurate. Therefore, in order to identify in an correct way the crack parameters, the consideration of the breathing crack effect is essential. On the other hand, the initial stress may prevent in some cases the opening of the crack. For this reason it is necessary to consider the initial stress effect to design conveniently the identification procedure.

An example was given respect to the crack identification which shows the efficiency of the proposed methodology.

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