XXXVI Ibero-Latin American Congress on Computational Methods in Engineering
CILAMCE
Rio de Janeiro, 22-25 Nov 2015

TOPOLOGY OPTIMIZATION METHOD APPLIED TO THE DESIGN OF A VIBRATIONAL PIEZO-STRUCTURE SUBJECTED TO ACTIVE CONTROL LAW

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Abstract. *Recent developments in structure design for transient analysis applying the TOM have covered the optimal distribution of piezoelectric material over a host structure to actuate it and satisfy an objective function. Often, the desired objective involves following a certain pattern of transient response what may account with an active control law based on the piezoelectric material. Yet covering dynamic applications, this work is devoted to the design of a piezoelectric vibrational structure optimized to reduce its displacement response in time under an applied transient load. In this case, two piezo-ceramics regions, the sensor and actuator ones, are specified within the domain while the metallic structure layout is designed through the TOM aiming to minimize the overall displacement in time of a measuring point. Electrical charges generated by the structure deformation over the sensor layer are converted into an isopotential input for the electrode on the actuator layer by means of a specified feedback gain. The TOM is implemented based on the SIMP material model. The sensitivity analysis is calculated by using the adjoint method. Examples are restricted to two-dimensional models and the influence of different control gains over each of the designed structures is discussed regarding their effect in the objective function value.*

Keywords: *Topology Optimization Method, Piezoactuator, Active control law, Transient analysis*

1 INTRODUCTION

The piezo-structure designed in this paper consists of a ceramic-aluminum-ceramic stack with a fixed end to which one aims to reduce its vibrating displacement when the structure is subjected to a step-like point load. Such assembling of smart materials has useful applications in electronics these days for transduction and actuation devices. A positioning device disturbed by an external load often requires a smooth restoration to its initial position, and this is where the control law for a flexible structure takes place.

The control problem for structure stabilization has long been a concern on the vibration engineering research field. Balas (1978) developed a feedback control for a class of flexible systems described by the generalized wave equation with damping. The controller was developed by a finite number of modes, therefore control and observation spillover due to the residual, or uncontrolled, modes emerged. Fard & Sagatun (2001) studied the exponentially stabilization of a free transversely vibrating beam by applying a boundary control with no model truncation and showed an exponentially decrease of the mechanical energy of the system by using a Lyapunov functional.

Only recently has emerged the approach of active control formulated for the Topology Optimization (TO) problem. Zhang & Kang (2014) studied the vibration suppression in a cantilever plate by top-bottom attaching to it two piezoceramic layers, for sensing and actuation purposes, and optimizing their layout while maintaining intact a middle structure layer. da Silveira et al. (2015) also designed actuators for the vibration control of flexible structures in the frequency domain, focusing on the controllability Gramian of a LQR control system to state the optimization problem.

In this work, we want to implement a velocity feedback control to a cantilever solid structure analysis in time domain in order to alter its structural damping and attenuate the displacement in time of a target point when a transient load is applied. The TOM is evaluated over the middle metallic layer, while the top and bottom piezoceramic layers don't change throughout the iterations, but their elasticity is considered in the model.

2 FINITE ELEMENT FORMULATION

In order to iteratively evaluate an objective function, which will be stated later in the text, at Section 4, it is necessary to discretize the initial domain into Finite Elements (FE). For the FE Method approach, the discretization is based on the 4-node rectangular element with 3 degrees of freedom per node, being the x, y displacements and the electrical potential ϕ. Following the discretization, the dynamic equilibrium equation (1) is evaluated through the *a-Newmark* time integration scheme.

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{\Phi}}_f \\ \ddot{\mathbf{\Phi}}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{\Phi}}_f \\ \dot{\mathbf{\Phi}}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi_f} & \mathbf{K}_{u\phi_p} \\ \mathbf{K}_{u\phi_f}^T & -\mathbf{K}_{\phi_f\phi_f} & -\mathbf{K}_{\phi_f\phi_p} \\ \mathbf{K}_{u\phi_p}^T & -\mathbf{K}_{\phi_f\phi_p}^T & -\mathbf{K}_{\phi_p\phi_p} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{\Phi}_f \\ \mathbf{\Phi}_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{Q}_f \\ \mathbf{Q}_p \end{Bmatrix}, \quad (1)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}_{uu}$, $\mathbf{K}_{u\phi}$ and $\mathbf{K}_{\phi\phi}$ are derived from the weak form of the linear elasticity problem and the vectors $\ddot{\mathbf{U}}$, $\dot{\mathbf{U}}$, $\mathbf{U}$ and $\mathbf{\Phi}$ are defined for the isoparametric element.

Yet, the $\mathbf{C}$ matrix is the *Rayleigh* damping matrix obtained as fractions of the mass matrix $\mathbf{M}$ and the elastic matrix $\mathbf{K}_{uu}$ according to the expression:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}_{uu}, \quad (2)$$

Cook et al. (2007).

3 CONTROL PROBLEM FORMULATION

For the control problem formulation, it is necessary to qualify the piezoceramic nodes into two groups, the prescribed nodes and the free nodes. The latter, denoted by a subscript f, are the nodes whose electrical charges are calculated given the equilibrium Eq. (1), and the former, denoted by a subscript p, are the nodes located at the edge of piezoelectric domain, or the nodes belonging to the electrodes, as it is seen in Fig. 1.

The electrodes on the top ceramic layer, the sensor layer identified by s, is short-circuited,

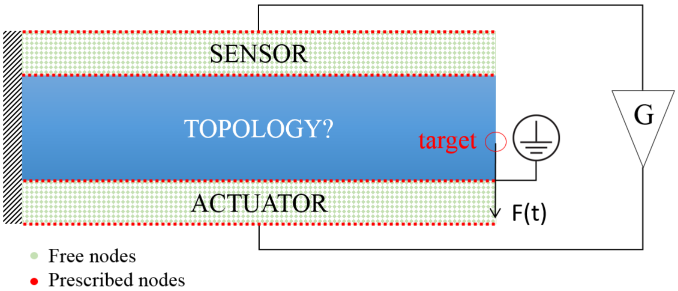


Figure 1: Design domain and boundary conditions

$\mathbf{\Phi}_{p_s} = 0$, and we know that the interior nodes on the ceramic domain have null charges, $\mathbf{Q}_{f_s} = 0$. Therefore, the electrical potential on the free nodes of the sensor layer is given by the expression:

$$\mathbf{\Phi}_{f_s} = \mathbf{K}^{-1}_{\phi_{f_s}\phi_{f_s}} \mathbf{K}^{T}_{u_s\phi_{f_s}} \mathbf{U}_s. \tag{3}$$

By replacing Eq. (3) into the two other equations of the system Eq. (1), the output forces $\mathbf{F}$ and the measured charges $\mathbf{Q}_{p_s}$ at the sensor layer are rewritten:

$$\mathbf{F} = \mathbf{M}_s\ddot{\mathbf{U}}_s + \mathbf{C}_s\dot{\mathbf{U}}_s + \underbrace{\left(\mathbf{K}_{u_su_s} + \mathbf{K}_{u_s\phi_{f_s}}\mathbf{K}^{-1}_{\phi_{f_s}\phi_{f_s}}\mathbf{K}^{T}_{u_s\phi_{f_s}}\right)}_{\mathbf{H}_{u_su_s}}\mathbf{U}_s, \tag{4}$$

$$\mathbf{Q}_{p_s} = \underbrace{\left(\mathbf{K}^{T}_{u_s\phi_{p_s}} - \mathbf{K}^{T}_{\phi_{f_s}\phi_{p_s}}\mathbf{K}^{-1}_{\phi_{f_s}\phi_{f_s}}\mathbf{K}^{T}_{u_s\phi_{f_s}}\right)}_{\mathbf{H}^{T}_{u_s\phi_{p_s}}}\mathbf{U}_s \tag{5}$$

Based on the voltage definition for a current amplifier, the sensor output charges $\mathbf{Q}_{p_s}$ are differentiate in time and multiplied by a constant gain G_s to obtain the sensor voltage output

$$\mathbf{\Phi}_{p_s} = G_s\mathbf{H}^{T}_{u_s\phi_{p_s}}\dot{\mathbf{U}}_s. \tag{6}$$

Premultiplying Eq. (6) by a unit vector $\mathbf{I}_{p_s} = \begin{Bmatrix}1 & \dots & 1\end{Bmatrix}_{\phi_{p_s}}$, a scalar voltage is obtained, which is the prescribed input voltage at the bottom electrode of the actuator layer identified by a.

Rewriting Eq. (1) in H-form matrices with modified damping effect based on previous developments, the equation to be solved by a time integration method is stated:

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{U}}\\ \ddot{\mathbf{\Phi}}_f\end{Bmatrix} + \begin{bmatrix}\mathbf{C} + G_s\mathbf{K}_{u_a\phi_{p_a}}\mathbf{I}_{\phi_{p_a}}\mathbf{I}^{T}_{\phi_{p_s}}\mathbf{H}^{T}_{u_s\phi_{p_s}} & \mathbf{0}\\ G_s\mathbf{K}_{\phi_{f_a}\phi_{p_a}}\mathbf{I}_{\phi_{p_a}}\mathbf{I}^{T}_{\phi_{p_s}}\mathbf{H}^{T}_{u_s\phi_{p_s}} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{U}}\\ \dot{\mathbf{\Phi}}_f\end{Bmatrix} + \begin{bmatrix}\mathbf{K}_{uu} & \mathbf{K}_{u\phi_f}\\ \mathbf{K}^{T}_{u\phi_f} & -\mathbf{K}_{\phi_f\phi_f}\end{bmatrix}\begin{Bmatrix}\mathbf{U}\\ \mathbf{\Phi}_f\end{Bmatrix} = \begin{Bmatrix}\mathbf{F}\\ \mathbf{0}\end{Bmatrix} \tag{7}$$

4 TOM FOR AN ACTIVE CONTROL LAW

The TOM has been employed to design smart structures based on piezoelectric material such as actuadors Kögl & Silva (2005) and transducers Silva & Kikuchi (1999) for a static or quasi-static analysis. In this work TOM aims to extremize an objective function for a structure under a transient load and a velocity feedback control. The optimization problem is stated bellow for the modified damping matrix of Eq. (7) represented by $\mathbf{C}_{tm}$:

$$
\begin{aligned}
&\min_{\boldsymbol{\rho}} \quad f(\boldsymbol{\rho}) = \int_0^{t_f} u^2_{\text{target}}(t)dt \\
&\text{s.t.} \quad \begin{cases}
\mathbf{M}(\boldsymbol{\rho})\ddot{\mathbf{u}} + \mathbf{C}_{tm}(\boldsymbol{\rho})\dot{\mathbf{u}} + \mathbf{K}(\boldsymbol{\rho})\mathbf{u} = \mathbf{F}(t) \\
\dot{\mathbf{u}}|_{t=0} = \mathbf{v}_0 \\
\mathbf{u}|_{t=0} = \mathbf{u}_0 \\
\mathbf{u}^T\mathbf{K}\mathbf{u} \leq C_{max} \\
\sum_{e=1}^{N_e} \rho_e V_e \leq V_{\text{max}} \\
0 < \rho_{\text{min}} \leq \rho_e \leq 1
\end{cases}
\end{aligned} \tag{8}
$$

4.1 Material Model

In this work, the problem relaxation applied to the material distribution through the metallic microstrusture is the Solid Isotropic Material with Penalization (SIMP), wich is simple to implement and is in accordance with the FEM. This relaxation consists in distributing intermediate densities $\boldsymbol{\rho}$ from void to solid within the domain, Bendsøe (1989). Therefore, in our TO problem, pseudodensities ρ_e that come from SIMP concept are the design variables to be considered for each element of the discretized domain.

4.2 Sensitivity Analysis

The sensitivity analysis is crucial for the optimization method chosen to evaluate the cost function towards the right direction in order to extremize its value. The solution of a second-order system of linear equations depends continuously on its boundary conditions $[\mathbf{u}_0, \dot{\mathbf{u}}_0]$, on its initial condition t_0 and on its parameters. To state the continuous dependence on its parameters, we need a mathematical representation of the perturbation of the objective function f.

Considering the design problem as an optimization problem in the design variables only, the displacement field is given implicitly in terms of those variables through the equilibrium equation (7). In this case, the sensitivity analysis consists of finding the derivatives of the displacements with respect to the design variables.

The adjoint method, a procedure where the derivatives of the displacement are not explicitly calculated, is used. This method involves an adjoint vector $\boldsymbol{\lambda}(t)$ that is chosen as the solution to adjoint equations which are derived from the Lagrangian function $\mathcal{L}(t, \mathbf{u}, \boldsymbol{\lambda})$:

$$
f(t, \mathbf{u}, \boldsymbol{\lambda}) = \mathcal{L}(t, \mathbf{u}, \boldsymbol{\lambda}) = \int_0^{t_f} g(t, \mathbf{u}(t))dt + \int_0^{t_f} \boldsymbol{\lambda}^t \left[\mathbf{M}(\boldsymbol{\rho})\ddot{\mathbf{u}} + \mathbf{C}_{tm}(\boldsymbol{\rho})\dot{\mathbf{u}} + \mathbf{K}(\boldsymbol{\rho}) - \mathbf{F}(t)\right] dt \tag{9}
$$

And the sensitivity expression is given by:

$$\frac{\partial \mathcal{L}(t, \mathbf{u}, \mathbf{\Lambda})}{\partial \rho_e} = \int_0^{t_f} \mathbf{\Lambda}^T(t_f - t) \left[\frac{\partial \mathbf{M}(\boldsymbol{\rho})}{\partial \rho_e} \ddot{\mathbf{u}} + \frac{\partial \mathbf{C}_{tm}(\boldsymbol{\rho})}{\partial \rho_e} \dot{\mathbf{u}} + \frac{\partial \mathbf{K}(\boldsymbol{\rho})}{\partial \rho_e} \mathbf{u} \right] dt, \tag{10}$$

Jensen et al. (2014).

5 RESULTS

Given the initial domain in Fig. 2, a 40×32 meshsize with a pseudodensity vector initially set to $\boldsymbol{\rho} = 0.5$ at the metallic layer, the optimization problem, Eq. (8), was evaluated for three different feedback gains. The topologies obtained are shown bellow, Figs. 3a to 5a, each one

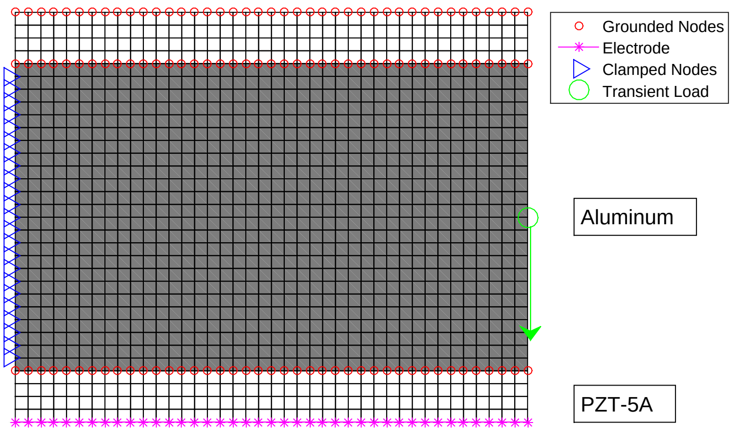
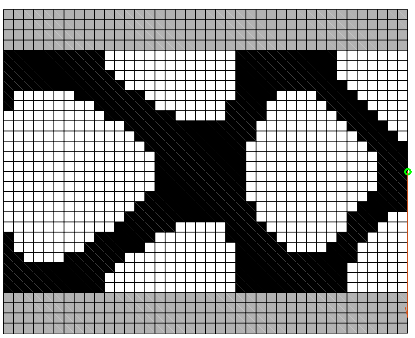
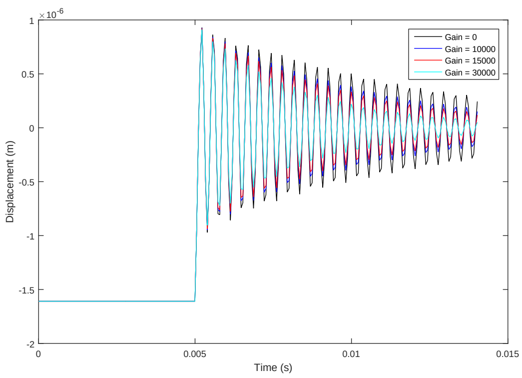


Figure 2: Initial Domain

along with its respective transient response, Figs. 3b to 5b when simulated for all three feedback gains. For $G_s = 30000$, Fig. 5, the system is unstable for most of the iteration steps but it

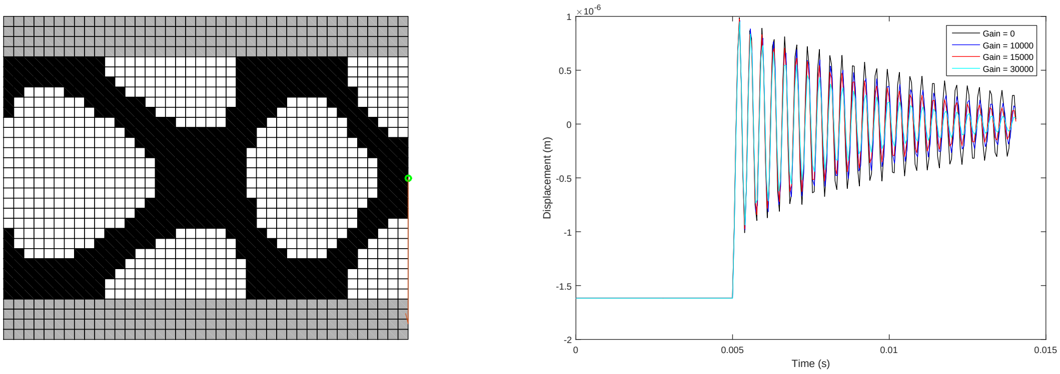


(a) Final Topology



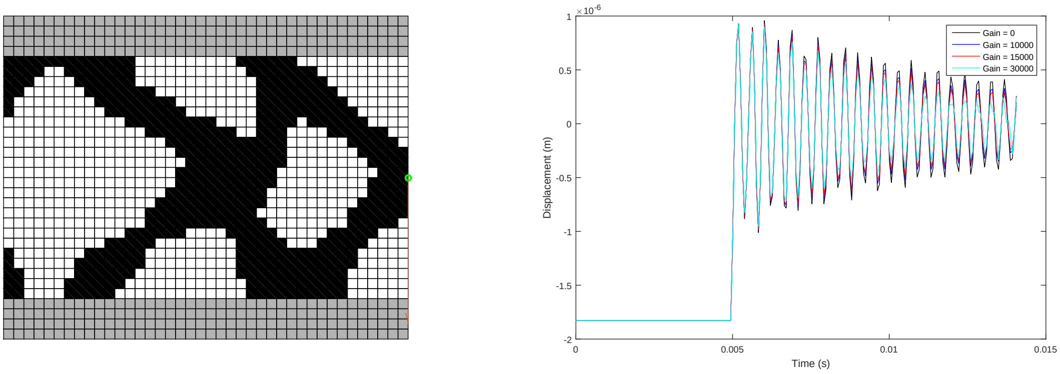
(b) Transient Response

Figure 3: Feedback Gain $G_s = 10000$



(a) Final Topology **(b) Transient Response**

Figure 4: Feedback Gain $G_s = 15000$



(a) Final Topology **(b) Transient Response**

Figure 5: Feedback Gain $G_s = 30000$

Table 1: Comparison of topologies designed for different gains and simulated for a range of gains.

	Design Gain	Simulation Gain	Obj. Func. Attenuation
Figure 3a	10000	10000	2.69%
Figure 3a	10000	15000	3.65%
Figure 3a	10000	30000	5.59%
Figure 4a	15000	15000	3.99%
Figure 4a	15000	30000	5.98%
Figure 5a	30000	30000	3.64%

ultimately converges, even though not having the best performance compared to the other two topologies simulated for the same feedback gain, see Table 1.

According to the results shown in Table 1, it is clear that the optimization process is crucial for the best performance of the system regarding the objective aimed. The largest attenuation percentages were obtained for the topology designed for that specific feedback gain, except for the last topology, Fig. 5, that had the worst performance due to the instabilities imposed to the problem formulation.

6 CONCLUSIONS

The TOM was applied to the design of a vibrational structure controlled by two piezoceramic layers aiming to reduce its displacement in time. The SIMP material model was adopted and the optimization problem was solved with a Sequential Linear Programming (SLP) algorithm. The results show that to all the velocity feedback gains chosen, the optimization problem converges and the greater improvement in displacement responses is seen for the gain to which the structure was designed. This pattern could not be observed for the last case though (Fig. 5), where the system is unstable for the first iteration steps. In future work, instabilities to this system will be analysed and the objective function and its constraints will be restated accordingly.

ACKNOWLEDGEMENTS

The author acknowledges CAPES for providing her with a doctoral scholarship.

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