FINITE ELEMENT ANALYSIS OF NOTCH-ROOT STRESS/STRAIN CONCENTRATION FACTOR APPLIED TO WELDED STRUCTURES

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Abstract. *Stress/strain concentration effects and residual stresses must be considered in mechanical analyses of welded structures. There is a vast literature on linear-elastic stress concentration factors* K_t*, which depend solely on the specimen/notch geometry and on the type of loading (Peterson, 1997). However, in the presence of plasticity at the notch root, the actual stress concentration factor* K_σ *becomes smaller than* K_t*, the stress gradient ahead of the notch root is affected by the stress redistribution at the yielding zone, and the strain concentration factor* K_ε*, which strongly affects the fatigue life predicted by the* εN *method, can become much larger than* K_t*. Several approximated models have been proposed to estimate the elastic-plastic behavior at the notch root, which in many cases may provide rough but reasonable* K_σ *and/or* K_ε *values. However, the differences among the fatigue life predicted by such* K_σ/K_ε *estimation rules can be unacceptably large for practical purposes. In addition, they do not account for the geometrical changes at the notch root under large displacements, leading to further errors. In this work, 2D finite element analyses are carried out to evaluate elastic-plastic stress and strain distributions around notch roots. The proposed methodology can be applied to improve the analyses of welded structures.*

Keywords: *Elastoplastic analysis of notches, Stress/strain concentration, Finite elements*

1 INTRODUCTION

The εN is a modern fatigue design method (Dowling, 1993; Fuchs and Stephens, 1980; Rice, 1988; Sandor, 1972) in which Neuber is the most used equation to correlate the nominal stress σ_n and strain ε_n ranges with the stress σ and strain ε ranges they induce at a notch root. The Neuber equation states that the product between the stress concentration factor K_s (defined as σ/σ_n) and the strain concentration factor K_e (defined as $\varepsilon/\varepsilon_n$) is constant and equal to the square of the geometric stress concentration factor K_t, thus

$$K_t^2 = \frac{\Delta\sigma \cdot \Delta\varepsilon}{\Delta\sigma_n \cdot \Delta\varepsilon_n} \tag{1}$$

Some authors prefer to use K_f, the fatigue concentration factor, instead of K_t in this Eq. (1) (Topper et al., 1969). When the nominal stresses are much lower than S_{Yc}, the cyclic yielding strength, it is common practice to model them as Hookean and, therefore, to use the Neuber equation in the simplified form:

$$K_t^2 = \frac{\Delta\sigma \cdot \Delta\varepsilon \cdot E}{\Delta\sigma_n^2} \tag{2}$$

Ramberg-Osgood is one of many empirical relations that can be used to model the cyclic response of the materials. Its main limitation is to not recognize a purely elastic behavior, and its main advantage is its mathematical simplicity. It can be used to describe the stress and strain ranges at the notch root by

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H_c}\right)^{1/h_c} \tag{3}$$

in which E is the Young's modulus, H_c is the hardening coefficient and h_c is the hardening exponent of the cyclically stabilized σε curve.

Eliminating $\Delta\varepsilon$ from Equations (2) and (3), σ_n is directly related to $\Delta\sigma$ by

$$(K_t \Delta\sigma_n)^2 = \Delta\sigma^2 + \left[2E/(2H_c)^{1/h_c}\right] \cdot \Delta\sigma^{(h_c+1)/h_c} \tag{4}$$

However, the above equation is logically incongruent, since it treats the very same material by two different constitutive models: Ramberg-Osgood at the notch root and Hooke at the nominal region. Moreover, this procedure can generate significant numerical errors even when the nominal stresses are much lower than the material cyclic yielding strength, as it will be discussed next.

2 CORRECTIONS FOR NON-LINEAR NOMINAL STRESS WITH RESIDUAL STRESS

To avoid the errors induced by the simplified (Hookean) Neuber approach, it is necessary to use the Ramberg-Osgood model to describe not only the stresses at the notch root, but also to describe the nominal stresses, writing (Castro and Meggiolaro, 2009)

$$\Delta\varepsilon_n = \frac{\Delta\sigma_n}{E} + 2\left(\frac{\Delta\sigma_n}{2H_c}\right)^{1/h_c} \tag{5}$$

In this case, given the nominal stress range $\Delta\sigma_n$, the stress range at the notch root $\Delta\sigma$ can be calculated from Equations (1), (3), and (5), yielding

$$K_t^2\left(\Delta\sigma_n^2 + \frac{2E\Delta\sigma_n^{(h_c+1)/h_c}}{(2H_c)^{1/h_c}}\right) = \Delta\sigma^2 + \frac{2E\Delta\sigma^{(h_c+1)/h_c}}{(2H_c)^{1/h_c}} \quad 6)$$

Figure (1) shows a comparison between the stress K_σ and strain K_ε concentration factors predictions made by the simplified Neuber approach using Eq. (4), and the general (corrected) ones obtained using Eq. (6), for a SAE 1009 steel when the notch root has a K_t of 3.

As it can be seen in the Fig. 1, for nominal stress amplitudes σ_n smaller than $0.5 \cdot S_{Yc}$ both predictions result in roughly the same concentration factors. However, for larger nominal stress values the predictions diverge, and the classical Neuber approach wrongfully predicts ever increasing strain concentration factors K_ε and even stress concentration factors K_σ smaller than unity, a complete non-sense.

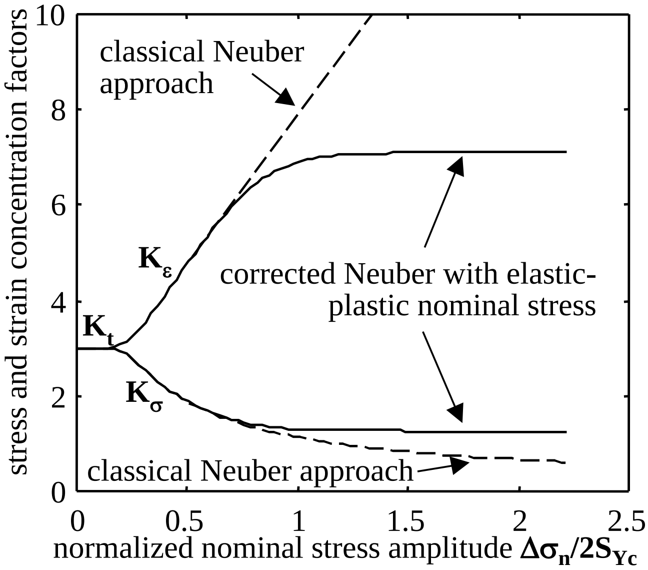

Figure 1. Stress and strain concentration factors calculated by both Neuber approaches (SAE 1009 steel, K_t = 3)

Note also that the general (elastic-plastic) Neuber formulation implies that both K_σ and K_ε tend to a constant value as the nominal stress amplitude is increased. According to Neuber's equation, any material that follows Ramberg-Osgood's equation presents this same behavior. These constant values can be calculated from Eq. (6), assuming that the elastic component of both nominal and notch-root strains are negligible compared to the respective plastic strain components, resulting in:

$$K_t^2\left(\frac{E\sigma_n^{(h_c+1)/h_c}}{(H_c)^{1/h_c}}\right) = \frac{E\sigma^{(h_c+1)/h_c}}{(H_c)^{1/h_c}} \Rightarrow K_\sigma = \frac{\sigma}{\sigma_n} = K_t^{2h_c/(1+h_c)} \tag{7}$$

From Eq. (7) and using that $K_\sigma \cdot K_\varepsilon = K_t^2$, then lower and upper bounds can be calculated for K_σ and K_ε

$$\begin{cases} K_t^{2h_c/(1+h_c)} \le K_\sigma \le K_t \\ K_t \le K_\varepsilon \le K_t^{2/(1+h_c)} \end{cases} \tag{8}$$

In addition, it is found that the errors in $\Delta\sigma$ are not a strong function of K_t, being mainly dependent on the nominal stress range $\Delta\sigma_n$. These errors tend to slightly decrease as K_t is increased, reaching a constant value for very high stress concentration factors. Therefore, the behavior shown for K_t equal to 3 can be extended to any stress concentration factor.

2.1 Residual Stress

Considering a residual stress (σ_{res}) presents on notch-root, it is not possible to add directly this residual stress with nominal stress to obtain the new nominal stress, because the stress/strain relationship must follow the Neuber equation. Instead, an equivalent nominal stress for residual stress (σ_{n_res}) must be obtained from equation:

$$K_t^2(\sigma_{n\_res}^2 + \frac{E\sigma_{n\_res}^{(h_c+1)/h_c}}{(H_c)^{1/h_c}}) = \sigma_{res}{}^2 + \frac{E\sigma^{(h_c+1)/h_c}}{(H_c)^{1/h_c}} \tag{9}$$

then the correct new nominal stress can be obtained:

$$\sigma_{n\_new} = \sigma_{n\_res} + \sigma_n \tag{10}$$

The σ_{n_res} will produce the same value of σ_{res} when the σ_{n_new} is loaded. If a fatigue analysis is performance, σ_{n_new} should be the first load applied in the material, representing an existing residual stress.

It is reasonable to assume that this interesting result can be efficiently verified by non-linear finite element (FE) calculations. However, this comparison must be carefully made, because even after considering elastic-plastic nominal stresses in the modeling, the numerical errors inherent to FE calculations can still lead to quite wrong predictions, as discussed next.

3 FINITE ELEMENT SIMULATIONS

In this section, numerical FE analyses are performed to calculate the notch-root stresses and strains. The details of the geometries are presented, and the results are plotted as K_σ and K_ε graphs to compare the FE analysis and Neuber predictions. Also, residual stresses are simulated around the notch.

In Miranda et. al. (2004), five geometries and four different constitutive models were employed to obtain FE results to compare to Eq. (6). First, a sensitivity analysis using the four constitutive models based on the Ramberg-Osgood equation was considered. These models have in common the same Young modulus of 210GPa and yielding strength of 400MPa. The monotonic hardening exponents, h, were fixed as 0.05, 0.10, 0.20 and 0.40, and the monotonic hardening coefficients, H, were computed as 546, 745, 1386 and 4804 MPa, respectively (Fig. 2). The authors have concluded the last constitutive model minimize the FE calculation errors. Figure 4 reproduces a result obtained by Miranda et al. (2004), using geometry of Fig. 3 and comparing to Eq. (6).

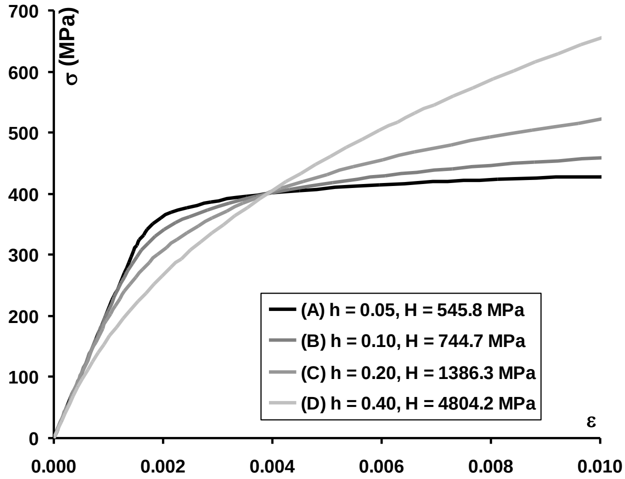

Figure 2. Constitutive models used in the sensitivity analysis in Miranda (2004)

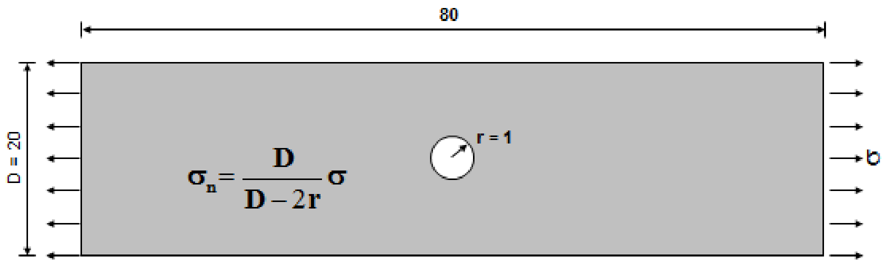

Figure 3. One of the geometry model used in Miranda (2004)

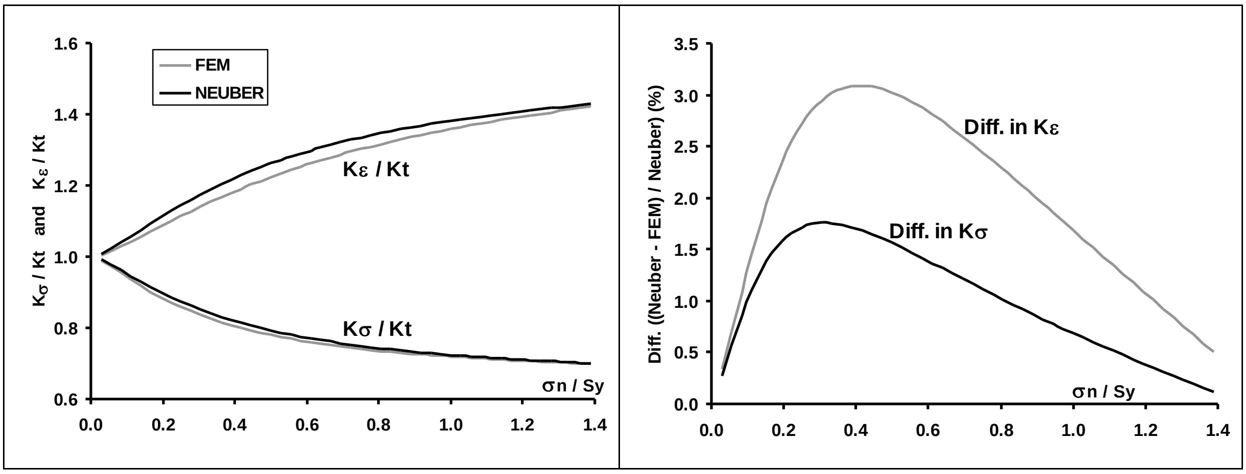

Figure 4. Concentration factors predicted using FE or Neuber's rule using Model (D) for Geometry in Fig. 3, and percentage differences between them (right) in Miranda (2004)

Once the finite element model is calibrated, the next step is to insert the residual stresses in the model and perform similar analysis presented above. To obtain some residual stresses, the analysis is performed in two steps. Initially a thermal steady state analysis is carried out with temperature equilibrium. This kind of simulation intends to simulate residual stress left by welding. Then this model is used to obtain a stress equilibrium that will result in an initial stress. These stresses are considered as residual stresses. Finally, a nominal stress is applied to the model to performance a static non-linear structural analysis. The results can then be compared with the equations of Section 2.1.

Two models were used in this work with different temperature variations. In the first model, model A, a temperature of 20^{o} C was applied on edges of hole (see Fig. 3), and 21^{o} C on external borders, considering an ambient temperature of 20^{o} C. In the second model, model B, was kept the temperature on edges of hole and imposed a 30^{o} C temperature on external borders. These two models resulted about 10 MPa and 90 MPa in stresses on the edge hole to model A and B, respectively. Residual stress distributions from the notch to the edge of the models are shown in Figure 5 and 6.

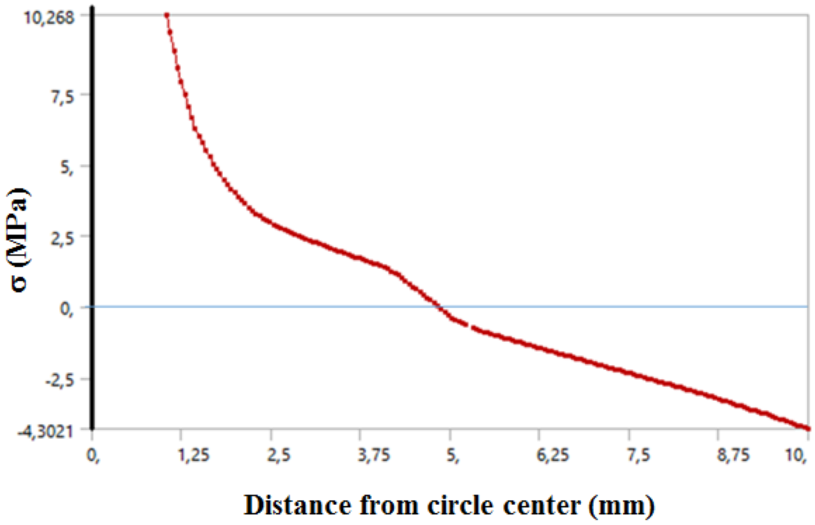

Figure 5. Residual stress distributions for model A

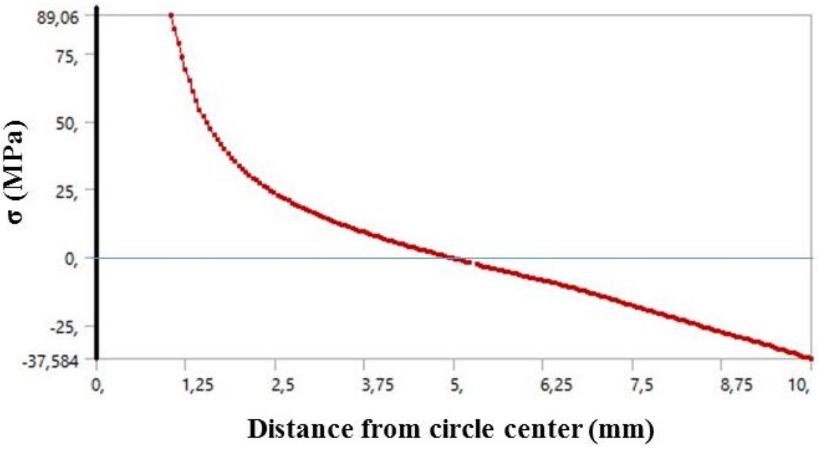

Figure 6. Residual stress distributions for model B

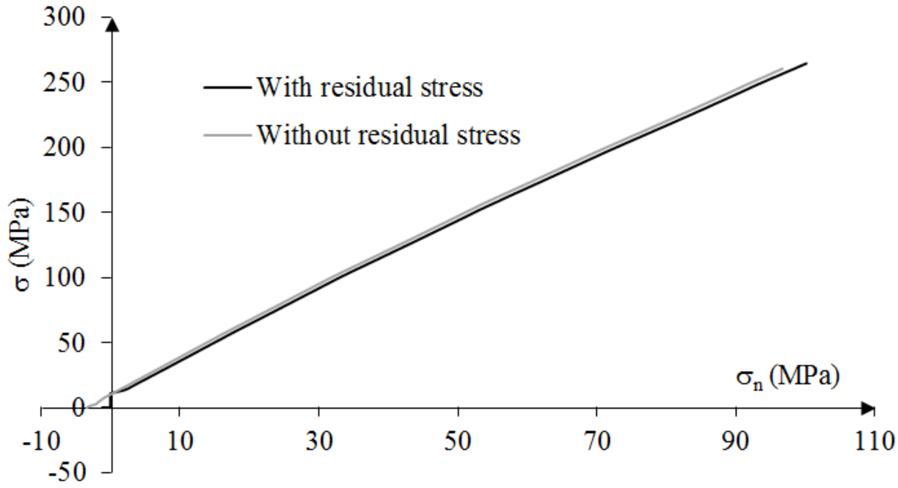

Figure 7. History of stress for model A

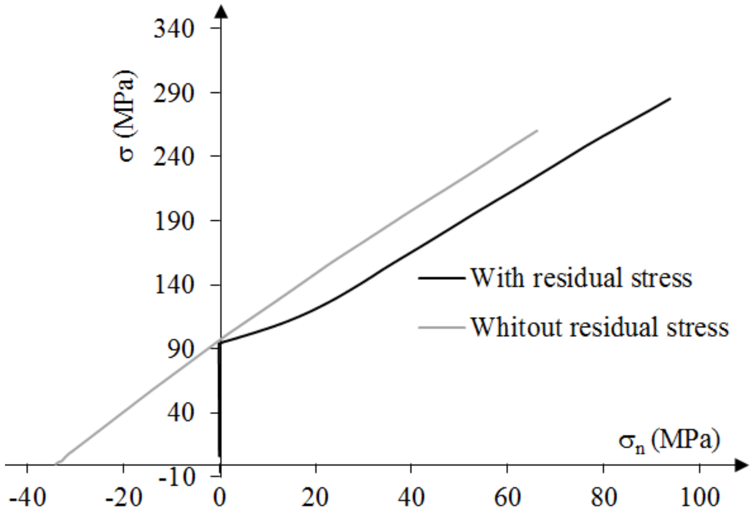

Figure 8. History of stress for model B

Using the models with residual stresses, a new tension is then applied to the models following the boundary condition shown in Fig. 3. Results are presented in 7 and 8. Note that, for small residual stress, the behavior of stress with and without residual stress is similar (Fig. 7). When the residual stress has a high value, there is a difference in the beginning of the curve and, after a point, the stress in the notch follows the same curve of stress without residual stress. The authors believe that this initial difference is because of a re-equilibrium of stress inside of the model.

4 CONCLUSIONS

In this work, numerical analyses were performed to compare predictions of notch root stress and strain concentration factors using Neuber's rule and FE calculations considering residual stress. Two plate geometries were considered, including a hole in its center. It was found that the correct use of Neuber's rule, considering elastic-plastic nominal stresses, is fundamental to obtain reasonable estimates of the concentration factors. Many claims of

Neuber's rule overestimating K_ε are in fact a result of inappropriate Hookean modeling of nominal stresses. The Neuber´s approach was extended to consider residual stress on the notch.

In addition, it is found that the reliable FE results under plane stress and high h_c agree well with Neuber's rule. The theoretical lower and upper bounds for K_σ and K_ε are verified in the calculations. This suggests that differences found in the literature between Neuber's and FE predictions might be in fact due to convergence problems in the FE analysis rather than to shortcomings of Neuber's strain concentration rule.

In the final, FE models shown that Neuber´s rule can be used with residual stresses when they involve small local plasticity. Otherwise, there is a re-distribution of stress that affects the history of stress on the notch. This re-distribution will be study soon.

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REFERENCES

Castro, J.T.P., Meggiolaro, M.A. *Fadiga - Técnicas e Práticas de Dimensionamento Estrutural sob Cargas Reais de Serviço, v. I: Iniciação de Trincas*. CreateSpace, Scotts Valley, CA.

Dowling, N., 1993. *Mechanical Behavior of Materials*. Prentice-Hall.

Fuchs, H., Stephens, R., 1980. *Metal Fatigue in Engineering*. Wiley.

Miranda, A.C.O., Lopes, A., Meggiolaro, M.A., Castro, J.T.P., Martha, L.F., 2004. Finite element analysis of notch-root stress and strain concentration factors under large deformations, *III Congresso Nacional de Engenharia Mecânica (CONEM),* Belém, Pará.

Neuber, H., 1961. Theory of Stress Concentration for Shear-Strained Prismatical Bodies With Arbitrary Nonlinear Stress-Strain Law. *Journal of Applied Mechanics*, Vol. 28, pp.544-550.

Peterson, R.E., 1974. *Stress Concentration Factors*. Ed. John Wiley & Sons, Inc., New York.

Rice, R. C. ed, 1988. *Fatigue Design Handbook*, *SAE*. Warrendale, PA.

Sandor, B., 1972. *Fundamentals of Cyclic Stress and Strain*. U. of Wisconsin.

Topper, T., Wetzel, R., Morrow, J., 1969. Neuber´s Rule Applied to Fatigue of Notched Specimens. *Journal of Materials*, vol. 4, No. 1, pp.200-209.