Comparison of Numerical Integration and Monte Carlo Simulation Methods for Structural Reliability based on the Failure Assessment Diagram

Alexandre Santos Francisco

Jorge Alberto Rodrigues Duran

Carolina Carvalho Vilela

Tiago do Nascimento Simoes

afrancisco@metal.eeimvr.uff.br

duran@metal.eeimvr.uff.br

carolkkrvalho@hotmail.com

tnsimoes@gmail.com

Universidade Federal Fluminense

Avenida dos Trabalhadores 420, 27255-125, Volta Redonda, Rio de Janeiro, Brazil

Abstract. *The Failure Assessment Diagram (FAD) can be used for assessing the reliability of crack-containing pressure vessels. In the FAD, there are two coordinates associated with the collapse and the fracture parameters. Realistically, these parameters must be considered as random variables in order to account for uncertainties in four quantities: crack size, applied stress, yield stress and fracture toughness. In this paper, it is applied just the marginal probability distribution functions for modeling the uncertainties, allowing to compute the probability of failure in pressure vessels, based on the FAD. The purpose is to compare results of probability of failure from the numerical integration and the Monte Carlo simulation methods .*

Keywords: *Fracture mechanics, Structural reliability, FAD analysis*

1 INTRODUCTION

The reliability assessment of structures has been carried out by applying one-criterion failure analysis method (Chuen-Horng et al., 1994), in which are considered brittle fracture or plastic collapse. Recently, the tendency in the assessment methods is to consider both the elastic and plastic nature of the failure mechanism (Guoshoa et al., 1996). Thus, there is a two-criteria failure analysis which takes into account the brittle fracture and the plastic collapse, simultaneously. These two failure mechanisms are taken as coordinate axes of a two-dimentional diagram, called Failure Assessment Diagram (FAD). In the FAD, there is a region delimited by the failure limit state function, which assigns crack-containing pressure vessels to a failure state.

The fracture and collapse criteria in the FAD are functions of four quantities: crack size, applied stress, yield stress and fracture toughness. The reliability assessment takes account the uncertainty of all these parameters. After, probability distributions are associated to the FAD parameters. The failure probability can be computed by integrating the corresponding distribution functions over the failure state region.

Alternatively, the FAD reliability assessment can be performed by simulating randomly failure conditions. This procedure is called Monte Carlo simulation. In this paper, it is presented a comparison of results of the integration method and the Monte Carlo simulation. Once the Monte Carlo simulation is validated, it is possible to apply such simulation for assessing the probability of failure of a pressure vessel containing cracks.

In the Section 2, it is presented the failure assessment diagram methodology. The numerical integration method is described in the Section 3. Next, a summary of the Monte Carlo simulation method is showed in the Section 4. The methods are compared by results of failure probability in the Section 5. And conclusions are drawn from the results in the Section 6.

2 FAILURE ASSESSMENT DIAGRAM

The FAD allowed to develop a two-criteria failure analysis method which expresses probabilistically the proximity to failure, taking the fracture parameter (Kr) and the collapse parameter (Lr) as coordinates of the diagram. Kr stands for a normalized crack driving force, and Lr stands for a normalized mechanical strength.

The formulae of Kr and Lr can be stated as (Shu-Feng, 1982):

$$Kr = \frac{\sigma(\pi a)^{1/2} Mm}{K_{IC}.Qo}, \tag{1}$$

and

$$Lr = \frac{\sigma}{\sigma_y(1 - Cw)}, \tag{2}$$

where a is the crack depth, σ is the applied stress, K_{IC} is the fracture toughness, and σ_y is the yield stress. Mm and Qo stand for revising factors (here, $Mm = 1.1$ and $Qo = 2.5$). Cw stands for another revising factor given by

$$Cw = \frac{a.c}{4T(c + T)}, \tag{3}$$

where c is the crack half length, and T is the wall thickness.

In the FAD, the failure state region is defined by the failure limit state function ($FLSF$), which depends on the values of Kr and Lr. According to Milne et al. (1986) the $FLSF$ can be expressed as follows:

$$g(Lr, Kr) = (1 - 0.14Lr^2)[0.3 + 0.7exp(-0.65Lr^6)] - Kr = 0. \tag{4}$$

Based on the two-criteria failure analysis and considering the uncertainty of Lr and Kr, the failure probability of a pressure vessel containing cracks can be expressed as follows:

$$P_f = \int\int_{g(Lr,Kr)\leq 0} f_{Lr,Kr}(Lr, Kr).dLr.dKr, \tag{5}$$

where $f_{Lr,Kr}(Lr, Kr)$ is the joint probability density function for the fracture and collapse criteria.

3 THE NUMERICAL INTEGRATION METHOD

In the integration method, the failure probability is computed by integrating the marginal distribution functions of each FAD parameters over the failure state region. The failure probability of a pressure vessel containing cracks can be expressed as follows (Francisco et al., 2014):

$$P_f = \int_0^T f_A(a).G(a).da, \tag{6}$$

where $f_A(a)$ is the probability density function of the crack depth, and $G(a)$ stands for the failure probability at a given crack depth a. $G(a)$ is expressed as follows:

$$G(a) = \int_0^\infty f_\sigma(\sigma).H(a, \sigma).d\sigma, \tag{7}$$

where $f_\sigma(\sigma)$ is the probability density function of the applied stress, and $H(a, \sigma)$ stands for the failure probability at a given crack depth a and applied stress σ. $H(a, \sigma)$ is expressed as follows:

$$H(a, \sigma) = 1 - \int_0^{2,67} f_{Lr}(Lr) \int_0^{(1-0.14Lr^2)[0.3+0.7exp(-0.65Lr^6)]} f_{Kr}(Kr).dKr.dLr, \tag{8}$$

considering that the Lr and Kr are random variables statistically independents and the following probabilistic relations are hold: $Kr = Kr(K_{IC})$ and $Lr = Lr(\sigma_y)$, for a given value of a and σ.

The calculus of integration involved in all the above equations can be adequately approximated by using the Simpson method (Burden et al., 2004).

4 THE MONTE CARLO SIMULATION METHOD

Monte Carlo simulation method involves sampling the four quantities at random to simulate artificially a large number of experiments and to check if the $FLSF$ is violated, that means, the pressure vessel has failed (Melchers, 1987). If N trials are conducted, the failure probability of the pressure vessel contained cracks is given approximately by

$$P_f = \frac{n_g}{N}, \tag{9}$$

where n_g is the number of trials for which $g(Lr, Kr) \leq 0$. The number of trials N required for the Monte Carlo simulation is according to a reasonable estimate of P_f.

5 APPLICATION IN AN EXAMPLE

The example consists of the case of a pressure vessel containing semi-round inner surface cracks (see Figure 1). For the pressure vessel, its wall thickness T is $0.108m$. The applied stress σ is normally distributed, its mean is $308MPa$, and standard deviation is $19.17MPa$. Material fracture toughness Kr is fitted for log-normal distribution. Its mean is $49MNm^{-3/2}$, and standard deviation is $3.43MNm^{-3/2}$. Material yield stress σ_y is fitted for normal distribution with mean of $430MPa$ and standard deviation of $21.5MPa$. After performing non-destructive inspection, crack depth a is defined as normal distribution with a mean of $0.02m$ and a standard deviation of $0.005m$.

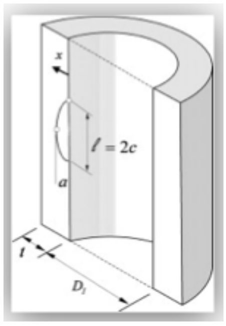

Figure 1: Illustration of the semi-round inner surface crack

Table 1: Computing results of the numerical integration

$a_i(\times 10^{-3}m)$	$f_A(a_i)$	$G(a_i)$	P_f
01.08	0.062042	0.0	0.0
02.16	0.137252	0.0	0.0
12.96	29.61100	0.000742	0.000019
14.04	39.21060	0.001649	0.000062
27.00	29.94650	0.279205	0.006596
28.08	21.62070	0.335043	0.065058
39.96	0.027636	0.855286	0.088734
41.04	0.011399	0.879915	0.088751
45.36	0.000207	0.946214	0.088762
47.52	0.000022	0.965028	0.088762

Failure probability computed by the numerical integration is shown in the Table 1. As larger crack depths are considered in the integrations, the value of $f_A(a_i)$ decreases closely to zero and the result for $G(a_i)$ tends to unity. Then, the value of 0.088762 is the asymptotic result obtained by this method. In the Monte Carlo simulation, simulations are performed with several number of trials in order to obtain a resonable estimate of the failure probability. The result for $N = 30000$ is the value of 0.0889, as shown in the Table 2. Both of the results are very close.

Table 2: Computing results of the Monte Carlo simulation

$Number\ of\ trials$	P_f
10000	0.0873
15000	0.0899
20000	0.0879
25000	0.0887
30000	0.0889

The value of failure probability of 8.9×10^{-2} for the pressure vessel containing cracks is considered greater than an acceptable one. This result is due to the high level of applied stress or to the small wall thickness considered at the pressure vessel.

Now an analysis is performed considering several values of wall thickness for the pressure vessel. Once the Monte Carlo simulation is validated, and being the simplest method, the failure probability is computed by using only this simulation. The results are shown in the Table 3. In these results, it can be noted that the failure probability values are in reasonable agreement with the range $10^{-4} - 10^{-6}$ as reported based on real-world statistics (Chuen-Horng et al., 1994).

In DNV (1992) are presented values of acceptable annual failure probabilities according to the class of failure and consequence of failure. In fact, the random variables involved in the problem are stochastic ones. However, in this paper is assumed that the failure probability is constant for one year in order to give the possibility of establishing a target reliability. Considering that there is a significant warning before the occurence of failure in the pressure vessel, if the consequence of failure is less serious we could have the wall thickness of $0.135m$; but if this consequence is serious the wall thickness should be $0.140m$.

Table 3: Computing results for several wall thickness

$Wall\ thickness\ (m)$	$Failure\ probability$
0.108	8.89×10^{-2}
0.120	7.06×10^{-3}
0.130	5.66×10^{-4}
0.135	2.33×10^{-4}
0.140	6.66×10^{-5}

6 CONCLUSIONS

At a practical level of applied stress, the result of failure probability is considered reasonable since it is in agreement with the values reported based on real-world statistics. The results of the numerical integration and Monte Carlo simulation are very close. That means that the number of trials in the Monte Carlo simulation is a reasonable for obtaining the failure probability.

Both of the methods in the present work is a promising procedure for assessing the structural reliability of pressure vessels containing cracks. It remains to be assessed the reliability using a first-order second-moment method.

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