WIND EFFECTS ON CONTAMINANT TRANSPORT IN SHALLOW LAKES

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Abstract. *In this work, a two-dimensional simulation of a contaminant which has been discharged in a shallow lake is considered. In order to model the lake, an incompressible Navier - Stokes equation coupled with a scalar transport equation is used. The effect of the wind in the lake flow is introduced through shear stresses via conservation of momentum. The resulting model is solved using the finite element library FEniCS, whereas the contour of the lake is obtained using hydrographic data from ArcGIS. The model is applied to study the behavior of the Ypacarai lake (in Paraguay), around where several industrial and tourist sites are settled. The results show critical regions, where the velocities are considerably low and the risk of contamination increases, as well as, allows to develop criteria depending on the fluctuation of the lake, the wind and season of the year, in order to take action in case of a possible contamination. This is a joint work with Derlis Ramirez , Jhabriel Varela and Christian Schaerer.*

Keywords: *Shallow lake, Wind stress, Finite element method, Hydrodynamics*

1 INTRODUCTION

The processes involved in the dynamics of shallow lakes have been studied in several works (Tsanis, 2006; Chen et al., 2012; Huang et al., 2014). Numerical models have been developed and used for research on flows in natural reservoirs such as lakes, coastal regions and rivers, and they were also applied in several engineering problems (Józsa, 2006; Wu et al., 2010; Chao, 2008). Shallow lakes are influenced by wind forces (wind wave effects), for instance, the frictional movement of wind blowing over water produces travelling surface waves (Tsanis, 2006).

In this work, the influence of wind forces in the dynamics of the Ypacarai lake is analyzed. This is a major water body located in Paraguay, of an average depth of 3 m. In recent years, the pollution of this lake has considerably increased, becoming the most polluted lake in the country.

In section 2, the wind effects on Navier - Stokes equations using mass and momentum conservation laws are introduced. Furthermore, a scalar (contaminant) transport equation is added in order to obtain the distribution of a contaminant in the lake. For the simulation, the Oasis solver is used to solve the equations (Mortensen, 2014). In section 3, the wind effect and contaminant transport are analyzed by contrasting the simulation results with *in situ* measurements. This information is extremely useful to identify regions of critical accumulation of the contaminant. These regions could eventually be different from the place where the contaminant is discharged. Therefore, the results can be used to make decisions about the right place to take action to reduce the contamination of the lake.

2 MODEL DESCRIPTION

In order to model the lake, the plane xy (at the surface) is considered and the effect of the bed of the lake is neglected. With these assumptions, the incompressible Navier - Stokes equations can be written as:

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \nu\nabla^2\mathbf{u} - \nabla p + \mathbf{f}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{2}$$

where t is the time, the operator ∇ is defined as $\nabla := (\partial_x, \partial_y)^T$ where $\mathbf{u} \in \mathbb{R}^2$ is the velocity defined as $\mathbf{u} = (u_1, u_2)^T$, p is the pressure, ν is the kinematic viscosity and $\mathbf{f}$ is the body force. The constant fluid density is incorporated into the pressure (Mortensen et al., 2015).

The wind shear stresses at the free surface are (Chao, 2008; Tsanis, 2006):

$$\tau_{wx} = \rho_a C_d u_w \sqrt{u_w^2 + v_w^2}, \quad \tau_{wy} = \rho_a C_d v_w \sqrt{u_w^2 + v_w^2}, \tag{3}$$

where τ_{wx} and τ_{wy} are the wind stresses at the direction x and y, respectively; u_w and v_w are the wind velocity components at 10 m elevation above the still water surface, in x and y, respectively. ρ_a is the air density. C_d is the drag coefficient, which is computed with $C_d = (0.8 + 0.065\|\mathbf{w}\|_2)\cdot 10^{-3}$ (Wu, 1980; Hsu,1988); where $\mathbf{w} = (u_w, v_w)^T$ is the wind velocity. The wind shear stress is introduced in the equation (1) as a body force in the term $\mathbf{f}$. In the context of this work, the wind velocity is taken as a constant.

The equation for the contaminant transport is based on the 2D - mass transport equation:

$$\partial_t c + \mathbf{u}\cdot\nabla c = D\nabla^2 c + f_c, \tag{4}$$

where $c(x, y, t)$ is the concentration of the contaminant and D its diffusivity, and a possible chemical reaction is denoted by f_c. This model can be extended to several chemical species.

3 NUMERICAL SIMULATION

The contour of the Ypacarai lake was obtained using hydrographic data from ArcGIS and the unstructured mesh grid was obtained using Hydrus (Šimůnek, 2012). The boundary of the lake, denoted as $\partial\Omega$, is divided as $\partial\Omega = \partial\Omega_t \cup \partial\Omega_D$ where $\partial\Omega_t$ is the part of the boundary associated to the tributaries and $\partial\Omega_D$ the rest of the boundary. The indexes 1, 2 and 3 ($\partial\Omega_1$, $\partial\Omega_2$ and $\partial\Omega_3$) correspond to the Salado, Pirayu and Yukyry streams, respectively (Fig. 1 (a)).

No-slip boundary condition at the contour the lake (except the tributaries):

$$\mathbf{u} = \mathbf{u}_{wall} \qquad \forall (x, y) \in \partial\Omega_D. \tag{5}$$

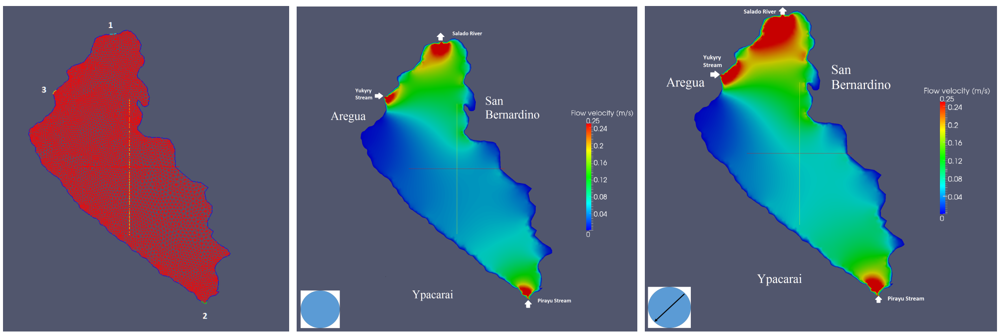
Neumann boundary conditions are used in the outflow tributaries, while Dirichlet boundary conditions are applied in the inflow:

$$\partial_\eta \mathbf{u} = g, \quad \forall (x, y) \in \partial\Omega_t \quad (outflow) \qquad \mathbf{u} = f, \quad \forall (x, y) \in \partial\Omega_t, \quad (inflow) \tag{6}$$

where η is the normal direction.

3.1 Wind analysis

A comparison of the lake condition in presence and absence of wind is presented. The inlet flows are placed in the Yukyry ($\partial\Omega_3$) and Pirayu ($\partial\Omega_2$) streams (average flows of 1.95 and 2.58 m^3/s, respectively) (Ruiz, 2012). The outlet flow is placed in the Salado River ($\partial\Omega_1$), with an average flow of 3.24 m^3/s. The viscosity is set to $\nu = 1.003 \cdot 10^{-3} m^2/s$, and the time step $\Delta t = 0.001s$. The wind speed magnitude is 15 km/h (DINAC, 2015).



(a) Ypacarai ssubdomains and grid

(b) Absence of wind

(c) Wind-induced currents. Direction: 230°

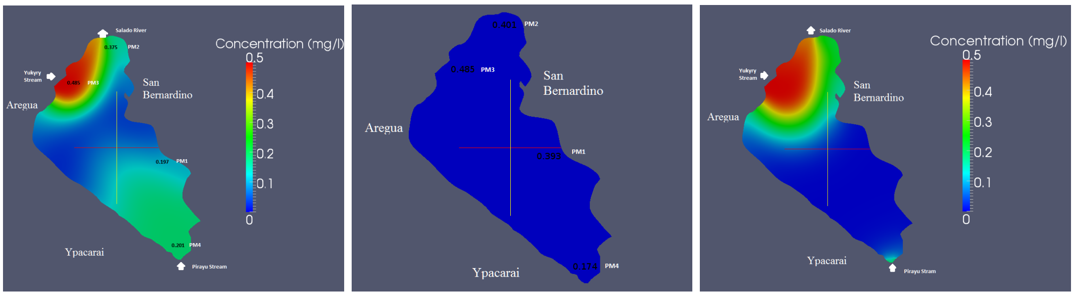
Figure 1: Flow velocities in the case studies. (a) Ypacarai ssubdomains and grid, (b) Absence of wind, (c) Wind-induced currents. Direction: 230°.

In Fig. 1, regions of the lake with very low velocity, which can be considered as dead zones, are observed (blue regions in Fig. 1). Hence, if a contaminant is discharged, it will remain there with small fluctuations. Regarding to the influence of the wind, the velocity profile in Fig. 1 (b) is larger than Fig. 1 (a) and also presents different distribution. In the regions of the lake that are not influenced enough by the wind, the circulation of water is not enough to extract a possible contamination. According to this, one of the most critical regions is Aregua, which agrees with the monitoring reports of the lake (CEMIT, 2015).

3.2 Contaminant transport analysis

The distribution of a contaminant in the lake is implemented through a non-reactive scalar equation model. Phosphorus compounds are considered as contaminants. The diffusivity value (used to compute the Schmidt number ν/D) is set to $D = 0.89 \cdot 10^{-9} m^2 s^{-1}$.

Figure 2 (a) presents the concentration of a contaminant after a discharge in the tributaries Yukyry ($\partial\Omega_3$) and Pirayu ($\partial\Omega_2$) simultaneously. It is observed that due to the velocity profile and the proximity of the outflow of the Salado river ($\partial\Omega_1$) to the source of contamination, most of the contaminant mass flows through out of it. Figure 2 (b) shows some points where total phosphorus data (mg/l) were obtained (CEMIT, 2015). Figure 2 (c), shows the influence of wind direction on the transport of the compound. Wind directions of 270° have the greatest influence on scalar transport to the area of Aregua. This point is taken into account because it has a very reduced flow rate, whereby accumulation is likely to occur.



(a) Tributaries income pollutant. (b) Measurement points. (c) Wind direction: 270°

Figure 2: Comparison between simulation results and field measurements.

Table 1: Measurement points references

Reference	Measurement Point	Numerical values (mg/l)	
		Real	Simulated
PM1	San Bernardino beach	0.393	0.197
PM2	Salado River output	0.401	0.375
PM3	Yukyry input stream	0.480	0.485
PM4	Pirayu input stream	0.174	0.201

Table 1, compares the values of contamination, obtained by the simulation and the measured values (CEMIT, 2015; SEAM, 2015). Qualitatively the range of values agrees between each other. However, there is a discrepancy between them in certain places, which is probably due to local contamination and non-modelled parts of the ecosystem which affects the flux, such as the non-modelled vegetation of the lake. In addition the measured data correspond to average values, while the simulation corresponds to values obtained with an ideal configuration of fluxes in the tributaries.

4 SUMMARY AND CONCLUSIONS

The numerical results showed that the transport of contaminants might be considerably affected by the wind. These results were compared with field measurement data. Even though the concentrations at those points do not match perfectly with the numerical results, we could be able to demonstrate a qualitative similarity, showing that the simulation captures the main real phenomenology of the lake. This is the most important information obtained, based on the limited information available. Another important feature of the model is that it can be used to analyze several configurations of boundary conditions, as well as, contaminant discharge. Despite that our model at the moment is not considering chemical reactions, neither all the dynamics present in the lake, such as the effect on the bottom of the lake in the simulation, it represents a starting point for future works which might consider these effects.

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