A REGULARIZED EVOLUTIONARY ALGORITHM FOR CONSTRAINED OPTIMIZATION

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Abstract. *Penalty techniques in conjunction with evolutionary algorithms applied on constrained optimization problems are usually adopted. However, this kind of modeling suffers with the difficulty to set adequate values for the penalization parameters. Dynamic and adaptive models are an attempt aiming to circumventing this drawback commonly used. Despite the quality obtained in constrained optimization using dynamic or adaptive models both suffers with the greediness phenomenon. When the problem has this characteristic the solution tends to be attracted to the infeasible region due to the balance between the objective function and penalty terms. In this work a regularized formulation is proposed in the context of a optimization evolutionary algorithm framework with the perspective to diminish the greediness effect. Tests in a suite of funtion optimization problems are carried out to evaluate the proposed strategy and its performance in relation to a similar non-regularized method.*

Keywords: *Constrained Optimization, Evolutionary Algorithm, Regularization*

1 INTRODUCTION

The handling of optimization problems with constraints is a necessity in several areas of the sciences. Tools come from mathematical programming has been used in last decades aiming to obtain reliable an efficient results avoiding local minimum and solutions in infeasible regions. Depending of the treated problems the adopted optimization tool could have major or minor difficulties to obtain the optimal solution. In practice, even the obtention of a simple feasible solution can be a challenger. The most common strategies to solve constraint optimization problems are transforming the constrained problem in unconstrained sub-problems that should converges to the solution. Again, difficulties in the parameters setting could be crucial for the quality of the obtained response.

In a complementary way in relation to the classical optimization methods, models based in evolutionary computation (EC) are obtaining interesting results in nonconvex, multimodal, multiobjective, among others optimization problems. However, these techniques are naturally constructed for unconstrained problems. Despite the adaptations for constrained problems are similar to the classical methods some particular properties of the evolutionary algorithms (EA) can be used to improve the developed model. The most adopted strategies to adapt an EA for constrained optimization are based on penalties function. Dynamic and adaptive methods to determine the penalty parameters bring robustness for this approach. In this work, a less usual method to solve a constrained problem based on augmented lagrangian method is introduced. The strategy is coupled with a traditional non-generational genetic algorithm (GA). Aiming to circumventing a characteristics of some problems with an extreme difficulty to obtain feasible solutions a regularization procedure is developed. Working together with the augmented lagrangian terms the method intends to guide the search to the feasible region more easily. Numerical experiments using a well know suite functions demonstrate the potential and efficiency of the methodology.

2 CONSTRAINED OPTIMIZATION PROBLEMS

Constraints optimization problems are presented in several fields, e.g., Biology, Economy, Engineering, etc. The goal is to obtain the values of the design variables that optimizes the problem in relation to a pre-defined objective function. Depending of the problem, constraints that restrict the search space to avoid solutions in undesirable regions are adopted. These constraints could be defined by equalities, inequalities or both types as presented in Belegundu (1982).

Optimization problems with constraints are usually described as:

$$\text{Minimize f}(\mathbf{x})$$

$$\text{Subject to:}$$

$$g_i(\mathbf{x}) \leq 0, \quad i = 1....m$$

$$h_j(\mathbf{x}) = 0, \quad j = 1...l$$

where $\mathbf{x}$ is the vector of design variable given by $\mathbf{x} = [x_1, x_2, ..., x_n]^T$ limited by box constraints. The function $f(\mathbf{x})$ is the objective function, $g(\mathbf{x})$ are the inequalities constraints and $h(\mathbf{x})$ the equalities constraints.

The objective function and the constraints depend directed of the design variables and could be linear or nonlinear functions.

2.1 Traditional techniques for resolution of constrained optimization

The transformation strategies are the most usual way to allow the solution of the constrained problem. These transformations aim to treat the constrained optimization as a sequence of unconstrained problem. For that, additional terms are coupled with the objective function.

Penalty function is one of the most common strategy to transform a constrained in a unconstrained problem. The violation of a constraint penalizes the objection function value using a penalty parameter associated with the amount of violation. The following equation is the usual form of the penalized function:

$$F(\mathbf{x}, r) = f(\mathbf{x}) + r\, P(\mathbf{x}) \tag{1}$$

with the function $F(\mathbf{x}, r)$ being the new objective function, r the penalty parameter and $P(\mathbf{x})$ the penalization function. These functions are constructed to avoid the constrain violation, forcing the search to the feasible region. Several penalty functions are used with the exterior penalty functions the most common can be written as:

$$P(\mathbf{x}) = \sum_{i=1}^{m} \max[0, g_i(\mathbf{x})]^2 + \sum_{j=1}^{l} [h_j(\mathbf{x})]^2 \tag{2}$$

When there are no constraint violated $P(\mathbf{x}) = 0$. Else the value of $P(\mathbf{x})$ is greater than zero. The definition of the penalty parameter is always a complex task due to the great influence of this choice in the quality of the results, Dynamic and adaptive models tries to circumvent this difficulty.

Lagrange multipliers method is another form to solve the constrained optimization. The introduction of multipliers associated with each constraint controls the violation by means of activation of the multipliers where the constraints are violated. The lagrangian function is defined as:

$$L(\mathbf{x}, \lambda, \mu) = \sum_{i=1}^{m} \lambda_i\, max[0, g_i(\mathbf{x})] + \sum_{j=1}^{l} \mu_j\, \|h_j(\mathbf{x})\| \tag{3}$$

where λ_i e μ_j are the multipliers related to the inequalities an equalities constraints, respectively.

The updating of the multipliers is obtained by a iterative process. The use of EC with penalty functions is very common. Nevertheless, the EC with Lagrange multipliers is less usual due to the difficulty to define a model to update the multiplies during the evolutionary process.

3 REGULARIZATION FOR THE GREEDINESS PROBLEM

3.1 Problem description

In this section a common problem that occurs in constraint optimization, mainly when strategies that need penalty parameters composing the objective function to be optimized are used. Usually, the penalized function is composed by the summation of the objective function $f(\mathbf{x})$ with a term related to the violated constraints multiplied by penalty parameters defined previously. So, the search of the optimum can be viewed as a manipulation of a function composed by two terms:

- The original objective function;
- The terms related to the unfeasible constraints adjusted by penalties parameters.

Considering the optimization process as the search for the solution that optimizes both terms of the penalized objective function, some behaviors could be expected due to this combination. Firstly, in the case of the penalty term has a magnitude very smaller than the objective function term the optimization process tends to give a major relevance to optimality in relation to feasibility. That is, the optimizing process consider an advantage to obtain solutions in regions of the search space where the objective function has optimum values even if the point is infeasible. This phenomenon occurs generally in the first iterations of the optimization, when low values for dynamic or adaptive penalty parameters are used. In this way, the optimum is not easily obtained because the solution is attracted for infeasible regions with excellent values for the objective function, indicating an indirect priority for the optimality of the objective function in detriment to the feasibility associated to the penalty terms. This behavior is known as greediness Castelani (2010); Castelani et al. (2012). The inverse behavior also happens when the parcel associated with the objective function has magnitude very smaller than the parcel of the penalization terms. The optimization tends to favour feasible solutions in detriment to optimum values for the objective function. That is, the feasibility will be preference in relation to optimality. This problem is less relevant since obtaining feasible solutions the influence of penalized terms is annulled. However, to occur this behavior elevated values for penalty parameters are necessary diminishing the chance of a well-posed problem. In optimization techniques using penalization functions the equilibrium of both terms is the goal to achieve a feasible solution with an optimum value for the objective function. Nevertheless, this control is not trivial for the most optimization problems. Basically, the main mechanism for this control is by means of penalty parameters setting. Several factors difficulty this definition adequately, as:

- The magnitude of objective function in relation to penalty terms is not previously evaluated easily;
- The values of objective function and the penalization terms vary too much during the optimization;
- The number of active constraints also changes too much during the optimization process.

The models with adaptive and dynamic penalties are considered an indirect way to circumventing this problem. However, problems with intense greediness characteristic or problems with reasonable difficulties to obtain feasible solutions could depend of a more specific strategy to obtain effectiveness in the optimization. One idea used with relative success in nonlinear optimization is the inclusion of an additional regularized term in the penalization function. Following, a regularization model aiming to control the greediness problem to obtain feasible solutions naturally is presented.

3.2 A strategy for regularization

One way to avoid the greediness problem is coupling an additional term to the optimized function. This new term has the incumbency of do not allow that the search deviates from feasible regions. The regularization term works penalizing solution points that are far from regions considered promising. In practice, acts as a penalization for the solution candidates which behavior indicates that they are moving for unfeasible regions. The regularization term can be defined as:

$$R(\mathbf{x}, \overline{\mathbf{x}}) = \alpha \, d(\mathbf{x}, \overline{\mathbf{x}}) \tag{4}$$

where α is the regularization parameter and $d(\cdot, \cdot)$ a distance measurement considered relevant calculated between the evaluated point **x** and a reference value $\overline{\mathbf{x}}$. A common choice for the measurement is the euclidian distance, generating:

$$R(\mathbf{x}, \overline{\mathbf{x}}) = \alpha \, \frac{||\mathbf{x} - \overline{\mathbf{x}}||^2}{2} \tag{5}$$

where the choice of squared distance facilitates optimization techniques that need derivative or gradients calculations. The reference point, as the name indicates, serves as a representant of the search space region considered relevant and, hence, where the search should be intensified. Thus, a point in the feasible region should be adopted as reference point what will keep the next iterative solutions near of this region. In some complex problems to obtain feasible points are not trivial generating the necessity of another strategy to define the value of the reference. After that, the regularization function R should be aggregated with the penalization function defined previously. The most common way used for the coupling is the direct sum of the terms, generating the penalized regularization function:

$$P_R(\mathbf{x}, \overline{\mathbf{x}}, r, \alpha) = P(\mathbf{x}, r) + R(\alpha, \mathbf{x}, \overline{\mathbf{x}}) \tag{6}$$

Despite the regularization function be presented as a strategy that enforce the obtainment of feasible points in constraint optimization it do no guarantee efficient results. The choice of the reference point in feasible regions of local optimum or the model used to update the reference point during the optimization could deviate the search of relevant feasible regions.

3.3 Considerations

Usually, a regularized model is adopted for problems with difficulty to obtain feasible points. The idea of regularization was initially presented to be used with classical constraint optimization techniques. Such techniques update a possible solution iteratively from a pre-defined initial value. Thus, the definition an updating of the reference value is determined by the generated sequence of solutions during the optimization process. Basically, if the iterative procedure generates a feasible solution with a better function value its substitute the previous reference point. In the case of only unfeasible points are obtained during the first iterations of the search the tendency is to use the less unfeasible point as reference point.

This work intends to use the regularization concepts combined with EAs. Based on population concept, the EAs bring an additional flexibility for the choice of the reference value, since all the individuals of the population are potential candidates. The expectation is to define an efficient model that avoids the stagnation of the search in feasible local optimum regions. In the case of EAs for constraint problems always is necessary the adaptation of the original AE adopted to consider the constraints efficiently. Usually the models constructed with EAs considering penalization functions differ reasonable from classical methods deriving from mathematical programming for nonlinear optimization. The main differences concerns to the population based and derivative free characteristics of the EAs. These differences bring a enormous potential to adjust the regularization term providing a possible enhancing in the performance of the optimization procedure. In this work the regularization term will be coupled with an augmented lagrangian technique. This kind of model includes in conjunction with penalty term an additional term associated with Lagrange multipliers. The expectation is a similar behavior of the regularization term working with the augmented lagrangian term in relation with models based only in penalization. Following, the EA with augmented lagrangian and the development of a specific regularization function are described. After that, the complete model is presented.

4 A REGULARIZED AUGMENTED LAGRANGIAN GENETIC ALGORITHM

A non-generational GA based on the augmented lagrangian method (ALGA) is the base for the development of the regularized EA. The choice of this model is related to the good performance obtained and described in the work of dos Santos Silva (2011). The augmented lagrangian method transforms a constrained optimization problem in a sequence of unconstrained optimization in the same way of the penalty models. Usually this kind of approach is applied on nonlinear optimization problems. The augmented lagrangian method was introduced by Hestenes (1969) and Powell (1969) for equality constraints problems. The extension for inequalities constraints was introduced in Rockafellar (1974).

In the classical lagrangian the updating procedure for the solution point and for the so called Lagrange multipliers and penalization parameters are defined to provide that the iterative solution converges to the optimal solution. Each constraint is associated with a multiplier and a penalization parameter. When a population based algorithm is used as optimizer, several potential solution candidates use the information of the same multiplies/penalizers associated to each constraint. In this way, the coupling connecting one candidate solution and the multiplier/penalizer of a constraint should be used by all candidates represented by the population. Besides, the updating of the individual solution is proceeded by the evolutionary operators, generating a reasonable difference to the original augmented lagrangian method. In the construction of the ALGA strategy for the updating of multipliers/penalizers during the evolutionary process is important to determines a model that presents a similar behavior of the traditional augmented lagrangian method. In dos Santos Silva (2011) an ALGA was introduced using non generational GA taking into account this necessary considerations. The algorithm is described following.

4.1 Non generational genetic algorithm using augmented lagrangian

The lagrangian function for inequalities constraint problems is given by:

$$L(\mathbf{x}, \lambda) = \sum_{i=1}^{m} \lambda_i \max[0, g_i(\mathbf{x})] \tag{7}$$

with $\lambda \in \Re^m$ being the Lagrange multipliers associated to the inequalities constraints. In practice, it is common to transform the equality constraint $h_j(\mathbf{x})$ in inequalities by means:

$$|h_j(\mathbf{x})| \leq \epsilon \tag{8}$$

where $\epsilon > 0$ is a tolerance parameter defined previously. So, without loss of generality, is possible to work only with inequalities constraints. For the development of the augmented lagrangian an extra term related to the penalization is added to the lagrangian function. Thus, the augmented lagrangian function is written:

$$F(\mathbf{x}, \lambda, r) = f(\mathbf{x}) + L(\mathbf{x}, \lambda) + P(\mathbf{x}, r) \tag{9}$$

where P is the penalty function with $r \in \Re^m$ the penalty parameters. The function $F(\mathbf{x}, \lambda, r)$ is called augmented lagrangian function with the standard model for inequalities constraint:

$$F(\mathbf{x}, \lambda, r) = f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i \max[0, g_i(\mathbf{x})] + \sum_{i=1}^{m} r_i \max[0, g_i(\mathbf{x})]^2 \tag{10}$$

The updating of multipliers and penalty parameters for this classical formulation can be seen in Pierre and Lowe (1975). Details about the behavior of the augmented lagrangian technique are presented in Srivastava and Deb (2010). One adaptation for use with EA is presented in Adeli and Cheng (1993).

As described early, when an EA is used as optimizer a reasonable level of flexibility could be consider in the definition of the updating strategy due its population based and evolutionary operators used that are different of the adopted in traditional mathematica programming algorithms dos Santos Silva (2011). The lagrangian function used is:

$$F(\mathbf{x}, \lambda, r) = f(\mathbf{x}) + \sum_{\forall g_i(\mathbf{x})>0} \lambda_i (1 + g_i(\mathbf{x})) + \sum_{\forall g_i(\mathbf{x})>0} r_i (1 + g_i(\mathbf{x}))^2 \tag{11}$$

with the multipliers initialized as zero. For the penalty parameters a direct relation of its value with the infeasible level of the population is considered for each constraint of the problem. The infeasible level of the population for a specific constraint is given by:

$$nig_i = \frac{\sum_{n=1}^{npop} \max[0, g_i(\mathbf{x})]_n}{iig_i} \tag{12}$$

where i is the i-th considered constraint, $npop$ the population size and iig_i the number of feasible individuals for the constraint $g_i(\mathbf{x})$ in the population. For the initial penalty values are assumed:

$$r_i^0 = nig_i \tag{13}$$

The updating of multipliers and penalty terms are determined based on the expected behavior of this variables in relation to the equilibrium between the objective function and the parcels of multipliers and penalization. The best individual of the current population (f_{better}) and the mean of the objective function of the population are used to determine the updating. Besides, normalization for the constraints that will be penalized is considered. The value nig_i represents the mean values of the constraint violation for the population. The idea is to allow a control for the value considered in the constraint taking as reference individual close to the mean of the constraint evaluated. So, the strategy for multipliers updating is defined as:

$$\lambda_i^{k+1} = \frac{(\|(f_{mean}^k - f_{better}^k)/f_{mean}^k\| + 1) \cdot (\|f_{better}^k\|) + 1)}{nig_i^k + 1} \tag{14}$$

A fundamental difference of this updating strategy in relation to the traditional augmented lagrangian model is the independence of the multiplier updating to the previous values of the multipliers and penalization parameters. The update depends of a global information of the population represented by f_{mean}, a local information of the same population defined by the value of f_{better} and a general information about the performance of the population considering the constraint given by the variable nig_i. In the case of penalty parameters the same ideas area adopted generating the formula:

$$r_i^{k+1} = \frac{\|(f_{media}^k - f_{melhor}^k)/f_{media}^k\| + 1}{nig_i^k + 1} \tag{15}$$

The unit value summed to nig_i is to avoid the increasing of the value of the operation when the variable nig_i is close to zero. The expectation is that during the evolutionary process using a non-generational EA the term $f_{mean} - f_{better}$ goes to zero, according to the expected behavior for the penalty component of an augmented lagrangian function. Following, the pseudo code of the ALGA is presented in Fig 1.

The Variable UPDT is responsible for the updating of the multipliers and penalty parameters. The value of three times the population size is adopted in dos Santos Silva (2011). The next step is to adapt the ALGA to generates a regularized model of ALGA (RALGA).

4.2 Regularized function composition

A regularized function considering the pattern presented in the equation (4) will be defined to be used in conjunction with the ALGA. Based on the regularization model previously presented the regularized lagrangian function takes the form:

$$F_R(\mathbf{x}) = F(\mathbf{x}) + R(\alpha, \mathbf{x}, \overline{\mathbf{x}}) \tag{16}$$

```
Algoritmo ALGA
  Begin
      Generate and evaluate the initial population
      Calculate the constraints violation of the individuals of the population
      Calculate nig_i for each constraint i
      Determine r^0 = nig_i, λ^0 = 0, for each constraint i
      Determine UPDT = false
      Calculate the lagrangian function given in (11) for the population
      Repeat
        Apply the genetic operators
        Evaluate new(s) individual(s)
        Calculate the lagrangian function given in (11)
        Select the survivor individual
        Allocate the new individual in the population
        Update variable UPDT
        If UPDT = true then
          Calculate nig_i
          Update λ based on (14)
          Update r based on (15)
          UPDT = false
        End if
      Until stop criteria is met
  End
```

Figure 1: ALGA pseudo-code

assuming:

$$F_R(\mathbf{x}) = f(\mathbf{x}) + \sum_{\forall g_i(\mathbf{x})>0} \lambda_i \left(1 + g_i(\mathbf{x})\right) + \sum_{\forall g_i(\mathbf{x})>0} r_i \left(1 + g_i(\mathbf{x})\right)^2 + \alpha \|\mathbf{x} - \overline{\mathbf{x}}\| \tag{17}$$

where $|| * ||$ defines a Euclidian norm. It is important to stand out that this regularized function is different from the function presented in equation (6). The difference is possible due to the baseline EAs used in the regularized model are zero order methods, that is, derivation of penalty function is not necessary. Thus, the regularized function adopted do not compromise the optimum search process. In sequence, the reference value needs to be defined. The determination of a reference value $\overline{\mathbf{x}}$ is done by means of an evaluation of the individuals of the current population of the non-generational GA. Initially, for the first value of $\overline{\mathbf{x}}$ an evaluation of the initial population is proceeded, with the $\overline{\mathbf{x}}$ value defined according to:

- There are feasible individuals in population: $\overline{\mathbf{x}}$ is the individual with the best objective function;
- There are no feasible individuals in population: $\overline{\mathbf{x}}$ is the less infeasible individual.

In the case of exists feasible individuals in initial population, the determination of $\overline{\mathbf{x}}$ is very direct and with no difficult. When there are no feasible individuals in the population, the definition of the less infeasible could be made by distinct manners. All constraints could be considered or only a sub-group of the constraint are chosen based on some strategy, e.g., the constraints more complexes or the worst constraint to be obeyed, etc. A definition is necessary and, in this work, the less infeasible individual or the individual that presents the minor value for the sum of all constraints without normalization is adopted as reference point. After that, a strategy to update the value of reference should be defined. The procedure used to update $\bar{\mathbf{x}}$ is very direct due to the based EA used is a non-generational GA. For this class of algorithm one or two individuals are usually generated in each iteration. If the individual deserves to be utilized, i.e., the individual will be inserted in the population, this individual also will be compared with the reference individual. The comparison indicates if the new individual should substitute the reference individual according to the rules:

- Reference individual is infeasible. The substitution is carried out if:
  - The new individual is feasible;
  - The new individual is ”less” infeasible.
- Reference individual is feasible. The substitution happens if:
  - The new feasible individual has a better objective function.

The main difficult using RALGA is the choice of the regularization parameter α. As in the case of penalization parameters the value of α influences greatly the quality of the obtained results of the regularized model. Initially, there is no constraint or directive for the definition of this parameter. In practice, the value of α should be chosen aiming that the regularization parcel becomes effective. The product of α by the distance of the individual to the reference value should have such magnitude that allows relevance in the final value of the function to guide the search to the region determined by the reference. Logically, the determination of a value that generates this behavior is not trivial. Mainly due to the lagrangian function is composed by the summation of parcels that involves values related to the objective function and the constraint violations, opposed to the regularized term that is calculated in the design variables space. Following, the experiments are presented.

5 NUMERICAL EXPERIMENTS

In this section numerical tests using the regularized algorithm presented are carried out with comparison when the regularization term is not adopted.

5.1 Description of the test suite

The mathematical functions used in the tests belong to the G-Suite proposed in Liang et al. (2006). Some functions taken to the experiments are presented in Table 1. Another properties of the chosen functions are described in Table 2. The adopted functions represent several types of constrained optimization problems, varying the type of objective functions, number of design variables, percentage of feasible space, among others.

5.2 Parameters used in the experiments

The parameters adopted in the experiments are described here, including the variables that evaluate the quality of the results.

In relation to the parameters for the algorithms the following configuration is assumed:

Table 1: G-Sute functions

Problem	Function	Optimum
G1	$5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i$	-15.00000
G2	$-\frac{\sum_{i=1}^{n} cos^4(x_i) - 2\prod_{i=1}^{n} cos^2(x_i)}{\sqrt{\sum_{i=1}^{n} ix_i^2}}$	-0.80361
G3	$-(\sqrt{n})^n \prod_{i=1}^{n} x_i$	-1.00050
G4	$5.3578547 \cdot x_3^2 + 0.8356891 x_1 x_5 + 37.293239 x_1 - 40792.141$	-3.06655e^4
G5	$3x_1 + 0.000001 x_1^3 + 2x_2 + (0.000002/3) x_2^3$	5126.49671

Table 2: Informations about the G-Sute functions

Problem	Variables	Type of function	p%	Constraints
G1	13	quadratic	0.0111	9
G2	20	non-linear	99.9971	2
G3	10	polinomial	0.0000	1
G4	5	quadratic	52.1230	6
G5	4	cubic	0.0000	5

- each algorithm runs a total of 25 times;
- the population size is 500 individuals;
- a total of 50 000 evaluations are realized in each run;
- the Wright recombination operator is used, with probability of 80%;
- the random mutation is used, with probability of 20%;
- value for the regularization parameter $\alpha = 0.8$.

In relation to the adopted variables used to monitoring the tested algorithms the following description defines their characteristics:

- **fcn:** indicates the number of the G-sute function used;
- **best:** shows the best solution obtained by the algorithm;

- **median:** correspond to the median of the 25 best solutions obtained;
- **mean:** presents the mean of the 25 best solutions obtained;
- **sd:** standard deviation of the 25 best solutions obtained;
- **worst:** indicates the worst obtained individual after 25 runs;
- **mne:** mean number of evaluation to obtain the best solution;
- **nb:** presents the number of times that the best is found.

5.3 Comparisons

Initially, the results using the ALGA without regularization for the used functions are depicted in Table 3.

Table 3: Results for ALGA

fcn	optimum	best	median	mean	sd	worst	nb	mne
G1	-15.000	-15.000	-12.000	-11.799	2.121	-9.000	25	36175
G2	-0.803	-0.6977	-0.5678	-0.4604	0.20059	-0.1798	25	32255
G3	-1.000	-0.686	-1.40$E-11$	-6.63$E-2$	0.197	-3.793$E-023$	12	40439
G4	-30665.538	-30665.538	-30665.422	-30547.997	262.374	-29770.815	25	47839
G5	5126.496	5126.498	5141.116	5164.188	78.351	5519.403	25	49778

The second result is obtained using the regularized model where the ALGAR uses the so called optimal reference point. In this case, the value of the reference $\bar{\mathbf{x}}$ is adopted as the solution of the optimization. This hypothetical experiments is important to assure that the algorithm has good performance using the best possible reference for $\bar{\mathbf{x}}$. Table 4 presents the results.

Table 4: Results for RALGA with optimal reference

fcn	optimum	best	median	mean	sd	worst	nb	mne
G1	-15.0000	-15.000	-15.000	-15.000	0.00	-15.000	25	37008
G2	-0.803	-0.804	-0.803	-0.803	1.124$E-3$	-0.799	25	27860
G3	-1.000	-1.000	-1.000	-1.000	4.540$E-16$	-1.000	23	5197
G4	-30665.538	-30665.538	-30665.538	-30665.538	1.560$E-3$	-30665.532	25	47341
G5	5126.496	5126.498	5126.508	5126.51	1.377$E-2$	5126.547	25	37317

Finally, the tests using the regularized model with the standard strategy described previously to define the value of the reference $\bar{\mathbf{x}}$ are performed. Table 5 shows the results attained.

In general, the results indicate that a regularization approach can guide the search for regions with best potential for the optimum obtainment. In the case of utilization the reference $\bar{\mathbf{x}}$ as the optimal reference the results are very good. This is a hypothetical test and can not be

Table 5: Results for RALGA with standard reference

fcn	optimum	best	median	mean	sd	worst	nb	mne
G1	-15.000	-15.000	-12.00	-11.799	2.121	-9.00	25	36175
G2	-0.803	-0.333	-0.241	-0.236	$3.137E{-2}$	-0.187	25	3556
G3	-1.000	-0.747	$-8.41E{-29}$	$-4.667E{-2}$	0.186	$-1.381E{-50}$	16	20508
G4	-30665.538	-30665.539	-30654.637	-30482.704	286.259	-29770.796	25	46540
G5	5126.496	5128.572	5177.262	5207.159	88.526	5545.788	25	45531

used in practice. However the results obtained show the potencial of a regularized function with adequate parameters.

Comparing the model without regularization and the regularized model is easy to observes that the regularization works for some functions and has no effect for others. The influence of the parameter α and a sub-optimum choice of the reference $\bar{\mathbf{x}}$ are responsible for that behavior. An adaptive model aiming to set the α value tends to increase the performance of the regularization strategy.

6 CONCLUSIONS

Constrained optimization problems present different levels of difficulty depending on the objective function that compounds the problem, the kind and number of constraints, the percentage of the feasible region in relation to the total search space, among others factors. In some problems it is hard even to obtain a feasible solution during the optimization process. For nonlinear optimization penalty strategies are usually adopted. When EA are used for constrained optimization adaptations are necessary to obtain an effective performance. These kind of adaptation by means of static, dynamic and adaptive models presents an enhancing in the optimization results. However, for very complex optimization problems to obtain feasible solutions is still a challenger. In this work, a regularization strategy to avoid the greediness phenomenon is developed. The regularization has the objective to facilitate the obtainment of feasible solution. The model is based on a reference value chosen based on its potential position in relation to the search space. Besides, the regularization model could be adapted for all types of EAs. A non-generational GA based on augmented lagrangian (ALGA) was adopted to receive the regularization model (RALGA).

Numerical tests were realized using a referential function suite for constrained optimization. Comparing the regularized model with the model without regularization, in general, for almost all measures used the regularized presents a better performance.

Despite of the potential demonstrated for the model with regularization new studies and developments are necessary to guarantee effectiveness and robustness of this approach applied on constrained optimization problems.

Acknowledgements The authors would like to express thanks the agencies CNPq , CAPES and FAPEMIG for the financial support for the accomplishment of this paper.

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