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LAMINATED PIEZOCOMPOSITES STRUCTURES (LAPS) DESIGNED FOR TOPOLOGY OPTIMIZATION CONSIDERING THE TRANSIENT RESPONSE

Ruben A. Salas

Emilio C. N. Silva

ruansava3748@yahoo.com, ruansava@usp.br

ecnsilva@usp.br

Department of Mechatronics and Mechanical Systems Engineering, Polytechnic School of University of São Paulo

Av. Prof. Mello Morais, 2231, 05508-030, São Paulo, SP, Brazil

J. N. Reddy

jnreddy@tamu.edu

[3]Department of Mechanical Engineering, Texas A&M University

401 Joe Routt Blvd, TX 77843-3123, College Station, TX, United States of America

Abstract. *LAPS are multi-layer structures composed by piezoelectric, metal and composite materials. This structures show superior features over conventional piezoelectric materials because their characteristics cannot be achieved by any of its components isolated. Thus, this work aims at the development of LAPS through the vibration modes design aiming at dynamic applications. Piezomotors can be mentioned as potential applications. Therefore, LAPS dynamic design can be systematized by using the Topology Optimization Method (TOM), which combines optimization algorithms and the finite element method (FEM). In this work, the TOM formulation points to determine simultaneously the optimal topology of the materials for different layers and the polarization sign of the piezoelectric material, in order to design a particular vibration mode by means of maximizing the vibration amplitude at certain points for a specified input. The TOM formulation includes the Simple Isotropic Material with Penalization (SIMP) for isotropic materials and the PiezoElectric MAterial with Penalization and Polarization (PEMAP-P) in order to describe polarization in piezoelectric material. Results are shown in order to illustrate the method.*

Keywords: *Topology optimization, Laminated piezocomposite materials, Transient analysis.*

1 INTRODUCTION

LAPS are composed by layers of different materials: piezoelectric, metal or composite, having some advantages over conventional pure piezoelectric structures, because of their superior features, which cannot be achieved by any of its components isolated. Potential applications include piezomotors (Koc, Cagatay et al. 2002; Friend, Nakamura et al. 2005). In the LAPS design have been used analytical and numerical approaches (Trajkov, Koppe et al. 2006; Balamurugan and Narayanan 2009), concluding the performance system is influenced by the distribution of material in the piezoelectric layer, having to be considered at the same time the amount and distribution of the material and optimal location of piezoceramics in the optimization problem.

The dynamic design of LAPS is complex; however, it can be systematized by using the Topology Optimization Method (TOM). The TOM is a method based on the distribution of material in a fixed design domain with the aim of extremizing a cost function subjected to constraints inherent to the problem by means of combining the optimization algorithms and the FEM. The TOM has been applied to piezoelectric devices design using static, modal and harmonic approaches maximizing vibrations amplitudes in actuators and output voltages in sensors (Nakasone and Silva 2010; Kiyono, Silva et al. 2012; Mello, Kiyono et al. 2014). In these approaches sinusoidal excitations are only considered in modeling, however in practice, other signal inputs are used for exciting. Therefore, transient approach has not been considered in LAPS design, being the motivation of this work.

The TOM formulation for the LAPS design considering the transient response aims to determine together the optimal topology of the materials for different layers and the polarization sign of the piezoelectric material, in order to design a particular vibration mode by means of maximizing the vibration amplitude at certain points for a specified input. The non-linear optimization problem is solved by SLP. Additionally, in order to contour mesh dependence and checkerboard a sensitivity filter was implemented. Results are presented to exhibit the proposed method.

This paper is organized as follows. In Section 2, the FEM formulation applied to the LAPS is introduced. In Section 3, the TOM, the optimization problem and the computation of sensitivities is discussed. In Section 4, implementation details are shown. Numerical results and concluding remarks are presented in Section 5 and Section 6, in that order.

2 FINITE ELEMENT FORMULATION

In the modeling, it is assumed that strains are small under a plane-stress condition given their reduced thickness and both the electric and displacement fields are linear. The structures are three-dimensional and non-dependent material properties with temperature are considered. In the elastic domain, it is considered damping effects using the Rayleigh model. The three-dimensional domain is discretized by means of eight-node isoparametric finite elements. The total number of elements in the discretization is N, while the total numbers of nodes is n. The FEM formulation is (Ahmad, Irons et al. 1970; Reddy 2004; Kogl and Bucalem 2005; Kogl and Bucalem 2005; Kiyono, Silva et al. 2012):

$$\begin{bmatrix} \boldsymbol{M}(\chi) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{U}}(\chi) \\ \ddot{\boldsymbol{\Phi}} \\ \ddot{\lambda} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}(\chi) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{U}}(\chi) \\ \dot{\boldsymbol{\Phi}} \\ \dot{\lambda} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{uu}(\chi) & \boldsymbol{K}_{u\phi}(\chi) & \boldsymbol{A}_G^T \\ \boldsymbol{K}_{u\phi}^T(\chi) & \boldsymbol{K}_{\phi\phi}(\chi) & \boldsymbol{0} \\ \boldsymbol{A}_G & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}(\chi) \\ \boldsymbol{\Phi}(\chi) \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{P} \\ \boldsymbol{Q} \\ \boldsymbol{0} \end{Bmatrix} \quad (1)$$

where $\boldsymbol{U}$, $\boldsymbol{P}$ (size: $5n$ x 1) and $\boldsymbol{\Phi}$, $\boldsymbol{Q}$ (size: $[n+N]$ x 1) are global vectors for the displacement, structural load, electric potential and electrical charge, respectively. $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K_{uu}}$ (size: $5n$ x $5n$), are the global mass, damping and structural stiffness matrices, respectively. $\boldsymbol{K_{u\phi}}$ (size: $5n$ x $[n+N]$) is the global piezoelectric stiffness matrix while $\boldsymbol{K_{\phi\phi}}$ (size: $[n+N]$ x $[n+N]$) is the global dielectric stiffness matrix. The matrix $\boldsymbol{A}_G$ (size: $3G$ x $5n$) corresponds to the bonding constraints between layers using the "layer wise" or "zig zag" theory (Kogl and Bucalem 2005), being G the constraints number. This coupling between layers is included in FEM model by means of the Lagrange multipliers method (Bathe 1996) where λ (size: $3G$) is a global Lagrange multipliers vector. The design variables vector is χ and it is defined by material model (see Section 3).

The global matrices are defined as:

$$\boldsymbol{M}(\boldsymbol{\chi}) = \mathop{A}_{e=1}^{N}\left[\rho(\chi_e)\int_{V_e}\boldsymbol{N}_u^T\boldsymbol{N}_u dV_e\right]_e \tag{2}$$

$$\boldsymbol{K_{uu}}(\boldsymbol{\chi}) = \mathop{A}_{e=1}^{N}\left[\int_{V_e}\boldsymbol{B}_u^T\boldsymbol{T}_\varepsilon^T\boldsymbol{C}(\chi_e)\boldsymbol{T}_\varepsilon\boldsymbol{B}_u dV_e\right]_e \tag{3}$$

$$\boldsymbol{C}(\boldsymbol{\chi}) = \alpha\boldsymbol{K_{uu}}(\boldsymbol{\chi}) + \beta\boldsymbol{M}(\boldsymbol{\chi}) \tag{4}$$

$$\boldsymbol{K}_{\phi\phi}(\boldsymbol{\chi}) = \mathop{A}_{e=1}^{N}\left[\int_{V_e}\boldsymbol{B}_\phi^T\boldsymbol{T}_\phi^T\epsilon(\chi_e)\boldsymbol{T}_\phi\boldsymbol{B}_\phi dV_e\right]_e \tag{5}$$

$$\boldsymbol{K}_{u\phi}(\boldsymbol{\chi}) = \mathop{A}_{e=1}^{N}\left[\int_{V_e}\boldsymbol{B}_u^T\boldsymbol{T}_\varepsilon^T\boldsymbol{e}(\chi_e)\boldsymbol{T}_\phi\boldsymbol{B}_\phi dV_e\right]_e \tag{6}$$

where A is a FEM assembly operator (Hughes 1987), index e refers to e^{th} finite element. $\rho(\chi_e)$ is the mass density and $\boldsymbol{C}(\chi_e),\epsilon(\chi_e)$ and $\boldsymbol{e}(\chi_e)$ are, elastic, permittivity and dielectric constitutive matrices, respectively. $\boldsymbol{N}_u, \boldsymbol{B}_u, \boldsymbol{T}_\varepsilon, \boldsymbol{B}_\phi$ and $\boldsymbol{T}_\phi$ are the structural shape functions, strain-displacement, strain transformation, electric potential gradient and electric potential transformation matrices, in that order. α and β are the structural damping coefficients and V_e is the finite element volume. The g^{th} bonding constraint for two nodes j and k belonging to two adjacent layers are defined as:

$$\boldsymbol{A}_G\boldsymbol{U}_G = \mathop{A}_{g=1}^{G}\begin{bmatrix}\boldsymbol{A}_j & \boldsymbol{A}_k\end{bmatrix}_g\begin{bmatrix}\boldsymbol{U}_j\\ \boldsymbol{U}_k\end{bmatrix}_g = 0 \tag{7}$$

where $\boldsymbol{A}_j, \boldsymbol{A}_k$ are the bonding matrices and $\boldsymbol{U}_j, \boldsymbol{U}_k$ are the displacement vectors for nodes j and k corresponding to g^{th} constraint, respectively. The FEM problem is solved by Generalized α-method (Chung and Hulbert 1993) and it is considered initial conditions starting from rest in both structural and electrical domains.

3 FORMULATION OF TOPOLOGY OPTIMIZATION

The TOM is a powerful structural optimization technique which determines a constrained material distribution in a given design domain in order to accomplish predefined optimization objectives. Constraints are related to the amount of material to be used, maximum allowed stresses, etc. Therefore, TOM solves an optimization problem combining the FEM with a material model and an optimization algorithm. In this work, the material model adopted is PEMAP-P (Kogl and Silva 2005), which is based on SIMP and adds a polarization variable which indicates positive or negative polarization in piezoelectric material. μ is a topological design variable and γ is the polarization design variable. Thus, the design variables vector for each finite element is:

$$\chi_e = \begin{Bmatrix} \mu_e \\ \gamma_e \end{Bmatrix} \tag{8}$$

Applying this interpolation model, the material properties at any finite element are governed by the functions:

$$\rho = \mu_e^{P_m} \rho_o \tag{9}$$

$$\boldsymbol{C} = \mu_e^{P_c} \boldsymbol{C}_o \tag{10}$$

$$\boldsymbol{\epsilon} = \mu_e^{P_\epsilon} \boldsymbol{\epsilon}_o \tag{11}$$

$$\boldsymbol{e} = \mu_e^{P_e} \left(2\gamma_e - 1\right)^{P_\gamma} \boldsymbol{e}_o \tag{12}$$

where $\mu_e = (0,1]$ and $\gamma_e = [0,1]$. ρ_o, $\boldsymbol{C}_o$, $\boldsymbol{\epsilon}_o$ and $\boldsymbol{e}_o$ are the base material properties defined in the e^{th} finite element. Observe that μ_e is higher than 0 to avoid singularities. The parameters P_m, P_c, P_ϵ and P_e are the corresponding penalty coefficients (Bendsøe and Sigmund 2003). The penalty coefficients are used to assure the final topology has or has not material and piezoelectric material has positive or negative polarization. In this work, the coefficients are arbitrarily chosen, following a heuristic criterion. For more elaborate criteria, Kim et al. (2010) is suggested.

3.1 Design problem formulation

The optimization problem is to distribute a given amount of material in a design domain considering a fixed time interval t_f considered as an input parameter whose valor is greater than the peak time. The peak time is a time instant where the first peak of the transient response is observed (Ogata 1997). The material distribution is performed in order to obtain a maximum output displacement u_{out} at certain points of the structure when on the piezoelectric layers is applied an electric potential difference, as a step excitation:

$$\Delta\phi(t) = \begin{cases} \Delta\phi, \forall\, 0 \le t \le t_f \\ 0, \forall\, t > t_f \end{cases} \tag{13}$$

This step represents a portion of a square wave with a specified frequency and amplitude independent of the design variables. Taking into account the transient response, the displacement is maximized progressively from the excitation instant until the final time t_f being possible improving the response speed. The optimization problem is defined as:

$$\max_{\chi} F_{\tau} = \int_{0}^{t_f} u_{out}\left(t, \chi\left(\mu, \gamma\right), \Delta\phi\left(t\right)\right) dt \tag{14}$$

$$s.t. \begin{cases} Equilibrium\,equations & \left(Eq.(1)\right) \\ \dot{u}_{out}\left(t_p\right) = 0 & \left(Zero\;peak\;velocity\;constraint\right) \\ V^*_{Min} \leq \sum_{e=1}^{N} \mu_e V_e \leq V^*_{Max} & \left(Material\;volume\;constraint\right) \\ 0 < \mu_{\min} \leq \mu \leq 1 & \left(Topological\;variables\;box\;constraint\right) \\ 0 \leq \gamma \leq 1 & \left(Polarization\;variables\;box\;constraint\right) \\ Symmetry\;conditions & \end{cases} \tag{15}$$

where V^*_{Min}, V^*_{Max} are the minimum and maximum constraints on material volume, respectively. The purpose of the zero peak velocity constraint is to ensure the first peak of the response is always observed. The symmetry conditions allow controlling the structure actuation (in plane or out of plane of the layers) and reducing design variables number.

3.2 Sensitivity Analysis

The sensitivities are calculated by adjoint method (Haftka and Gürdal 1992) with the aim of using them in the solution of the optimization problem (see Section 4). Equation (1) can be expressed in this way:

$$\bar{\boldsymbol{M}}\ddot{\boldsymbol{W}} + \bar{\boldsymbol{C}}\dot{\boldsymbol{W}} + \bar{\boldsymbol{K}}\boldsymbol{W} = \bar{\boldsymbol{F}} \therefore \boldsymbol{W} = \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{\Phi} \\ \boldsymbol{\lambda} \end{Bmatrix} \tag{16}$$

and the output displacement sensitivity as:

$$\frac{dF_{\tau}}{d\boldsymbol{\chi}} = \int_{0}^{t_f} \frac{du_{out}}{d\boldsymbol{\chi}} dt = \int_{0}^{t_f} \boldsymbol{L}^T \frac{d\boldsymbol{W}}{d\boldsymbol{\chi}} dt \tag{17}$$

where $\boldsymbol{L}^T$ is a vector consisting of zeros except for the position corresponding to the degree of freedom (DOF) of the output displacement direction, whose value is one. Using the adjoint method, the sensitivity in relation to design variables vector $\boldsymbol{\chi}$ is:

$$\frac{dF_{\tau}}{d\boldsymbol{\chi}} = \int_{0}^{t_f} \boldsymbol{L}^T \frac{d\boldsymbol{W}}{d\boldsymbol{\chi}} dt = \int_{0}^{t_f} \boldsymbol{\Lambda}^T \boldsymbol{R} dt \tag{18}$$

being $\boldsymbol{\Lambda}^T$ an adjoint vector that satisfies:

$$\bar{\boldsymbol{M}}\ddot{\boldsymbol{\Lambda}} + \bar{\boldsymbol{C}}\dot{\boldsymbol{\Lambda}} + \bar{\boldsymbol{K}}\boldsymbol{\Lambda} = \boldsymbol{L}\big|_{t_f - t} \tag{19}$$

$$\dot{\boldsymbol{\Lambda}}\left(t_f\right) = \boldsymbol{0} \tag{20}$$

$$\boldsymbol{\Lambda}\left(t_f\right) = \boldsymbol{0} \tag{21}$$

$\boldsymbol{R}$ is determined by:

$$\boldsymbol{R} = \frac{d\bar{\boldsymbol{F}}}{d\boldsymbol{\chi}} - \frac{d\bar{\boldsymbol{M}}}{d\boldsymbol{\chi}}\ddot{\boldsymbol{W}} - \frac{d\bar{\boldsymbol{C}}}{d\boldsymbol{\chi}}\dot{\boldsymbol{W}} - \frac{d\bar{\boldsymbol{K}}}{d\boldsymbol{\chi}}\boldsymbol{W} \tag{22}$$

Equation (19) is solved using Generalized α-method (Chung and Hulbert 1993) transforming the final value problem into an initial value problem with the variable substitution $\tau = t_f - t$.

4 NUMERICAL IMPLEMENTATION

Initially, the initial domain is discretized by finite elements and designs variables are defined with a uniform values guess and the transient FEM problem is solved. The non-linear optimization problem is solved iteratively using a Sequential Linear Programming (SLP) algorithm (Haftka and Gürdal 1992). For this process, at each SLP iteration, the objective function (Eq.(14)) is linearized around the current design point $\boldsymbol{\chi}$. This linearization is based on the objective function sensitivities calculated above (see Eq.(18) to Eq.(22)). The linearized problem is solved for obtaining a new approximation. To ensure an accurate approximation of the original non-linear problem solution, box constraints or moving limits for each design variable are applied in the linearized problem. The values range within the moving limits is reduced if the corresponding design variable oscillates or stagnates, and it is increased otherwise.

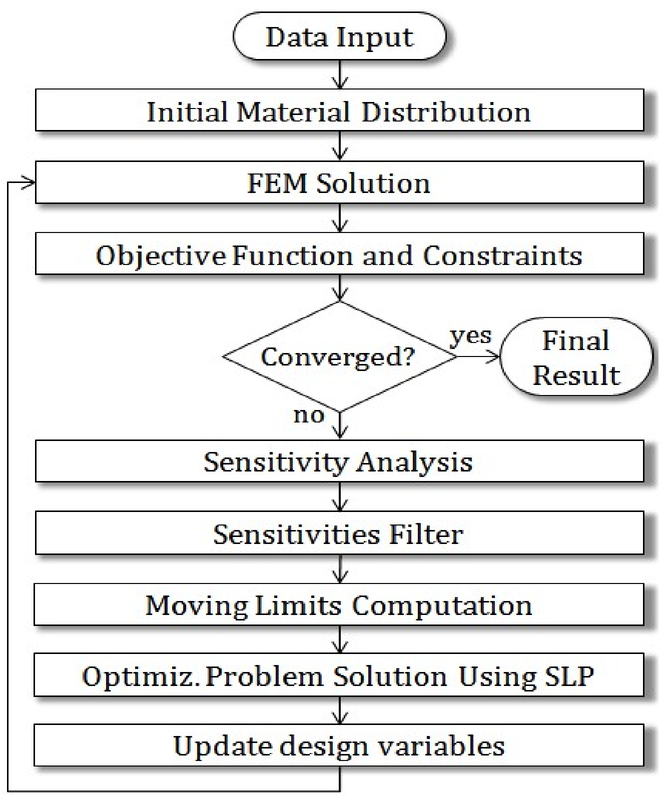

Figure 1. Flowchart of the optimization algorithm

Numerically regions with $\mu_e = \mu_{\min}$ can be considered void regions being defined $\mu_{\min} = 10^{-11}$ with the purpose of avoiding numerical problems or singularities in FEM. It is not necessary define a minimum value $\gamma_{\min}$, since the piezoelectric matrix $\boldsymbol{K}_{u\phi}$ is not part of the diagonal of the stiffness matrix $\overline{\boldsymbol{K}}$ and therefore it does not affect the stability FEM. When a new design variables set $\boldsymbol{\chi}$ is calculated, the design and LAPS transient response is updated. The SLP iterative process is continued until a convergence criterion is achieved. The flowchart of the algorithm used for the optimization process is shown in Fig. 1.

In order to avoid checkerboard patterns and mesh dependency (Bendsøe and Sigmund 2003) a sensitivities filter has been implemented. The filter (Andreassen, Clausen et al. 2011) imposes that each topological design variable depends on its neighbors, as a function of a pre-defined and constant radius. The continuation method (Bendsøe and Sigmund 2003) is employed for obtaining the final topologies due to the LAPS design problem is highly non-convex. Thus, it is minimized the multiple local maximum (or minimum) problem allowing TOM to approach to the global maximum (or minimum). In the continuation method, the penalty coefficients vary from 1, increasing 0.2 by iteration, until a maximum value is reached.

5 RESULTS

The example presented here illustrates the potential of the TOM in the LAPS dynamic design. The design domain consist of a 3 layers made of an aluminum substrate between two PZT-5A layers. The objective is to maximize the displacement at the free tip in the direction specified in Fig. 2 when one load is applied in the same place. The material properties are shown in Table 1 and remain necessary data are shown in Table 2.

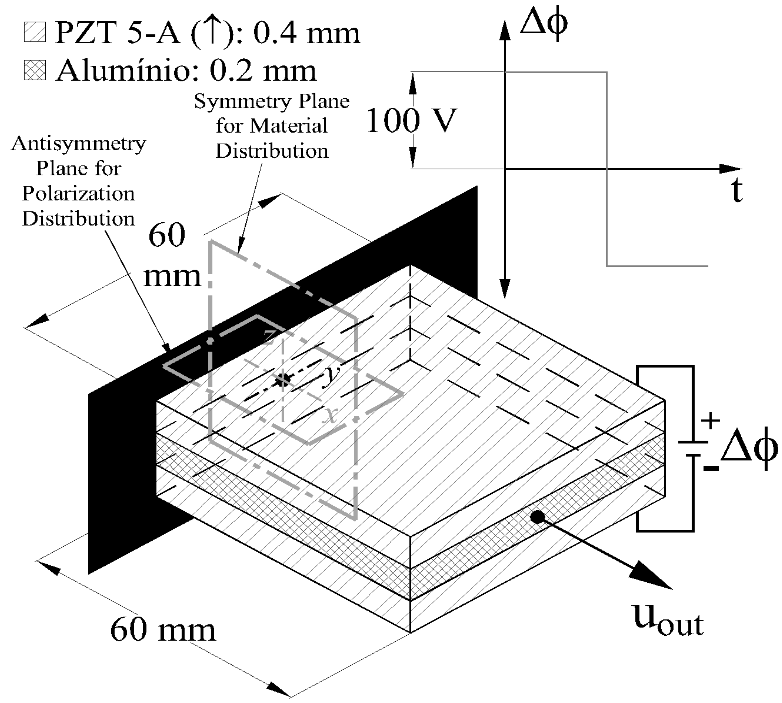

Figure 2. Design domain and boundary conditions

Table 1. Material properties

Material	Property (Units)		Value
Aluminum	Mass density (kg/m^3)		2700
	Young's modulus (GPa)		71
	Poisson's modulus		0.334
PZT-5A	Mass density (kg/m^3)		7800
	Elastic constants (GPa)	C_{11}^E	107.60
		C_{12}^E	63.12
		C_{13}^E	63.85
		C_{33}^E	100.40
		C_{44}^E	19.62
		C_{66}^E	22.24
	Piezoelectric constants (C/m^2)	e_{31}	-9.6
		e_{33}	15.1
		e_{15}	12.0
	Dielectric constants (F/m)	$\epsilon_{11}^{\varepsilon}$	$1936\,\epsilon_o$
		$\epsilon_{33}^{\varepsilon}$	$2109\,\epsilon_o$
		ϵ_o	$8.854187817 \times 10^{-12}$

Source: (Kogl and Bucalem 2005; Mohr, Taylor et al. 2008; Kiyono, Silva et al. 2012)

Table 2 – Date used in the numerical example

Description (Units)		Value
Input voltage (V)		100
Square wave frequency (kHz)		4
Transient time (ms)		0.12
Mesh	Elements	768
	Nodes	2499
	Elements per layer	256 (16x16)
Integration time steps		15
Aluminum volume constraint		$0.6 \leqslant V_{Alu} \leqslant 0.75$

Piezoelectric volume constraint		$V_{Pie} \leqslant 0.65$
Initial guess for topological design variables		0.7
Initial guess for polarization design variables		0.25 (negative electrode) 0.75 (positive electrode)
Filter radius (mm)		0.9375
	$P_m, P_\epsilon, P_\gamma$	1
Maximum penalty	P_c	3
	P_e	4

The non-intuitive final topology is shown in Fig. 3. In the same figure, the optimized topology is compared with a model with the initial domain fully covered. The features compared were rise time t_r, settling time t_s, stabilization displacement U_s, maximum overshoot S_p. Besides, aluminum V_{Alu}, piezoelectric V_{Pie} and total volume used V_{Tot}. Arrows indicate the desired magnitude.

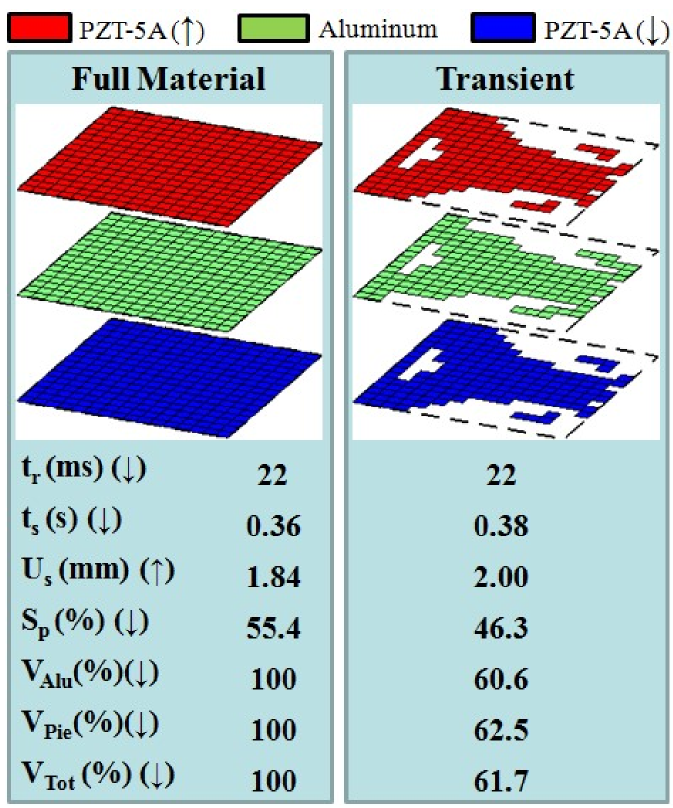

Figure 3. Design domain and boundary conditions

It can be seen that time performance is low for optimized topology. However, this topology has a better performance related with displacements and provides the lower amounts of total material. The speed response practically was not changed while the output displacement increased by 8%. The amount of material saved in aluminum, piezoelectric and total material was 39.4%, 37.5% and 38.3%, respectively.

6 CONCLUSIONS

The TOM was applied to LAPS design considering the transient response. The model material adopted was PEMAP-P and an algorithm based on SLP solved the optimization problem. Besides, a sensitivity filter was implemented as a solution control, reducing inherent problems in TOM.

It was shown that TOM successfully improved the displacement while fulfilling the imposed constraints. An increase in the output displacement of 8% was obtained in the case study. It could be attributed to the fact that transient optimization uses every time step to maximize the response gradually.

The objective function used in this work does not guarantee that maximum displacement and to minimize the rise time be achieved simultaneously because the optimization problem seeks to maximize the area under the curve displacement in time subject to satisfy the equilibrium equations. It is recommended to include in the objective function a term that seeks to maximize of displacement in rise time instant.

In future works, our intention is to include composite material with optimal fiber orientation and extends the TOM to optimal harvesters design.

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REFERENCES

Ahmad, S., B. M. Irons, et al. (1970). "Analysis of thick and thin shell structures by curved finite elements." *International Journal for Numerical Methods in Engineering* 2(3): 419-451.

Andreassen, E., A. Clausen, et al. (2011). "Efficient topology optimization in Matlab using 88 lines of code." *Structural and Multidisciplinary Optimization* 43(1): 1-16.

Balamurugan, V. and S. Narayanan (2009). "Multilayer higher order piezolaminated smart composite shell finite element and its application to active vibration control." *Journal of Intelligent Material Systems and Structures* 20(4): 425-441.

Bathe, K. J. (1996). *Finite Element Procedures*. Upper Saddle River, N. J., Prentice-Hall.

Bendsøe, M. P. and O. Sigmund (2003). *Topology Optimization: Theory, Methods and Applications*. Berlin, Springer.

Chung, J. and G. M. Hulbert (1993). "A Time Integration Algorithm for Structural Dynamics With Improved Numerical Dissipation: The Generalized-α Method." *Journal of Applied Mechanics* 60(2): 371-375.

Friend, J., K. Nakamura, et al. (2005). "A traveling-wave, modified ring linear piezoelectric microactuator with enclosed piezoelectric elements - The "scream" actuator." *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control* 52(8): 1343-1353.

Haftka, R. T. and Z. Gürdal (1992). *Elements of Structural Optimization*. Dordrecht, Kluwer Academic Publishers.

Hughes, T. J. R. (1987). *The Finite Element Method*. Englewood Cliffs, N. Y., Prentice-Hall.

Kim, J. E., D. S. Kim, et al. (2010). "Multi-physics interpolation for the topology optimization of piezoelectric systems." *Computer Methods in Applied Mechanics and Engineering* 199(49-52): 3153-3168.

Kiyono, C. Y., E. C. N. Silva, et al. (2012). "Design of laminated piezocomposite shell transducers with arbitrary fiber orientation using topology optimization approach." *International Journal for Numerical Methods in Engineering* 90(12): 1452-1484.

Koc, B., S. Cagatay, et al. (2002). "A piezoelectric motor using two orthogonal bending modes of a hollow cylinder." *IEEE Transactions on Ultrasonics Ferroelectrics and Frequency Control* 49(4): 495-500.

Kogl, M. and M. L. Bucalem (2005). "Analysis of smart laminates using piezoelectric MITC plate and shell elements." *Computers & Structures* 83(15-16): 1153-1163.

Kogl, M. and M. L. Bucalem (2005). "A family of piezoelectric MITC plate elements." *Computers & Structures* 83(15-16): 1277-1297.

Kogl, M. and E. C. N. Silva (2005). "Topology optimization of smart structures: design of piezoelectric plate and shell actuators." *Smart Materials & Structures* 14(2): 387-399.

Mello, L. A. M., C. Y. Kiyono, et al. (2014). "Design of quasi-static piezoelectric plate based transducers by using topology optimization." *Smart Materials and Structures* 23(2).

Mohr, P. J., B. N. Taylor, et al. (2008). "CODATA recommended values of the fundamental physical constants: 2006." *Reviews of Modern Physics* 80(2): 633-730.

Nakasone, P. H. and E. C. N. Silva (2010). "Dynamic design of piezoelectric laminated sensors and actuators using topology optimization." *Journal of Intelligent Material Systems and Structures* 21(16): 1627-1652.

Ogata, K. (1997). *Modern Control Engineering*. New York, Prentice-Hall.

Reddy, J. N. (2004). *An Introduction to Nonlinear Finite Element Analysis*. New York, Oxford University Press.

Trajkov, T. N., H. Koppe, et al. (2006). "Vibration control of a funnel-shaped shell structure with distributed piezoelectric actuators and sensors." *Smart Materials & Structures* 15(4): 1119-1132.