CONSTRUCTAL DESIGN OF ISOTHERMAL DOUBLE-T SHAPED CAVITY BY MEANS OF SIMULATED ANNEALING

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Abstract. *In this work it is employed the Constructal Design (CD) and the Simulated Annealing (SA) algorithm to the geometric optimization of an isothermal double-T shaped cavity. The problem consists in a solid with internal uniform heat generation and its outer surfaces are in adiabatic conditions. The heat only can be removed by the cavity that is maintained at a minimal temperature. The objective is to minimize the maximum excess of temperature (θ_{max}) in the solid domain through geometric optimization of the cavity. The cavity geometry is optimized varying three degrees of freedom (H_2/L_2, H_1/L_1 and S_1/H_0), but* *the ratios H_2/L_2 and H_1/L_1 are varied at the same interval. The search for the optimal geometry is performed by Exhaustive Search (ES), which are used to validate the results of SA algorithm. In this study was also investigated the influence of Cooling Schedule (CS) parameter of the SA algorithm in the results of optimization process. The results shown that the use of SA reduce at five percent the simulations required by ES. The lowest values of the ratios H_2/L_2 and H_1/L_1 conduct to optimal geometry. For the ratio S_1/H_0 the optimal value is the one that represents the intermediate position between the branches of double-T shaped cavity.*

Keywords: *Constructal Design, Exhaustive Search, Heat Transfer, Simulated Annealing*

INTRODUCTION

According to the Constructal Theory, similar geometries that emerge in different objects of nature are not a coincidence or a random cause (Bejan and Zane, 2012), but follows a physical principle and led the flux systems to deterministic patterns. These objects consist or have a flow system that molds its geometry towards facilitating the passage of the flux that flows through it. So, the result is the optimal shape that minimizes the resistance to the flow. The similarity between the geometry of the tree roots or their branches and the geometry of thunderbolts is an example. Bejan (1996) demonstrate and explains these similarities as a physical phenomenon named Constructal Law “For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides easier access to the currents that flow through it”. The same law that governs the optimal shapes presented in nature is also used to find the optimal configuration in engineering problems like the cooling of small electronic packages. This method that applied the Constructal Theory to optimization of the engineering flows systems is called Constructal Design (CD).

The application of Constructal Law in projects of the engineering developed and created the Constructal Design (CD), which is a method based on objectives and constraints for evaluation of flow systems. The CD has helped the research of cavity geometry influence in thermal performance of fins array (Biserni et al., 2004).The study in fin array improve the techniques of heat transfer in many engineering applications, like heat exchanger or cooling of electronic package (Kraus, 1999). In Biserni et al. (2004), it was performed the optimization of the first construction of the elementary C-shaped cavity and second construction with lateral T-shaped cavity, which showed a better thermal performance than first construction. In Biserni et al. (2007) the optimal H-shaped performance was better than T-shaped. The need to minimize the thermal resistance in the same volume leads to increases the geometry complexity and numbers of degrees of freedom (DOF) of the cavity. The increase of DOF needs more computational effort and precludes the Exhaustive Search (ES) aproach. In this sense, the study of Lorenzini et al. (2014a) was the pioneer study to employ the meta-heuristic method Genetic Algorithm (GA) to the geometric optimization of Y-shaped cavity in association to CD.

In this study, we applied the CD combined with SA algorithm for geometric optimization of a heat transfer problem. The problem has similarity with previous study of Lorenzini et al. (2014), but is concerned with the employment of SA and study of a not evaluated geometry (double-T shaped cavity). The cavity is intruded into a solid conducting wall that has uniform internal heat generation and its heat flux can be removed only by the cavity. Therefore, the geometry of cavity influence the temperature fields. So, the main goal is to find the optimal geometry that minimize the thermal resistance and to investigate a theoretical recommendation about performance of the double-T shaped cavity. The optimization process is subject to constraints and the objective that are determined by CD. First, two DOF are optimized (H_1/L_1 and S_1/H_0) and the search by optimal shapes is carried out by ES method and their results are used to validate those obtained with SA algorithm. Next, the three DOF (H_2/L_2, H_1/L_1 and S_1/H_0) are optimized and the results of different versions of SA algorithm are compared with ES method.

The SA algorithm is employed in different versions that distinguish itself by a single parameter called Cooling Schedule (CS). This parameters control the temperature of the SA, a parameter that influence the behavior of the algorithm (Kirkpatrick et al. 1983; Eglese, 1990). five parameters of CS are used, the traditional parameters already implemented in the MATLAB environment (Exponential and Boltz), and more three hybrid parameters purposed in Gonzales et. al. (2014a; 2014b) (BoltzExp, ConstExp1 and ConstExp2). The aim is try to improve the robustness of SA algorithm applying different version, as well as, find the better version of algorithm for future studies, like the complete geometry cavity optimization and investigate the effects of its constraints.

MATHEMATICAL MODEL

Figure 1 shows the conducting body in the two-dimensional configuration, with the third dimension W perpendicular to the plane of the figure. The solid domain is represented by the gray region in the Fig. 1, which has a constant and uniform heat generation at the volumetric rate given by q''' (W·m^{-3}), with area A. The solid has a constant conductivity k. The outer surfaces of the solid are perfectly insulated, adiabatic conditions. In this case the heat flow only can be removed by the double-T shaped cavity kept at a minimum temperature (T_{min}). The minimal temperature of the cavity simulates the flow of refrigerant fluid in change of phase and at low temperature flowing through the cavity. For the sake of simplicity, the heat transfer coefficient on the cavity wall is assumed so large that the convective resistance can be neglected in comparison to the solid conduction resistance.

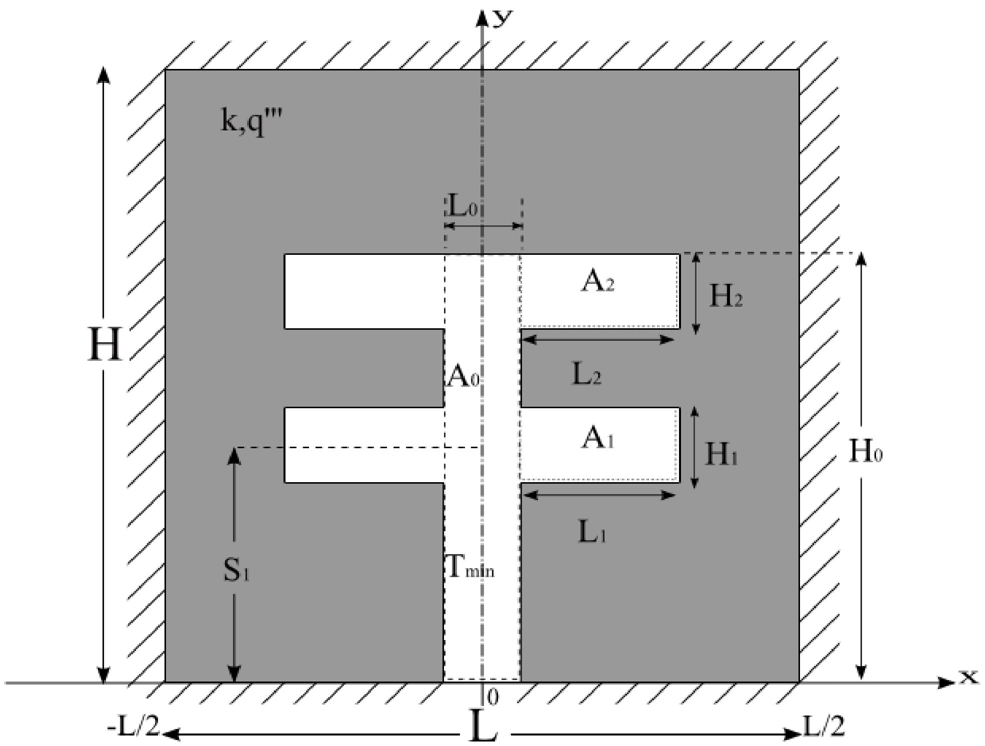


Figure 1. Computational Domain of Double-T Shaped Cavity into a solid with internal heat generation.

The objective of the analysis is to determine the optimal geometry (H/L, H_0/L_0, H_2/L_2, H_1/L_1 and S_1/H_0) that is characterized by the minimum global thermal resistance $(\theta_{max} - \theta_{min})/(q'''A)$. According to CD this optimization can be subjected to the constraints of the total area and cavity area, represented respectively by following equations:

$$A = HL \tag{1}$$

$$A_c = A_0 + 2A_1 + 2A_2 \tag{2}$$

Eq.(1) can be expressed as a fraction of the cavity area in relation to total area giving by:

$$\phi_c = A_c / A \tag{3}$$

For the determination of the temperature field as function of the geometry in the solid domains, it is need to solve numerically the heat conduction equation giving by:

$$\frac{\partial^2 \tilde{\theta}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2} + 1 = 0 \tag{4}$$

where the dimensionless variables are:

$$\tilde{\theta} = \frac{\theta - \theta_{min}}{q''' \cdot \frac{A}{k}} \tag{5}$$

$$\tilde{x}, \tilde{y}, \tilde{H}_0, \tilde{H}_1, \tilde{H}_2, \tilde{L}_0, \tilde{L}_1, \tilde{L}_2, \tilde{H}, \tilde{L}, \tilde{S}_1 = \frac{x, y, H_0, H_1, H_2, L_0, L_1, L_2, H, L, S_1}{A^{1/2}} \tag{6}$$

The boundary conditions of null flux in the solid outer surfaces, as well as, the boundary conditions of minimal temperature in the cavity wall are represented by following equations:

$$\frac{\partial \tilde{\theta}}{\partial \tilde{x}} = 0 \quad \text{em} \quad \tilde{x} = -\frac{\tilde{L}}{2} \quad \text{ou} \quad \tilde{x} = \frac{\tilde{L}}{2} \quad \text{e} \quad 0 \le \tilde{y} \le \tilde{H} \tag{7}$$

$$\frac{\partial \tilde{\theta}}{\partial \tilde{y}} = 0 \quad \text{em} \quad \tilde{y} = 0 \quad \text{e} \quad -\frac{\tilde{L}}{2} \le \tilde{x} \le -\frac{\tilde{t}_1}{2} \quad \text{ou} \quad \frac{\tilde{t}_1}{2} \le \tilde{x} \le \frac{\tilde{L}}{2} \tag{8}$$

$$\frac{\partial \tilde{\theta}}{\partial \tilde{y}} = 0 \quad \text{em} \quad \tilde{y} = \tilde{H} \quad \text{e} \quad -\frac{\tilde{L}}{2} \le \mathrm{x} \le \frac{\tilde{L}}{2} \tag{9}$$

$$\tilde{\theta} = \tilde{\theta}_{\min} \quad \text{at} \quad \tilde{x} = -\tilde{L}_0/2 \quad \text{or} \quad \tilde{x} = \tilde{L}_0/2 \quad \text{or} \quad 0 \le \tilde{y} \le \tilde{S}_1 - \tilde{H}_1/2 \tag{10}$$

$$\tilde{\theta} = \tilde{\theta}_{\min} \quad \text{at} \quad \tilde{x} = -\tilde{L}_0/2 \quad \text{or} \quad \tilde{x} = \tilde{L}_0/2 \quad \text{and} \quad \tilde{S}_1 + \tilde{H}_1/2 \le \tilde{y} \le \tilde{H}_0 - \tilde{H}_2 \tag{11}$$

$$\tilde{\theta}=\tilde{\theta}_{\min} \quad \text{at} \quad \tilde{x}=-(\tilde{L}_1+\tilde{L}_0/2) \ \text{ or } \ \tilde{x}=\tilde{L}_1+\tilde{L}_0/2 \quad \text{and} \quad \tilde{S}_1-\tilde{H}_1/2 \le \tilde{y} \le \tilde{S}_1+\tilde{H}_1/2 \tag{12}$$

$$\tilde{\theta}=\tilde{\theta}_{\min} \quad \text{at} \quad \tilde{x}=-(\tilde{L}_1+\tilde{L}_0/2) \ \text{ or } \ \tilde{x}=\tilde{L}_1+\tilde{L}_0/2 \quad \text{and} \quad \tilde{H}_0-\tilde{H}_2 \le \tilde{y} \le \tilde{H}_0 \tag{13}$$

$$\tilde{\theta}=\tilde{\theta}_{\min} \quad \text{at} \quad \tilde{y}=\tilde{S}_1-\tilde{H}_1/2 \ \text{ or } \ \tilde{y}=\tilde{S}_1+\tilde{H}_1/2 \ \text{ or } \ \tilde{y}=\tilde{H}_0-\tilde{H}_2 \quad \text{and} \quad -(\tilde{L}_1+\tilde{L}_0/2) \le \tilde{x} \le -\tilde{L}_0/2 \tag{14}$$

$$\tilde{\theta}=\tilde{\theta}_{\min} \quad \text{at} \quad \tilde{y}=\tilde{S}_1-\tilde{H}_1/2 \ \text{ or } \ \tilde{y}=\tilde{S}_1+\tilde{H}_1/2 \ \text{ or } \ \tilde{y}=\tilde{H}_0-\tilde{H}_2 \quad \text{and} \quad \tilde{L}_0/2 \le \tilde{x} \le \tilde{L}_1+\tilde{L}_0/2 \tag{15}$$

$$\tilde{\theta}=\tilde{\theta}_{\min} \quad \text{at} \quad \tilde{y}=\tilde{H}_0 \quad \text{and} \quad -(\tilde{L}_2+\tilde{L}_0/2) \le \tilde{x} \le \tilde{L}_2+\tilde{L}_0/2 \tag{16}$$

The dimensionless form of the Eqs. (1-2) are represented by the following equations:

$$1=\tilde{H}\tilde{L} \tag{17}$$

$$\phi_c=\tilde{H}_0\tilde{L}_0+2\phi_1+2\phi_2 \tag{18}$$

$$\phi_1=\tilde{H}_1\tilde{L}_1 \tag{19}$$

$$\phi_2=\tilde{H}_2\tilde{L}_2 \tag{20}$$

The aim is to minimize the maximal excess of temperature $\tilde{\theta}_{max}$ represented by the follow equation:

$$\theta_{max}=\frac{\theta_{max}-\theta_{min}}{q''' \cdot \dfrac{A}{k}} \tag{21}$$

For determination of $\tilde{\theta}_{max}$ it is needed to optimize the five degrees of freedom (H/L, H_0/L_0, H_2/L_2, H_1/L_1 and S_1/H_0) submitted at the followed constraints of the cavity area (ϕ_c, ϕ_1 e ϕ_2) and the total solid area. In this paper is optimized just three DOF (H_2/L_2, H_1/L_1 and S_1/H_0).

NUMERICAL MODEL

The function represented by Eq. (21) is determined numerically by solving Eq. (4) for the temperature field in every assumed configurations (H/L, H_0/L_0, H_2/L_2, H_1/L_1 and S_1/H_0) and calculating $\tilde{\theta}_{max}$ to see whether $\tilde{\theta}_{max}$ can be minimized by varying the configuration. The numerical solution is performed with the Finite Element Method (FEM)(Reddy and Gartling, 1994), based on linear triangular elements, developed in the MATLAB environment, precisely the PDE (partial-differential-equations) toolbox (MATLAB, 2000). The grid was non-uniform in both x and y directions, and varied from one geometry to the next.

The grid was non-uniform in both x and y directions, and varied from one geometry to the next. The appropriate mesh size was determined by successive refinements (h-adaptively), increasing the number of elements four times from the current mesh size to the next one, until the following criterion be satisfied:

$$|(\tilde{\theta}^{i}_{max} - \tilde{\theta}^{i+1}_{max})/\tilde{\theta}^{i}_{max}| < 5\times10^{-4} \quad (22)$$

This criterion was based on the studies of C-Shaped and T-Shaped cavities in Biserni et al. (2004). Here $\theta^{j}_{\max}$ represents the maximum temperature calculated using the current mesh size, and $\theta^{j+1}_{\max}$ correspond to the maximum temperature using the next mesh, where the number of elements was increased by four times. Table 1 shows how grid independence was achieved. The following results were performed by using a range between approximately 1,260 and 322,560 triangular elements. The search for grid independence was performed for geometry with following configuration: $\phi_c = 0.1$; $\phi_1 = 0.015$; $\phi_2 = 0.015$; $H/L = 1$; $H_0/L_0 = 0.6$; $H_1/L_1 = 0.4$; $H_2/L_2 = 0.4$ and $S_1/H_0 = 0.5$.

Table 1. The grid independence test.

Elements Number	$\tilde{\theta}_{max}$	$\lvert(\tilde{\theta}^{i}_{max} - \tilde{\theta}^{i+1}_{max})/\tilde{\theta}^{i}_{max}\rvert$
1260	0.16646	5.565×10^{-3}
5040	0.16739	2.224×10^{-3}
20160	0.16776	8.614×10^{-4}
80640(Independent Mesh)	0.16791	3.435×10^{-4}
322560	0.16797	-------------

GEOMETRIC OPTIMIZATION

The optimization process was oriented by the CD method in the definition of the objective and constraints, as well as, the definition of the degrees of freedom (DOF) that are varying in the seek for the optimal geometry. In this work three DOF was optimized (H_2/L_2, H_1/L_1 and S_1/H_0), and the others DOF was maintained fixed in H/L = 1 and H_0/L_0 = 6. The four study constraints are keep in A = 1, ϕ_c=0.1, ϕ_1=0.015 and ϕ_2=0.015. The ratios H_2/L_2 and H_1/L_1 are varied in the same interval, thus kept equal the branches of the double-T shaped cavity. After the determination of the DOF and constraints imposed to the problem, the search for the optimal geometry was achieved in two ways. First is employed the Exhaustive Search (ES) method to vary the geometry configuration (H_2/L_2, H_1/L_1 and S_1/H_0), simulating all the combinations possibility of the DOF. In the second moment, the Simulated Annealing (SA) algorithm was used to optimization process. Finally, the results of the ES method are used to validate the results obtained by the SA algorithm.

Different versions of the SA algorithm was applied in the optimization process. The main difference between the SA versions is the Cooling Schedule (CS) parameter, This algorithm parameter was already investigated in the works of the Gonzales et al. (2014a) and Gonzales et al.(2014b), but for a Y-shaped cavity optimization. The second goal here is detect the algorithm version that have more probability in lead to the optimal shaped and use this method in the future complete optimization. Then, the results of the SA algorithms are compared with ES method results, because this validation just few DOF are optimized in this work.

Constructal Design of the Shaped Cavity Double-T

The problem study here is concerned with the maximization of the heat transfer between the solid and the double-T shaped cavity, i.e. , to achieve the cavity geometry that minimizes the thermal resistance of the domain. The geometry cavity variation is subjected to the constraints and the main constraint is the fraction cavity area in relation to the total area (ϕ_c) represented by Eq. (18). The geometry of the problem domain studied here has nine variables (H, L, H_0, L_0, H_1, L_1, H_2, L_2 e S_1), and four constraints (A, ϕ_c, ϕ_1 and ϕ_2), then is needed five degrees of freedom to complete the equations and determination of the geometry. In this study only three DOF are optimized (H_2/L_2, H_1/L_1 and S_1/H_0) and others are kept with fixed values. The three DOF receive values defined by the search space discretization during the optimization process. The following equations show variable definition as function of various DOFs.

$$\tilde{L}=\left[\frac{1}{\left(\frac{H}{L}\right)}\right]^{1/2} \tag{23}$$

$$\tilde{H}=\left(\frac{H}{L}\right)\cdot\tilde{L} \tag{24}$$

$$\tilde{L}_0=\left[\frac{\phi_c-2\phi_1-2\phi_2}{\left(\frac{H_0}{L_0}\right)}\right]^{1/2} \tag{25}$$

$$\tilde{H}_0=\left(\frac{H_0}{L_0}\right)\cdot\tilde{L}_0 \tag{26}$$

$$\tilde{L}_1=\left[\frac{\phi_1}{\left(\frac{H_1}{L_1}\right)}\right]^{1/2} \tag{27}$$

$$\tilde{H}_1=\left(\frac{H_1}{L_1}\right)\cdot\tilde{L}_1 \tag{28}$$

$$\tilde{L}_2=\left[\frac{\phi_2}{\left(\frac{H_2}{L_2}\right)}\right]^{1/2} \tag{29}$$

$$\widetilde{H}_2 = \left(\frac{H_2}{L_2}\right) \cdot \widetilde{L}_2 \tag{30}$$

$$\widetilde{S}_1 = \left(\frac{S_1}{H_0}\right) \cdot \widetilde{H}_0 \tag{31}$$

With the five DOF defined (H/L, H_0/L_0, H_1/L_1, H_2/L_2 e S_1/H_0) is possible to continues with the optimization process determining the search space of the problem, i. e., the amount of values that the DOF can be assumed and combine them self to generate several geometry configurations that will be simulated. In the optimization process with ES method one DOF is varied and the others are maintained with fixed values, but with SA algorithm the DOF combinations are generated randomly from the discretized search space.

Using Simulated Annealing to Geometric Optimization

With the ES method, all combination of the DOF are simulated, and in the final process the results are compared to find the geometry that minimize the thermal resistance. However this method requires more computational effort when the number of DOF increases. Them, the alternative is the use of meta-heuristic algorithm methods, like can be see in Lorenzini et al (2014a, 2014b) and Gonzales et al.(2014a, 2014b). The meta-heuristic algorithm performed in this study is the SA algorithm. The SA algorithm does not simulates all combination of DOF in the search space, instead, the algorithm use a stochastic method to explore the search space, decreasing considerably the computational effort.

The SA algorithm is based on a thermodynamic process of the annealing of materials (Kirkpatrick et al. ,1983; Eglese 1990). The SA was firstly implemented by Kirkpatrick et al. (1983), applying the algorithm show in Metropolis et al. (1953), for different levels of temperature. The search behavior of the SA algorithm is controlled by a virtual temperature that simulates the physics processes. In the high temperatures the algorithm have more probability to accept bad solution, but in the lower temperatures the probability is much smaller and only better solution is accepted. The probability of solution acceptance is given by the follow equation:

$$\mathrm{P} = \frac{1}{1 + \exp\left(\frac{\Delta\, OF}{T}\right)} \tag{32}$$

where Δ is the difference between the value returned by Objective Function (OF) of the new configuration in relation to OF value for the best evaluated solution. More precisely, when a new geometry neighbor is proposed in comparison with the current solution, the value of OF, which it is intended to be minimized, is calculated for this new geometry (neighbor). Afterwards, it is calculated Δ subtracting the value of OF for the neighbor solution and OF for the current best solution. The variable T in the Eq.(32) is the temperature in the current iteration of the SA algorithm and its value changes in the optimization process influencing the acceptance probability (Bertsimas and Tsitsiklis, 1993). The numerical model for acceptance probability used here is the same of the default function in MATLAB environment (MATLAB,2000).

The Cooling Schedule (CS) is the way that the temperature *T,* Eq (32), is decreased during the algorithm iterations. In the literature there are several studies comparing the CS parameter and variants of classical SA (Szu e Hartley, 1987; Hajek, 1988; Ingber, 1989; Ingber, 1993; Nourani e Andresen, 1998; El-Boury et al., 2007; Schneider e Puchta, 2010; Kastanya, 2013; Brusco, 2014), but each research is valid for a only kind of problem investigated, the results do not ensure the general effectiveness of the methods. So, according to Kirkpatric et al (1983), the choice of the CS parameters is experimental. In this research, was applied the CS parameter investigated in the studies of Gonzales et al.(2014a, 2014b).

To apply the SA algorithm in the geometric optimization of the double-T shaped cavity, it is used the MATLAB environment, more precisely the Global Optimization Toolbox with a specific function named *simulannealbnd*. By default, the SA solver works in the continuous search space, then it was needed an adaptation in the *annealing* and *acceptance* functions to discrete search space. For each DOFs optimized here, was created a vector with the ratios that will be combined with others DOFs in the geometry variation during the optimization process. This adaptation also can be realized in the works of the Lorenzini et al. (2014a, 2014b) for GA and Gonzales et al.(2014a, 2014b) applying SA algorithm, transforming the exhaustive optimization problem in a combinatorial optimization problem.

Firstly the optimization process is given by ES, simulating all possibility, and its results was used for validated the results of SA algorithm. For the optimization of two DOFs, it is investigated the values of H_1/L_1, H_2/L_2 and S_1/H_0. The values of H_1/L_1 and H_2/L_2 were kept equal, their values were varied equally in the search space of the interval 0.4 to 2, with step of 0.4. For each combination of H_1/L_1 or H_2/L_2, the search space of S_1/H_0 changed, then their search space was determined according to the variation of the variable S_1, respecting the minimal and maximum values of the S_1 for each combination of H_1/L_1 and H_2/L_2. The minimal and maximum values of S_1 and the search space for H_1/L_1 and H_2/L_2 are represented by following equation:

$$S_{1min}=(H_1/2)+\Delta S_1 \tag{33}$$

$$S_{1max}=H_0-H_2-H_1/2-\Delta S_1 \tag{34}$$

$$H_1/L_1=H_2/L_2=[0{,}4 \quad 0{,}8 \quad 1{,}2 \quad 1{,}6 \quad 2{,}0] \tag{35}$$

where ΔS_1 is the variation of S_1variable with a step of 0.001.

The application of SA algorithm was realized in the same search space used in the ES method. Were applied three versions of the SA algorithm, each one with a different parameter of CS (BoltzExp, ConstExp1 and ConstExp2), this parameters were studied in the works of Gonzales et al.(2014a, 2014b). The *annealing* function of the SA algorithm, that is responsible to generate a new geometric configuration or solution in the search space, was adapted for the two DOF optimization. The others DOF of the double-T shaped cavity was kept in H/L = 1 and H_0/L_0 = 6, as well as, the cavity constraints ϕ_c= 0.1 and ϕ_1=ϕ_2=0.015.

The three DOF optimization (H_1/L_1, H_2/L_2 and S_1/H_0) was realized in the same search space of the two DOF optimization process, the only exception is the variation of the DOF H_1/L_1 in different values of the DOF H_2/L_2. Then, the H_1/L_1 and S_1/H_0 were investigated forming distinct combinations between the ratios of S_1/H_0 and H_1/L_1 for each value of H_2/L_2, valued in the interval of $0.2 \leq H_2/L_2 \leq 2.0$.

The ES method was applied again to the validate of the SA algorithm results. So, it is simulated all possible combination between the values of S_1/H_0 and H_1/L_1 for each value of H_2/L_2. Finally, it is achieved the minimized maximum excess of temperature for each value of the H_2/L_2. For the execution of the SA algorithm with the MATLAB solver, it is necessary an adaptation in the annealing function. In this function version, are generated random values for two DOF (S_1/H_0 and H_1/L_1), creating random combination to the geometry configuration.

The hybrid CS, present in the study of Gonzales et al.(2014b) is employed together with with the traditional parameters of CS in MATLAB environment. According to the results and recommendation of Gonzales et al.(2014b), the CS called *Fast* wasn't used here because this parameter tends to achieve several local optimums. Therefore, different versions of the SA algorithm were analyzed comparing their results to the three times optimized geometry.

RESULTS AND DISCUSSION

The two DOF optimization is achieved with ES method and different versions of SA algorithm with hybrid CS. The optimization results for $H_1/L_1 = H_2/L_2 = 0.4$ can be observed in the Figure 2. The gray points represents the results and all values simulated by ES method with a total of 363 simulations. In red color it can be seem the best results of the version of SA algorithm with hybrid CS ConstExp2, with a total of 126 simulations, minor than half number of simulations necessary for the ES method. The two methods achieved the optimal geometry and the minimum maximum excess temperature is represented by the green triangle. In Figure 2 it is also possible to see a large number of points near the optimal region in the seek performed with SA.

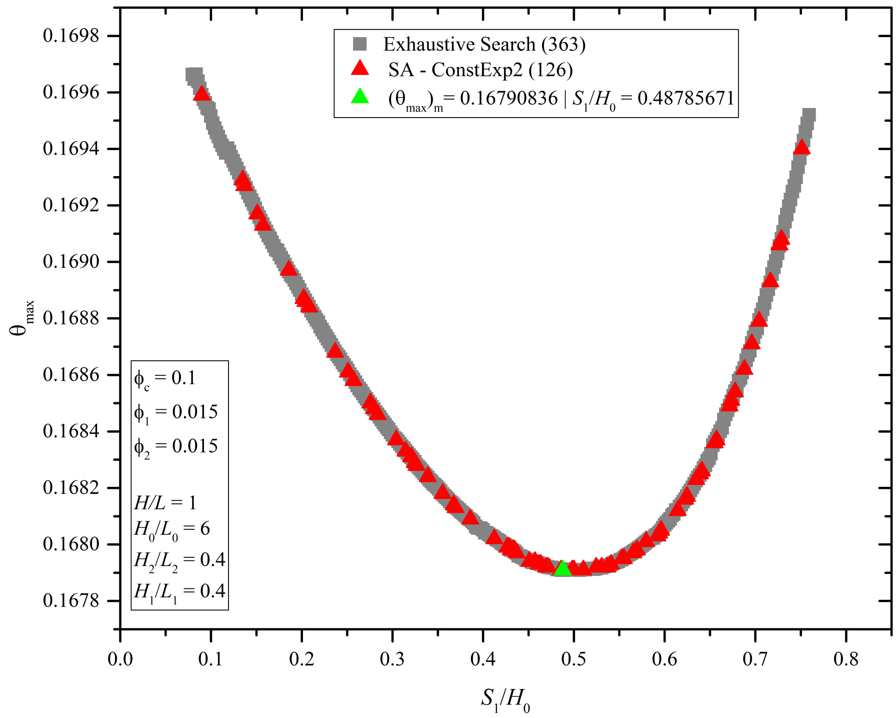


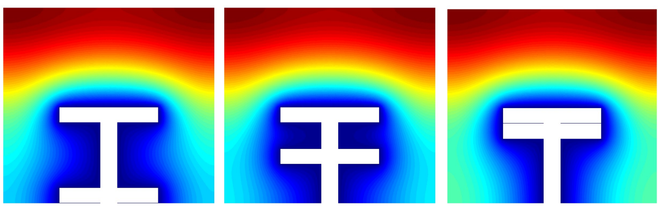
Figure 2 Comparison between the results of ES method and SA algorithm for the effect of S_1/H_0 over $(\theta_{max})_m$

Table 2 shows the results of others versions of the SA algorithm configured with different CS parameter. The parameter of the *StallIterLimit* also is represented in Table 2, this parameter of SA algorithm determine the stop criterion of the algorithm and is the number of the iterations without changes in the current best solution. For example, when the *StallIterLimit* is equal 100, the algorithm is stopped with hundred iterations without changes in the optimal solution. Other SA parameter is the *Reannealing* that define when the algorithm must be grow up the temperature to the initial value, in this case, this parameter was configured with 50 iterations without changes in the best solutions. It is possible see that with the elevation of the *StallIterLimit* parameter from 50 to 100, all versions of the SA algorithm returned the optimal value of the S_1/H_0. Thereafter, the increase of stop criterion value, *StallIterLimit*, even increase the total number iterations of the algorithm and the performed simulations. The SA version with CS *BoltzExp* was the algorithm that needed the lowest number of iterations (Iter.), i.e., a total of 170 simulations, when the *StallIterLimit* is equal 100.

Table 2. Results of ES method and different versions of SA algorithm for two DOF optimization in H_2/L_2=0.4.

Iter.	$(\theta_{max})_m$	S_1/H_0	**Method**	**Time(s)**	
363	0.1679083	0.487	Exhaustive Search	~1400	StallIterLimit
126	0.1679083	0.487	SA ConstExp2	533	50
83	0.1679093	0.485	SA BoltExp	331	50
98	0.1679100	0.500	SA ConstExp1	418	50
170	0.1679083	0.487	SA BoltExp	693	100
176	0.1679083	0.487	SA ConstExp2	730	100
212	0.1679083	0.487	SA ConstExp1	901	100

Figure 3 shows the temperature fields for the lowest extreme of the search space of S_1/H_0, equal 0.08, Fig. 3(a), for the ratio one times minimized $(S_1/H_0)_o$ = 0.488, Fig 3(b), and the superior limit of S_1/H_0 = 0.759, Fig 3(c). In general, for the fixed geometry parameters (ϕ_c = 0,1; $\phi_1 = \phi_2$ = 0,015; H/L = 1,0; H_0/L_0 = 6,0 e $H_1/L_1 = H_2/L_2$ = 0,4), one can observe that the DOF S_1/H_0 has almost no influence over the magnitude of the dimensionless maximum excess of temperature. This behavior occurs, mainly, because the variation of the intermediate branches that emerge in the solid domain region that already cooled by the cavity. As can be see, the temperature field with hot spots has a similar distribution in the superior region of the solid domain for the three geometries shown in Figure 3.



(a) (b) (c)

(a)	**(b)**	**(c)**
H/L = 1.0	H/L = 1.0	H/L = 1.0
H_0/L_0 = 6.0	H_0/L_0 = 6.0	H_0/L_0 = 6.0
$H_1/L_1 = H_2/L_2$ = 0.4	$H_1/L_1 = H_2/L_2$ = 0.4	$H_1/L_1 = H_2/L_2$ = 0.4
S_1/H_0 = 0.0796	$(S_1/H_0)_o$ = 0.4879	S_1/H_0 = 0.7593
θ_{max} = 0.1697	$(\theta_{max})_m$ = 0.1679	θ_{max} = 0.1695

Figure 3 Comparison between the optimal geometry and two others geometries of the Figure 2.

The Table 3 shows the optimization results for the others values of the H_2/L_2 ($0.8 \leq H_2/L_2 \leq 2.0$). The optimization process was performed with the ES method and different versions of the SA algorithm. In most results, the SA algorithm with hybrid CS requires only half of iterations than ES method for achievement of optimal geometry. The total execution time of the algorithm was approximated according to the number of iterations, considering the time of the four seconds for each simulation.

Throug the results shown in the Table 3, it can be see the great influence of the ratio of the cavity branches (H_1/L_1 and H_2/L_2), for the values of H/L = 1.0 e H_0/L_0 = 6.0, over the thermal resistence of the solid domain. The once minimized values of θ_{max} are achieved for the ratios of H_1/L_1 and H_2/L_2 equal to 0.4. This behavior is in agreement with preliminary results where Constructal Design was employed in the optimization of the several cavities (C, T, H, Y, complex cavities and multiple simple cavities) (Biserni et al., 2004; Biserni et al. 2007; Xie et al., 2010; Lorenzini et al., 2011, 2012, 2013; Hajmohammadi et al., 2013). For example, in the problem of the T-Shaped cavity, the best thermal performance (lowest θ_{max}) occurred when the cavity had more penetration in the solid domain. In the case studied here, this occurs to minor ratios of H_1/L_1 and H_2/L_2.

Table 3. Comparison of the results reached with different versions of SA algorithm and ES method for two degrees of freedom.

Method	$(\theta_{max})_m$	$(S_1/H_0)_o$	$H_2/L_2=H_1/L_1$	**Iterations**	**Time ~(s)**
ES	0.1879	0.4266	0.8	315	1260
ConstExp2	0.1879	0.4245	0.8	104	439
ConstExp1	0.1879	0.4245	0.8	107	431
BoltzExp	0.1879	0.4204	0.8	190	796
ES	0.1985	0.4000	1.2	278	1112
BoltzExp	0.1985	0.4000	1.2	100	463
ConstExp2	0.1985	0.4000	1.2	173	745
ConstExp1	0.1985	0.4000	1.2	192	838
ES	0.2054	0.3572	1.6	246	984
ConstExp1	0.2054	0.3572	1.6	123	517
BoltzExp	0.2054	0.3572	1.6	129	638
ConstExp2	0.2054	0.3572	1.6	170	696
ES	0.2103	0.3265	2.0	220	880
ConstExp2	0.2103	0.3265	2.0	106	424
ConstExp1	0.2103	0.3245	2.0	125	500
BoltzExp	0.2103	0.3265	2.0	209	1000

The Figure 4 shows the effects of S_1/H_0 over the dimensionless maximum excess temperature, for different values of H_1/L_1 and H_2/L_2. As can be seen in all cases there is an intermediate optimal geometry for S_1/H_0 that conduct to $(\theta_{max})_m$. This optimal geometry is influenced by the ratios H_1/L_1 and H_2/L_2. It can also be seen that the lowest values of the H_2/L_2 leads to lowest magnitudes of the θ_{max} over all studied values of S_1/H_0. So it can be concluded that the degrees of freedom H_1/L_1 and H_2/L_2 for double-T cavity shape has a considerably influence over thermal performance of the present problem.

The optimal geometries obtained in Figure 3 are compiled in Figure 5. More precisely it is measured the effect of H_2/L_2 over the dimensionless maximum excess of temperature once minimized, $(\theta_{max})_m$, and their optimal geometries, i. e., $(S_1/H_0)_o$. Results suggested that there is an increase, almost linear, of $(\theta_{max})_m$ with the increases of H_2/L_2. It is also observed that the rise of H_2/L_2 leads to a decrease of the ratio S_1/H_0, i. e., for more slender lateral cavities, the best performance occurs when the intermediate cavities are located nearest to the lower base area of the solid.

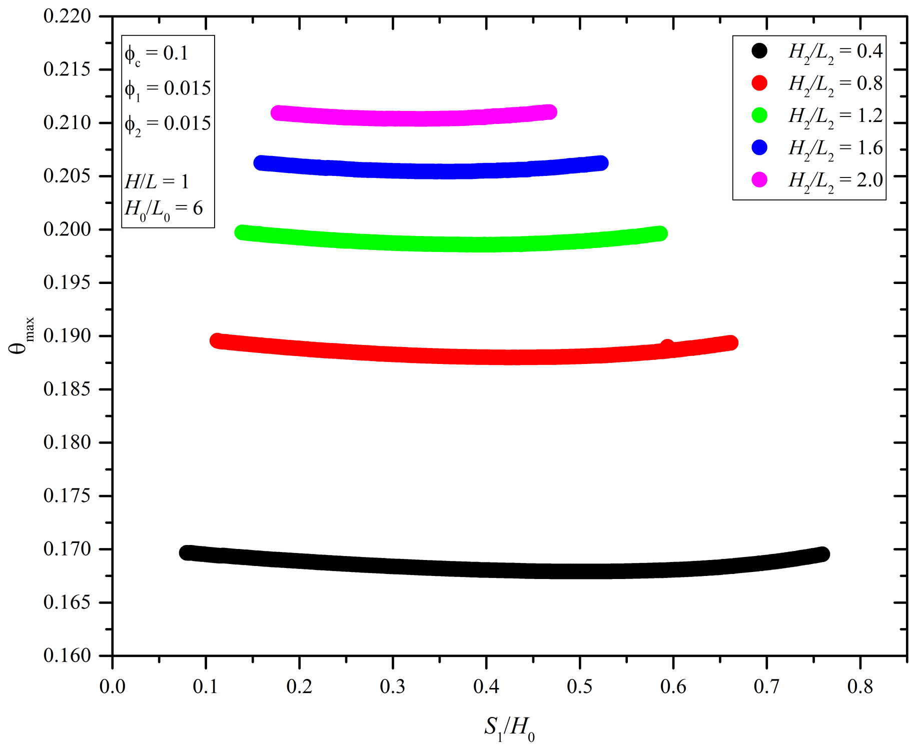


Figure 4 Comparison between the effect of $(S_1/H_0)_o$ over $(\theta_{max})_m$ for different values of H_2/L_2.

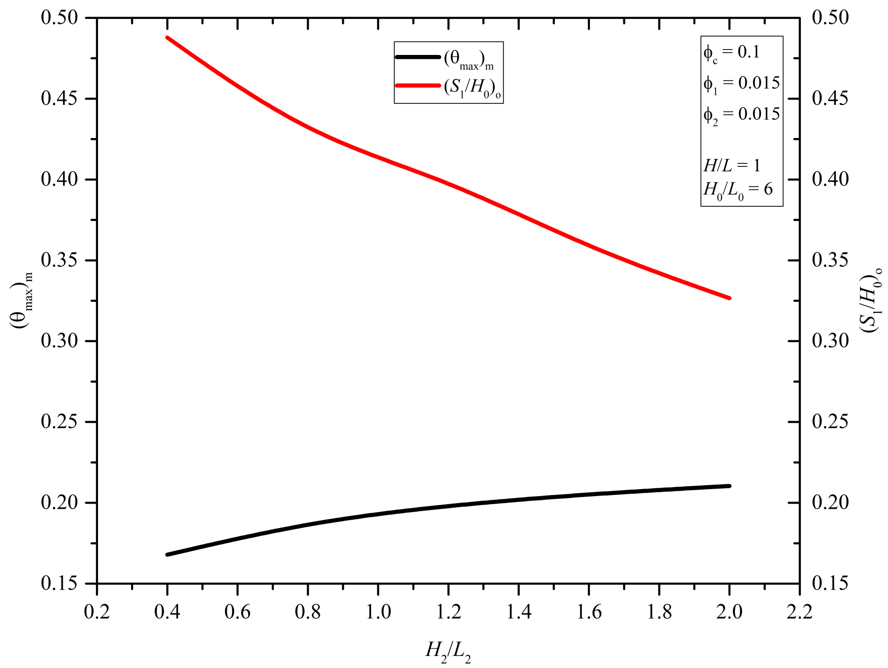


Figure 5 Effect of H_2/L_2 over $(\theta_{max})_m$ and $(S_1/H_0)_o$ for H/L = 1.0 and H_0/L_0 = 6.0.

Figure 6 shows the twice optimized geometry of the shaped cavity double-T inside of the investigated search space.

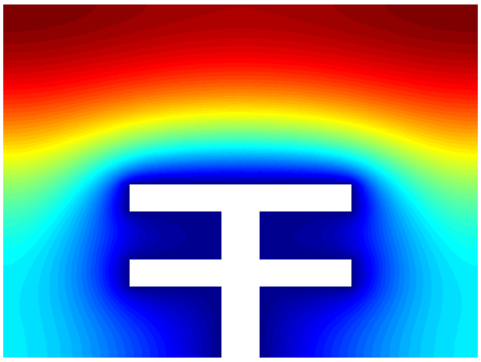


Figure 6.The twice optimized geometry of the shaped cavity double-T.

For the optimization of three DOF, the value of H_1/L_1 was varied together with S_1/H_0 for different values of H_2/L_2. The search space of the H_1/L_1 and H_2/L_2 is between the ratios 0.2 and 2.0 with the step equal to 0.2. The search space of S_1/H_0 changes according to the combination of the values of H_1/L_1 and H_2/L_2. The ratio H/L was kept constant in H/L =1, as well as, the ratio H_0/L_0 = 6. The constraints also were fixed in the following values: $\phi_c = 0.1$; $\phi_1 = 0.015$ e $\phi_2 = 0.015$.

Firstly, it is employed the ES method as a validation of SA algorithm results. The Table 4 shows the optimization results with ES method for the effect of H_2/L_2 over the dimensionless maximum excess of temperature twice minimized, $(\theta_{max})_{mm}$, and their optimal geometry: $(H_1/L_1)_o$ and $(S_1/H_0)_{oo}$. According to the Table 6 results, all optimal geometries were achieved with the ratio of $(H_1/L_1)_o = 0.2$, i. e., the lowest ratio improves the distribution of the cavity geometry along of the domain area and consequently reduce the thermal resistance. From the geometry viewpoint, lowest reasons of H_1/L_1 and H_2/L_2 denotes lateral cavities more thinner and elongated. Therefore, the optimal geometries are more concentrated in the lowest reasons of H_1/L_1 and H_2/L_2.

Table 4. Results of the three degrees of freedom optimization with the ES method

$(\theta_{max})_{mm}$	$(S_1/H_0)_{oo}$	$(H_1/L_1)_o$	H_2/L_2	Iterations
0.14736	0.57155	0.2	0.2	358
0.16061	0.75526	0.2	0.4	336
0.16745	0.71443	0.2	0.6	318
0.17189	0.69402	0.2	0.8	304
0.17555	0.67361	0.2	1.0	291
0.17957	0.63278	0.2	1.2	278
0.18240	0.61237	0.2	1.4	268
0.18488	0.59196	0.2	1.6	258
0.18722	0.57155	0.2	1.8	248
0.18938	0.55114	0.2	2.0	239
		Total Iterations:		2898

Figure 7 shows the curves of effect of the H_2/L_2 over the $(\theta_{max})_{mm}$ and the degrees of freedom $(H_1/L_1)_o$ and $(S_1/H_0)_{oo}$. In this figure one can be seen that the ratio $(H_1/L_1)_o$ is insensible to the variation of H_2/L_2. Meanwhile, the ratio $(S_1/H_0)_{oo}$ has a maximum value in the intermediated region of H_2/L_2. Regarding to $(\theta_{max})_{mm}$ there is a greatest increase of temperature in the lowest values of ratio H_2/L_2.

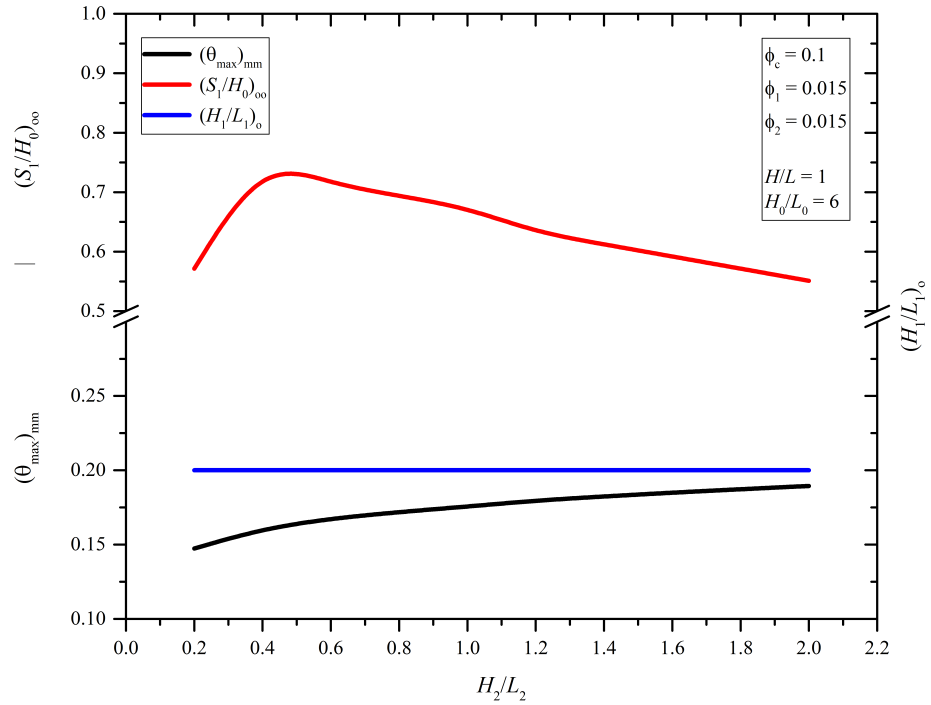
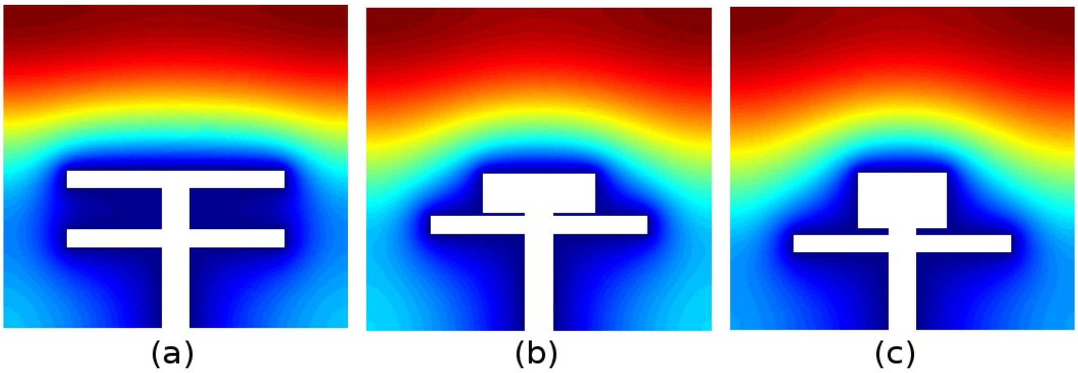


Figure 7 Effect of H_2/L_2 over $(\theta_{max})_{mm}$ and degrees of freedom $(H_1/L_1)_o$ and $(S_1/H_0)_{oo}$.

The Figure 8 shows a comparison between the twice optimized geometry and the three times optimized geometry, i.e., these geometry that returned the lowest value of $(\theta_{max})_{mm}$ between the all optimal geometry for each value of H_2/L_2. In all geometries it is possible to observe that the ratio $(H_1/L_1)_o = 0.2$ remained constant for all values of H_2/L_2 included to the values of H_2/L_2= 1.0 and H_2/L_2 = 2.0, as can be seen in Figs. 8(b) and 8(c). The geometry three times optimized showing in the Fig. 8(a), kept equal the ratio of the double-T branches (H_1/L_1 and H_2/L_2). Note that optimal geometry is observed when lateral cavities (horizontals) deeply penetrates the solid domain. In this specific case, the lowest penetration of the superior lateral cavities , as can be seen in the Figs. 8(b) and 8(c), leads to the temperature peaks concentrating in the top corners of the solid domain. Then, in these cases, the temperature fields distribution are less homogeneous. In the cavity of the Fig. 8(a) the distribution of the hot spots is more homogeneous in the superior region of the solid domain, i.e., the better thermal performance occurs according to the Constructal principle of "optimal distribution of imperfection"(Bejan, 1996).



$(H_2/L_2)_o = 0.2$	$H_2/L_2 = 1.0$	$H_2/L_2 = 2.0$
$(H_1/L_1)_{oo} = 0.2$	$(H_1/L_1)_o = 0.2$	$(H_1/L_1)_o = 0.2$
$(S_1/H_0)_{ooo} = 0.5716$	$(S_1/H_0)_{oo} = 0.6736$	$(S_1/H_0)_{oo} = 0.5511$
$(\theta_{max})_{mmm} = 0.14736$	$(\theta_{max})_{mm} = 0.17555$	$(\theta_{max})_{mm} = 0.18938$

Figure 8 Comparison between the three times optimized geometry and twice optimized geometries for the followin fixed parameters: ($\phi_c = 0.1$; $\phi_1 = \phi_2 = 0.015$; $H/L = 1$; $H_0/L_0 = 6$).

With the results of ES method it is possible to analize the SA algorithm results with different parameters of CS and *StallIterLimit*. The parameter of *Reannealing* is kept constant with value 100 for all versions of SA algorithm. It is investigated three values for the *StallIterLimit* parameter for each CS parameter, i.e., it is observed the results of fiften different versions of SA algorithm for the optimization of three degrees of freedom of shaped cavity double-T.

The Table 5 summarizes the results of all versions of the SA algorithm comparing with ES method, the main comparison is the number of iteration (simulations request) of each method. The first column distinguish the different methods, the name SA *Exp* (150) , means that the SA algorithm was employed with CS parameter *Exponencial* and with the stop criterion (*StallIterLimit*) defined to one hundred and fifty iteration without changes in the best solution found. The second column of the Table 5 shows the number of necessary iterations for the optimization with value of $H_2/L_2 = 0.2$, the column immediately beside exposes the total iteration for optimization of DOFs $(H_1/L_1)_o$ and $(S_1/H_0)_{oo}$ for each ten values of ratio H_2/L_2 ($0.2 \leq H_2/L_2 \leq 2.0$). All versions of SA algorithm converged to the minimized dimensionless maximum excess of temperature $(\theta_{max})_{mm} = 0.1474$ in the ratio $H_2/L_2 = 0.2$. But, for the others ratios of H_2/L_2 some versions of SA algorithm leads to the local minimal, i.e., do not converged to the global optimal geometric configuration related by the ES method. Therefore, the colunm called *Errors* indicates the amount of times that it is achieved local minimal for each version of SA algorithm and for all values of H_2/L_2 search space.

According to the Table 5, the SA versions with hybrids CS (*ConstExp1* and *ConstExp2*) and *StallIterLimit* equal to 100, obtained the best performance, because they converged to the global optimal geometries in all cases of H_2/L_2 investigated, representing properly the effect of H_2/L_2 over $(\theta_{max})_{mm}$. The Figure 9 shows a graphic comparing the curve of the ES method results for $(\theta_{max})_{mm}$ in function of H_2/L_2 with the points of convergence for each version of SA algorithm. Figure 5 illustrates that only the SA versions with CS *ConstExp1* and *ConstExp2* achieved the dimensionless maximum excess of temperature three times minimized for all values of the ratio H_2/L_2.

Table 5. Comparison between ES method and different versions of SA algorithm.

Method	*Iterations $H_2/L_2 = 0.2$*	*Total Iterations*	*Errors*
ES method	358	2898	-
SA Boltz (50)	178	1122	2
SA Boltz (100)	228	1368	3
SA Boltz (150)	278	1967	4
SA Exp (50)	104	1338	2
SA Exp (100)	154	826	4
SA Exp (150)	204	2025	3
SA BoltzExp (50)	156	886	4
SA BoltzExp (100)	206	1414	2
SA BoltzExp (150)	256	2167	3
SA ConstExp1(50)	142	965	5
SA ConstExp1(100)	192	1786	0
SA ConstExp1(150)	242	2497	0
SA ConstExp2(50)	130	858	5
SA ConstExp2(100)	180	1776	0
SA ConstExp2(150)	230	2246	0

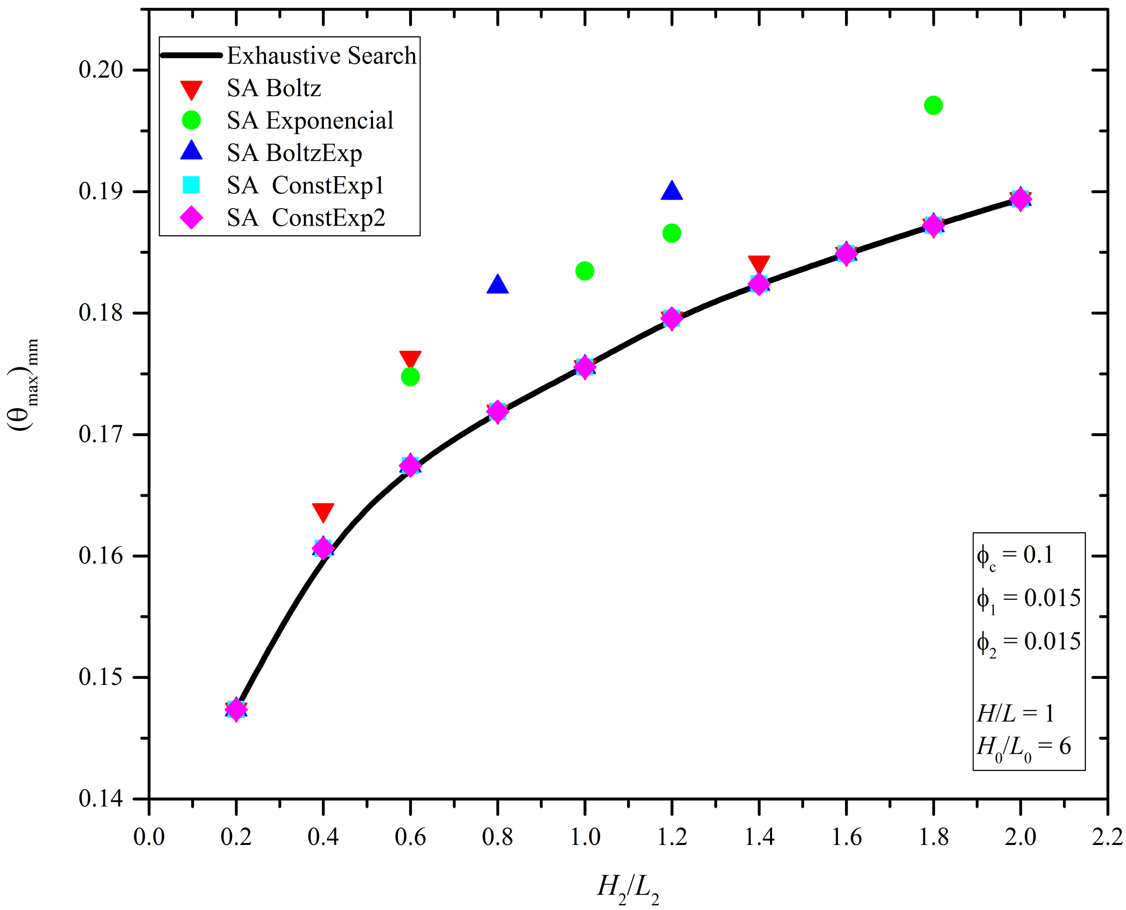


Figure 9 Comparison between the Exhaustive Search method results and the results of different versions of Simulated Annealing.

CONCLUSION

This study investigated the use of Constructal Design (CD) combined with the meta-heuristic algorithm Simulated Annealing (SA) for partial geometric optimization of a heat transfer problem. The problem is regarded with the geometric optimization of a shaped cavity double-T with five Degrees of Freedom (DOF) into a solid domain with constant and uniform heat generation. The search of best geometry with different versions of SA algorithm was compared with results obtained with ES method for two an three DOF. The study has as aim to find the best geometry of the shaped cavity double-T that minimized the thermal resistance , as well as, evaluate the best version of SA algorithm applied to the partial optimization that can be used for future studies of complete geometric optimization, even as, evaluate effects of the constraints.

For the partial optimization, two and three DOF of the shaped cavity double-T the ES method was employed to validate the SA results. For two DOF, all versions of SA algorithm achieved the optimal geometry, validating the use of SA algorithm with hybrid models of Cooling Schedule (CS). Additionally, the number of iteration was also similar. Sometimes, in the optimization process, any version of SA algorithm showed few or high number of iterations, but this behavior is a characteristic of the non-deterinistic nature of the stochastic algorithm. For the optimization of three degrees of freedom of the double-T shaped cavity the hybrid models of CS parameter , ConstExp1 and ConstExp2, with StallIterLimite equal to 100, were the SA versions that achieved less number of local optimal geometries. These SA versions lead to the optimal results related by ES method and with half of required simulations.

Concerning to the results of SA algorithm, the results achieved here agreed with the results of studies of Gonzales et. al. (2014a; 2014b) when the hybrid parameters of CS improve the robustness of the algorithm. The results showed that is possible the use of the SA algorithm with hybrid parameters to the complete optimization of shaped cavity double-T, as well as, evaluate the effects of constraints and degrees of freedom. This is a important conclusion, mainly when the computational effort do not permits the application of ES method, in order to the large number of simulations required.

ACKNOWLEDGEMENTS

Prof. L. A. Isoldi, E. D. dos Santos and L. A. O. Rocha thanks CNPq (Brasília, DF, Brazil) for research Grant and for financial support in Universal project (Process: 445095/2014-8).

REFERENCES

Bejan, A., 1996, "Constructal-theory network of conducting paths for cooling a heat generating volume ", *Int. J. Heat Mass Transfer*, vol. 40, n. 4, pp. 799–816.

Bejan, A., Zane, J. P., 2012, *"Design in Nature"*, Doubleday, New York, 1st Ed.

Bertsimas, D., Tsitsiklis, J., 1993, "Simulated Annealing",*Statistical Science*, v. 8,n.1, pp 10-15;

Biserni, C., Rocha, L. A. O., Bejan, A.,2004, Inverted Fins: Geometric Optimization of the Intrusion into a Conducting Wall, *Int. J. Heat Mass Transfer*, 47, pp. 2577-2586.

Biserni, C., Rocha, L. A. O., Stanescu, G., Lorenzini, E.,2007,Constructal H-shaped cavities according to Bejan's Theory, Int. J. Heat Mass Transfer, 50,pp. 2132-2138.

Brusco M. J.,2014, “A comparison of simulated annealing algorithms for variable selection in principal component analysis and discriminant analysis”, *Computational Statistics and Data Analysis*, v. 77, pp. 38–53.

Eglese, R. W.,1990, “Simulated Annealing: A tool for Operational Research”, *European Journal of Operational Research*, p.271-281.

El-Bouri A., Azizi N., Zolfaghari S., 2007, “A comparative study of a new heuristic based on adaptive memory programming and simulated annealing: The case of job shop scheduling”, *European Journal of Operational Research*, v. 177,pp. 1894–1910.

Gonzales, G. V. ; Emmendorfer, L. R. ; Santos, E. D. ; Rocha, L. A. O. ; Isoldi, L. A. ; Estrada, E. S. D., 2014a, “Constructal Design and Simulated Annealing Employed for Geometric Optimization of a Y-Shaped Cavity Intruded into Conductive Wall”. In: *VI Southern Conference on Computational Modeling*, Rio Grande, v. 1, p. 280-290.

Gonzales, G. V. ; Santos, E. D. ; Rocha, L. A. O. ; Estrada, E. S. D. ; Emmendorfer, L. R. ; Isoldi, L. A., 2014b, “A Comparative Study of Simulated Annealing with different Cooling Schedules for Geometric Optimization of a Heat Transfer Problem According to Constructal Design”. In: *VI Southern Conference on Computational Modeling*, Rio Grande -RS, v. 1, p. 426-435.

Hajek B., 1988, “Cooling Schedules for Optimal Annealing”, *Mathematics of Operations Research*, v. 13,n. 2,p. 311-129.

Hajmohammadi, M. R., Poozesh, S., Campo, A., and Nourazar, S. S., 2013, “Valuable Reconsideration in the Constructal Design of Cavities” *Energy Convers. Manage.*, 66, pp. 33–40.

Ingber L.,1989,“Very Fast Simulated Re-annealing”, *Math. Comput. Modelling*, v. 12,n. 8, pp.967-973.

Ingber L., 1993, “Simulated Annealing: Patrice versus Theory”, *Math. Comput. Modeling* Vol. 18 ,No 11,pp-29-57.

Kastanya D., 2013, “Evaluating the effects of cooling schedule on the performance of ADORE”, *Annals of Nuclear Energy*, v. 62, pp. 483–486.

Kirkpatrick, S.,Gelatt, C. D.,Vecchi, M., P.,1983, “Optimization by Simulated Annealing”, *Science*, New Series, Vol 220, No 4598, pp 671-680.

Kraus, A. D., 1999, Developments in the Analysis of Finned Arrays, in: Donald Q. Kern Award Lecture, *National Heat Transfer Conference*, Baltimore., MD, August 11., 1997, *Int. J. Transport Phenom*., ed. 1, pp. 141-164.

Lorenzini, G., Biserni, C., Isoldi, L. A., Dos Santos, E. D., Rocha, L. A. O., 2011 “Constructal Design Applied to the Geometric Optimization of Y-shaped Cavities Embedded in a Conducting Medium”, *J. Electronic Packaging*, vol. 133, pp. 041008-1 – 041008-8.

Lorenzini, G. ; Garcia, F.L. ; dos Santos, E.D. ; Biserni, C. ; Rocha, L.A.O. 2012 “Constructal design applied to the optimization of complex geometries: T-Y-shaped cavities with two additional lateral intrusions cooled by convection”. *International Journal of Heat and Mass Transfer*, v. 55, p. 1505-1512.

Lorenzini, G., Biserni, C., Link, F. B., Isoldi, L. A., dos Santos, E. D., and Rocha, L. A. O., 2013, "Constructal Design of T-Shaped Cavity for Several Convective Fluxes Imposed at the Cavity Surfaces" J. Eng. Thermophys., 22, pp. 309–321.

Lorenzini, G. ; Biserni, C. ; Estrada, E. da S. D. ; Dos Santos, E. D. ; Isoldi, L. A. ; Rocha, L. A. O., 2014a, "Genetic Algorithm Applied to Geometric Optimization of Isothermal Y-Shaped Cavities". *Journal of Electronic Packaging*, v. 136, p. 031011-1-031011-9 .

Lorenzini, G. ; Biserni, C. ; Estrada, E. D. ; Isoldi, L. A. ; Dos Santos, E. D. ; Rocha, L. A. O., 2014b, "Constructal Design of Convective Y-Shaped Cavities by Means of Genetic Algorithm." *Journal of Heat Transfer*, v. 136, p. 071702-071702-10.

MATLAB *User's Guide*, 2000, Version 6.0.088, Release 12, The Mathworks Inc., Natick, MA.

Metropolis N, Rosenbluth AW, Rosenbluth MN, Teller AH., 1953, "Equation of State Calculations by Fast Computing Machines",*The Journal Of Chemical Physics*, v 21, p 1088-1092.

Nourani Y., Andresen B., 1998, "A comparison of simulated annealing cooling strategies",*J. Phys. A: Math. Gen.*, v. 31,pp. 8373–8385.

Reddy JN, Gartling DK. *The Finite Element Method in Heat Transfer and Fluid Dynamics*. CRC, Boca Raton, Florida,USA;1994.

Schneider J. J., Puchta M., 2010, "Investigation of acceptance simulated annealing — A simplified approach to adaptive cooling schedules", *Physica A*, v. 389, pp. 5822–5831.

Szu H., Hartley R.,1987, "Fast Simulated Annealing", *Physics Letters A*, v. 122, p. 157-162.

Xie, Z., Chen, L., Sun, F., 2010, "Geometry Optimization of T-Shaped Cavities According to Constructal Theory" *Math. Comput. Model.*, 52, pp. 1538–1546.