EVALUATION OF THE SENSITIVITY OF SEISMIC INVERSION ALGORITHMS TO DIFFERENT WAVELETS

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Abstract. *Seismic wavelet estimation is an important step in processing and analysis of seismic data. Inversion methods as Narrow band and Sparse spike require information about it so that the inversion solution, once it is not unique, may be restricted by comparing the real seismic trace with the synthetic generated by convolution of the estimated reflectivity and wavelet. Besides helping in seismic inversion, a good estimate of the wavelet enables an inverse filter with less uncertainty to be produced in the deconvolution step and while tying well logs, a better correlation between the seismic trace and well log. Depending on the use or not of well log information, the methods of wavelet estimation can be divided into two classes: statistical and deterministic. This work aimed to test the sensitivity of seismic inversion algorithms to different wavelets. An analysis of the wavelets estimated with two methods and the inversion results allows one to relate the uncertainty in the estimated wavelet and in the inversion results. All methods used here were implemented in MATLAB and synthetic data were used.*

Keywords: *Seismic wavelet estimation, Seismic inversion, Narrow band, Sparse spike*

1 INTRODUCTION

Geophysical techniques are used to investigate indirectly the structures and properties associated with the geology of the subsurface. Depending on the features of the desired target, a set of techniques will be chosen. Among the existing methods, the seismic is the most used, once it has good resolution even for great depths and allows obtaining the acoustic properties of the medium by inverting the data.

The purpose of seismic acoustic inversion is to generate a consistent impedance (product of density and velocity) model. There are several methods of seismic inversion, which may, following Russell (1988), be divided into two classes: pre and post-stacking. In this work, only the latter will be treated. More specifically, the types: Narrow Band and Sparse Spike.

Narrow band type, occasionally referred to as SEISLOG or VERILOG, is the simplest seismic inversion method and, in accordance with Cooke and Schneider (1983) and Russell (1988), also the most popular. Sparse Spike in turn, includes more robust methods which allow introduction of *a priori* information and admits more consistent assumptions about reflectivity. Several papers propose methods of this second kind, and often they differ only in the deconvolution technique used to reflectivity estimation. As examples of these methods, there are the ones that perform deconvolution using maximum likelihood algorithms (Kormylo and Mendel, 1982; Chi et al., 1985; Ursin and Holberg, 1985), the L1 norm (Oldenburg et al., 1983; Sacchi, 1997), in particular the work of Wang (2011), and Bayesian methods as those discussed in Ulrych and Sacchi (2005).

Usually, inversion algorithms need information about the seismic pulse signature and its behavior during propagation in the subsurface. Combination of this signature and the effects on wave propagation is called seismic wavelet. Thus, its estimative from the treated data is a fundamental step to ensure good results in the inversion process.

There exists a vast range of work dealing with wavelet estimation methods, so that it is possible to divide them between deterministic and statistical, their difference being related to the introduction of well log information. Much of the work in the area until 1996 can be found in Osman and Robinson (1996). Robinson (1954, 1957) attack this problem using the Kolmogorov method which is based on the assumption that the wavelet has minimum phase and that the phase spectrum can be obtained by the Hilbert transform of the natural logarithm of the estimated amplitude spectrum. Ulrych (1971) seeks an estimate using the theory of homomorphic systems presented in Oppenheim (1965). The work of Velis and Ulrych (1996) seeks a solution to the problem discussed here using a strategy based on a simulated annealing algorithm. More recent studies discuss about more efficient methods, though also more complex as: Zheng et al. (2013), Yi et al. (2013) and Wang et al. (2014).

Sensitivity of the inversion algorithms treated here, was analyzed regarding wavelets estimated using the methods: Kolmogorov or Hilbert Transform (as done in Robinson (1954)) and a combination of smoothing the seismogram's amplitude spectrum and guessing of a linear model to the phase spectrum, similar to the methodology used in Oliveira and Lupinacci (2013). After estimation, a correction in the phase spectrum similar to the one proposed in Levy and Oldenburg (1987) was applied in order to ensure less uncertainty in the estimated wavelet.

In this work it was possible to analyze the behavior of the seismic inversion methods: Narrow band and Bayesian, regarding wavelets estimated by the aforementioned methods and noise. In general the second got far superior results, admitting more robust estimates of the impedance in all tests. For wavelet estimation methods, the second (smoothing) obtained better results, although for both, the phase estimates were not good.

2 METODOLOGY

The methodology used for sensitivity testing, in summary involved the creation of a synthetic seismogram from an input model, an estimated wavelet and finally the inversion.

For modeling of the seismogram the convolution of reflectivity with a Ricker wavelet with center frequency of 10 Hz was used. This model for subsurface, in its simplest form, was originally described in Robinson (1954) and is valid for several steps of seismic processing. Its validity depends on some assumptions, two of which are: (1) the reflectivity is a random process and (2) the wavelet has minimum phase. Considering the reflectivity r and wavelet w, the seismogram s can be mathematically represented as:

$$s_t = \sum_{k=0}^{\infty} w_t r_{t-k} \qquad \text{or} \qquad s(t) = w(t) * r(t). \tag{1}$$

It is emphasized that the seismic wavelet is an estimate of the source signature and the dispersive effects acting upon the signal propagation. Then, what is sought is an approach that encompasses all effects affecting the seismic pulse over time.

Results of the wavelet estimation were analyzed for synthetic models generated with minimum and zero phase wavelets, both tests considering introduction of noise. In the inversion analysis, zero phase wavelets were used in creation of the synthetic seismogram and existence of additive noise was also considered. The noise used in both analyses is random and has standard deviation $\sigma = 10^{-3}$.

Inversion and wavelet estimation methods used here are discussed below.

2.1 Wavelet Estimation Methods

As said previously, wavelet estimation used two methods: Hilbert Transform and one involving smoothing the seismogram's amplitude spectrum, where the latter will be, from now on, referred to as Smooth spectra. A theoretical introduction concerning these is presented in the following subsections.

The Hilbert Transform or Kolmogorov factorization Method. This method is able to estimate the amplitude spectrum of the minimum phase wavelet associated with the analysed data, based on its autocorrelation. The actual phase of the wavelet however, might not be minimum, so the phase spectrum must be also estimated.

Using the convolutional model and consequently assuming reflectivity as a random series, the autocorrelation of the wavelet can be approximated by the seismogram's one, differing only by a scale factor equal to the total energy of reflectivity, as shown in Yilmaz (2001).

As stated in Robinson and Treitel (2000), the calculation of spectral density of a signal can be accomplished by Fourier Transform (FT) of its autocorrelation, and also by the square of the magnitude of its frequency spectrum. Following this idea, the method discussed here estimates an amplitude spectrum for the minimum phase wavelet ($W(\omega)$) of the data from the relationship:

$$|W(\omega)| = \sqrt{|\phi(\omega)|}, \tag{2}$$

where $\phi(\omega)$ is the FT of the seismogram's autocorrelation.

The phase spectrum can be obtained from the equation:

$$\sphericalangle W(\omega) = -2\sum_{1}^{\infty}[\sin(\omega t)]\left[\frac{1}{2\pi}\int_{0}^{\pi}\cos\omega t \ln|W(\omega)|d\omega\right], \tag{3}$$

derived in Robinson (1954).

In Lines and Ulrych (1977) the authors state that the derivation of the above equation can be performed from the Hilbert transform of the logarithm of the wavelet's amplitude spectrum, justifying the method's name.

Smoothing Spectra Method. The theory behind this method is simpler than the previous one. In short, this involves the choice of a smoothing operator to the amplitude spectrum of the seismogram and a minimum , mixed or maximum phase guess for the spectra, where the most common is the selection of a constant value. The parameter that most influences the final result, is the operator's size, which can be a simple moving average filter.

As the true wavelet's phase spectrum is not known, to guess it is not the most appropriate way to solve the problem, hence we introduced the APC correction (Automatic Phase Correction), similar to the one presented in Levy and Oldenburg (1987), where the phase spectrum is rotated and a synthetic seismogram calculated until an entropy norm is maximized. Here, it was sought to minimize the norm of the error between calculated synthetic and the real seismogram. This correction is applied to wavelets estimated by the two methods used herein.

2.2 Seismic Inversion Methods

For seismic data inversion, Narrow band and Bayesian methods were used, these are presented below.

Narrow Band Inversion. The inversion treated here is a recursive method for the estimation of an impedance model from a wavelet, the impedance value of the first layer and the reflectivity.

The equation for estimation of the impedance Z using recursive inversion is obtained from the reflectivity formula:

$$r_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}, \tag{4}$$

where i represents iteration.

Starting from Eq. (4) we have:

$$Z_{i+1} = Z_i \frac{1 + r_i}{1 - r_i}, \tag{5}$$

which represents the recursive process of seismic data inversion.

The big difference between the method herein and other recursive methods involves choosing the deconvolution technique used to estimate the reflectivity model. For this case, the techniques known as classic, that use the theory behind the Wiener filtering (Wiener, 1966), are adopted. Filter implementation was made in an adaptative way, allowing better results in the presence of good estimates of signal to noise ratio.

Seismic deconvolution comes from image processing theory, described according to Yilmaz (2001), as being responsible for estimating the reflectivity by compressing the wavelet and attenuating reverberations and short period multiples, hence providing greater temporal resolution.

The computational cost involved in the deconvolution technique used here is very low. Still, for inversion purposes it represents the worst alternative due to the limited bandwidth property, characteristic of the estimated reflectivity and from where the method's name comes. Low frequency loss represents absence of information regarding the main trend of the geological model of subsurface, while the loss of high frequency components, leads to reduction in temporal resolution. This problem is extensively discussed in literature, e.g. Russell (1988), for that reason here, in order to work around the loss of low frequency content, which will control the main trend of the estimated impedance model, we use a source with a 10 hertz center frequency .

Bayesian Inversion (Sparse Spike) . Seeking to overcome the limited bandwidth problem discussed above, the Sparse Spike methods emerged. Considering the deconvolution algorithms used, they will try to estimate a reflectivity comprised of sparse deltas, what will ensure an increase in the estimated frequency band (Ulrych and Sacchi, 2005). Sparsity will be provided through minimization of a norm different from the $L2$, which is used by the above deconvolution method seeking to whiten the spectrum, i.e., estimate a random series.

Considering the inversion methods, a bigger band will lead to less uncertaninty results. In addition to the increase in bandwidth related to sparsity in the estimated reflectivity, the method discussed can increase the band through *a priori* information introduction. This bandwidth increase is associated with higher complexity algorithms and consequently a rise in computational cost, which is often irrelevant compared to the robustness of results.

Ulrych and Sacchi (2005) introduced the method which involves searching for a blocky impedance model through minimization of the following objective function:

$$\boldsymbol{J} = \kappa \boldsymbol{J}_c + \frac{1}{2} \| \frac{1}{\sigma} (\boldsymbol{W}\boldsymbol{r} - \boldsymbol{s}) \|^2 + \frac{1}{2} \| \boldsymbol{N}^{-1} (\boldsymbol{C}\boldsymbol{r} - \boldsymbol{\mathfrak{N}}) \|^2, \tag{6}$$

where κ ponders the sparsity level of reflectivity, $\boldsymbol{J_c}$ is the term responsible for regulating it, σ is an estimate of the standard deviation of the noise in the data, $\boldsymbol{W}$ is the convolution matrix representing the wavelet, $\boldsymbol{r}$ is the reflectivity vector, $\boldsymbol{s}$ is the trace, $\boldsymbol{C}$ is an integration operator and $\mathfrak{N}$ is the natural logarithm of the normalized impedance or double the cumulative sum of reflectivity. The term $\boldsymbol{N}$ is the diagonal matrix $\boldsymbol{N}_{(k,k)} = \varsigma_k$ responsible for introduction of the restrictions such that information reliability must be pondered by associating large (unreliable) or small (great confidence) values for ς_k, so that the latter can be seen as a vector of the uncertainties associated with *a priori* information.

In the original work, there are four options for choosing $\boldsymbol{J_c}$, the one used here was the Cauchy norm, given by:

$$\boldsymbol{J}_{Cauchy} = \frac{1}{2}\sum_k \ln(1 + \frac{\boldsymbol{r}_k^2}{\varsigma_k^2}). \tag{7}$$

Analyzing the Eq. (6) it can be noted that its right side has three terms. The first concerns the sought sparse reflectivity, the second ensures that the estimated seismogram is similar to the real and the last ensures that the *a priori* information is honored.

Minimization of the objective function can be accomplished through optimization. Here the latter was adopted.

Finally it is noteworthy that the choice of parameters κ and σ, as well as the initial model must be carefully made in order to ensure convergence to the solution or to the global minimum of the objective function.

3 RESULTS AND DISCUSSION

The results can be divided into two stages, the first was analysis of the wavelet estimation methods the second of inversion techniques.

For the tests, a synthetic model was used, specifically the Marmousi, shown in Fig. 1. It was created in 1988 by the French Petroleum Institute, based on a geological profile in the Kwanza basin in North Quenguela. It presents a complex geological environment and is one of the most widespread models for seismic analysis, modelling and inversion, and for that reason it was chosen.

In the following subsections the results will be presented together with a discussion about them.

Wavelet estimation. As stated above, the tests to estimate the wavelet were performed assuming data modeled with minimum and zero phase Ricker wavelets. Figure 2 represents the results for when noise is absent.

For the Hilbert transform method, the results are shown in figures 3 and 4. As expected, better results are reached for the minimum phase situation. For the zero phase case even after APC the estimated wavelet does not have a shape similar to the true. It was discussed in the subsection 2.1 that this method depends on the assumption that the wavelet is minimum phase, which justifies the better results.

Noise introduction to the trace worsens the algorithm's performance, generating artifacts in the estimated wavelet shape and for the zero phase case, it leads to unsatisfactory results.

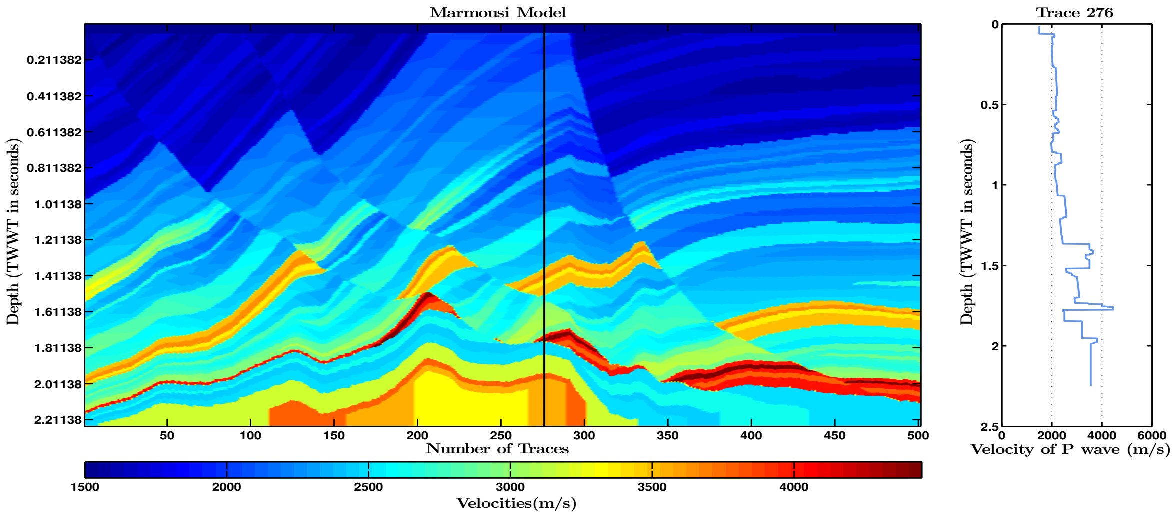

Figure 1: Marmousi model and trace 276 in time depth.

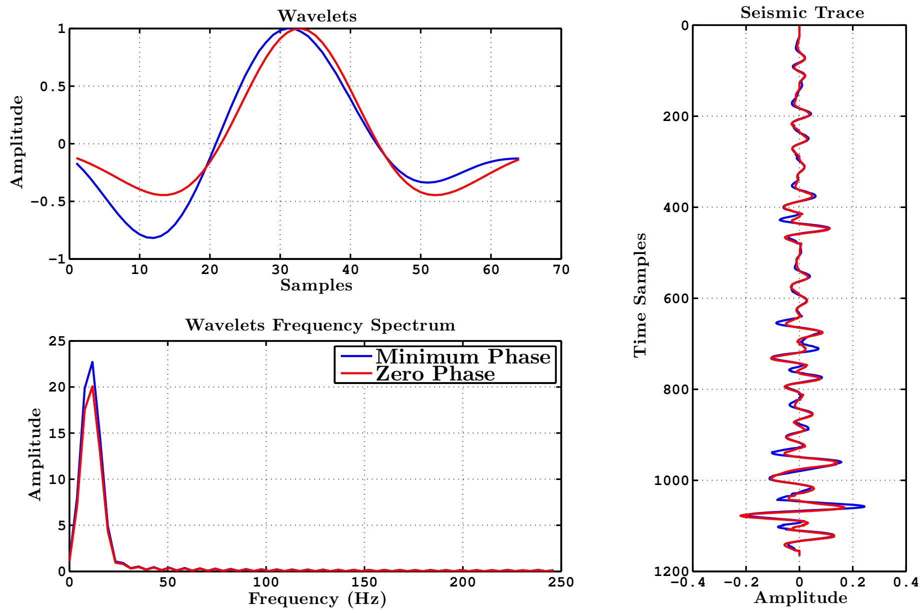

Figure 2: Ricker wavelets with zero and minimum phase, their correspondent frequency spectra and the modelled traces.

Analyzing the results of estimation with Smoothing method, shown in Figs. 5 and 6, it can be noted that the presence of noise doesn't have as much influence on the results as for the previous method. Also, as there is not a dependency of results with assumptions about the phase, the APC correction proved more effective to this technique. However, it can be seen that the best estimates for the shape and phase spectrum correspond to the zero phase case.

It was observed that better performance of the method is achieved when guessing a linear phase spectrum for the estimated wavelet before APC. The best results for the zero phase case were obtained when considering small values inside the interval [1,2] for the phase spectrum; for the minimum phase case, larger values achieved more success.

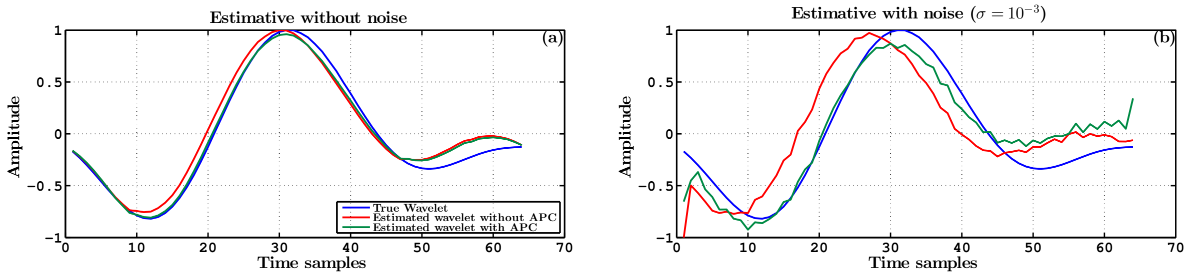

Figure 3: Minimum phase Ricker wavelet estimation with Hilbert transform method. (a) Without and (b) with noise.

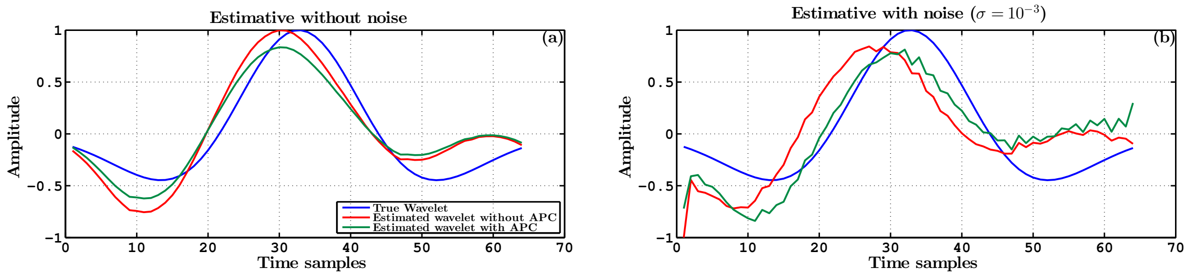

Figure 4: Zero phase Ricker wavelet estimation with Hilbert transform method. (a) Without and (b) with noise.

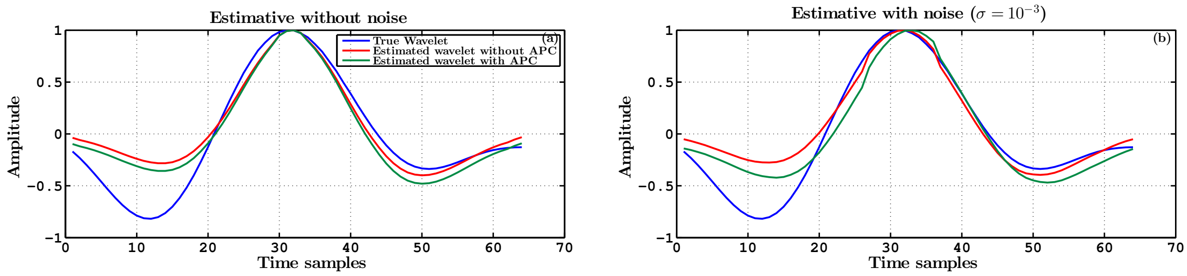

Figure 5: Minimum phase Ricker wavelet estimation with Smooth spectra method. (a) Without and (b) with noise.

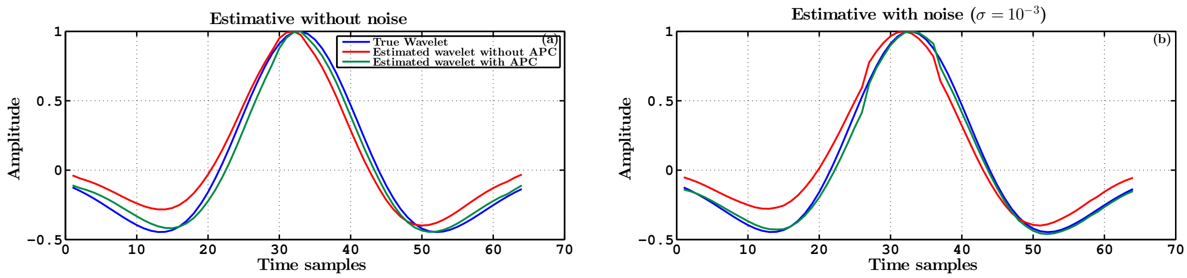

Figure 6: Zero phase Ricker wavelet estimation with Smooth spectra method. (a) Without and (b) with noise.

Use of a moving average filter for spectral smoothing is largely responsible for attenuation of noise effects as the shape artifacts, seen in the results of the previous method.

In general it can be said by looking at the results that the second method can achieve an estimate with less uncertainty for the wavelet when analyzed with respect to noise and the situations of minimum and zero phase. An option for improvement of the results of both techniques would be to use a more suitable algorithm for correction and estimation of the phase spectrum as the good estimatives were obtained for the amplitude spectrum, as shown in Figs. 7, 8, 9 and 10.

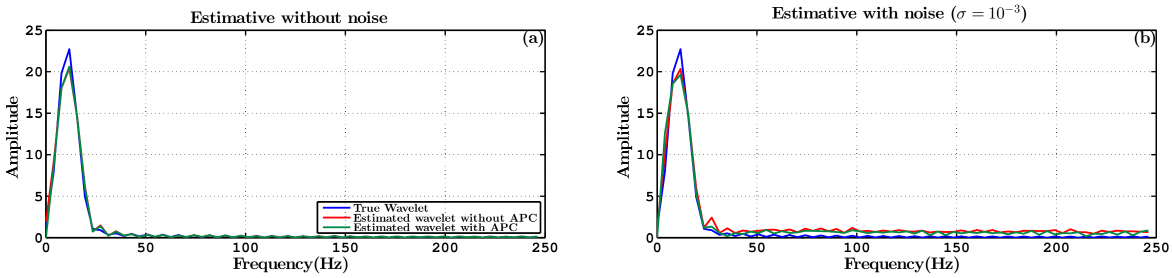

Figure 7: Amplitude spectrum of minimum phase Ricker wavelet estimated with Hilbert transform method. (a) Without noise and (b) with.

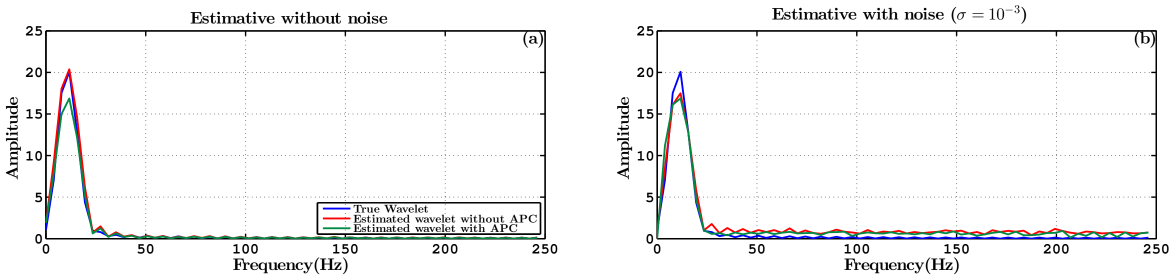

Figure 8: Amplitude spectrum of zero phase Ricker wavelet estimated with Hilbert transform method. (a) Without noise and (b) with.

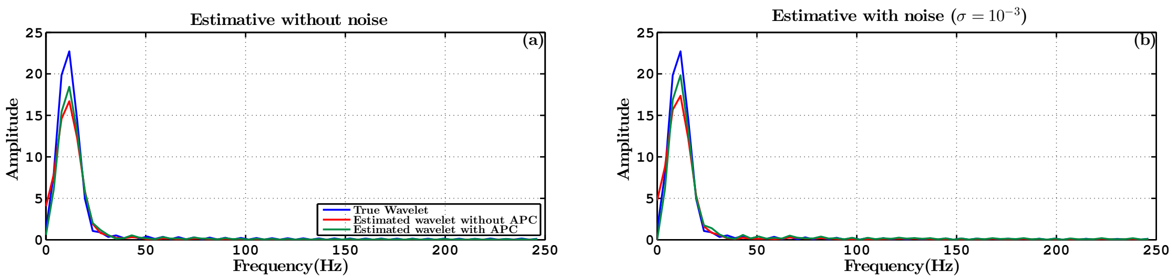

Figure 9: Amplitude spectrum of minimum phase Ricker wavelet estimated with Smooth spectra method. (a) Without noise and (b) with.

Inversion. Synthetics used to analyze the performance and sensitivity of inversion algorithms were generated using a zero phase wavelet as previously stated. For simplicity, density was assumed to be constant so that the estimates, even when referred to as impedances, are the velocities of wave propagation P. Results are presented and discussed below. First the performance of the methods for situations when noise is present is analyzed, then their sensitivity to the estimated wavelets is observed.

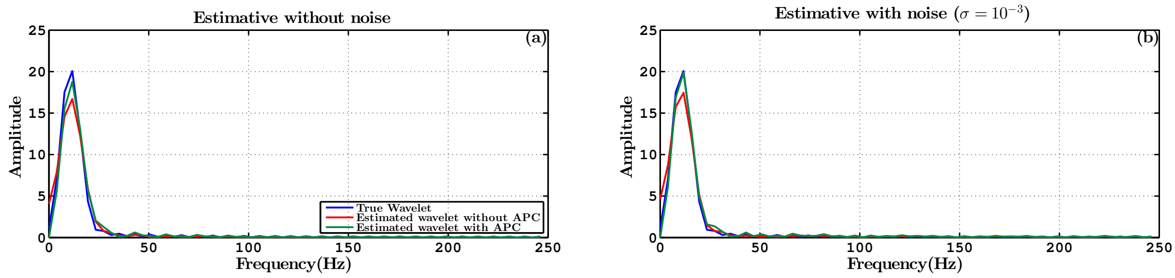

Figure 10: Amplitude spectrum of zero phase Ricker wavelet estimated with Smooth spectra method. (a) Without noise and (b) with.

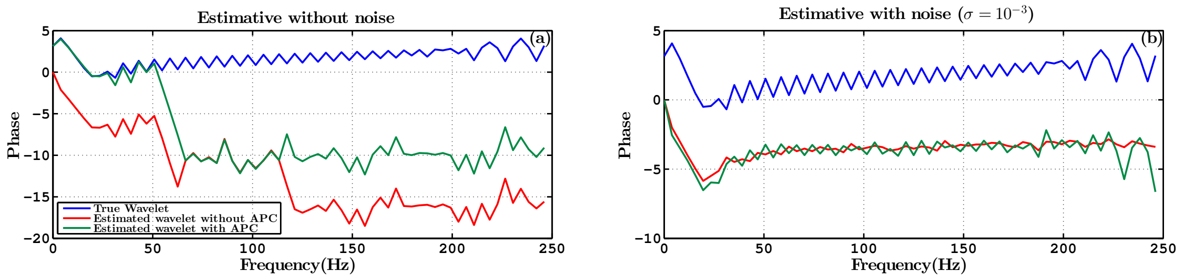

Figure 11: Phase spectrum of minimum phase Ricker wavelet estimated with Hilbert transform method. (a) Without noise and (b) with.

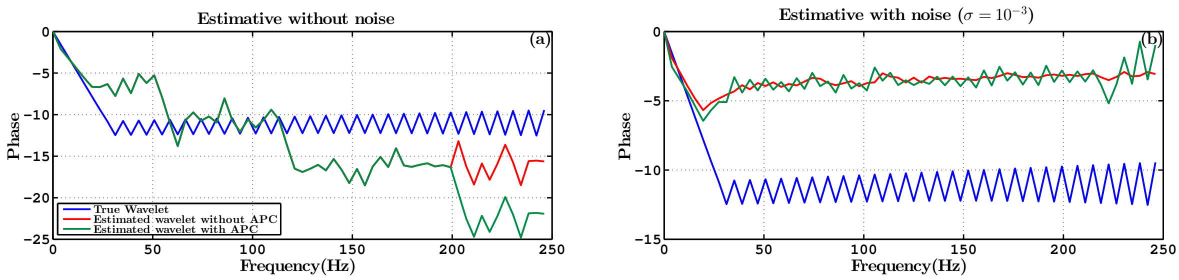

Figure 12: Phase spectrum of zero phase Ricker wavelet estimated with Hilbert transform method. (a) Without noise and (b) with.

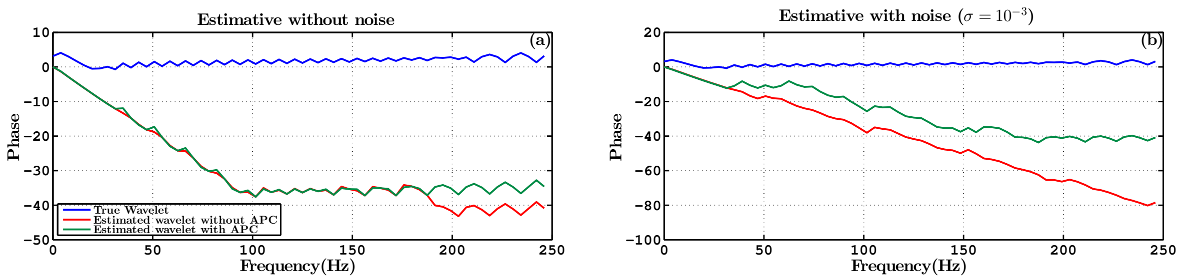

Figure 13: Phase spectrum of minimum phase Ricker wavelet estimated with Smooth spectra method. (a) Without noise and (b) with.

Considering the Narrow band method for the case where the real wavelet is used, the Figs. 15 and 16 show that problem is solved even in the presence of noise, although the results are not as good as in the absence of it. It can be concluded that if the wavelet is well estimated,

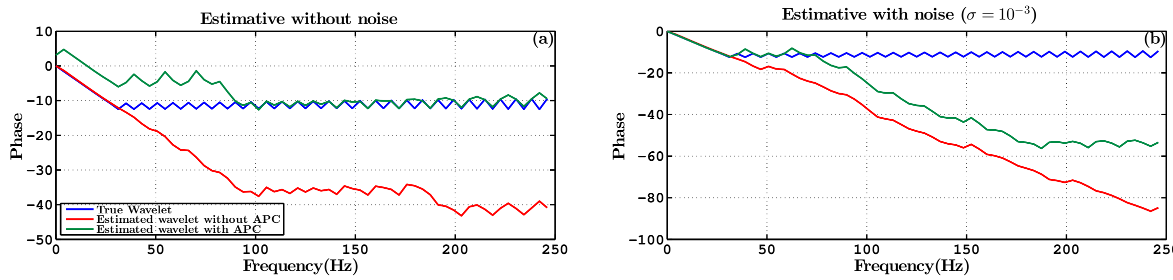

Figure 14: Phase spectrum of zero phase Ricker wavelet estimated with Smooth spectra method. (a) Without noise and (b) with.

even in the presence of noise good results are obtained. In Fig. 16 this idea is well represented, showing the similarity between the estimated and synthetic models.

It is necessary to discuss here the parameter μ, which represents introduction of information about the noise. A good estimate of it will guarantee results with less uncertainty. One can think of μ as a regularization parameter to the inversion since the results depend on it. It was observed that for the tests with noise addition $\sigma = 10^{-3}$, a value which produces good results is $\mu = 0.1$. This constant will also act on situations with estimated wavelets, even in the absence of noise, as will be seen ahead, once these wavelets carry an amount of uncertainty associated with the estimation process which will be interpreted as noise by the algorithm.

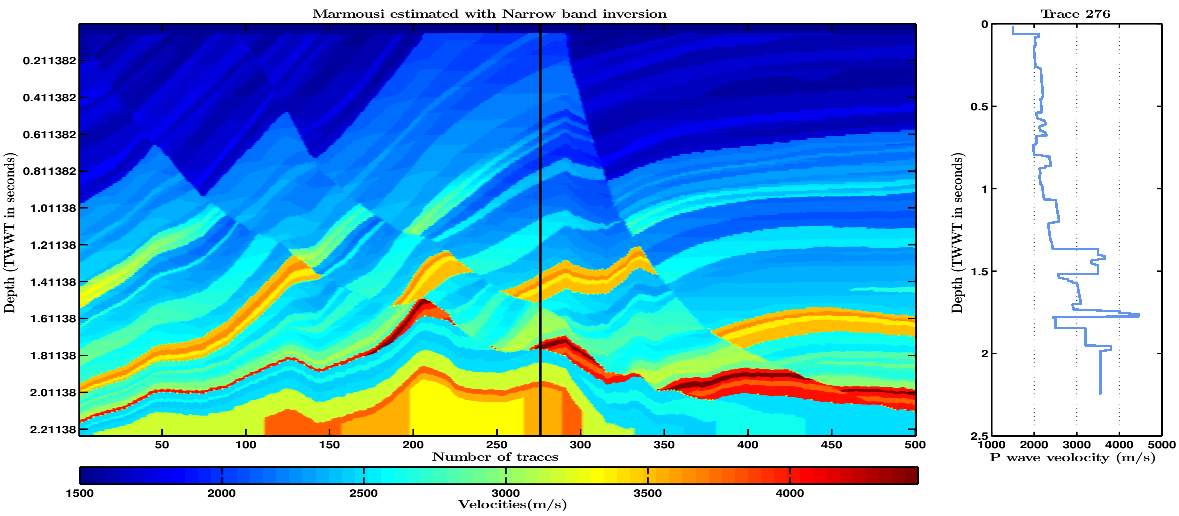

Figure 15: Narrow band inversion with true wavelet and without noise addition.

The Use of an estimated wavelet, contrary to the above results, will not produce such reliable results as previously discussed. Figures 17, 18, 19 and 20 show the results to the sensitivity of Narrow band algorithm to estimated wavelets. The striking effect in all cases is the discrepancy between amplitudes, although interfaces have been well delimitated.

This effect on the other hand is related to the previously mentioned problem of lack of low frequencies, in fact, during the tests it was observed that lowering the central frequency of the wavelet for $5Hz$ produces better results. It must be noted that the method has not failed, it succeeded in identifying the most significant subsurface interfaces.

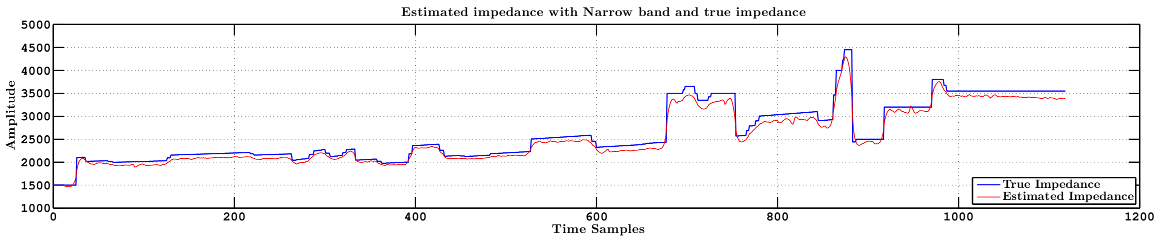

Figure 16: Narrow band inversion with true wavelet and with noise ($\sigma = 10^{-3}$) addition for trace 276.

For results of the tests with introduction of noise together with an estimated wavelet, the noise does not show significative influence when a good estimate for μ was used. This confirms the earlier statement that the method can solve effects related to the presence of noise. However, the results generated by the use of a wavelet estimated by Hilbert method are more sensitive to the noise, requiring larger values for μ.

Comparison of the results using different estimation techniques again shows that the results of Smooth are superior (Figs. 17 and 19). As a final statement regarding Narrow band inversion results, it must be pointed out the need to estimate a value for μ even in the absence of noise. This relates to numerical noise introduced by the use of the Discrete Fourier Transform algorithm (calculation of the deconvolution step is performed in the frequency domain).

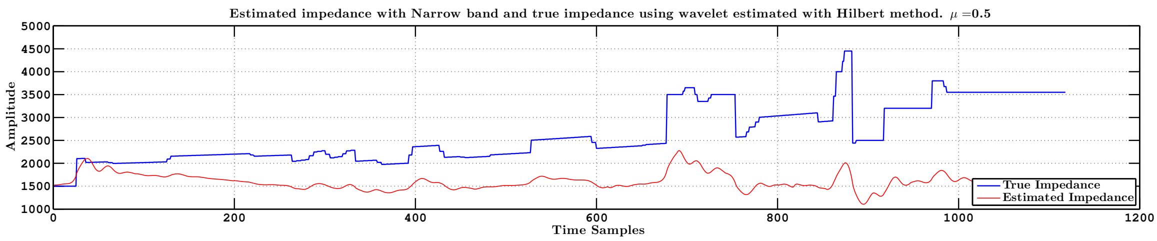

Figure 17: Narrow band inversion with wavelet estimated using Hilbert Transform method for trace 276.

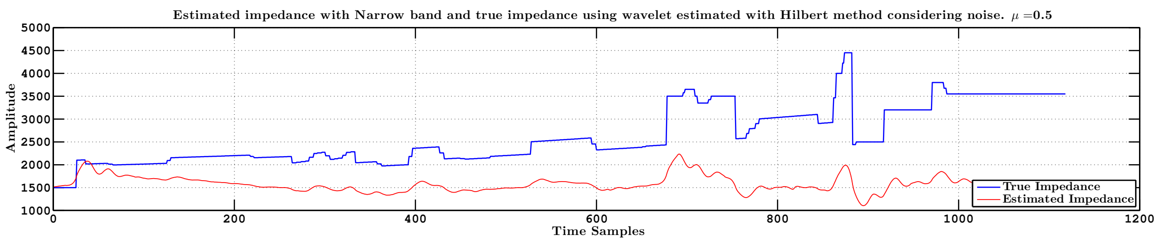

Figure 18: Narrow band inversion with wavelet estimated using Hilbert Transform method and noise ($\sigma = 10^{-3}$) addition for trace 276.

Looking now at the results of the Bayesian method, Figs. 21, 22 and 23 show the inversion results using the true wavelet. One can see that the method can obtain good estimates for impedance even if noise is present. In general, the algorithm converged well to all methods.

The first discussion regarding the Bayesian inversion must be about the parameters chosen for the purpose of ensuring better results. In Eqs. (6) and (7) all parameters involved are

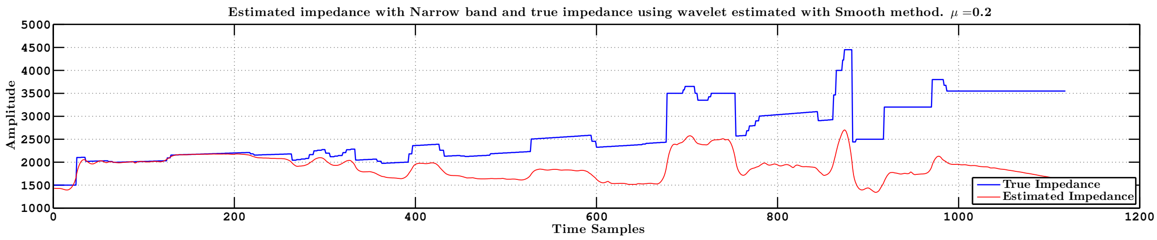

Figure 19: Narrow band inversion with wavelet estimated using Smooth method for trace 276.

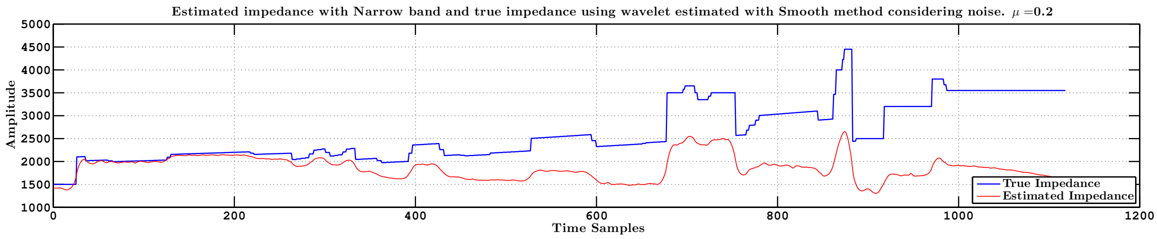

Figure 20: Narrow band inversion with wavelet estimated using Smooth method and noise ($\sigma = 10^{-3}$) addition for trace 276.

shown, however the ones actually used as input to the algorithm are: an estimate of the standard noise deviation (σ) and the uncertainty vector associated with the *a priori* information (ς). Nonetheless, for each test performed a new set of parameters should be selected once each setting possesses different levels of uncertainty.

Complementing the discussion in the paragraph above, it is necessary to talk about the use of an initial impedance model. The one used here was generated by smoothing the true model. It is responsible for introducing *a priori* information and consequently increase bandwidth, therefore ensuring better convergence. As previously discussed, the low frequency components carry relevant information about trends of subsurface geology and thereby, use of an initial model allowed incorporation of these components and hence of the actual impedance trend, constraining the results as observed in the tests.

By observing the influence of noise, Figs. 23, 26 and 27 prove the method as more efficient than the previous to deal with these situations, even when using estimated wavelets. A characteristic effect of noise are the artifacts with oscillatory behavior in the calculated impedance, easily noticeable in the last two above-mentioned figures which however, does not prevent the identification of the main interfaces. Finally it is reiterated that the good performance of the method is dependent on the above mentioned parameters so that, good results can only be achieved through a good estimate of these.

Regarding the performance tests with estimated wavelets, again this method was superior to the Narrow Band, achieving good results for the two estimation techniques. In tests with the noise, however, the presence of the artifacts discussed is significant, especially when using wavelets estimated by Hilbert (Fig. 26). By observing the results, once again the Smooth appeared as the best alternative, although in the tests discrepancies were not as evident, showing that the Bayesian can solve for uncertainties associated with the estimation of the wavelet. Figures 24 and 25 illustrate the differences for results with both estimated wavelets.

As a final discussion about this method, the artifacts in Fig. 21 will be treated. They emerge as a consequence of using optimization methods, in this case Conjugate Gradient, to search for solutions. As the inversion is performed trace by trace, situations where the method cannot converge well or when errors associated with numerical approximation appear, might happen, justifying the artifacts.

Comparing the above results, it was noted that Bayesian inversion has guaranteed less estimation errors, proving superior to Narrow band for all tests. However, it is necessary to mention that its computational cost is greater, in some cases requiring more than 100 iterations so that convergence to acceptable results is achieved. In any case, its sensitivity to the uncertainties introduced by noise and estimated wavelet is smaller, what can justify its choice over the Narrow band.

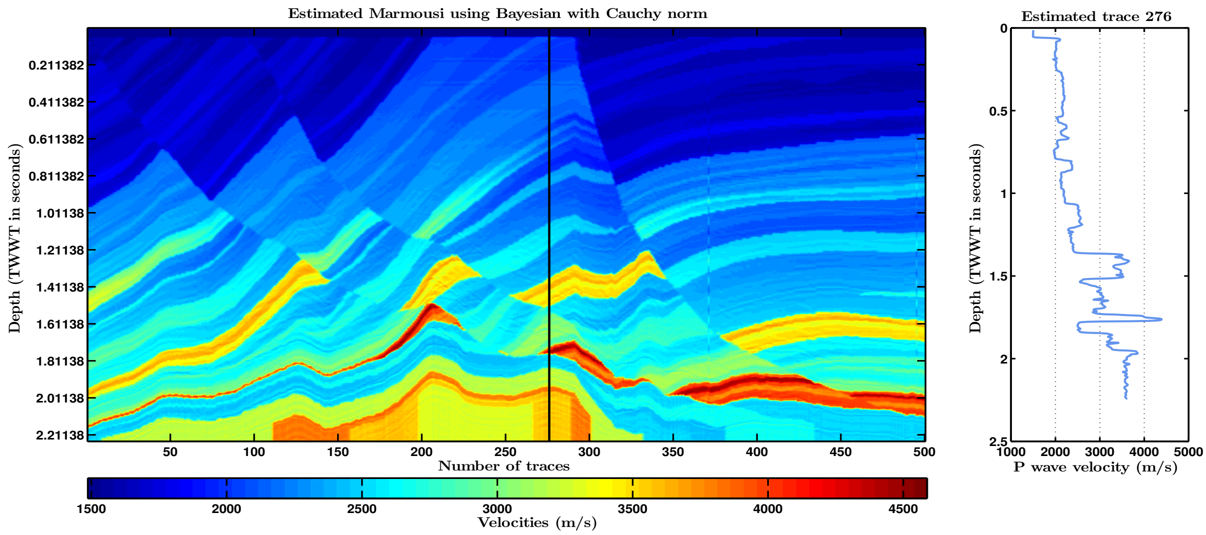

Figure 21: Marmousi estimated with Bayesian inversion for full seismogram and 1000 iterations per trace.

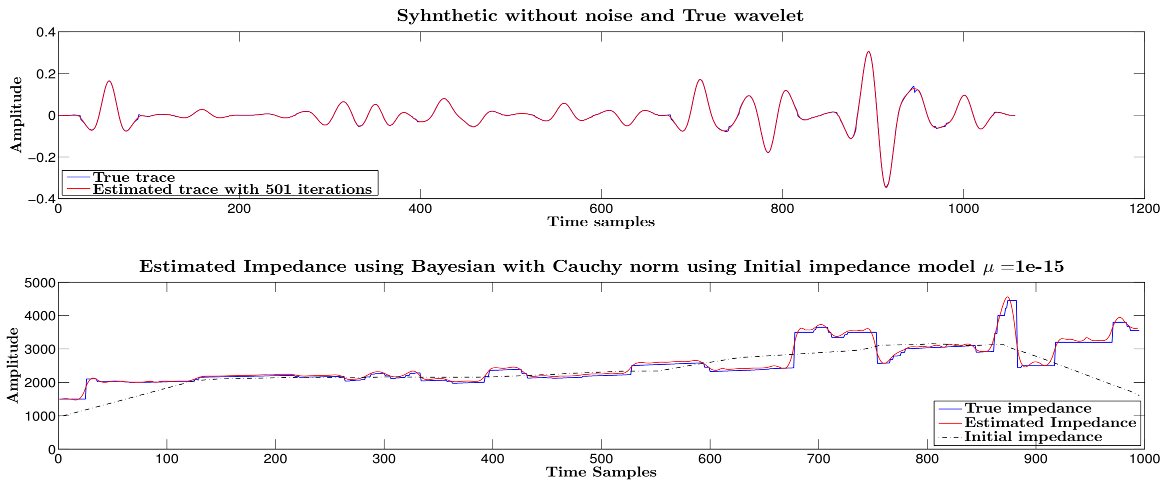

Figure 22: Marmousi estimated with Bayesian inversion.

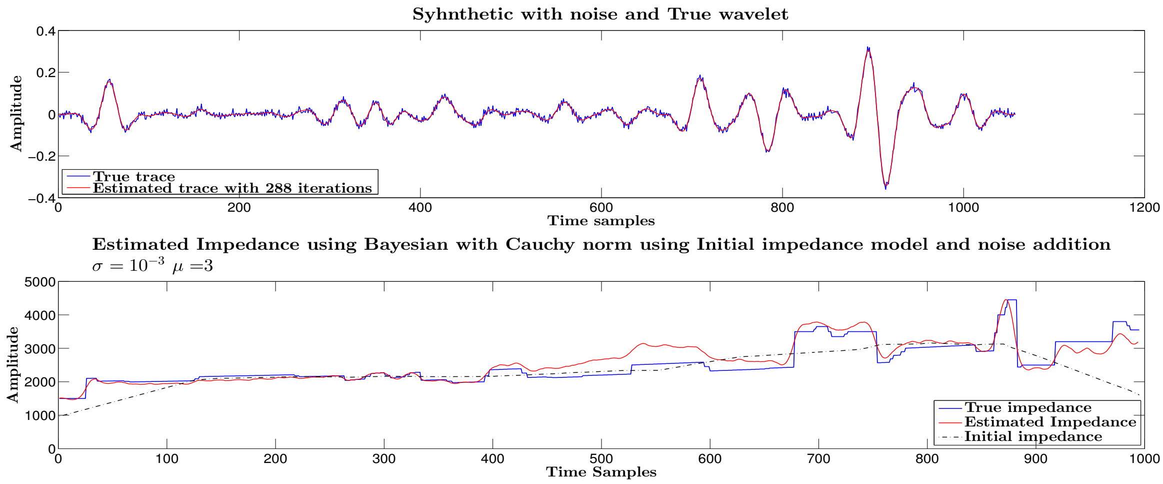

Figure 23: Marmousi estimated with Bayesian inversion and noise $\sigma = 10^{-3}$.

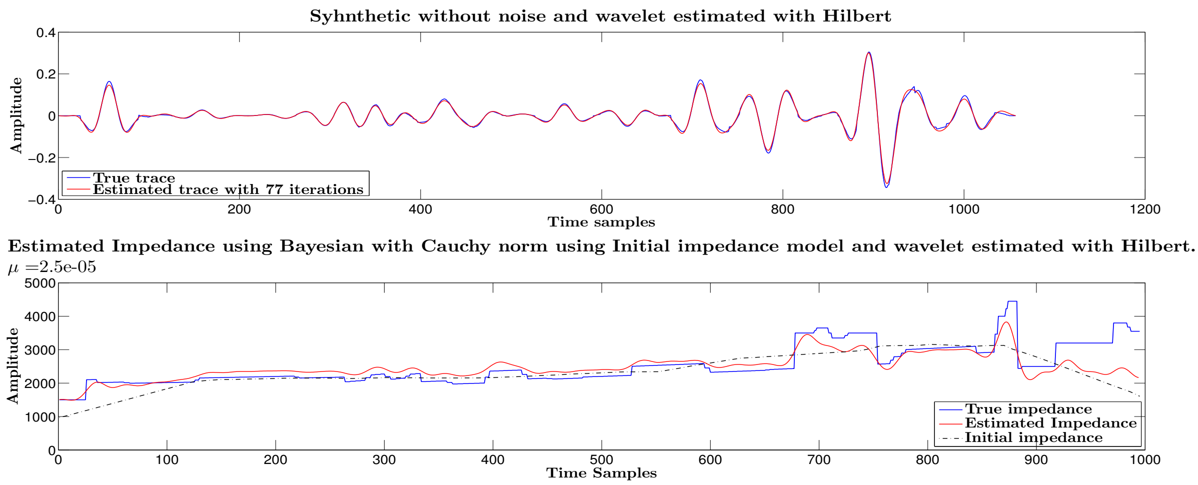

Figure 24: Marmousi estimated with Bayesian inversion using Hilbert wavelet estimation method without noise.

4 CONCLUSIONS

Arising from the need to obtain estimates of the parameters associated with subsurface geology mainly with the purpose of exploration, geophysical methods have increasingly gained space once it is an indirect way and therefore more economical.

In the case of the seismic method, probably the most popular, information on acoustic properties can be obtained using seismic inversion algorithms, among which two were discussed here, Narrow band and Bayesian.

The proposed methodology aimed to test the sensitivity of the inversion methods considering estimated wavelets and noise.

Comparison of the wavelet estimation methods showed that Smooth produces better results. However, the amplitude spectrum of the estimated wavelets showed that both can solve the problem. Phase spectrum analysis indicates otherwise, so that even after APC phase correction the spectrum was not well approximated. Techniques that are able to produce better estimates

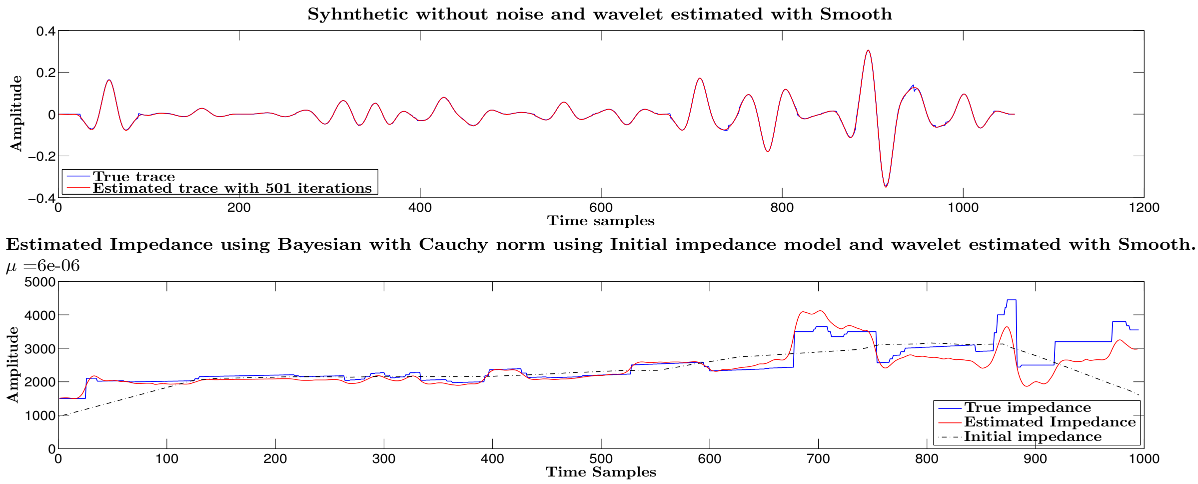

Figure 25: Marmousi estimated with Bayesian inversion using Smooth wavelet estimation method without noise.

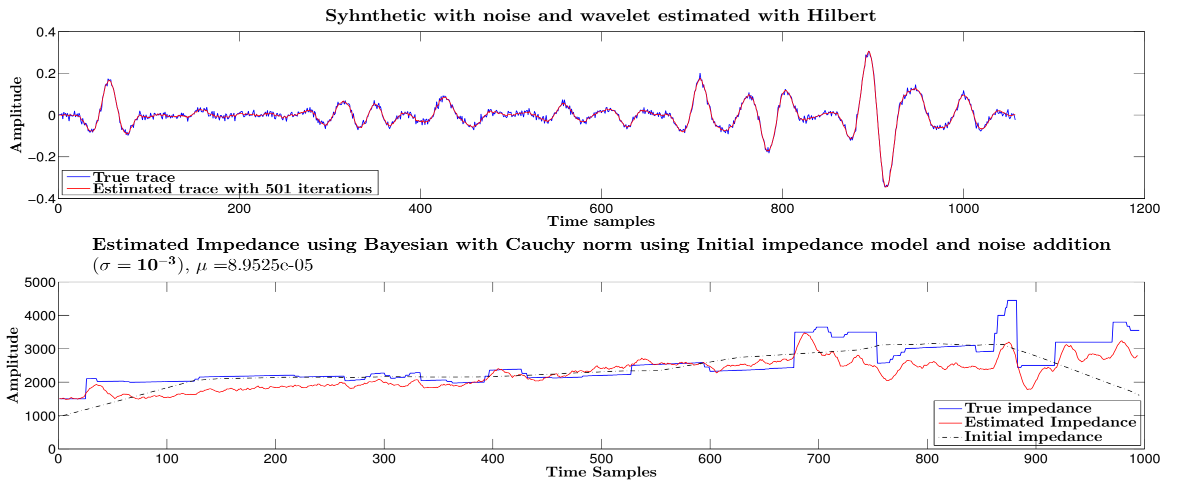

Figure 26: Marmousi estimated with Bayesian inversion using Hilbert wavelet estimation method with noise ($\sigma = 10^{-3}$).

to the phase spectrum would obtain in this way, superior results.

For inversion, the tests indicated Bayesian as the most efficient once it converged to the real model in all tests, even in the most complex situation with wavelet estimated and noise, though artifacts appeared in the inverted data. Anyway this method produces good estimates of the impedance model, delimiting all the interfaces and solving the uncertainties related with both the noise and an estimated wavelet.

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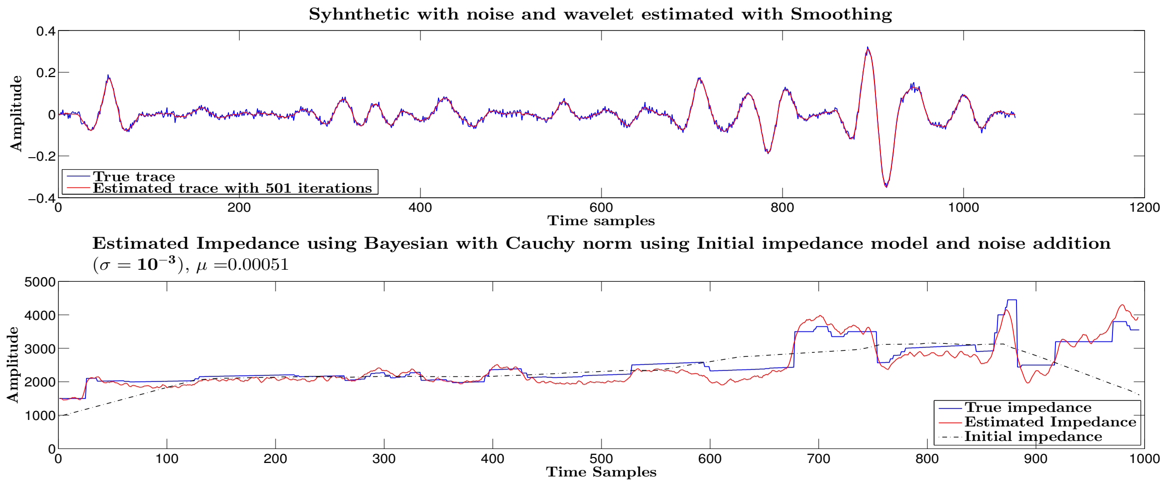

Figure 27: Marmousi estimated with Bayesian inversion using Smooth wavelet estimation method with noise $(\sigma = 10^{-3})$.

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