NUMERICAL EXPERIENCE WITH AN ENSEMBLE-BASED METHOD FOR CONSTRAINED WATERFLOODING OPTIMIZATION

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Abstract. *An implementation of an ensemble-based method for constrained waterflooding optimization is presented. Net present value (NPV) is maximized subject to constraints on total production and injection rates. Tuning of algorithm parameters is discussed. An ensemble-based method is implemented for waterflooding optimization. The objective function is maximized using the sequential quadratic programming algorithm. Control variables are the flow rates of the wells. An approximate gradient is computed based on the covariance matrix of control vectors of ensemble realizations as well as the cross-covariance of the corresponding NPV values obtained through simulations of the realizations. The realizations contained in the ensemble are perturbations of previous controls during the optimization process. Filters were developed to enforce feasibility of realizations with respect to problem constraints. The proposed methodology was applied to a reservoir model with 12 wells and 6 control cycles. The optimization problem is subject to constraints on total production and injection rates. We conducted an extensive parametric study the assess influence of the various algorithmic parameters. Ensemble size is one of the major parameters studied since it is essential to use small number of realizations in the ensembles. It was found that results obtained using an ensemble size of 5.5% of the total number of controls*

Keywords: *Waterflooding optimization; Ensemble-based methods and Constrained optimization*

1 INTRODUCTION

The focus of reservoir management in the oil industry is typically to optimize sweep such that water production is minimized, or oil recovery or profitability. The waterflooding is a method used to improved recovery in which water is inject into a reservoir to recover additional quantities of oil that have been left behind after primary recovery.

In this paper we focus on production optimization with constraints. The aim of production optimization is to find the optimal operation of wells that maximize the NPV of reservoir. Constraints are imposed to guarantee the physical feasibility of the solution that does not violate common engineering practices. Therefore, constrained optimization problems need to be handled

For production optimization in oil reservoir, the most efficient approach involves computation of the gradient of an objective function with respect to the control variable (Chen and Oliver, 2012). The gradient vector of the objective function is efficiently calculated using an adjoint method (Brouwer and Jansen, 2004), it requires only one forward and one adjoint simulation to compute the gradient regardless of the number of controls, however, implementation of the adjoint formulation is complicated and requires access to the simulation source code which is not possible when using a commercial simulator.

Chen et al., 2009 introduced one stochastic technique, ensemble-based optimization, as an alternative way to the adjoint as it is relatively easy to implement and independent of the reservoir simulator. The gradient computed by the ensemble method is a nonlocal approximation over the space spanned by the realization of the control variables (Chen and Oliver, 2012).

In this paper we investigate and discuss several aspects for approximate calculation of the gradient vector for waterflooding problems, Spherical and Gaussian correlation functions, control perturbation size, and imposition or not of feasibility on the realizations.

To solve the optimization problem we use sequential quadratic programming SQP algorithm for constraint nonlinear production optimization. This approach has been used in production optimization (Lorentzen et al., 2009; Dehdari and Oliver, 2012; Horowitz et al., 2013, Tueros et al., 2014).

The numerical experience with ensembles methods was obtained through the use of the Brush Canyon outcrop based reservoir with a total twelve wells, seven producers and five injectors.

2 OPTIMIZATION PROBLEM

The optimization process is to find the best solution (or optimal) a set of solutions to a problem. The optimization techniques should be used when there is no simple and direct way to compute the problem solution. This usually occurs when the problem structure is complex, and there are large numbers of possible feasible solutions.

2.1 Formulation

The general problem to solve is:

$$Maximize\ f(\boldsymbol{x})\ subject\ to \begin{cases} h_i(\boldsymbol{x}) = 0, \ i = 1 \ldots m \\ g_j(\boldsymbol{x}) \leq 0, \ j = 1 \ldots p \end{cases} \tag{1}$$

where $\boldsymbol{x} = \left[x_1, x_2 \cdots x_n\right]^T$ is the vector of control variables, $f(\boldsymbol{x})$ is the objective function, $h_i(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$ are constraint functions assumed to be continuous and have continuous second partial derivatives. In addition, the feasible region of this problem is assumed to be nonempty. To compute the final solution of this problem, which is denoted by $\boldsymbol{x}^*$, the control variables are updated in each iteration. The vector x must satisfy the Karush-Kuhn-Tucker (KKT) conditions that are necessary and sufficient for a local minimum [see, e.g., Antoniou and Lu, (2009)]:

$$
\begin{array}{ll}
Feasibility & h_i(\boldsymbol{x}) = 0,\ 1 \dots m. \text{ and } g_j(\boldsymbol{x}) = 0,\ j = 1 \dots p \\
Stationary & \nabla f(\boldsymbol{x}) + \sum_{j=1}^{p} \lambda_j \nabla g_j(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(\boldsymbol{x}) = 0 \\
Complementarity & \lambda_j g_j(\boldsymbol{x}) = 0,\ \text{with } \lambda_j \geq 0,\ j = 1 \dots p
\end{array}
\tag{2}
$$

In Eq.(2) λ is the vector of Lagrange multipliers

2.2 Sequential Quadratic Programming

The sequential quadratic programming (SQP) algorithm is applied to non-linear problems (Eldersveld, 1992). This algorithm uses a method based on a one-dimensional search scheme that involves iterative processes:

1. Internal iterative process solves a quadratic subproblem to determine the search direction.
2. External iterative process calculates a new approximation of the Hessian of the Lagrange function, which is updated using Quasi-Newton BFGS (Broyden-Fletcher-Golfarb-Shanno) technique (Broyden, 1967).

The sequences of this approximations generated by the external iterative process converge to a point that satisfies the conditions of Eq. (2). At the beginning of the algorithm a starting point, x_0, is provided. The step size along the direction d is found through a line search that uses an exact penalty function that balances the influences of the objective and constraint functions.

In SQP algorithm solves a sequence of quadratic subproblems and the objective function is a quadratic approximation to the Lagrange function. This algorithm uses the information of second order of the function to optimize, so the function expanded to the second order Taylor series around, $\overline{\boldsymbol{x}}$:

$$
f(\boldsymbol{x}) = f(\overline{\boldsymbol{x}}) + d^T \nabla f(\overline{\boldsymbol{x}}) + \frac{1}{2} d^T \nabla^2 L(\overline{\boldsymbol{x}}) d \tag{3}
$$

where; $d = \boldsymbol{x} - \overline{\boldsymbol{x}}$ is the search direction, $\nabla f(\overline{\boldsymbol{x}})$ is the gradient vector, the elements are defined by:

$$\nabla f\left(\boldsymbol{x}\right)=\frac{\partial f}{\partial x_i} \tag{4}$$

and $\nabla^2 L$ is Hessian of the Lagrange function, which is updated at every iteration. Below we present the steps involved in the SQP algorithm:

1. Establish an initial solution x_0;
2. Set up an initial approximation to the Hessian matrix of the quadratic terms of the objective solution;
3. Solve the subproblem to find the search direction d;
4. Conduct a linear search to determine step size α in the direction d;
5. Update the solution by referring it to the indicated position;
6. Check the converge of the SQP algorithm:
   6.1. If the local minimum was found the process ends.
   6.2. Otherwise the Hessian is updated with the BFGS scheme and return to stop 3.

3 FORMULATION OF THE WATERFLOONDING PROBLEM

Mathematically the waterflooding can be formulated as follows:

$$\begin{aligned}
&Maximize \quad NPV = f(\boldsymbol{q})=\left[\sum_{t=1}^{n_t}\frac{1}{(1+d)^{\tau_t}}F(\boldsymbol{q}_t)\right]\\
&subject\ to: \quad \sum_{p\in P} q_{p,t} \le Q_{l,\max},\ t=1\ldots n_t\\
&\qquad\qquad\quad \sum_{p\in I} q_{p,t} \le Q_{inj,\max},\ t=1\ldots n_t\\
&\qquad\qquad\quad q_{p,t}^{l} \le q_{p,t} \le q_{p,t}^{u},\ t=1\ldots n_t \quad and \quad p=1\ldots n_w
\end{aligned} \tag{5}$$

in the eq. (5) $\boldsymbol{q}=\left[q_1^T, q_2^T, \cdots q_{n_t}^T\right]^T$ is the vector of well rates for all control cycles; $\boldsymbol{q}_t=\left[q_{1,t}, q_{2,t}, \cdots q_{n_w,t}\right]^T$, is the vector of well rates at control cycle t, $q_{p,t}$ is the liquid rate of well p at control cycle t; n_t is the total number control cycle. In the objective function d, is the discount rate and τ_t, is the time at the end of $t\mathrm{th}$, control cycle.

The cash flow at control cycles t, which represents the oil revenue minus the cost of water injection and water production, is given by:

$$f(\boldsymbol{q})=\Delta\tau_t\left[\sum_{p\in P}(r_o q_{p,t}^{o}-c_w q_{p,t}^{w})-\sum_{p\in I}(c_{w_i} q_{p,t})\right] \tag{6}$$

where $\Delta\tau_t$, is the time size of the $t\text{th}$ control cycle; P and I are the sets of production and injection well, respectively; $q^o_{p,t}$, and $q^w_{p,t}$ are the average oil and water rates at the $p\text{th}$, production well at $t\text{th}$ control cycle ; r_o, is the oil price; c_w and c_{w_i} the cost of producing and injection water (Horowitz et al., 2013).

$Q_{l,\max}$, is the maximum allowed total production liquid rate and $Q_{inj,\max}$ is the maximum allowed total injection rate of the field. Superscripts l and u denote respectively the lower and upper bounds of design variable. Superscript o and w denote respectively oil and water phases.

This problem is to subdivide the concession period into a number of control cycles, n_t, is the time fixed. The design variables are the well rates in each control cycles. Let well rate be scaled by their respective maximum allowable field rates

$$x_{p,t} = \frac{q_{p,t}}{Q_{l,\max}}, \;\; p \in P; \quad x_{p,t} = \frac{q_{p,t}}{Q_{inj,\max}}, \;\; p \in I \; . \tag{7}$$

Design variables, $x_{p,t}$ are then the allocated rate for well p, at time cycle t. We consider the following formulation for this problem:

$$\begin{aligned} &Maximize \quad NPV = f(\boldsymbol{x}) = \left[\sum_{t=1}^{n_t} \frac{1}{(1+d)^{\tau_t}} F(\boldsymbol{x}_t) \right] \\ &subject\ to: \quad \sum_{p\in P} x_{p,t} \le 1, \; t = 1 \ldots n_t \\ &\qquad\qquad\quad \sum_{p\in I} x_{p,t} \le 1, \; t = 1 \ldots n_t \\ &\qquad\qquad\quad x^l_{p,t} \le x_{p,t} \le x^u_{p,t}, \; t = 1 \ldots n_t \quad and \quad p = 1 \ldots n_w \end{aligned} \tag{8}$$

where $\boldsymbol{x} = \left[x_1^T, x_2^T, \cdots x_{n_t}^T \right]^T$ is the vector of scaled well rates for all cycles; $\boldsymbol{x}_t = \left[x_{1,t}^T, x_{2,t}^T, \cdots x_{n_w,t}^T \right]^T$, is the vector of scaled well rates for cycle t.

The calculation of the objective function (NPV) in waterflooding problems needs a complete run of reservoir simulator and is one of the largest time consuming tasks in the computation of the gradient vector.

4 THE GRADIENT

Computation of the gradient vector is one of the most expensive steps to carry out in the optimization process. In this paper, we approximate the gradient vector by an ensemble based method. The gradient computed by the ensemble method is a nonlocal approximation over parameter space spanned by the realizations of control variables (Chen and Oliver, 2012). For this reason we need to generate different realizations which are simulated to compute the Net Present Value (NPV) of each realization. This makes it possible to estimate the cross-covariance between the control variables and NPV which multiplier by pseudo-inverse of the covariance matrix of the control variables to obtain the approximate gradient vector.

4.1 Motivation of the approximate gradient vector computation

Let $x \in R^n$; such that $f(x) = \alpha + x^T g$ with $g \in R^n$ and $\alpha \in R$. An ensemble of the vector $\left(x_1, x_2 \cdots x_{N_e}\right)$ of N_e realizations with mean $\bar{x} = \frac{1}{N_e}\sum_{i=1}^{N_e} x_i$, then g is the constant gradient vector of *f*(*x*).

Given that $f(\bar{x}) = \alpha + (\bar{x})^T g$, then the difference between $f(x) - f(\bar{x}) = (x - \bar{x})^T g$. Since $\sum_{i=1}^{N_e} f(x_i) = N_e f(\bar{x})$, then $\bar{f} = f(\bar{x})$ and $f(x) = \bar{f} + (x - \bar{x})^T g$.

The cross covariance vector:

$C_{xf} = \frac{1}{N_e - 1}\sum_{i=1}^{N_e}(x_i - \bar{x})(f_i - \bar{f}) = \frac{1}{N_e - 1}\sum_{i=1}^{N_e}(x_i - \bar{x})(x_i - \bar{x})^T g = C_{xx} g$. Therefore pre-multiplication by Pseudo inverse of the covariance matrix gives: $g = (C_{xx})^{-1} C_{xf}$.

4.2 Generation of realizations of the ensemble

The following procedure was used to generate the realization:

1. Generate Gaussian or Spherical covariance function for regularization of control variable and avoid abrupt change of controls in time.

$$C(t_{ij}) = exp\left(\frac{3\left(t_i - t_j\right)^2}{s^2}\right). \tag{9}$$

$$C(t_{ij}) = \begin{cases} 1 - \frac{3}{2}\left(\frac{\left|t_i - t_j\right|}{s}\right) + \frac{1}{2}\left(\frac{\left|t_i - t_j\right|}{s}\right)^3 & if \quad 0 \le \left|t_i - t_j\right| \in s \\ 0 & if \quad \left|t_i - t_j\right| > s \end{cases}. \tag{10}$$

In this Eq. (9) and (10) $t_i - t_j$ is the difference between time step i and j, and s is the correlation length size.

2. Compute the Cholesky decomposition of the covariance function $LL^T = C$.
3. Inform initial values for the control variables x. The starting point is only informed for the first iteration, in the following iterations we use the previous iterand.
4. Generate a vector of independent random variable Z of Gaussian distribution with zero mean and unit variance.

5. Generate an ensemble of N_e realization of the control variable from the following expression:

$$r = x + \sigma LZ. \tag{11}$$

Where σ is the standard deviation, used to adjust the magnitude of the size of perturbations.

6. Check the maximum and minimum limits of the control variable and also constraints of the problem using filters if necessary.

The following procedure was used to filters:

While $\sum_{i=1}^{N_w} x_j^i > 1, \quad j = 1 \ldots N_c$ separate the control variable in:

$$P_I = \left\{ x_j^i, \, such \; that \; x^l \leq x_j^i < x^u \right\}_{i=1}^{L} \;\; and \;\; P_J = \left\{ x_j^i = x^u, \, such \; that \; x^u \leq x_j^i \right\}_{i=L}^{N_w}. \tag{12}$$

compute a multiplier:

$$\lambda = \frac{1 - \sum_{i=L}^{N_w} x_j^i}{\sum_{i=1}^{L} x_j^i}. \tag{13}$$

The Eq.11, we have $P_I = \lambda P_I = \left\{ \lambda x_j^i \; such \; that \; x^l \leq x_j^i < x^u \right\}_{i=1}^{L}$.

The set of all realizations form an ensemble of realizations.

4.3 Computation of gradient vector for the waterflooding problem

There are several options to calculate the gradient vector for a waterflooding problem, for example, the Finite Difference Method (not feasible for problem thousand variables); the adjoint method (Brouwer and Jansen , 2004; Sarma et al., 2005), simultaneous perturbation stochastic approximation (SPSA) (Gao et al., 2007 and Wang et al., 2009), and also approximating gradient from ensembles (Nwaozo, 2006; Chen et al., 2009, Su and Oliver, 2010, Dehdari and Oliver, 2012). In this paper, we approximate the gradient vector from ensembles of realizations. For this reason, we need to generate different realization from an ensemble of realizations and which are simulated in reservoir simulator to calculate NPV of the members of the ensemble.

The calculation of the gradient vector for ensemble is obtained by the following steps:

1. The matrix of the ensemble is defined by:

$$\boldsymbol{X} = \left[x_1, x_2, \cdots, x_i, \cdots x_{N_e} \right]_{(N_x, N_e)} \tag{14}$$

Where

$$x_i = \left[\left(x_1^i\right)^1, \cdots, \left(x_1^i\right)^w, \cdots\cdots, \left(x_{n_t}^i\right)^1, \cdots, \left(x_{n_t}^i\right)^w \right]_{(1,N_x)}^T \tag{15}$$

2. The ensemble mean is:

$$\bar{x} = \frac{1}{N_e} \sum_{i=1}^{N_e} x_i \tag{16}$$

3. Compute the matrix of deviations between the ensemble and its mean:

$$\Delta \boldsymbol{X} = \boldsymbol{X} - \bar{x} \tag{17}$$

4. Create the covariance matrix:

$$C_{xx} = \boldsymbol{A}\boldsymbol{A}^T; \qquad \boldsymbol{A} = \sqrt{\frac{1}{N_e - 1}} \Delta X \tag{18}$$

5. Find C_{xx}^{-1} using the pseudo-inverse method:

First we need to decompose matrix $\boldsymbol{A}$, using singular value decomposition method (SVD):

$$\left[U\right]_{m,m} \left[\sum\ \right]_{m,n} \left[V\right]_{n,n}^T = \left[A\right]_{m,n} \tag{19}$$

$$\left(\sum\ \right)_{ij} = \begin{cases} 0 & if \quad i \neq j \\ \sigma > 0 & if \quad i = j \end{cases} \Rightarrow \sigma \ is\ a\ singular\ of\ A. \tag{20}$$

Then the pseudo inverse is:

$$C_{xx}^{-1} = \sum_{i=1}^{k} \frac{1}{\sigma^2} u_i u_j^T \tag{21}$$

6. Compute the cross-covariance between the control variables and the NVP

$$C_{xf} = \frac{1}{N_e - 1} \sum_{i=1}^{N_e} (x_i - \bar{x})(f(x_i) - f(\bar{x})) \tag{22}$$

7. Approximate the gradient as:

$$g = C_{xx}^{-1} C_{xf} \tag{23}$$

The efficiency of the ensemble based methods depends on the size of the ensemble. Generally, with the increasing number of the realizations, we can increase the final NPV, but selecting large number of realizations will increase the number of simulator runs, and that is undesirable for us (Dehdari, 2010).

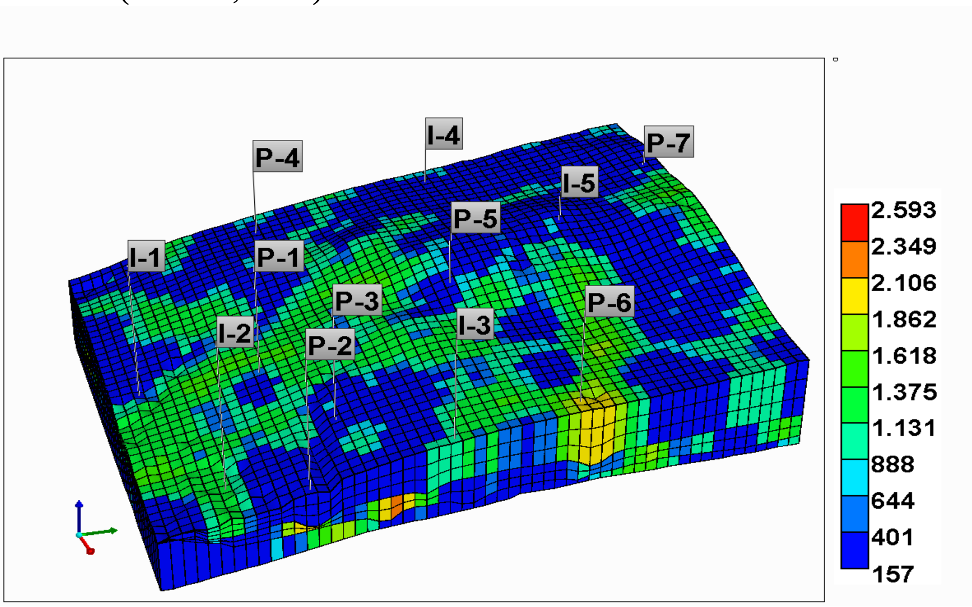


Figure 1. Permeability field

5 EXAMPLE

In this section the tools described previously are employed in the model of Brush Canyon outcrop synthetic reservoir case. The studies are conducted using Matlab Optimization Toolbox. The reservoir numerical simulations are performed using IMEX of the Computer Modeling Group (CMG, 2014).

Figure 1 show the permeability field. This model was presented in Oliveira, (2006) and Horowitz et al., 2013. The number total well is twelve, seven producers and five injectors. Main characteristics of the model reservoir are shown in the Table1.

In this model six control cycles was used (Horowitz et al., 2013) involving 72 independent design variables.

Normalized producer rates for the initial design point for all strategies are: $x_{p,t}$=0.14 $t = 1 \ldots n_t \quad p \in P$, while for all injectors, $x_{p,t}$=0.20 $t = 1 \ldots n_t \quad p \in I$, the corresponding NPV value is $2.5652 \, \mathbf{x} 10^8$.

5.1 Results and discussion

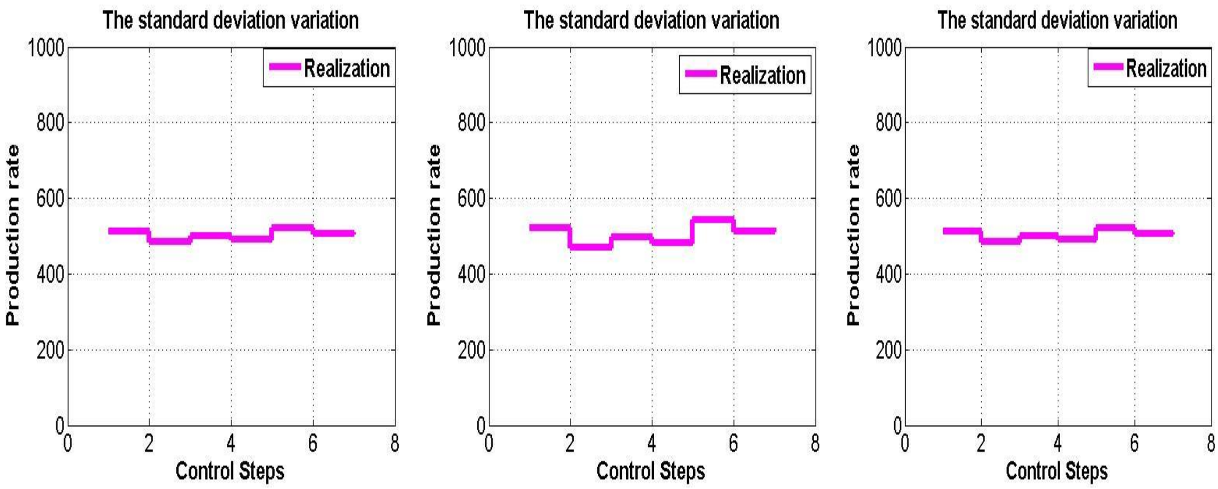
In these results, we use an ensemble size of 5.5% of the total number of control variables. We make comparisons between Gaussian and Spherical correlation functions, the control perturbation size and imposition or not of feasibility on the realizations.

The standard deviation plays very important role in the size of the perturbation of the control variables. This work will use standard deviation as a percentage of the difference of the upper and lower limits of the control variables.

Figure 2 are shown with different sizes of perturbations. In this paper we used size of 5%, 7% and 10%.

Table 1. Model reservoir characteristics

Simulation mesh	43x55x6
Porosity	16-28%
Horizontal permeability	157-2592 mD
Vertical permeability	30% of k_h
Viscosity	0.11 cP
GOR	78.1 $m^3 / m^3 std$
Maximum platform liquid production rate	5000 m^3 / dia
Maximum platform injection production rate	5750 m^3 / dia
Maximum liquid rate at producers	900 m^3 / dia
Maximum water rate at injectors	1500 m^3 / dia



(a) $\sigma = 5\%$ (b) $\sigma = 7\%$ (c) $\sigma = 10\%$

Figure 2 Standard deviation of different sizes

One can see that the larger the size of standard deviation the larger is the size of perturbation of the control variables. Using a large standard deviation has produced abrupt changes in controlling variables.

The results of the optimization process are obtained are shown in Table 2.

Table 2. Optimization results

Realization	Correlation	s	σ	Feasibility	Imex	Iterations	$NPV \mathbf{x} 10^8$
5	Gaussian	6	7	Yes	192	21	3.7323
5	Gaussian	5	7	Yes	93	11	3.7035
4	Spherical	4	10	Yes	105	13	3.6987
4	Spherical	6	10	Not	52	6	3.6628

The numerical experience has shown that best results are obtained using a Gaussian correlation function with temporal correlation length 6, standard deviation 7, with imposition of lower bounds on control variables and total capacities constraints for the realizations. The best NPV obtained is $3.7323\mathbf{x}10^8$ in 21 iterations with 192 reservoir simulation calls. A total of 21 ensembles of realizations were necessary, each with five realizations of control variables for gradient vector computation. Another approximate result is obtained using a temporal correlation length of 5, with NPV was $3.7035\mathbf{x}10^8$ needing 11 iterations with 93 simulator calls.

The lowest number of simulation calls, 52, was obtained using ensemble size of four, spherical correlation function with temporal correlation length 6, and not imposing feasibility of total capacity constraints on the ensemble of realizations. The NPV of $3.6628\mathbf{x}10^8$ is only 2% lower than our best result. Ensemble size is the most impacting parameter on the optimization computational cost and has to be used very wisely on large problems.

Rates of production wells corresponding to our best result as shown in Table 3 and rates of injection wells are shown in Table 4.

Table 3 Producer rates

P1	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**
564.5	505.5	721	856.5	670.5	800	881.5
470	621.5	887.5	806.5	513	835	866
492.5	814.5	892.5	701	549	804	746
516.5	856	864.5	844.5	492	726	675.5
501	888.5	754	855.5	436	764	639
716	457	600.5	900	666	780.5	880

Table 3 Injector rates

I1	I2	I3	I4	I5
813.62	27.6	6.325	1500	1499
1173.6	51.75	37.95	1500	1492.1
1369.1	330.05	551.42	1500	1487
1274.8	447.35	880.9	1500	1480.6
1212.7	174.8	1409.9	1204.6	1424.3
682.52	77.625	1.725	1500	1484.1

5.2 Conclusions

In this paper, we investigate the tuning of parameters used in ensemble based methods for the computation of approximate gradients that are necessary during the constrained optimization process of the waterflooding problem. The parameters studied were the ensemble size, Spherical and Gaussian correlations functions, size of control perturbations and imposition or not of feasibility on the ensemble realizations.

It was found that using the Gaussian correlation function works better for larger ensembles while Spherical correlation function is more effective for smaller ensemble sizes. Smaller ensemble sizes are desirable since they drastically reduce the cost of the optimization process. Ensemble size in the order of 5% of the total number of the control produced results only 2% lower than those obtained with larger ensembles.

As a rule-of-thumb we recommend perturbation size in the controls of 7%, with imposition of feasibility on the realizations, and temporal correlation length of 6. Our numerical experience with ensemble based methods for constrained waterflooding problems demonstrate that they are very effective with the added important benefit of being independent of the simulator.

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