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MANUFACTURING PROCEDURE OF MULTI-SCALE STRUCTURES OBTAINED USING BESO OF CELLULAR MATERIALS

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Abstract. *With the advances in additive manufacturing techniques, development of high performance materials became a reality for engineers. In addition, Topology Optimization may be used to design new materials. In this article, a Multi-scale Topology Optimization analysis using an evolutionary technique is applied to develop a manufacturing procedure of structures with cellular materials. This procedure consists in performing a numerical validation of microstructures obtained by the Topology Optimization method, and then using additive manufacturing techniques. Cellular materials are defined as microstructures that have only one type of isotropic material and voids. For the multi-scale model, the homogenization theory is used in order to obtain the equivalent elastic properties of the micro-scale domain. For the optimization problem, the Bi-directional Evolutionary Structural Optimization (BESO) method is applied in this work for designing microstructures of cellular materials. The objective function is to minimize the structural mean compliance. Numerical examples are presented to validate the algorithm for different boundary conditions. Finally, this article shows the final 3D product of the manufacturing procedure and experimental results obtained for a double-clamped beam, and some details of the manufacturing equipments used are also presented.*

Keywords: *Topology Optimization, Multi-scale, Manufacture, BESO, Cellular materials*

1 INTRODUCTION

Additive manufacturing (AM) technology has been researched and developed for more than 20 years. Rather than removing materials, AM processes make three-dimensional parts directly from CAD models by adding materials layer by layer, offering the beneficial ability to build parts with high geometric and material complexities (Guo and Leu, 2013). Specifically AM technology can be used to develop cellular materials with desired properties, as we addressed in this work. Cellular materials are defined as microstructures that have only one type of isotropic material and voids. Their main applications include thermal insulation, sound and energy absorption, vibration dampening and structures lightweighting.

On the other hand topology optimization techniques have been used extensively to solve the design problems not only for macroscopic structures, but also for microstructures of materials/composites in recent years (Huang et al. 2013). Thus, considering the AM manufacturing advances and the applications of multi-scale topology optimization in materials design, a methodology to manufacture structures with cellular materials is constructed. This approach was developed analyzing optimized structures for mean compliance minimization with cellular materials.

In order to apply the topology optimization method at the microscopic model, the homogenization theory, as presented first in Bendsøe and Kikuchi (1988) and reviewed in Hassani and Hinton (1998a; 1998b), is used. Through this theory, the effective properties of the heterogeneous material are obtained and integrated into the macroscopic analysis. However, the homogenization theory can only be used for the multi-scale modeling if structure periodicity exists and the base cell size is infinitesimal compared with the macrostructure. For practical purposes, there is a minimum base cell size that meets the homogenization theory and also favors structure fabrication. This methodology looks for the real size of the base cell that achieves this adequate macro/macro scales relationship (to consider cell size as almost infinitesimal) through an objective function convergence for process validation with different boundary conditions. Finally, this article shows the final 3D product of the manufacturing procedure for a double-clamped beam, and some manufacture equipments details for the overall process.

2 OPTIMIZATION FORMULATION

Topology optimization problem considered in this work consists in maximize the structural stiffness, or minimize the mean compliance. The mean compliance C is defined as the total strain energy of the structure. The static problem can be formulated as follows:

$$\text{Find:} \quad \mathbf{x} \tag{1}$$

$$\text{that minimize:} \quad C = \frac{1}{2}\mathbf{F}^{\mathrm{T}}\mathbf{U} \tag{2}$$

$$\text{subject to:} \quad \mathbf{KU} = \mathbf{F} \tag{3}$$

$$V^* - \sum_{j=1}^{N} V_j x_j = 0 \tag{4}$$

$$x_j = 0 \text{ or } 1 \tag{5}$$

where x_j is the jth design variable of the micro model and V^* is the target volume. **U** is the nodal displacement field vector of the macrostructure. **K** and **F** are respectively the global stiffness matrix and the global force vector of the macro model.

The material of an element in the base cell is modeled considering an isotropic material behavior. Its physical properties are assumed to be a function of the elemental density x_j. The material interpolation scheme named SIMP (Solid Isotropic Material with Penalization) (Huang and Xie, 2010) is adopted, and the following approximation is used:

$$\mathbf{D}(x_j) = x_j^p \mathbf{D} \qquad \text{where} \quad x_j = x_{min} \text{ or } 1 \tag{6}$$

where p is the exponent of penalization. In this paper, $p = 3$ is used. The soft-kill approach indicates that no element is allowed to be completely removed from the design domain ($x_{min} = 0.001$) (Huang and Xie, 2010).

3 MULTI-SCALE MODEL

For the multi-scale modeling, the concepts of the homogenization theory was used considering a two-scale problem. According to this theory, if a material presents low periodic inhomogeneities, the homogenized properties of the base cell can be obtained (Hassani and Hinton, 1998a; Hassani and Hinton, 1998b). Using asymptotic analysis, as a result, we can express the mechanical macroscopic properties as a function of the microstructural configuration. The Figure 1 shows the micro and macro scales considered in the homogenization theory for a structure with periodic base cell. At the macroscopic domain Ω at Fig. 1, **f** and Γ_d represent body forces and constraints, respectively.

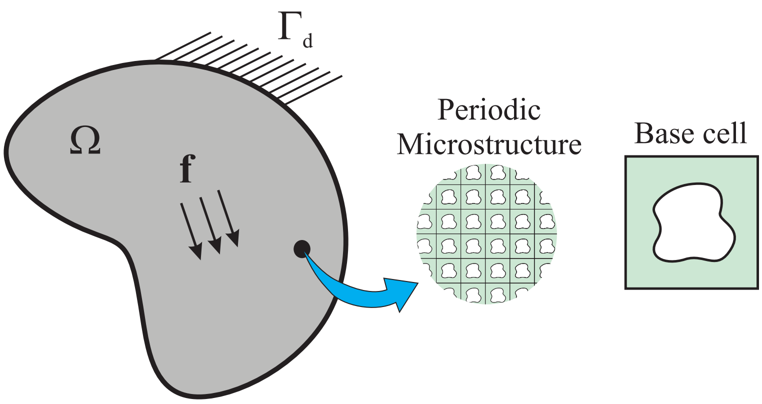


Figure 1: Homogenization Theory: Macro and micro scales

In order to use the homogenization theory, the microscopic scale should be much smaller than the macroscopic scale.

Using the finite element method, the multi-scale model should have two different meshes, one for each scale or model. In the microscopic scale, applying the periodic boundary condition and the homogenization conditions, the equilibrium equation can be written as (Hassani and Hinton, 1998a; Hassani and Hinton, 1998b):

$$\int_Y \mathbf{b}^T \mathbf{D} \mathbf{b} dY \mathbf{u} = \int_Y \mathbf{b}^T \mathbf{D} dY \tag{7}$$

where **b** is the strain-displacement matrix, **D** is the elasticity matrix of the material, **u** is the displacement fields in the micro scale model and Y is the volume of the base cell. At the left side of Eq. 7, $\mathbf{k} = \int_Y \mathbf{b}^T \mathbf{D} \mathbf{b} dY$ is the stiffness matrix of the microstructure level. At the Eq. 7, the right side represents the external forces caused by three uniform strain fields $\{1, 0, 0\}^T$, $\{0, 1, 0\}^T$ and $\{0, 0, 1\}^T$ for 2D problems (Hassani and Hinton, 1998a; Hassani and Hinton, 1998b).

Finally, the homogenized elasticity matrix $\mathbf{D}^H$ for the microstructure is calculated using the following expression:

$$\mathbf{D}^H = \frac{1}{|Y|} \int_Y \mathbf{D}(\mathbf{I} - \mathbf{bu}) dY \tag{8}$$

where **I** is an identity matrix and **u** denotes the displacement fields of the unit cell caused by the uniform strain fields.

4 SENSITIVIY ANALYSIS

The sensitivity number represents the importance of each element in the objective function. In order to maximize the global structural stiffness, we need to calculate the derivative of the mean compliance in the macro scale with respect to the design variable of the micro scale. Initially, at the macroscopic domain, the derivative of the objective function is given by (Huang and Xie, 2010):

$$\frac{dC}{dX_i} = \frac{1}{2} \mathbf{F}^T \frac{\partial \mathbf{U}}{\partial X_i} = -\frac{1}{2} \mathbf{U}_i^T \frac{\partial \mathbf{K}_i}{\partial X_i} \mathbf{U}_i \tag{9}$$

where X_i and $\mathbf{K}_i$ are the design variable and the stiffness matrix of a ith element at the macro scale, respectively.

At the micro scale, the change of each element density x_j affects the sensitivity of all elements at the macro scale (Huang et al., 2013). Thus, the sensitivity number of each element x_j can be written as:

$$\frac{dC}{dx_j} = -\frac{1}{2} \sum_{i=1}^{N} \mathbf{U}_i^T \frac{\partial \mathbf{K}_i}{\partial x_j} \mathbf{U}_i \tag{10}$$

where N is the number of elements of the macroscopic mesh. The derivative of elemental stiffness matrix with respect to the microscopic design variable is obtained and the previous equation becomes:

$$\frac{dC}{dx_j} = -\frac{1}{2} \sum_{i=1}^{N} \mathbf{U}_i^T \left(\int_{\Omega_e} \mathbf{B}_e^T \frac{\partial \mathbf{D}^H}{\partial x_j} \mathbf{B}_e d\Omega \right) \mathbf{U}_i \tag{11}$$

where $\mathbf{B}_e$ is the elemental strain-displacement matrix and Ω_e is the element domain of the macrostructure.

Applying the adjoint variable method (Haug et al., 1986) at the multi-scale model, the derivative of the homogenized elastic matrix with respect to x_j can be written as:

$$\frac{\partial \mathbf{D}^H}{\partial x_j} = \frac{1}{|Y|} \int_Y (\mathbf{I} - \mathbf{bu})^T \left(\frac{\partial \mathbf{D}}{\partial x_j} \right) (\mathbf{I} - \mathbf{bu}) dY \tag{12}$$

Replacing the Eq. (12) into Eq. (11) and applying the material interpolation model shown in Eq. (6), the sensitivity of the mean compliance is rewritten as:

$$\frac{dC}{dx_j} = -\frac{px_j^{p-1}}{2|Y|} \sum_{i=1}^{N} \mathbf{U}_i^T \left(\int_{\Omega_e} \mathbf{B}_e^T \left[\int_Y (\mathbf{I} - \mathbf{bu})^T \mathbf{D} (\mathbf{I} - \mathbf{bu}) dY \right] \mathbf{B}_e d\Omega \right) \mathbf{U}_i \tag{13}$$

Finally, the sensitivity number of each finite element of the microscopic domain is obtained by the following expression (Huang et al., 2013):

$$\alpha_j = -\frac{1}{p} \frac{dC}{dx_j} \tag{14}$$

$$\alpha_j = \frac{x^{p-1}}{2|Y|} \sum_{i=1}^{N} \mathbf{U}_i^T \left\{ \int_{\Omega_e} \mathbf{B}_e^T \left[\int_Y (\mathbf{I} - \mathbf{bu})^T \mathbf{D} (\mathbf{I} - \mathbf{bu}) dY \right] \mathbf{B}_e d\Omega \right\} \mathbf{U}_i \tag{15}$$

where the term between braces represents the derivative of elemental stiffness matrix $\left(\frac{\partial \mathbf{K}_i}{\partial x_j}\right)$.

5 BI-DIRECTIONAL EVOLUTIONARY STRUCTURAL OPTIMIZATION PROCEDURE

The optimization procedure applied in this work is the BESO method. It consists in gradually removes and adds material on the finite element mesh. The evolutionary optimization uses a discrete design variable, which means that the material can only be void or solid. A mesh-independent filter was used to avoid numerical instabilities. A history averaging scheme is used to improve the convergence of the solutions. The method details are presented by Huang and Xie (2010).

1. Define the macro/micro domains, target volume V^* as presented in Eq. (4) and BESO parameters;
2. Solve microscopic problem in Eq. (7) and calculate the homogenized elasticity matrix $\mathbf{D}^H$ using Eq. (8);
3. Applying $\mathbf{D}^H$, solve the macroscopic equilibrium equation in Eq. (3) and obtain the objective function, as presented in Eq. (2);
4. Using Eq. (15), calculate the elemental sensitivity at the micro scale;
5. Apply the mesh-independent filter and average the sensitivity number with its history information;
6. Update the base cell topology according to the sensitivity number;
7. Repeat steps 2-6 until the target volume (V^*) is obtained and the convergence of the objective function (C) is analyzed;

Summary of the evolutionary optimization procedure is presented in the flowchart of Figure 2. The algorithm was implemented using MATLAB and the numerical results shall be presented in the next section.

The nomenclature of the BESO parameters used in this work is equivalent to the one defined by Huang and Xie (2010), Picelli et al. (2014) and William et al. (2015).

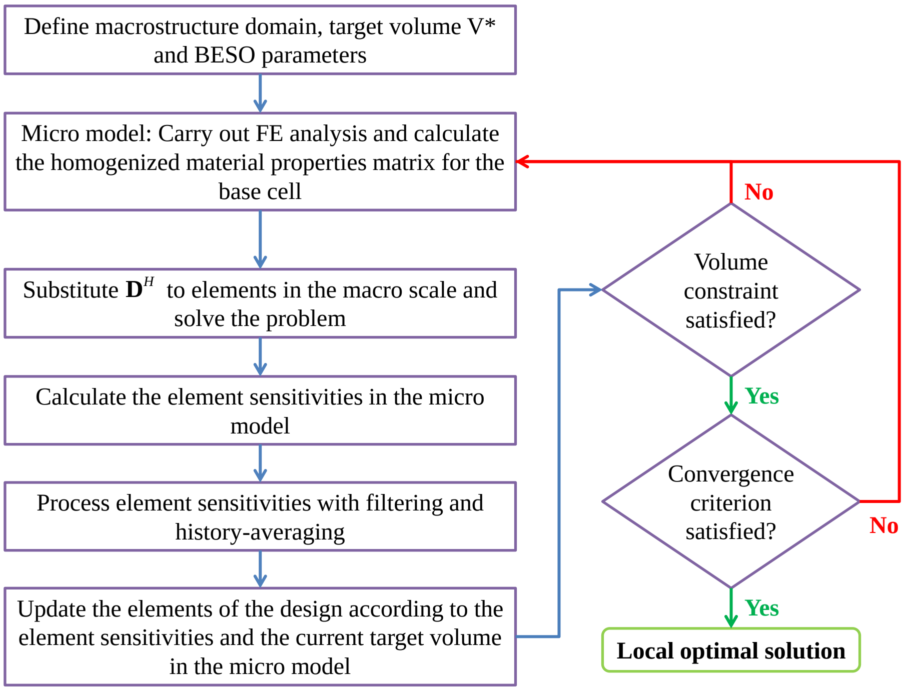


Figure 2: BESO procedure for multiscale optimization problem

6 RESULTS AND DISCUSSION

In this section, we will present some multi-scale topology optimization results for mean compliance minimization. After that, the numerical solution will be applied to the manufacture procedure proposed. At the end, a periodic structure prototype is presented using additive manufacturing techniques.

6.1 Multi-scale Topology Optimization

Initially, the multi-scale evolutionary topology optimization is applied to obtain the optimized microstructure for different macroscopic boundary conditions. For all analyses, the macroscopic domain remains constant and only the microstructural topology is optimized. Three boundary conditions were applied: cantilever, double-supported and double-clamped.

The material properties considered for these numerical simulations are: Young's modulus $E = 210\text{GPa}$ and Poisson's ratio $\nu = 0.3$. The BESO parameters applied are: $ER = 2\%$, $AR_{max} = 2\%$, $r_{min} = 0.07$ (cantilever), $r_{min} = 0.04$ (double-supported), $r_{min} = 0.2$ (double-clamped), $\tau = 0.01\%$ and $N = 5$. The final volume V^* predefined is 40%. The unit cell considered has dimensions 1×1. The microstructural mesh uses 100×100 finite elements, while the macrostructural mesh presents finite elements with size $1\text{mm} \times 1\text{mm}$. The external force applied is $F = 100\text{N}$. The initial design of the microscopic mesh is full with solid material ($V_i = 100\%$), except for four void elements at the center of the base cell.

Optimized topologies obtained for stiffness maximization are shown at Fig. 3. The Figure 3 presents the boundary condition, the final base cell, the homogenized elastic matrix and the 4x4 microestrutural mesh.

For a cantilever beam with dimensions $40\text{mm} \times 20\text{mm}$, the results indicate that x-direction structural stiffness is higher than y-direction. This was expected since the normal stress caused

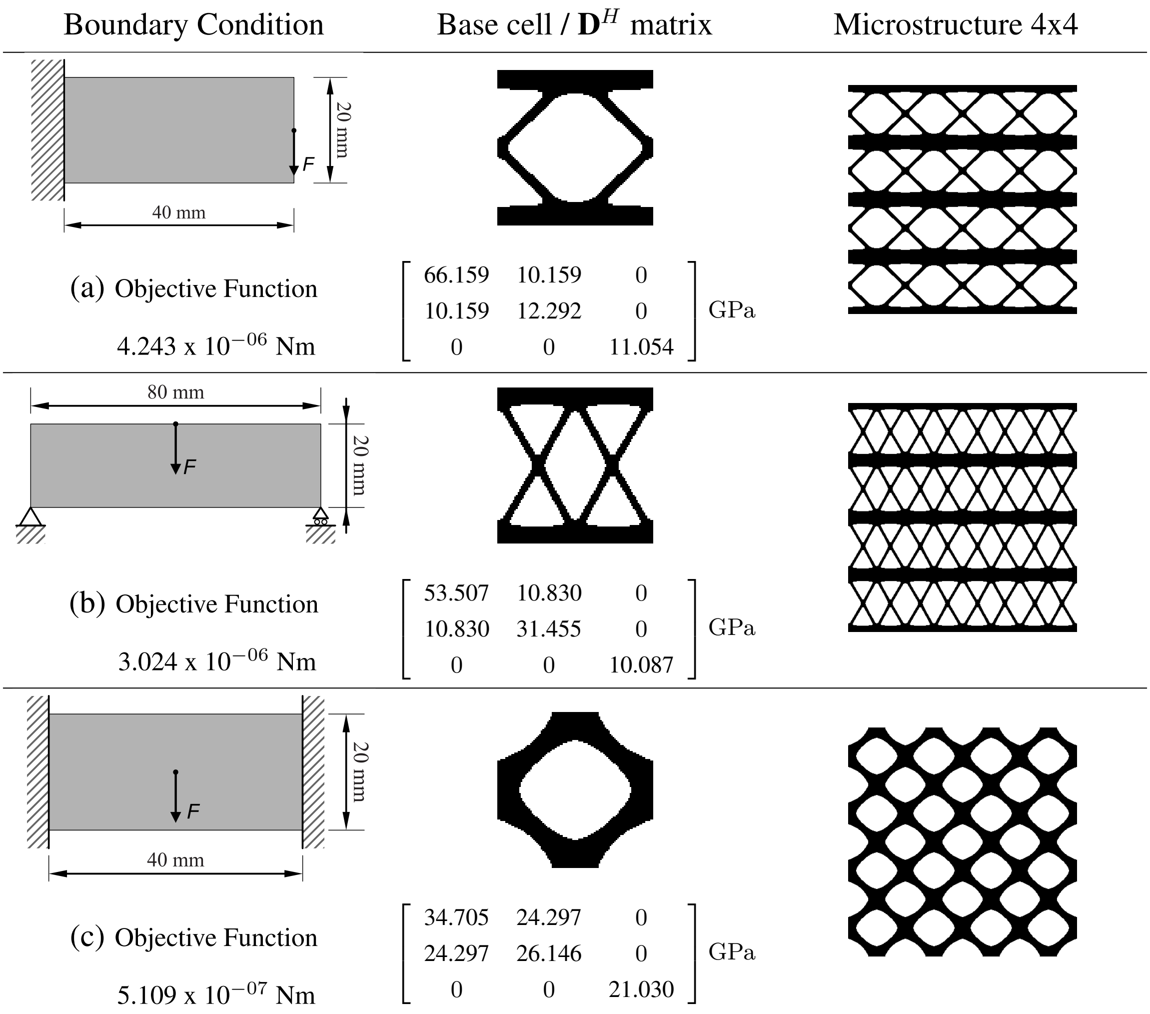


Figure 3: Optimized topologies obtained for mean compliance minimization in different boundary conditions: (a) Cantilever beam; (b) Double-supported beam; (c) Double-clamped beam

by bending is higher than the shear stress for a cantilever beam with these dimensions. The numerical results presented at Fig. 3(a) show good agreement with the base cell obtained in Huang et al. (2013).

The double-supported beam with dimensions $80\text{mm} \times 20\text{mm}$ is fully discretized in all domain and the final base cell is also symmetric. The results are presented at Fig. 3(b). It can be seen that the stiffness in y-direction is higher compared to the cantilever beam case.

Finally, the Figure 3(c) presents numerical results for a double-clamped beam with dimensions $40\text{mm} \times 20\text{mm}$. In this case, the final structure shear stress is more relevant than results for the other boundary conditions, reducing the material normal stiffness. This can be observed at the final homogenized matrix $\mathbf{D}^H$.

For all boundary conditions analyzed by this algorithm, the final base cells obtained are symmetric, obtaining orthotropic materials. This behavior is verified by the homogenized elasticity matrix. It can also be analyzed the filter radius r_{min} influence at the base cell final topologies. Comparing topologies at Fig. 3, it can be seen that r_{min} larger values result in thicker microstructures. This was already presented for macroscopic topologies by Huang and Xie

(2010).

After presenting numerical results for different boundary conditions, the multi-scale evolutionary topology optimization is applied to fabricate optimized microstructures. In order to facilitate experimental analysis, the macroscopic constraint used was a double-clamped beam with dimensions $100\mathrm{mm} \times 50\mathrm{mm}$, as presented in Fig. 4.

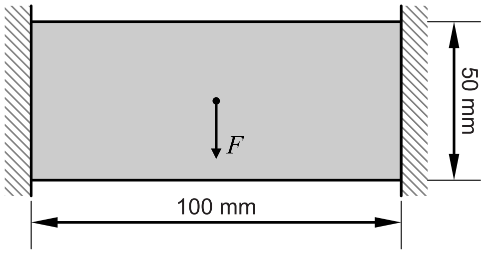


Figure 4: Double-clamped beam used to the macrostructural domain

The material chosen for numerical simulation was the polyamide SLS (Selective Laser Sintering), which is widely used for fabrication with additive manufacturing. The solid material isotropic properties considered were: Young's modulus $E = 1.568\mathrm{GPa}$ and Poisson's ratio $\nu = 0.38$.

For multi-scale analysis, the structure final volume is 50%. BESO parameters used were:

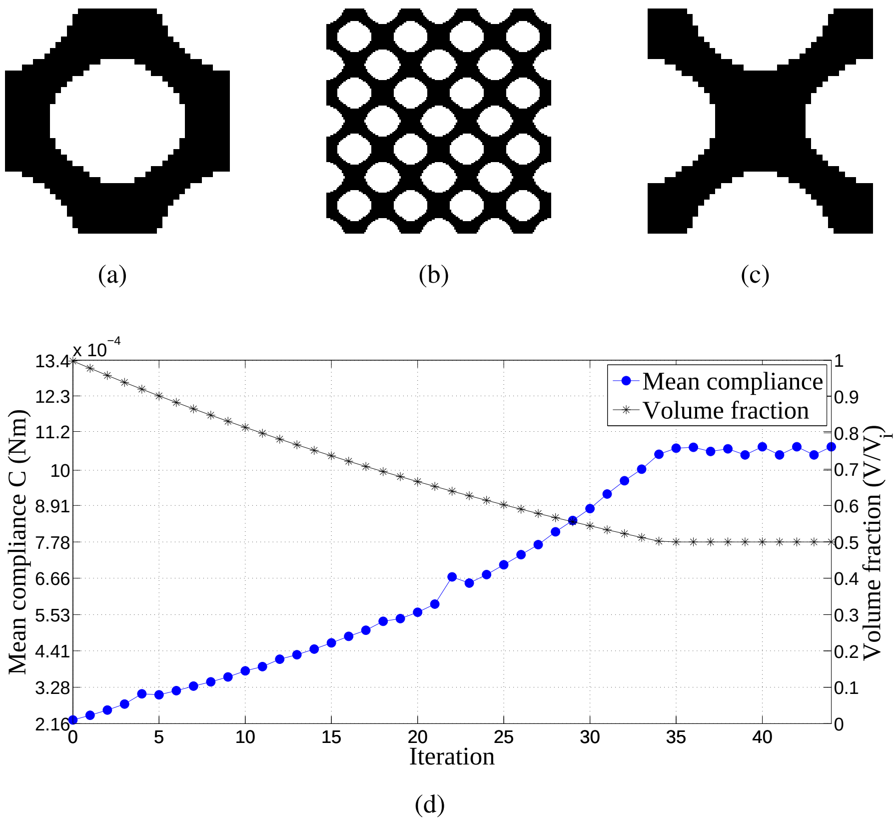


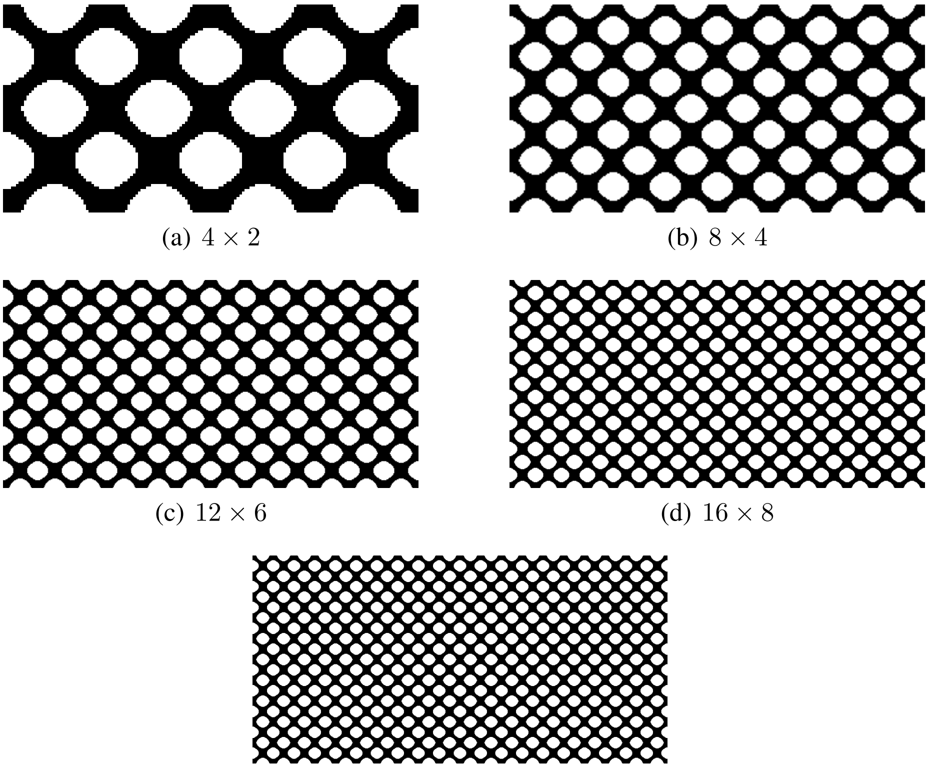
Figure 5: Optimized microstructure (40x40) obtained to double-clamped beam with central force: (a) base cell, (b) 4x4 unit cells, (c) moved cell, (d) optimization history of the mean compiance and volume fraction

$ER = 2\%$, $AR_{max} = 2\%$, $r_{min}^{mic} = 0.2$, $\tau = 0.1\%$ and $N = 5$. Due to manufacturing equipment restrictions, a finite element mesh of 40×40 four-node quadrilateral elements is used for the base cell. Macroscopic domain is discretized into 40×20 finite elements. The applied force at the structure's center is $F = 500$N. Same initial microscopic mesh design from previous examples was used for this simulation.

The solution obtained, considering the boundary condition shown at Fig. 4 and the structural stiffness maximization problem, is presented at Fig. 5. The histories of mean compliance and volume fraction of the solid phase during the evolutionary process are shown at Fig. 5(d).

6.2 Periodic structure reproduction

Using final topology obtained by the homogenization method, a 40×40 microstructural mesh was reproduced. In order to maintain square unit cell consistency, the following macrostructural meshes were analyzed: 4×2, 8×4, 12×6, 16×8 and 20×10. To ensure that there is a force application point at the structure's center (as illustrated at Fig. 4), it is used the base cell displaced as shown at Fig. 5(c), which results in the same regular microstructure and homogenized elastic matrices obtained with the base cell of Fig. 5(a). The meshes used are presented at Fig. 6.



(a) 4×2 (b) 8×4

(c) 12×6 (d) 16×8

(e) 20×10

Figure 6: Reproduced meshes at the macroscopic domain

Once the structures for simulation are obtained, the finite element problem is solved for static case and the structural mean compliance is calculated. To obtain the numerical solution, commercial finite element program ANSYS is used. For 2D quadrilateral element modeling, element PLANE42 is adopted. The Figure 7 shows the mean compliance results for each macroscopic mesh. At the convergence analysis, the objective function obtained by the multi-scale optimization algorithm is also presented, considering an infinitesimal theoretical microscopic mesh.

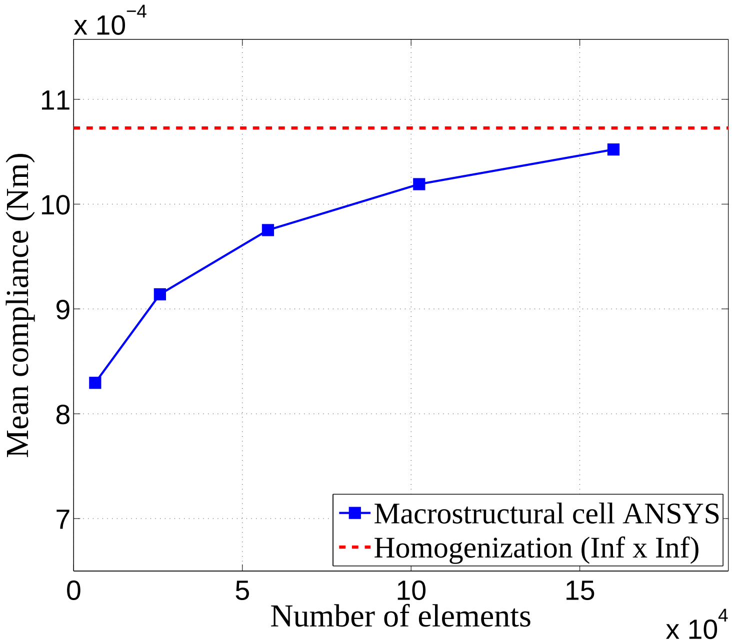


Figure 7: Convergence analysis of mean compliance for different meshes: 4×2**,** 8×4**,** 12×6**,** 16×8 **and** 20×10

From Fig. 7, it is possible to see that, increasing the base cell discretization in the macrostructural mesh, the objective function tends to converge to the value obtained by the multi-scale algorithm. In this work, a 20×10 unit cells mesh is used to represent the theoretical multi-scale problem, due to the fact that the percentage error of the objective function becomes small (less than 2%) for that discretization value.

Then, to fabricate the structure using additive manufacture, it is necessary to construct a computational virtual model in CAD (Computer aided design). For that, a .STL three-dimensional mesh is generated using APDL programming language (ANSYS Parametric Design Language). Given that a two-dimensional domain under plane stresses is assumed, a structural thickness should be considered for manufacture. This work used a thickness of $10\mathrm{mm}$. The left side of Fig. 8 presents the virtual model generated by this analysis.

In order to make boundary condition more realistic in experimental tests, the supports are modeled as solid blocks of continuum material attached to the structure, fabricated also using additive manufacturing.

The 3D printing was carried out by Industrial Program ProInd (Three-dimensional Technology Division at Industry) which uses the rapid prototyping technology. From a virtual model, the physical model or prototype is built with the aid of machines which produces models by depositing sequential layers of specific materials. This infrastructure belongs to the Three-dimensional Technologies Division at Industry (DT3D) the CTI (Information Technology Center) at Campinas, Sao Paulo. Pictures of the final structure obtained by 3D printing are also

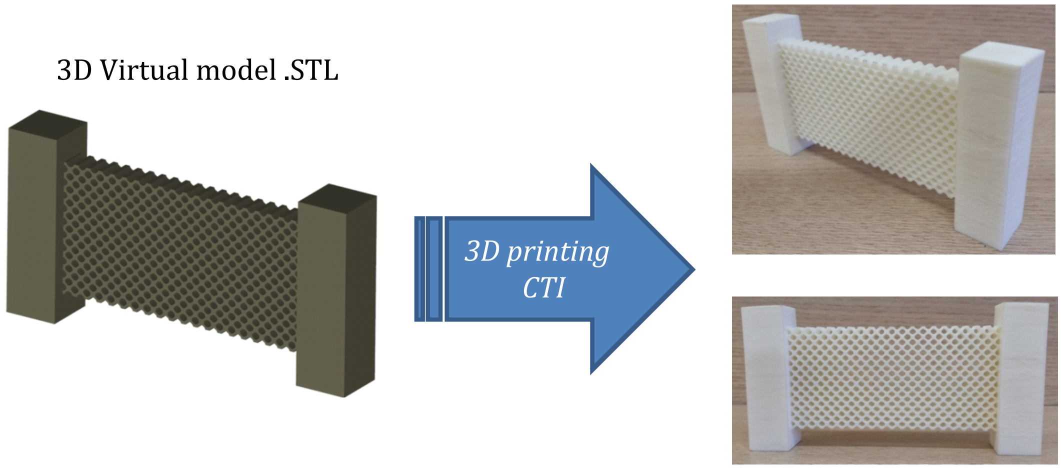


Figure 8: 3D virtual model in .STL and the final product of the additive manufacturing

shown at Fig. 8.

6.3 Manufacturing procedure

Structures manufacturing procedure using multi-scale optimization algorithm can be summarized as follows:

1. Problem definition: analysis type, boundary conditions and applied loads.
2. Obtaining the optimum base cell by the multi-scale evolutionary topology optimization algorithm.
3. Unit cell reproduction obtained to construct the macrostructure and solving the finite element problem.
4. Convergence analysis of the objective function with the increase of base cells at the macroscopic domain.
5. 3D virtual model construction in .STL for manufacture.
6. Manufacturing the final multi-scale structure using rapid prototyping technology.

The procedure proposed in this work is ilustrated at Fig. 9. It can be applied not only for mean compliance minimization problems, but also for dynamic problems (fundamental frequency maximization).

The next step of this work is an experimental analysis of fabricated structures. Thus, it can be compared the multi-scale structure real stiffness with the obtained by the numerical model. Errors are expected because of the difficulty of boundary conditions reproduction (force application, supports), the material and geometric properties variability, and the additive manufacturing process influence related to the material properties.

7 CONCLUSION

This paper has analyzed multi-scale evolutionary topology optimization to design cellular material base cells so that the global stiffness of the structure is maximized. Optimization

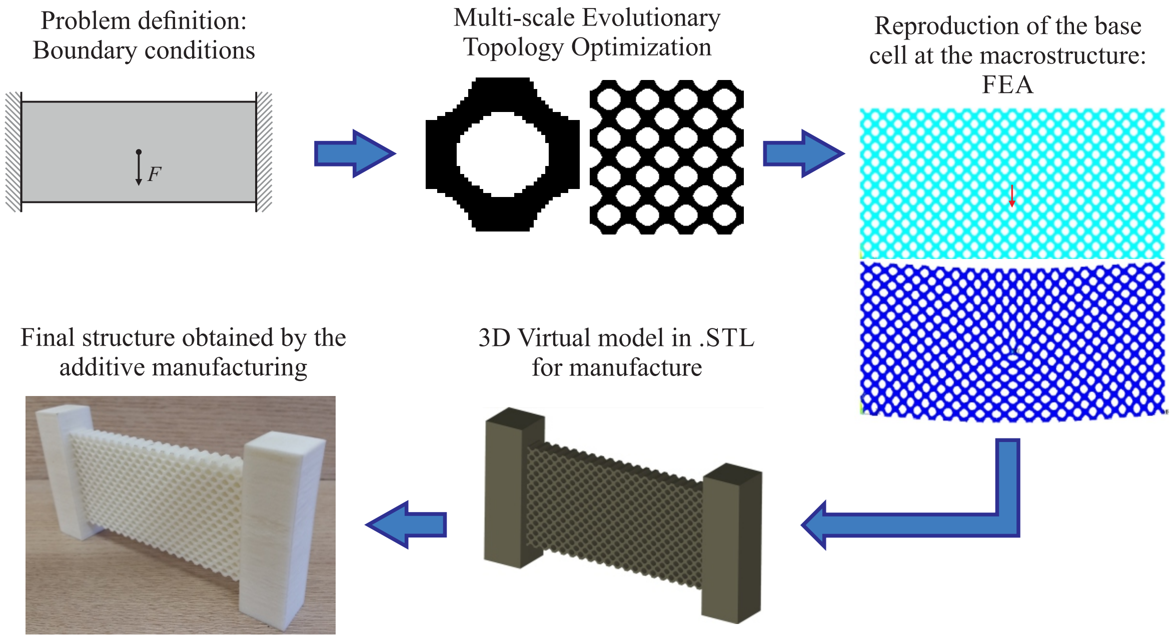


Figure 9: Manufacturing procedure of multi-scale structures

problem was formulated for mean compliance minimization and multi-scale model is based on homogenization theory.

Numerical results were presented for different boundary conditions. Difference between the final base cells obtained was pointed out, and also the r_{min} influence at the final microstructure was analyzed.

It was also presented a manufacturing procedure for multi-scale structures using BESO with cellular materials. A double-clamped beam was used to verify the procedure and the final product obtained by additive manufacturing was presented. This procedure can be also applied to others optimization problems (different objective functions and analysis types). It means that this procedure is a powerful tool for multi-scale structures validation and fabrication. Experimental results are necessary to compare the stiffness with numerical results.

ACKNOWLEDGEMENTS

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