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NUMERICAL MODELING OF THE MECHANICAL BEHAVIOR OF STEEL FIBER REINFORCED CONCRETE PIPES

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Abstract. *The use of steel fiber reinforced concrete (SFRC) as a reinforcing material in concrete pipes used for water drainage and sewerage in partial or total substitution of steel meshes and bars may have a positive impact in product optimization inside the precast industry from a technical and economical point of view. The main effect of the fiber addition is the control of cracking processes, resulting in significant increases in composite toughness, as well as additional benefits related to its strength. In this paper, as part of a initial stage of an on course investigation, the mechanical behavior of SFRC pipes is assessed by means of a simple numerical model which the SFRC is modeled as a homogeneous composite (macro scale). For numerical simulation working within continuum mechanics and a damage-plasticity model is used. The equations are solved with the Finite Elements Model and the results are compared with experimental data obtained by other authors.*

Keywords: *Numerical modeling, Steel fiber reinforced concrete, concrete pipes, non lineal model*

1 INTRODUCTION

The use of fibers as reinforcement dates since ancient times, when horsehair and thatch were used for the manufacturing of mortar and bricks respectively. Later there appear, different types of industrialized fibers appear, like carbon, aramid, glass and polymeric fibers among others. Within them, steel fibers are the most popular fiber used as concrete reinforcement. Initially they were used for nonstructural purposes namely for the prevention and control of shrinkage. Some further research reveals that fiber addition significantly increases flexural stiffness and energy absorption capacity of the concrete, hence reducing the cracking and resulting in a durability improvement. (Altun et al, 2006)

SFRC is a relatively emerging material in the market, which has come up with the purpose of offering a technical and economically sustainable alternative to the plain concrete.Like conventional concretes, it is made up of a hydraulic binder, portland cement in most cases, aggregates with different granulometry, water and steel fibers. These discrete fibers randomly distributed in the concrete mass replace the strengthening function provided by steel bars.

Despite that fact that only twenty years have passed sice its introduction, the SFRC has numerous applications among which the precast products are found, for instance concrete pipes . However, though several experimental campaigns has been carried out and published in the scientific literature, the generalized use of SFRC has not been put in practice yet due to several reasons. (De la Fuente et. al., 2010).In addition, the absence of suitable numerical models that reflects the failure mechanism of SFRC and quantifies the contribution of the fibers, represents another restriction for the consolidation of SFRC as a competent alternative in the current market.

Is for that reason that a non-lineal damage plasticity model capable of recreates the failure process taking into account irreversible inelastic processes associated to material instability is exposed in this work.

2 MECHANICAL PROPERTIES OF SFRC

The most important effect in the mechanical behavior of the concrete, because of the presence of fibers, occurs in the post-cracking tension resistance. This also affects many others mechanical properties as the reinforcement adherence, shear resistance, fatigue, etc, to name a few. (Massicote, 2001) It is worth mentioning that, the fibers slightly stiffen the response at pre-crack stage respect to common concrete and substancially provide a residual resistance capacity in the post-crack stage. The latter is due to the sewing effect that the fibers achieved between the crack lips. See Figure 1. (Mármol Salazar, 2010)

In fact, adding a 1.5% in volume of fibers to concretes or mortars, increases of between 30-40% in tensile resistance are obtained (ACI 544.1R-96, 2009). With respect to flexural resistance, the addition of steel fibers considerably intensifies it, even more than the one experiencied in tension, because of the ductile behavior of SFRC in the cracked tensile zone, which is produced by residuals resistance. (Hannant, 1978)

In general, the addition of fibers to the concrete, does not lead to any significant increase in the compressive resistance of the concrete. (Yazici et al, 2007).

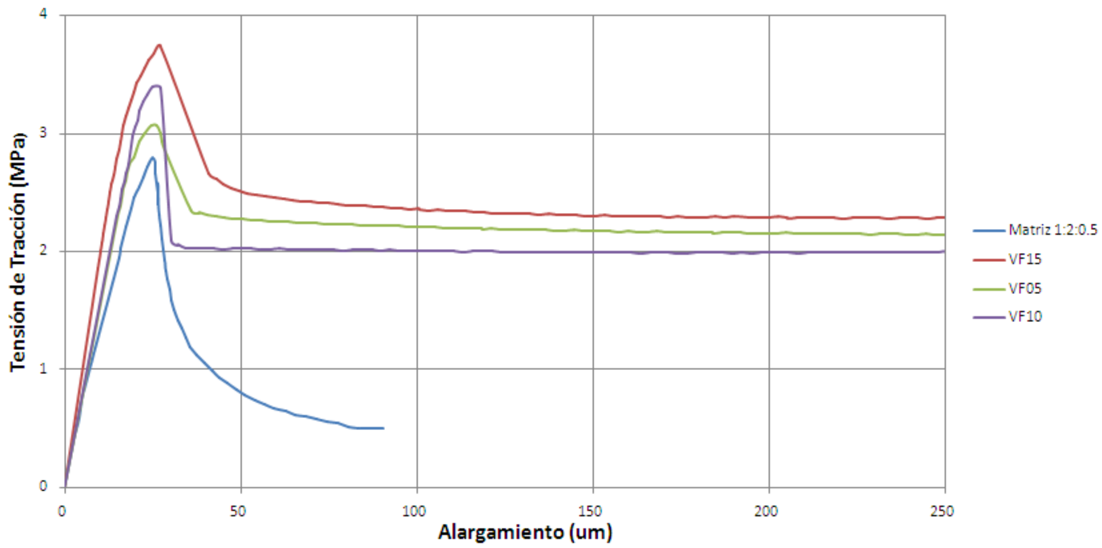

Figure 1: Tension-stretching curves in concretes in function of volume of fibers. (ACI 544.1R-96, 2009)

3 USE OF SFRC IN CONCRETE PIPES

Concrete pipes are the most efficient, economic and ecological system for sanitary systems networks. Among its main advantages as structural element, its good response to dynamic and impact loads, proper flexibility to eventual ground displacements through the use of expansion joints and a minimal maintenance after being put into service should be mentioned

The use of concrete pipes between 400 mm and 1200 mm of diameter is the most widespread option in urban drainage. In fact, some concrete aqueducts built ,by the Romans, 2000 years ago are still in use, which demonstrates that the concrete based drainage infrastructure gather suitable durability with a good structural efficiency. (Peyvandi et al, 2013)

The use of SFRC for the manufacturing of concrete pipes involves a series of improvements related to the performance of this composite which have been described above, but more specifically, it should be pointed out that manufacturing the steel reinforcement cage for the pipe requires a special bending and the use of specific machinery for its collocation which is expensive and takes time. Thus, the use of fibers as the main reinforcement in place of those cages could overcome this disadvantage without affecting the overall quality of the product. (Mohamed et al, 2014)

4 MODELING COMPOSITES AND MODELING SCALES

Concretes, like others cementitious materials, are multiphase materials commonly named in the literature as “composites”. Each component of the composite has its constitutive law that conditions the behavior of the material depending on its volumetric proportion and the morphological distribution. (Molina et al, 2009)

In a macro level of observation, they can be consided as an homogeneous continuous material, while at lower levels of observation, they must be considerate as multiphase materials.

There are several constitutive models related to SFRC currently available in the literature which are classified according to the observation scale considered:

- Macroscale models: the materials with a cementitious base can be idealized considering them as a continuum, and modelled in a theoretical frame that includes the classical formulations for the continuum e.g. localized deformations and distributed cracks.
- Mesoscale models: a better understanding of the failure mechanism of SFRC subjected to external loads can be obtained considering the mesoscalar behavior of the material. The meso scale of the concrete can be idealized considering the different phases of the composite. Thus, the interaction between those different phases (i.e. fibers, matrix, coarse aggregate and its interfaces) are explicitly considered.

  The mesoscale models look upon the phenomenon represented by the fiber-matrix bond, where each fiber is modeled as a discrete entity. These approaches provide a much more powerful description of the material behavior, modeling the cracking processes with special accuracy. The apparent macroscopic behavior observed is a direct consequence of a more complex mesoscopic phenomenon that takes place at the level of the material heterogeneity.

  The main disadvantages of these approaches are related on one hand, to the elevated computational cost and on the other hand to the great number of variables involved in the formulation of the model.
- Microscale models: they are based on a level of observation where the cementitious paste is described in terms of its chemical constituents. The typical models aim to model the microstructure of the composite where the cementitious paste is represented as a "net" of cement particles. The latter reacts together with a variety of components leading to obtain hydrated products. The numerical modeling of this kind of hardened composites requires a wide scale of mathematical formulations and numerical methods related to hydration/dehydration concepts that happen at the microstructure level (van Breugel, 1991).

5 CONSTITUTIVE MODELS OF SFRC

Unlike what occurs in compression, the tensile behavior of SFRC is quite different from the common concrete. For that reason, one of the most important challenges is to develop a consti- tutive equation model that allows a proper characterization of the tensile behavior of this material.

SFRC should be understood as a concrete that includes discrete and short fibers in its composition, randomly distributed in its mass, which gives to the material a marked anisotropy and a non lineal behavior. Once the concrete has cracked, the loss of adherence and fiber pull-out dissipate a great amount of energy, which leads to an important increase in the toughness (Barros and Figueiras, 1999).

The SFRC should be considered on one side in the elastic range of deformations and, on the other side, its behavior beyond the elastic limit. The SFRC structures should be calculated using a plastic analysis that allows taking advantage of the greater stress redistribution capacity provided by the contribution of fibers. However, it is not known whether a constitutive model related to the tensile behavior of the SFRC that can be considered better than others exists..(Alvarez, et al 2010).

Additionally, the behaviour of SFRC can be obtained by homogenization techniques based on the properties of constituents (concrete and fibers) together with the shape, volume proportion, orientation and distribution of the fibers or, alternatively, they can be explicitly modeled using different types of discrete elements. The main disadvantage of this approach is the computational cost refered to capacity and time of calculation.

5.1 Composite material model

The SFRC can be considered as a composite material with a concrete matrix and short fibers with different orientations dispersed in the volume. A simple way for modeling the behaviour of composite materials is the ”mixtures theory”. The mixtures theory for composite materials enables consideration of the simultaneous behaviour of all the phases of a composite and, from the combination of them, to obtain the behaviour of the whole. Each one of the basic substances can have its own constitutive law, isotropic or anisotropic.

In the case of SFRC, it is considered as a composite material formed by a concrete matrix and fibers oriented in 3 directions (x, y, z). The interface fiber/matrix is not explicitly considered. It Is known that the slipping of the fibers in relation to the matrix plays an important role in the behaviour of the SFRC. To take into account this slipping without modeling the interface, the model of the fibers is modified (Ruano Sandoval, 2010).

5.1.1 Concrete model

Experimentally, it has been observed that the collapse mechanism in frictional materials like concrete is preceded by a non lineal dissipative process characterized by the concentration of deformations in strips with very little thickness compared with the global dimensions of the structure.Irreversible processes of degradation and dissipation of energy (damage- plasticity) are concentrated in them, while the rest of the solid undergoes elastic unloading. The occurrence of this strain localization mechanism implies directly or indirectly the beginning of the material failure process, hence the importance of developing numerical models capable of simulating the phenomenon efficiently. The strips with highly localized strains can become apparent under different forms among which cracking processes in structures built of mortar are found. (Sanchez, 2006)

A damage-plasticity constitutive model for the modeling of SFRC allows taking the qualities of two failure mechanisms: by the side of the plasticity mechanism, it allows store inelastic strains (permanent) associated to a load-unload process of the material, and by the side of dam- age mechanism it enables to generate a degradation of the elastic constants based on the evolution law of the damage variable. (Caicedo Silva, 2010

For concrete, a modified damage-plasticity model is used (Luccioni, Rougier, 2005). By one hand, the plastic behavior is obtained as a generalization of the classic plasticity theory. As properties, the elastic properties of concrete base are used, changing only the tensile hardening curve to define the plastic threshold. The limit of the elastic behavior is defined by a yield function:

$$F^p(\sigma_{ij};\kappa^p) = f^p(\sigma_{ij} - K^p(\sigma_{ij};\alpha^k) \leq 0 \qquad (1)$$

where $f^p(\sigma_{ij})$is the equivalent stress defined in the stresses space that could take the form of any of the yield functions of the classic plasticity (Tresca, Von Mises, Mohr Coulomb, Drucker Prager, etc.).

K$(\sigma_{ij};\alpha_k)$ is the yield threshold and κ^p eis the plastic damage variable or isotropic hardening variable.

The following evolution rule for the plastic strains is defined:

$$\dot{\epsilon}^p_{ij} = \dot{\lambda}\frac{\partial G(\sigma_{mn};\kappa^p)}{\partial \sigma_{ij}}. \tag{2}$$

Where $\dot{\lambda}$ is the plastic consistency factor and G is the potential function.

The plastic hardening variable κ^p is obtained normalizing the dissipated energy in the plastic process to the unity and its varies between 0 for the virgin material and 1 when has been plastically dissipated all the energy that the material is capable to dissipate in this form.

The following evolution rule is used for the equivalent yield threshold:

$$K^p(\sigma_{ij};\kappa^p) = rR^{op}\sigma_t(\kappa^p) + (1-r)\sigma_c(\kappa^p) \tag{3}$$

Where $\sigma_t(\kappa^p)$ y $\sigma_c(\kappa^p)$ represents the evolution of the yield threshold in tension and compression uniaxial tests respectively, while R^{op} is the ratio between the yield thresholds in unixaial compression and unixial tension. The load/unload conditions are derivate from the Kuhn-Tucker relationships formulated for problems with unilateral constrains.

$$\dot{\lambda} \geq 0, \quad F^p \leq 0, \quad \dot{\lambda}F^p = 0. \tag{4}$$

Considering now the damage process, and in analogous manner to the limit of plastic behavior, a limit or threshold damage is defined, which is described by a function with the following form:

$$F^d = f^d(\sigma_{ij} - K^d(\sigma_{ij});\kappa^d) \leq 0 \tag{5}$$

Where, similarly to that seen in the plastic process $f^d(\sigma_{ij})$ is the equivalent stress defined in the stresses space, $K^d(\sigma_{ij},\kappa^d)$ is the equivalent threshold damage and κ^d is the hardening damage variable.

The equivalent stress $f^d(\sigma_{ij})$ could be assessed using known functions of the plasticity theory as already exposed (Tresca, Von-Mises, Mohr-Coulomb or Drucker-Prager) or any other function especially developed for damage. The hardening damage variable varies between 0 for the virgin material to 1 for the material completely damaged. Is obtained normalizing the dissipated energy by damage to the unity. The following equation for the equivalent damage threshold is proposed:

$$K^d(\sigma_{ij};\kappa^d) = r\sigma_t(\kappa^p) + (1-r)\sigma_c(\kappa^d) \tag{6}$$

where $\sigma_t(\kappa^d) y\ \sigma_c(\kappa^d)$ represents the evolution of the thresholds damage in uniaxial tension and compression respectively.

The load/unload conditions are derivate from the Kuhn-Tucker relationships and are analogous to the corresponding to the plastic process.

$$d \geq 0 \quad F^d \leq 0 \quad dF^d = 0 \tag{7}$$

The scalar damage variable d varies from 0 to d_c, that is $0 \leq d \leq d_c$ where $0 \leq d_c \leq 1$ is the level of damage correspondent to the material failure.

The evolution of the plastic strains and damage is obtained from the simultaneous solution of the following equations named problem consistency conditions:

$$\begin{cases} F^p = 0 \\ F^d = 0 \end{cases} \tag{8}$$

These consistency equations are two lineal equations in λ and δ that can be easily solved.

5.1.2 Fibers model

The classical mixtures theory suppose that all the components have the same deformation. This works relatively good in composite materials with long fibers in which the direction of the load is the same as the direction of the fibers. However, according to whether the fibers have less slenderness, the effect on the ends becomes progressively more signficant

A way to simulate the behaviour of SFRC using the mixtures theory is modifying the constitutive equations of the fibers (Luccioni & López, 2002); (Luccioni, et al., 2005); (Isla Calderón & Luccioni, 2009). Taking into account that in this case the fibers slip before the fluence, the conventional isotropic elastoplastic model of the steel is modified reducing the yield stress in the axial direction of the fibers. Thus, the plasticity threshold in this direction in fact represents the slipping threshold of the fibers in relation to the concrete.

So, for the steel fibers a elastoplastic model for orthotropic materials that take into account not only the elastic orthotropy own of geometry of the fiber that only contributes to resist stresses in the axial direction, but also the orthotropy in the thresholds of the inelastic behaviour, smaller to the yield stress, in the axial direction of the fibers can be used.

To consider the orthotropy, the model starts from the hypothesis that there exist two spaces (Luccioni, et al., 1996): a) a real anisotropic space and b) a fictitious isotropic space. The problem is solved in the fictitious isotropic space which allows to use elastoplastic and damage models developed for isotropic materials.

5.2 Equivalent homogeneous model

The equivalent homogeneous material model is a concrete model with identical elastic and no lineal compression properties that the concrete that forms the SFRC but with a hardening/softening modified curve in tension due to the contribution of the fibers.The curve that represents the tensile behavior of the SFRC can be obtained by tensile tests, indirectly by flexural tests or alternately by numerical methods. For it to take place, we resort to a "plastic damage model" whose basics allows to simulate the multiaxial behavior of the concrete considering the phenomenon of stiffness degradation produced during plastic and elastic quasi-static load processes since the beginning of cracking process. (Oller et. al, 1988)

As mentioned above and highlighting the fact that this work is part of an ongoing research, a first approach for the simulation of SFRC is done considering it as a homogeneous material, which among other things reduces its computational cost and the amount of variables involved in the formulation of the model.

6 NUMERICAL SIMULATION

The mechanical behaviour and the structural performance of the pipes were analyzed by the three edge bearing test according to standard ASTM C497. The set up of the elements for the execution of the test , according to the mentioned standard, is shown in Figure 2.

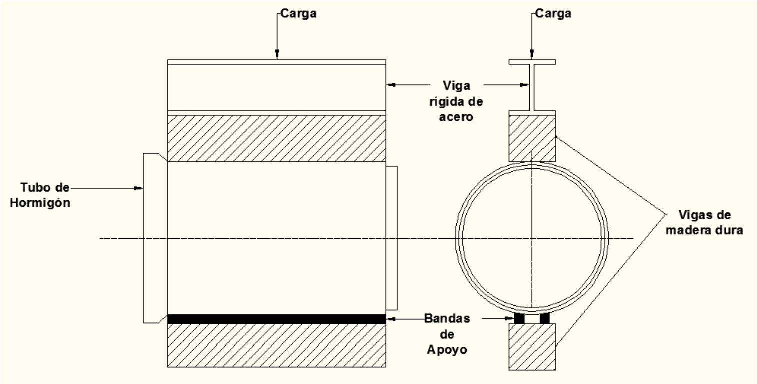

Figure 2: Set up of Three Edge Bearing Test according to ASTM C497

The adopted dimensions are the following: Internal diameter: 300mm, wall thickness: 69mm (type C according standard ASTM C76).

A macromodel for the SFRC was used, where the material was represented as a equivalent homogeneous material. This model is implemented in a non-lineal finite elements code.

Triangular finite elements with three nodes and six degrees of freedom in plain strain state were used. In the Figure 3, the mesh adopted is depicted.

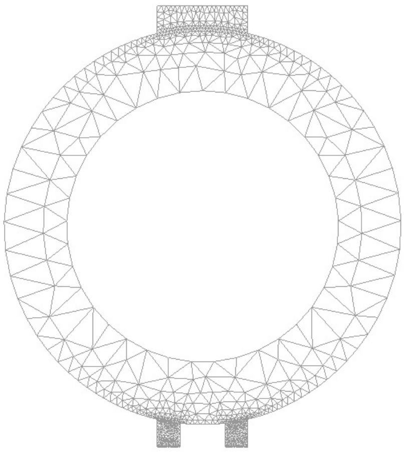

Figure 3: Finite elements mesh used

Both the pipe with the dimensions abovementioned and the lower and upper supports that are materialized with hard rubber strips, were modeled.

The obtained results show a concordance with the cracking mode of this type of pipes, where the higher tension stresses are focused in the inner sectors of the crown and the base. This stresses are those which finally cause the failure.Is has been mentioned here that the model used is not capable to reproduce the behaviour up to the failure load.This problem could be overcome by a micro-mechanic approach that consider the fibers separately and randomly disperse in the concrete(Ruano Sandoval, 2013).

In the Figure 4 a normal stresses distribution diagram coincident with the coordinates axis is shown.

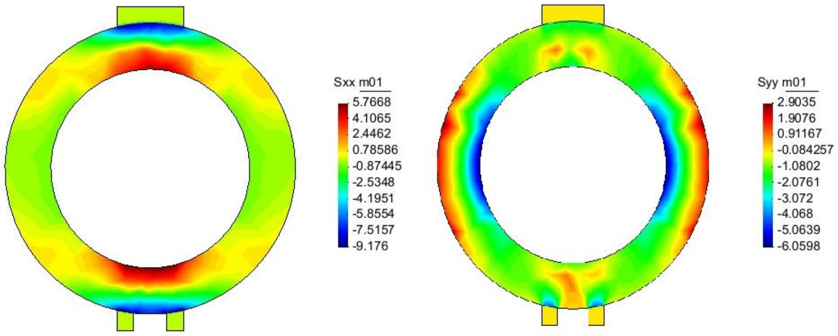

Figure 4: Normal stresses distribution diagram σ_{xx} and σ_{yy}

To verify and contrast the results obtained, they are compared with the found by other autors in an experimental way (Mohamed et. al 2014).The parameters used in the simulation were taken from the Mohamed's work for the SFRC. A plain concrete mix without fibers (PC) and three SFRC mixes with different dosage of 20, 40 and 60kg/m^3 (SFRC20, SFRC40 and SFRC60) were modeled.

In the Table 1 the properties of each mix are exposed. The mechanical properties of the hard rubber are presented in Table 2.

Mix	PC	SFRC20	SFRC40	SFRC60
Young Modulus (Mpa)	35.79	34.85	37.77	34.37
Poisson's Ratio	0.5	0.5	0.5	0.5
Compression ultimate strength(MPa)	45.8	52.88	58.02	61.64
Tension ultimate strength (MPa)	5.59	6.76	7.32	7.92
Yield threshold (Mpa)	34.35	40	43.5	46.23
Initial compression/tension strength ratio	8.2	7.8	7.93	7.78
Plastic hardening variable, κ_p	0.2			
Fracture energy (N/mm)	0.1	2.58	5.4	8.55
Crushing energy (N/mm)	6.72	156.9	339.6	517.5
Fluence criterion	Lubliner - Oller			
Potential criterion	Lubliner - Oller			

Table 1: Mixes mechanical properties

Hard rubber properties	
Young Modulus (MPa)	2600
Poisson's Ratio x-x	0.15
Poisson's Ratio x-z	0.015
Poisson's Ratio z-y	0.15
Compression ultimate strength (MPa)	11
Tebsion ultimate strength (MPa)	11
Maximum elongation (%)	300
Initial compression/tension strength ratio	1
Fluence criterion	Tresca

Table 2: Hard rubber mechanical properties

The numerical values obtained as well as the experimental values for cracking loads (P_f) for different dosage together with the mix without fibers are summarized in Table 3.

Fibers dosage ($\mathrm{kg/m^3}$)	$\mathrm{P_f}$ Experimental (N/mm)	$\mathrm{P_f}$ Numeric (N/mm)
No fibers	136	132
20kg/m^3	151	160
40kg/m^3	160	171
60kg/m^3	157	185

Table 3: Cracking loads according experimental tests and numerical model

As can be observed, the values numerically obtained have a correlation with the experimental ones, with a difference that does not exceed 10% overestimating the experimental value. As a further stage, it is suggested that other dosages be included to confirm or not the trend observed in the numerical results.

7 CONCLUSIONS

The mechanical behaviour of SFRC pipes was simulated by a simple model, as a first stage of a ongoing research, where the SFRC is modeled as a homogeneous composite (macro scale). The governing equations were solved by the Finite Elements Method, and the results were compared with those obtained experimentally by others authors.

It is observed that the addition of steel fibers improves the mechanical behaviour of concrete pipes increasing the load that they can resist up to the cracking. While the numerical model used has a good correlation with the experimental results up to the moment of cracking, it seems to overestimate the effects of the fibers at the pre-crack stage, because always values higher to the experimental ones were obtained.

Nowadays, work is being done on the implementation of a composite material model that explicitely considers the fibers as discrete entities and independent of the cementitious paste, in view of their interaction with the matrix considering pull-out and slipping events. This model takes the mechanical properties of the fiber, its volumetric proportion in relation to the total of the mix being able to reproduce the behaviour of SFRC up to the failure.

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