REDUCTION MODEL METHODS APPLIED OF MECHANICAL NONLINEAR SYSTEM

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Abstract. *Complex engineering problems, automotive, aerospace, among others, subject to nonlinearities requires big efforts numerical-computational. Dynamic systems are to a large majority, modeled by finite element (FE), numerical analysis technique designed to obtain approximate solutions in problems governed by equations differentials due to better accuracy in the results. Model flexibility and reliability, require at discretization process, arrays of higher dimensions and consequently large processing capacity of computers and higher computational cost. In this context, the use of reduced models is a good alternative. Based on the use of transformation matrices, the size of the system is reduced while preserving the maximum of its dynamic characteristics. The objective of this work has been to explore the efficacy of strategy reduction and improvement for structural dynamic models having nonlinearities. The following techniques and some improvements will be analyzed: Iterative*

Improved Reduction System (IIRS), Guyan-like, System Equivalent Reduction Expansion Process (SEREP) and Modal Base (BM). The Euler-Bernoulli beam is modeled by finite elements, extern loads and the coupling the nonlinear springs. Transient analysis of the full and reduced systems were performed utilizing the numerical integration of Newmark method.

Keywords: *Nonlinear mechanical system, Finite element method, Model reduction methods.*

1 INTRODUCTION

In modern projects in automobilist or aerospace engineering, there has been a growing interest in the study of fine or moderately fine mechanical and higher quality structures safely and cost favorable benefit. Analysis behavior of a mechanical structure is important but it is a costly task of modeling complexity and significant resolution due to non-linear phenomenon motivating the use of numerical analysis techniques. In recent decades, the finite element method, has excelled in the academic and industrial environment for structural analysis due to its efficiency and practicality. (Guedri, 2006; Rade, 2011).

Complex structures of industrial interest exhibit nonlinear properties, in this cases the FE models are usually constitute by a large number of degrees of freedom (DoFs), hundreds of thousand or even millions DoFs, fact that can hamper the calculation the response directly in the time domain, uses numerical integration Newmark method. In this context, it is necessary to develop and use reduction techniques to reduce the dimensions of the system and the associated computational load without compromising the structural integrity.

The first condensation technique, directly applied to models originating from structural mechanics, is Static or Guyan (Guyan and Irons, 1965), here, the inertia terms contributing to the models dynamics are ignored. Thus, this technique is exact only for static problems. Along the years several variations of the static condensation followed, which contributed to taking into account the dynamic effects of the model and generating qualitatively better model reduction.

O’Callahan (1989) proposed the Improved Reduced Ssytem (IRS) technique by including inertia force of the structures as pseudo-static force in the transformation matrix, however, this technique, according Gordis (1994), found stiffer than the Guyan-reduced system and was less suitable for checking the orthogonality of modes. Friswell et al. (1995, 1998) extended the IRS method by using the dynamic reduction technique and applied to an iterative scheme, improvement suggested in Xia et al. (2004).

O’Callahan et al. (1989) introduced the System Equivalent Reduction Expansion Process (SEREP) based on the modes of undamped analytical model and the latter reported its applications to undamped non-gyroscopic systems, a similar approach can be seen in Kammer (1987). Friswell and Inman (1999), used SEREP to reduce the model in element level with only undamped analytical modes. Friswell et al. (2000) used the technique to a rotor-shaft system with gyroscopic effect by constructing the transformation matrix without the left eigenvectors, which are, however, required for maintaining bi-orthogonality of analytical modes for these systems. Other simple computational implementation techniques are reported in Burton and Rhee (2000), Burton and Young (1994), Young (1994), Rhee (2000), Burton et al. (2000) and Kim et al. (2002) applied in mass-springs systems witch non-linear models such as Coulomb friction and static dead band and bang-bang effects.

A strategy commonly used in various problems of mechanical structures is to generate a set of vectors of Ritz capable of portraying precisely the behavior of the structure to undergo major structural changes (Masson et al., 2006), in which the transformation matrix or set of base vectors is obtained from the truncation of normal modes. In some cases, the selection of the reduction base is a delicate process since this should be able to portray changes occurred in the dynamic behavior of the system.

Masson et al. (2006) proposed improvements to the standard reduction methods superelement, see Craig and Bampton (1968). Here, developing a single enriched reduction transformation model while preserving the accuracy of the reduced substructure model for optimization of processes, in which the base pattern is extended condensation by a set of optimized static residual and secondly, in eliminating the associate coordinates from the reduced system. De Lima et al. (2015) proposed an adaptation of a method reduction in the time domain based on enrichment of a reduction base for treatment of linear and nonlinear viscoelastic systems subject to structural changes, presenting good results and showing computational gains. In this approach, the enrichment of base reduction is based on priori information of the viscoelastic damping forces, wherein the use of such static residues shown to be sufficient to represent the viscoelastic behavior of the system with satisfactory precision.

The objective of this work was to explore the efficacy of reduction strategies in dynamic models discretized by finite element procedures with nonlinearities located, using response simulations via numerical integration of the model's differential equations. Some improvements in different methods of reducing studied here have been proposed to improve the representativeness of the reduced model in response to the proposed problem.

2 MODEL REDUCTION METHODOLOGY

The general idea of reduction methods is to reduce the system order to reduce the computational effort involved in the solutions of the problem. The reduction of the original system is obtained by approximation of the discrete variables vector of the system by a small number of global approximation functions by transformation matrices.

The equation of motion of multiple degree of freedom of structural model with nonlinearity located can be written as:

$$[M]\{\ddot{x}(t)\}+[D]\{\dot{x}(t)\}+[K_0]\{x(t)\}+[K_{nonlinear}]\{x^3(t)\}=F(t) \quad (1)$$

where $[M]$, $[D]$, $[K_0]=[K]+[K_{linear}]$ and $[K_{nonlinear}]$, order $n \times n$, are the mass, damping, stiffness and nonlinearity stiffness matrices, respectively, and $f(t)$, order $n \times 1$, is a load vector varying time. Assumed the proportional Rayleigh, $[D]=0.005*[\mathrm{K}_0]$.

According Koutsovasilis et al. (2007), the concept of model reduction is to find a low dimension subspace $[T]$, order $n \times R$, with $R << n$, to approximate the state vector x, i.e.:

$$\{x_n\} \approx [T]\{x_R\} \quad (2)$$

By substituting Eq. (2) in Eq. (1) and projecting Eq. (1) on the subspace $[T]$, a new differential equation so second order and lower dimension is obtained:

$$[\overline{M}]\{\ddot{x}_R(t)\}+[\overline{D}]\{\dot{x}_R(t)\}+[\overline{K}_0]\{x_R(t)\}+[\overline{K}_{nonlinear}]\{x_R^3(t)\}=\overline{F}(t) \tag{3a}$$

where

$$\overline{M}=T^T MT \quad \overline{D}=T^T DT \quad \overline{K}_0=T^T K_0 T \quad \overline{K}_{nonlinear}=T^T K_{nonlinear} T \quad \overline{F}=T^T F \tag{3b}$$

In general, a correct selection of degrees of freedom considered and the particularities of each reduction technique are relevant to a sufficiently good predictive power of the numerical models.

In the subsections below, are presented the main stages of the transformation matrix formulation for some reduction techniques the Modal Base, IIRS, Guyan-like and SEREP that are based in modal truncation, substructuring and static condensation.. Improvements are proposed for best results in the analysis of dynamic nonlinear model presented in this work.

2.1 Modal base (BM)

The calculation of the natural modes of integration is not possible in finite element models of industrial type. Thus, one of the methods commonly used in different structures is the method of modal basis. As part of a dynamic analysis conventional low frequency, is used only the firsts modes for to form a truncated modal basis. The reduced equations are obtained in this method using a basis set that includes only a truncated set of vibration modes. Thus, the reduced base vector can be obtained by the resolution of the eigenvalue problem:

$$\begin{aligned}&(K_0-\lambda_i M)\phi_i=0\\&\phi_0=[\phi_1 \quad \phi_2 \quad \ldots \quad \phi_r], \quad \Lambda_0=diag(\lambda_1,\lambda_2,\ldots,\lambda_r)\end{aligned} \tag{4}$$

where $i=1,\ldots,n$.

In models with non-linearities, this base may result in some errors due to truncation of the database (Guedri, 2006). Indeed, the basis T, is enriched by introducing the following residues formed by the static displacements associated to external excitations (de Lima et al., 2010):

$$\Delta R=K_0^{-1}f(t) \tag{5}$$

The enriched reduced base vector for the system is given as:

$$T=[\phi_0 \quad \Delta R] \tag{6}$$

2.2 Iterative improved reduction system (IIRS)

According Friswell et al. (1995, 1998) and Xia et al. (2004), the generalized eigenvalue problem of a system is described as follows, in the sub-structuring form, in terms the chosen master DoFs (retained) and slave DoFs (removed),

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{bmatrix} \Phi_{mm} \\ \Phi_{sm} \end{bmatrix} = \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{bmatrix} \Phi_{mm} \\ \Phi_{sm} \end{bmatrix} \Lambda_{mm} \tag{7}$$

where K and M, order $n \times n$, are symmetric stiffness and mass matrices; Φ consists of the mass-normalized eigenvectors and $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_n)$. Only the first r modes are included in the above equation. The subscripts “m” and “s” represent the master and slave DoFs, respectively; the sizes of master and slave DoFs are assumes to be m and s with $n = m + s$; and the eigenvalues are arranged in ascending order. From the second set of the above equation, Φ_{sm} hence can be expressed as,

$$\Phi_{sm} = -K_{ss}^{-1} K_{sm} \Phi_{mm} + K_{ss}^{-1} \left(M_{sm} \Phi_{mm} \Lambda_{mm} + M_{ss} \Phi_{sm} \Lambda_{mm} \right) \tag{8}$$

Being t the transformation matrix between Φ_{mm} and Φ_{sm}, which takes the form of

$$t = t_{Guyan} + K_{ss}^{-1} \left(M_{sm} + M_{ss} t \right) \Phi_{mm} \Lambda_{mm} \Phi_{mm}^{-1} \tag{9}$$

where t_{Guyan} represents the transformation matrix of the Guyan reduction technique (Guyan, 1965).

We write Eq. (9) as

$$t = t_{Guyan} + t_d \,, \tag{10a}$$

$$t_d = K_{ss}^{-1} \left(M_{sm} + M_{ss} t \right) \Phi_{mm} \Lambda_{mm} \Phi_{mm}^{-1} \tag{10b}$$

and

$$T = \begin{bmatrix} I_{mm} \\ t \end{bmatrix} = \begin{bmatrix} I_{mm} \\ t_{Guyan} + t_d \end{bmatrix} = T_{Guyan} + \begin{bmatrix} 0 \\ t_d \end{bmatrix} \tag{11}$$

where I_{mm} is the unit matrix of size $m \times m$.

Substituting Eq. (11) into $K_R = T^T K T$ and $M_R = T^T M T$, one get the reduced matrices of stiffness and mass and algebraic manipulations a new relationship can be obtained:

$$\Phi_{mm} \Lambda_{mm} \Phi_{mm}^{-1} = M_d^{-1} K_{Guyan} \tag{13}$$

where,

$$M_d = M_{Guyan} + \left(M_{ms} + t_{Guyan}^T M_{ss} \right) t_d \tag{14}$$

and substitute it into Eq. (9),

$$t = t_{Guyan} + K_{ss}^{-1} \left(M_{sm} + M_{ss} t \right) M_d^{-1} K_{Guyan} \tag{15}$$

This leads to a the iterative scheme, in which the formulae are

$$t^{(k)} = t_{Guyan} + K_{ss}^{-1}\left[M_{sm} + M_{ss}t^{(k-1)}\right]\left[M_d^{(k-1)}\right]^{-1} K_{Guyan}, \tag{16a}$$

$$T^{(k)} = \begin{bmatrix} I_{mm} \\ t^{(k)} \end{bmatrix}, \tag{16b}$$

$$M_d^{(k-1)} = M_{Guyan} + \left(M_{sm} + t_{Guyan}^T M_{ss}\right) t_d^{(k-1)}, \tag{17}$$

$$K_R^{(k)} = \left[T^{(k)}\right]^T K T^{(k)}, \tag{18a}$$

$$M_R^{(k)} = \left[T^{(k)}\right]^T M T^{(k)}, \tag{18b}$$

and

$$K_R^{(k)} \Phi_m^{(k)} = M_R^{(k)} \Phi_m^{(k)} \Lambda_m^{(k)}. \tag{19}$$

2.3 Guyan-like

The formulation of the reduced model done following the steps summarized below (Burton and Young, 1994; Burton, Hermez, and Rhee, 2000; Burton and Rhee, 2000; Kim and Burton, 2002).

First, assume that the system coordinate vector $\{x_n\}$ is partitioned into master and slave sub-vectors:

$$\{x_n\} = \begin{Bmatrix} x_s \\ x_m \end{Bmatrix} \tag{20}$$

where $\{x_m\}$ is an master vector; $\{x_s\}$ is an slave vector; and $s + m = n$.

Being m the lowest frequency undamped mode shapes, determined by solving the eigenvalue problem of the version linear, without damping and unforced of Eq. (1), see Eq. (4), the associated modal matrix is then partitioned as follows:

$$[\phi]_{n\times m} = \begin{bmatrix} [\phi_s] \\ [\phi_m] \end{bmatrix} \tag{21}$$

where the $[\phi_s]_{s\times m}$ is associated with the slave DoFs, and $[\phi_m]_{m\times m}$ is associated with the master DoFs. Based on these matrices, a relationship matrix between the master and slave coordinates can be obtained in the Guyan-like form,

$$\{x_s\} = [R]_{s\times m}\{x_m\} \tag{22}$$

where $[R] = [\phi_s][\phi_m]^{-1}$. Thus, the reduction matrix $[T]$ is then defined so that

$$\{x\} = [T]\{x_m\}; \quad [T] = \begin{bmatrix} [R] \\ [I_m] \end{bmatrix} \tag{23}$$

For a thorough representation of non-linearities in the model, improvements are proposed. Pre-multiplying the Eq. (23) by the new matrix has

$$[T_0] = [S_0]\begin{bmatrix}[R]\\ [I_m]\end{bmatrix} \tag{24}$$

where $[S_0] = [K]^{-1}$, and considering the enrichment of the proposed basic vector in Eq. (5), a new transformation matrix for this method is obtained

$$[T] = [T_0 \quad \Delta R] \tag{25}$$

2.4 System equivalent reduction expansion process (SEREP)

According O'Callahan et al. (1989), Friswell and Inman (1999), Friswell et al. (2000), Koutsovasilis et al. (2007), Das and Dutt (2008), the methodology basis this reduction strategy is the modal matrix containing the eigenvectors organized in a specific column format. Similarly Guyan-like, in the formulation for this reduction strategy assuming that m to be the number of dominant modes (generally, the modes lowest frequencies), determined by solving the eigenvalue problem of the version linear, without damping and unforced, the truncated solution of the eigenvalue problem may be expressed as:

$$\{x_n\} = [\phi]\{p\} \tag{26}$$

where $[\phi]_{n\times m}$ is the eigenvector matrix and $\{p\}_{m\times 1}$ is the modal participation vector.

In the second stage of reduction, all the states of the system are classified into two categories: the active (or master) states and omitted (or slave) states. There isn't criterion of selecting the active states, however, the number of active states is $\leq n$. Let a be the number of active states of a system and $d = n - a$ be the number of omitted states. Based on the division into master and slave sub-states of the system, the Eq. (26) can be rewrite as

$$\begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = \begin{bmatrix} [\phi]_{a\times m} \\ [\phi]_{d\times m} \end{bmatrix}\{p\}_{m\times 1} \tag{27}$$

Now, there are two different conditions depending on the number 'm' of the retained modes and the number 'a' of the active states viz. (1) $a \geq m$, which is the case commonly obtained in the structural dynamic problems and (2) $a < m$, which is a rare case but not impossible (O'Callahan et al., 1989).

For the case (1), $a \geq m$, and according to the first part of Eq. (27) the modal participation vector can be expressed in terms of the master DoF coordinates,

$$\{p\} = [\phi^{+}_{a\times m}]\{x_a\} = \left[[\phi^{T}_{a\times m}]\ [\phi_{m\times a}]\right]^{-1}[\phi^{T}_{a\times m}]\{x_a\} \tag{28}$$

where the subscript $+$ signifies the pseudo-inverse. By substituting the relationship of Eq. (28) in Eq. (27), the coordinate transformation matrix T is defined,

$$\begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = \begin{bmatrix} [\phi]_{a\times m} \\ [\phi]_{d\times m} \end{bmatrix}[\phi^{+}_{a\times m}]\{x_a\} = [T]\{x_a\} \tag{29}$$

For this model reduction strategy it is proposed an improvement, similar to the previous case. Pre-multiplying the Eq. (23) by the new matrix and considering the enrichment of the proposed basic vector in Eq. (5), a new transformation matrix for this method is obtained,

$$[T_{new}] = [[S][T] \quad \Delta R] \tag{30}$$

where $[S] = -[K]^{-1}[M]$.

3 NUMERICAL ANALISYS OF THE NONLINEAR MECHANICAL SYSTEM

The results presented in this section are based on numerical simulations for the non linear system in forced vibrations, described by Eq. (1). This analysis is based on the check of the dynamic behavior of the system through the development of system response time and phase diagram. The program used for modeling and numerical simulations is the Matlab®.

In numerical simulations, two case studies for a beam structure Euler-Bernoulli discretized by finite procedures according to Hamilton's principle, were considered. Thus, the external loads applied, the amount and position of the non-linear springs are modified, the springs stiffness values are considered fixed: $k_{linear} = 100 \, N/m^2$; and $k_{nonlinear} = 1e05 \, N/m^2$. To better describe the dynamic behavior of the model, for each of the case studies also varied the excitation force.

The equations of motion was solving by Newmark integration method. A time interval for simulation of 0 to 0.5 seconds with a time step of $0.0001s$ was used, the system being at rest at the beginning of the process, with the excitation force of the $F_e(t) = F\sin(\omega t)$ with frequency excitation coincident the first natural frequency of the beam.

The physical and geometrical properties of the beam to the two case studies are shown in Tab. 1.

Table 1. Beam's physical and geometrical properties.

Aluminum Beam		
Geometrical Properties	Length $[m]$	0.5
	Width $[m]$	0.01
	Tickness $[m]$	0.003
Physical Properties	Volume density $\left[kg/m^3\right]$	2710
	Modulus of Elasticity $[Mpa]$	7e+10

3.1 Case 1

The Fig. 1 illustrate one scheme of the beam modeled for the first case considered. It considered the discretized beam 10 elements in the direction of its length, with external load $F_0 = 1N$ applied in the last node. It was felt also springs coupled in the second, fourth and tenth node.

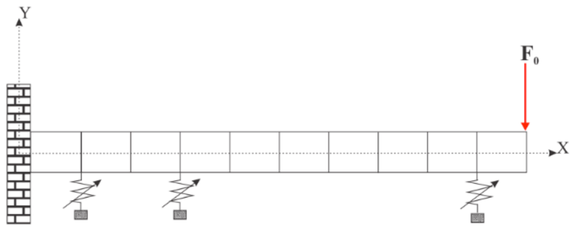

Figure 1. Scheme of the beam modeled to case 1.

The Fig. 2(a) and 2(b) illustrate the dynamic response and phase diagram of the beam for excitation force of $F_e = 20N$ and $F_e = 70N$. It was found that the dynamic behavior is closely related to the excitation force applied to the system. For a small excitation force, $20N$, the system exhibits little nonlinearity. As for a higher excitation strength, $70N$, the non-linearity is noticeably stronger and the system starts to have chaotic characteristics, according to the phase plane of Fig. 2(b), below. This behavior is due to non-linearity of the model, expressed by the cubic term in the springs.

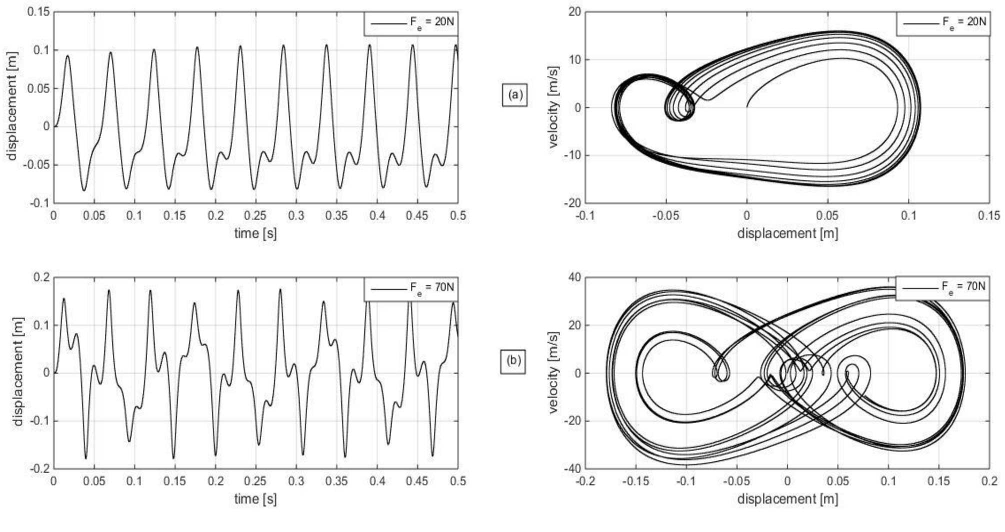

Figure 2. Dynamic response and phase diagram for 20N (a) and 70N (b).

3.2 Case 2

The Fig. 3 illustrate one scheme of the beam modeled for the second case considered. Here, we considered the discretized beam 20 elements with multiple external loads, the ninth to the fourteenth node, being $F_0 = F_1 = F_4 = F_5 = 1N$ and $F_2 = F_3 = 5N$, and still, the springs are coupled in the 2nd, 4th,6th, 11th, 12th, 17th, 19th and last node.

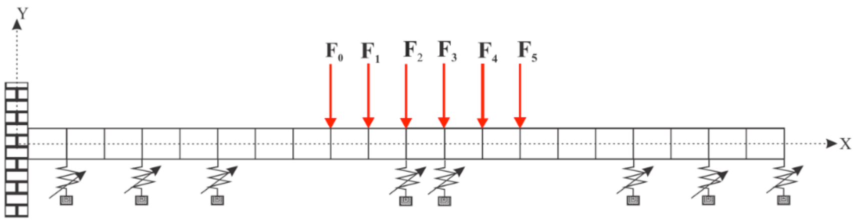

Figure 3. Scheme of the beam modeled to case 2.

The Fig. 4(a) and 4(b) illustrate the dynamic response and phase diagram of the beam for excitation force of $F_e = 10N$ and $F_e = 20N$. As in the first case, the dynamic behavior is closely related to the excitation force applied to the system. However, the nonlinearities is greater than the previous case due to the external loads that are distributed in the central part of the beam with different strength values and, the number of nonlinear springs coupling coupled at both ends and the central part of the beam. In this case the system is strongly non-linear for $10N$ and $20N$, this can be seen in the phase plane (Fig.4).

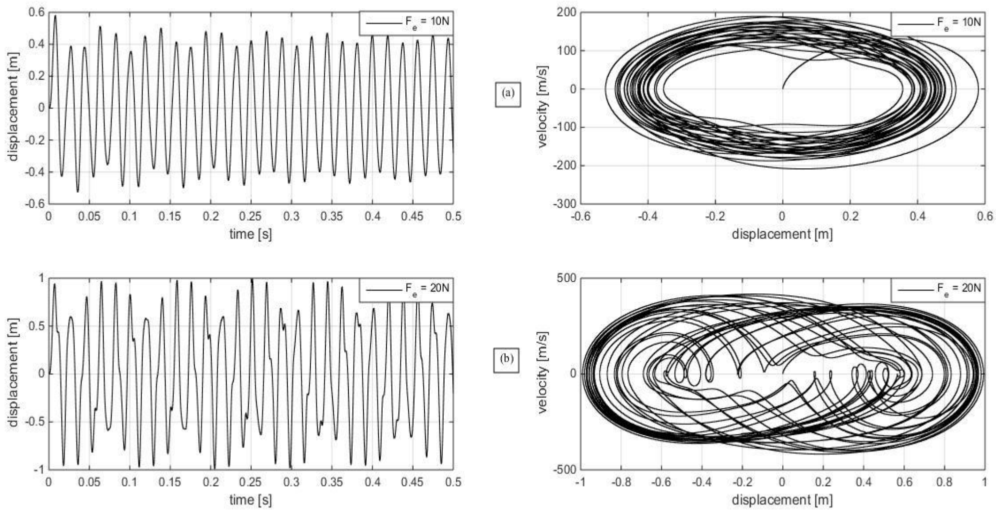

Figure 4. Dynamic response and phase diagram for 10N (a) and 20N (b).

4 MODEL REDUCTION APPLIED OF NONLINEAR SYSTEM

As mentioned earlier, one of the great advantages of using models reduction strategies in nonlinear systems is to reduce system size and the processing load associated in the process of solving problems. The following will be presented numerical simulations of the dynamic behavior of the original systems for the two case studies above, and their systems reduced grounded in reduction strategies described in this paper.

4.1 Numerical analysis of the model reduction methods for case 1

The Fig. 5 and Fig. 6 illustrates the dynamic response of the system described in Section 3.1 for excitation force of $20N$ and $70N$, respectively, using reduction strategies.

For this case, for the SEREP Modified (SEREPMod) and BME used 10 degrees of freedom masters or retained and the static basic residues for enrichment. The methods based on the sub-structuring, for the most part, are highly dependent on proper choice of degrees of freedom masters and slaves. In IIRS and Guyan-Like modified (GUYANLIKEMod) methods, we used 10 degrees of freedom masters in the latter it was considered five dominant modes and base enrichment.

It was considered an excitation force of $20N$, in which the non-linearity present in the system is still small. According to the graphics shown in Fig. 5 (a), 5 (b), 5 (c) and 5 (d), it is clear that all reduction methods used were effective and had good approach when confronted with the system original (reference model).

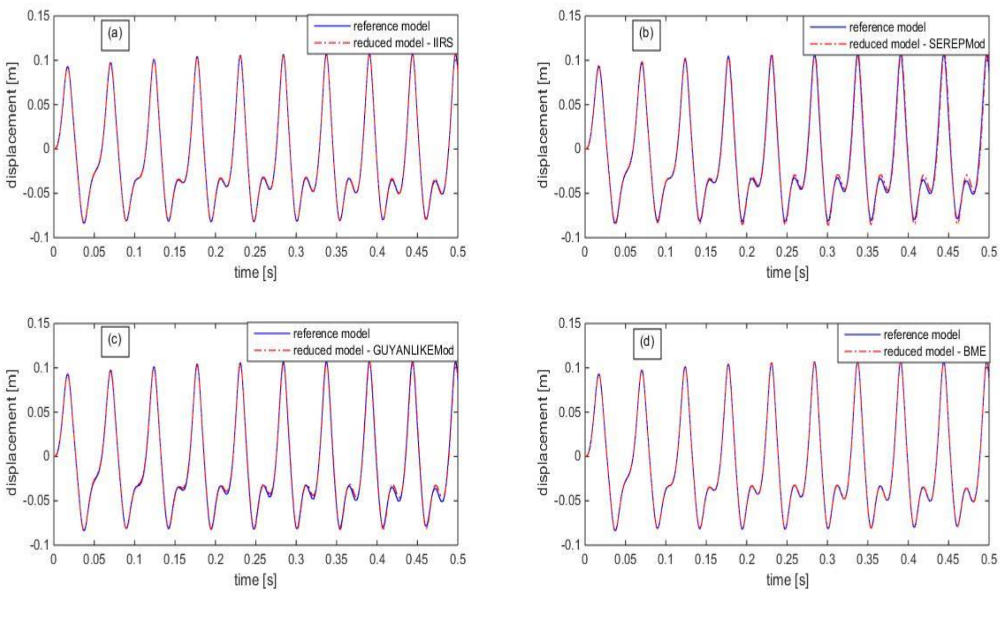

Figure 5. Dynamic response of original system and reduced – (a) IIRS, (b) SEREPMod, (c) GUYANLIKEMod and (d) BME – for excitation force of 20N.

The Fig. 6(a), 6(b), 6(c) and 6(d) illustrate the dynamic response of original system and reduced for excitation force of $70N$ In the second case, for the models reduction methods is considered the same characteristics as the previous case. Here, it changes only to excitation force applied to the system. The value of the excitation force tends to greatly increase the non-linearity of the system. However, this fact did not affect the representativeness of reduction methods.

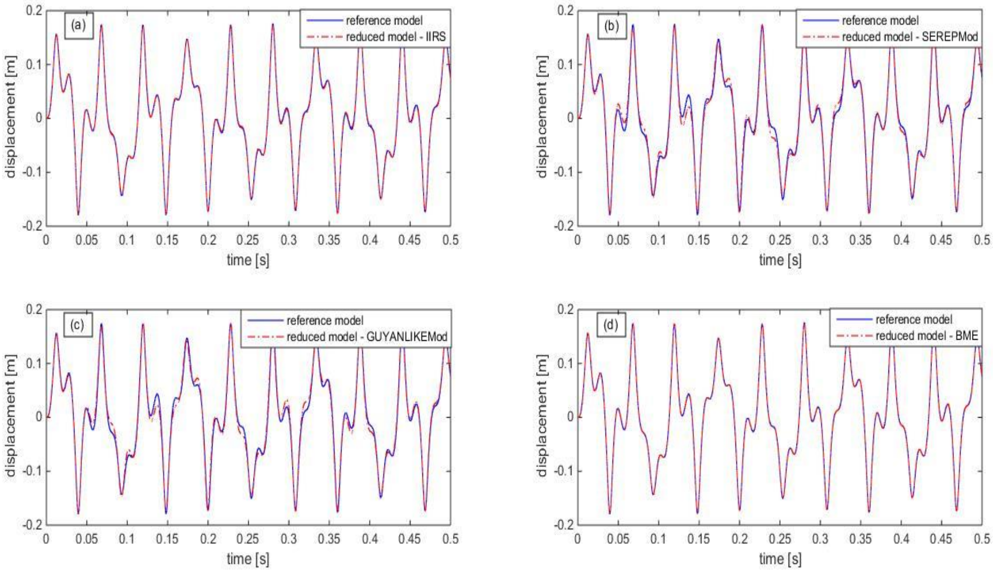

Figure 6. Dynamic response of original system and reduced – (a) IIRS, (b) SEREPMod, (c) GUYANLIKEMod and (d) BME – for excitation force of 70N.

The numerical simulations by using reduction techniques enable the resolution of systems based on a limited number of degrees of freedom and consequently, matrices of smaller dimensions, are factors that favor a drastic decrease of the associated computational load on solving the problems.

The Fig. 7 shows the computational time spent in resolution the reduced systems. Regarding the time in solving the original systems, for the two cases discussed with excitation force of 20N and 70N, using reduction techniques was obtained time savings. It can be seen for the time spent in IIRS, GUYANLIKEMod and BME methods, the number of degrees of freedom and retained the basic enrichment are directly related to the performance of each reduction method and the computational gain.

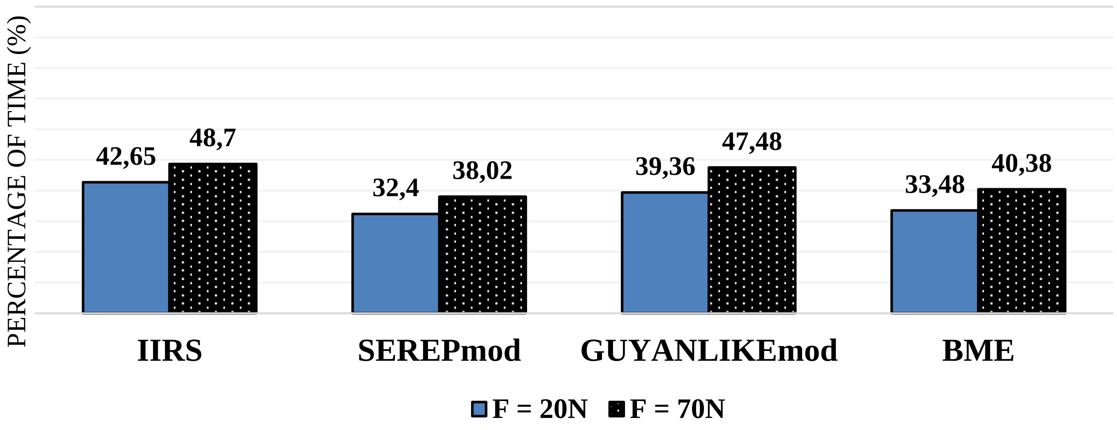

Figure 7. Computational time of the reduction techniques from the original systems, for excitation force of 20N and 70N.

4.2 Numerical analysis of the model reduction methods for Case 2

The Fig. 8 and Fig. 9 illustrates the dynamic response of the system described in Section 3.2 for excitation force of $10N$ and $20N$, respectively, using reduction strategies.

According the Fig. 8(a), 8(b), 8(c) and 8(d), for an excitation force of $10N$, there was good representation of the reduction techniques.

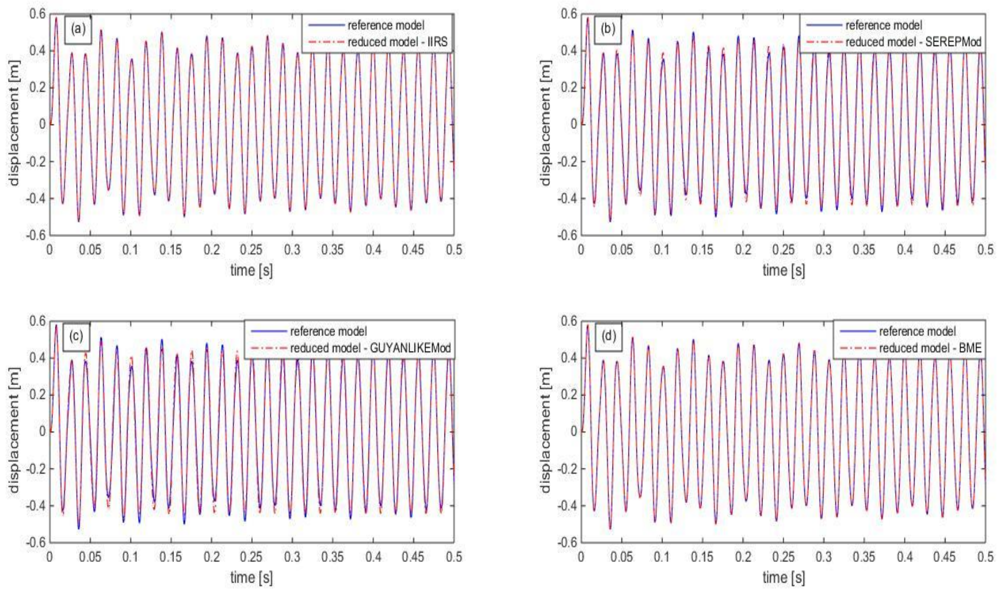

Figure 8. Dynamic response of original system and reduced – (a) IIRS, (b) SEREPMod, (c) GUYANLIKEMod and (d) BME – for excitation force of 10N.

The non-linearity of the system is greater than the previous case due to the coupling of a greater number of non-linear springs to the beam and to the applied loads, is considered 10 degrees of freedom retained for the SEREP Modified and BME method and used the static basic residues for enrichment. In IIRS and Guyan-Like modified methods, we used 10 degrees of freedom masters in the latter it was considered five dominant modes and base enrichment.

As shown in Fig. 9(a), 9(b), 9(c) and 9(d), increasing the excitation force to 20N, the representativeness of the abatement technology is still plausible. Here, the strong non-linearity affect the representative capacity of the reduced models. For most reduction methods used, except for BME the extent that the time interval increases, the approximation of these techniques is decreases.

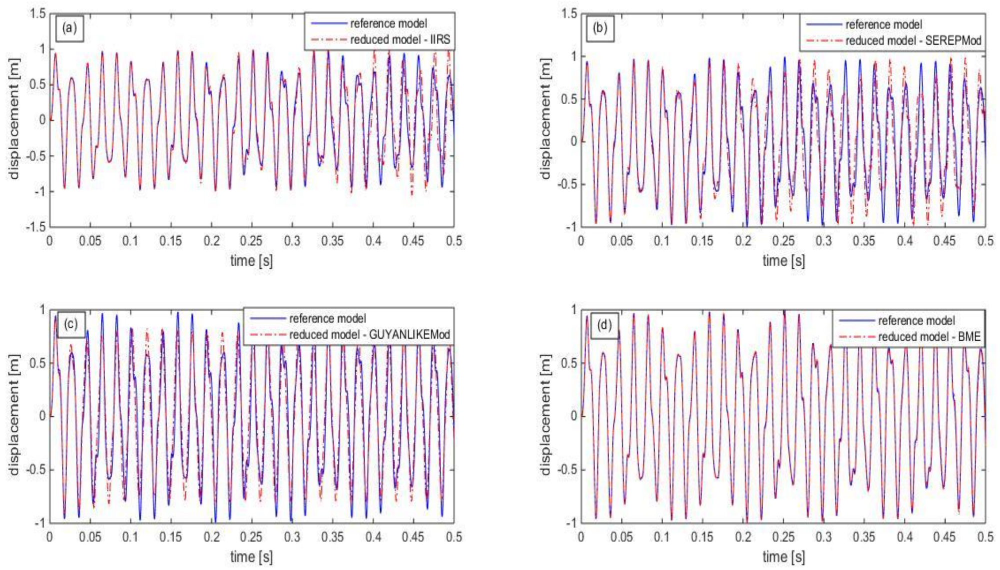

Figure 9. Dynamic response of original system and reduced – (a) IIRS, (b) SEREPMod, (c) GUYANLIKEMod and (d) BME – for excitation force of 20N.

The Fig. 10 shows the computational time spent in resolution the reduced systems for case 2, with excitation force of 10N and 20N. In this case, the time savings was higher that the studied previously (F = 10 N), where the non-linearity is less. The computational gain is directly related to the number of elements considered for the structure. In numerical analysis for the model described in case 1, it was considered 10 degrees of freedom masters or retained for a discretized structure in 10 finite elements, now considered to be the same number of degrees of retained freedom, but for a discretized structure 20 elements.

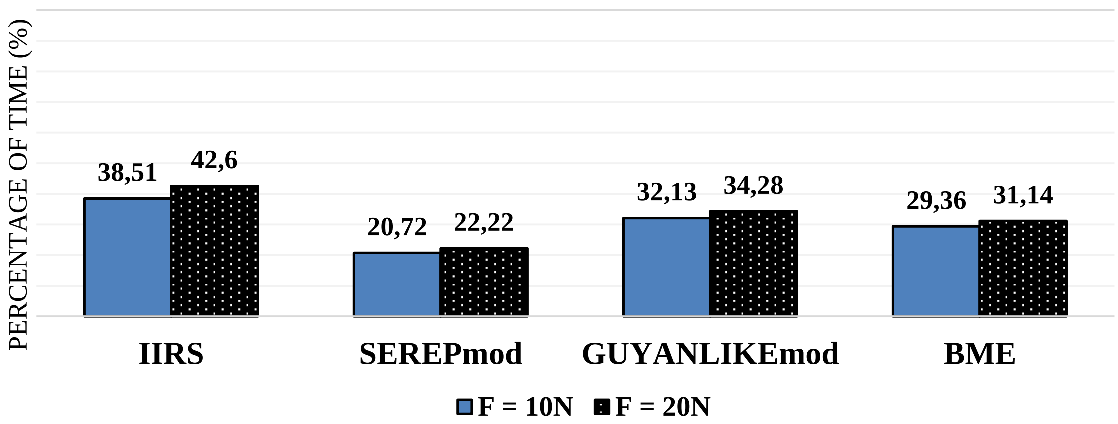

Figure 10. Computational time of the reduction techniques from the original systems, for excitation force of 10N and 20N.

5 CONCLUSION

In this work, the modeling of dynamic structures with nonlinearities located discretized by finite element procedures and the application of reduced models for numerical analysis techniques based on responses in the time domain were implemented. Based on numerical simulations, the nonlinearity of the presented system is influenced by the excitation force applied. As it increases the external force applied to the system, the non-linear characteristics become more evident in the time domain response.

The applied of reduction of models is an important tool in the representation of nonlinear dynamical systems. The reduction techniques and the changes proposed in this work, allowed numerical analysis based in models reduced equivalents to the reference model. The computational load associated to the analysis of nonlinear systems was decreased. In the numerical simulation, non-linear systems reduced in relation to the reference system, saved about 50% of the total time, in particular, the method SEREP modified had the best performance with greater than 75% savings in some applications.

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