UNCERTAINTY ANALYSIS IN A ROTOR SYSTEM BY USING THE FUZZY LOGIC APPROACH

Wagner José G da Silva Pinto

wagnerjgsp@aero.ufu.br

Arinan Dourado Guerra Silva

arinandourado@doutorado.ufu.br

Aldemir Ap Cavalini Jr

aacjunior@mecanica.ufu.br

Valder Steffen Jr

vsteffen@mecanica.ufu.br

LMEst - Laboratory of Mechanics and Structures, Federal University of Uberlândia, School of Mechanical Engineering

Av. João Naves de Ávila, 2121, Uberlândia, MG, 38408-196, Brazil.

Abstract. *This paper is dedicated to the uncertainty analysis of the dynamic response of a simple supported rotor system composed by a horizontal flexible shaft, one rigid disc, and one additional ball bearing. A mathematical model of the rotor is derived from the Lagrange's equation by using the Rayleigh-Ritz method. Some simplifications are adopted in order to obtain the responses of the rotor system considered. The inherent uncertainties of the bearing are modeled through the fuzzy logic technique. Triangular fuzzy numbers are used to represent the uncertain stiffness of the rotor's bearing. A heuristic optimization algorithm is used in the α-level optimization to map the fuzzy inputs. A defuzzification process is carried out to estimate the stiffness of the bearing regarding the given uncertain interval. The analysis is confined to the frequency domain so that the envelopes of the rotor unbalance responses are evaluated.*

Keywords: *rotor dynamics, simplified rotor model, fuzzy logic, defuzzification, optimization.*

1 INTRODUCTION

Nowadays, the industrial applications require mechanical systems working with optimal performance subject to specific operational conditions, which demands: high reliability, robustness against environmental conditions and low operating requirements. Consequently, it is necessary to develop reliable numerical models that take into account uncertain parameters, and allow the prediction of the dynamic behavior of the system under realistic conditions.

The components of flexible rotors are subjected to uncertainties. The main sources of uncertainties include the variation of mechanical properties, such as damping, stiffness, and geometrical imperfections (Lara-Molina et al., 2014). The dynamic behavior of rotors is consequently affected by the uncertain parameters of the system. Consequently, the quantification of parametric uncertainties is necessary to predict how the dynamic response varies by means of numerical models of industrial applications and research testbeds (Chowdhury and Adhikari, 2012).

Uncertainty analysis of flexible rotors has been studied by applying stochastic approaches based on the stochastic finite element method (Ghanem and Spanos, 1991). Didier et al. (2011) quantified the uncertainties effects in the response of flexible rotors based on the Chaos Polynomial theory. Koroishi et al. (2012) represented the uncertainties in the rotor parameters by using Gaussian homogeneous stochastic fields discretized by Karhunen-Loève expansion. In this work, the dynamic response of the system with uncertain parameters was characterized through Hypercube Latin sampling and Monte Carlo simulation. Lara-Molina et al. (2014) used a fuzzy stochastic finite element method to quantify the effects of high order uncertain parameters on the response of a rotating machine.

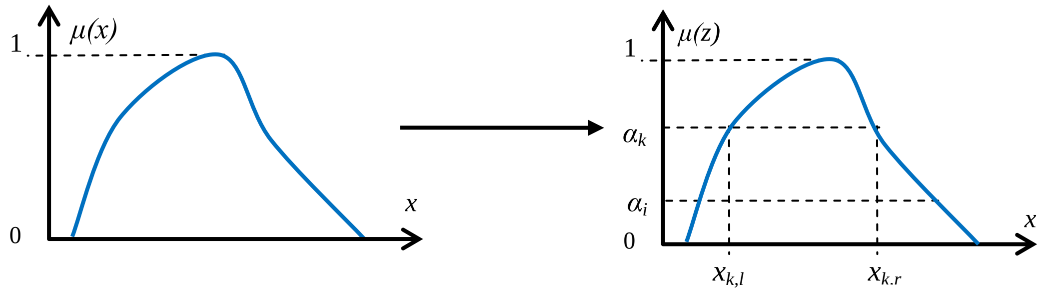
In agreement with the fuzzy approach, the present work proposes the application of a straightforward approach to simulate the dynamic response of a flexible rotor with uncertain parameters by performing a fuzzy dynamic analysis. For this purpose, the fuzzy uncertain parameters are mapped onto the model with the aid of the so-called α-level optimization (Moller and Beer, 2004). Additionally, the Differential Evolution algorithm is used to solve the optimization problem in the fuzzy analysis (Price et al., 2005). The uncertainties were introduced on the stiffness coefficient acting along the vertical direction of a considered bearing.

2 FUZZY ANALYSIS

According to Lara-Molina et al. (2014), the uncertain parameters of rotating machines can be modeled by using fuzzy theory as an alternative approach to the stochastic methods. By using the fuzzy theory, it is possible to describe incomplete and inaccurate information. The theory of fuzzy sets was initially formulated by Zadeh (1965) to characterize vague aspects of information. Thereafter, it was developed a different approach for fuzzy sets that can be compared to the theory of possibilities to deal with the uncertainty of information (Zadeh, 1978). Both theories are connected, so that the uncertainties are modeled by means of the theory of fuzzy sets for the cases in which the stochastic process that describes the random variables is unknown (Moens and Hanss, 2011; Waltz and Hanss, 2013). The basic concepts of the fuzzy variables are revisited next.

2.1 Fuzzy variables

Let X be an universal classical set of objects whose generic elements are denoted by x. The subset A ($A \in X$) is defined by the classical membership function μ_A: $X \to \{0,1\}$, shown in Fig. 1. Furthermore, a fuzzy set $\tilde{A}$ is defined by means of the membership function μ_A: $X \to [0,1]$, being [0,1] a continuous interval. The membership function indicates the degree of compatibility between the element x and the fuzzy set $\tilde{A}$. The closer is the value of $\mu_A(x)$ to 1, more x belongs to $\tilde{A}$.



a) Fuzzy set b) α-levels

Figure 1. Fuzzy set and α-level representation

Thus, the fuzzy set is completely defined by (where $0 \leq \mu_A \leq 1$):

$$\tilde{A} = \left\{ \left(x, \mu_A(x) \right) \middle| x \in X \right\} \tag{1}$$

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets that are denominated α-levels. These subsets, which correspond to real and continuous intervals, are defined by $A_{\alpha k}$ (Fig. 1), thus:

$$A_{\alpha k} = \left\{ x \in X, \mu_A(\mathrm{x}) \geq \alpha_k \right\} \tag{2}$$

The α-level subsets of $\tilde{A}$ have the property:

$$\underline{A}_{\alpha k} \subseteq \underline{A}_{\alpha i} \forall \alpha_i, \alpha_k \in (0,1] \tag{3}$$

with $\alpha_i \leq \alpha_k$. If the fuzzy set is convex for the unidimensional case, each α-level subset $A_{\alpha k}$ corresponds to the interval [$x_{\alpha kl}$, $x_{\alpha kr}$], shown in Eq. (4).

$$\begin{aligned} x_{\alpha kl} &= \min \left[x \in X \middle| \mu_A(x) \geq \alpha_k \right] \\ x_{\alpha kr} &= \max \left[x \in X \middle| \mu_A(x) \geq \alpha_k \right] \end{aligned} \tag{4}$$

2.2 Dynamic models with fuzzy parameters

In this work, the dynamic model describes the behavior of the rotor by means of a set of differential equations. The relationship between the inputs $\mathbf{x}$ and outputs $\mathbf{z}$ of an specific dynamic model M_f is characterized by f, which represents the set of differential equations of the model in Eq. (5).

$$M_f : \mathbf{z}(\tau) = f(\mathbf{x}) \tag{5}$$

Therefore, the function f maps the inputs $\mathbf{x}$ onto the outputs $\mathbf{z}(\tau)$. Thus, $\mathbf{x} \rightarrow \mathbf{z}(\tau)$, where τ is the independent variable of the dynamic response that may represent time, frequency or spacial coordinates. Considering the inputs of the model as fuzzy variables $\tilde{\mathbf{x}}$ or fuzzy functions $\tilde{\mathbf{x}}(\tau)$, the dynamic response of the system corresponds to the resulting fuzzy functions $\mathbf{z}(\tau)$. These fuzzy functions result from the mapping, thus $\mathbf{x} \rightarrow \tilde{\mathbf{z}}(\tau)$.

2.3 Fuzzy dynamic analysis

The fuzzy dynamic analysis is an appropriate method to map a fuzzy input vector $\tilde{\mathbf{x}}$ onto the output $\tilde{\mathbf{z}}(\tau)$ of a numerical model by using the deterministic model given by Eq. (5). In structural analysis, the combination of uncertainties modeled as fuzzy variables with the deterministic model based on the finite element method is denominated fuzzy finite element method. The fuzzy dynamic analysis includes two stages, as shown in Fig. 2.

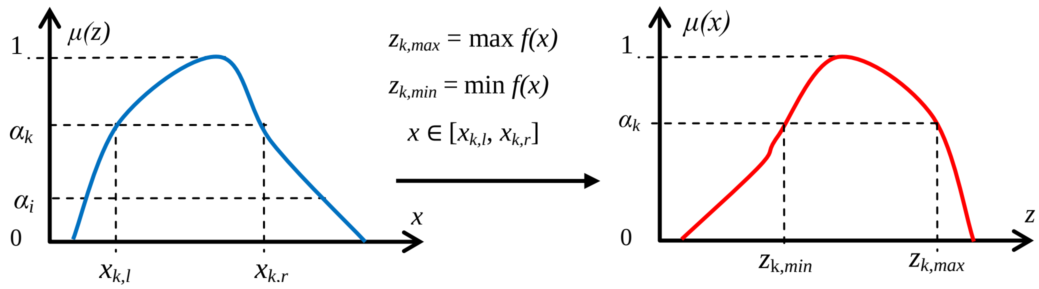


Figure 2. The α-Level optimization

In the first stage, for computational purposes, the input vector that corresponds to the fuzzy parameter is discretized by means of the α-level representation, presented in the Eq. (2) and Fig. 2. Thus, each element of the fuzzy parameters vector $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n)$ is considered as an interval $X_{i\alpha k} = [x_{i\alpha kl}\ , x_{i\alpha kr}]$, where $\alpha_k \in (0,1]$. Consequently, the sub-space $\mathbf{X}_{\alpha k}$ is defined so that $\mathbf{X}_{\alpha k} = (\mathbf{X}_{1\alpha k}\ , \ldots, \mathbf{X}_{n\alpha k})$, where $\mathbf{X}_{\alpha k} \in \mathrm{R}^n$.

The second stage is related to solving an optimization problem. This optimization problem consists in finding the maximum or minimum value of the output, at each evaluated τ, for the mapping model $M_f : \mathbf{z}(\tau) = f(\mathbf{x})$, thus:

$$z_{\alpha kl} = \min_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x})$$

$$z_{\alpha kr} = \max_{\mathbf{x} \in \mathbf{X}_{\alpha k}} f(\mathbf{x}) \tag{6}$$

where $z_{\alpha kl}$ and $z_{\alpha kr}$ correspond to the upper and lower bounds of the interval $z_{\alpha k} = [z_{\alpha kl}\ , z_{\alpha kr}]$ in the α-level α_k. The set of discretized intervals $[z_{\alpha kl}\ , z_{\alpha kr}]$ for $\alpha_k \in (0,1]$ composes the whole fuzzy resulting variable $\tilde{z}$.

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems regarding all α-level of interest for each considered time step. Each upper and lower bounds of the system analysis at a given time instant is obtained from the Differential Evolution optimization algorithm (Price et al., 2005). The output value of the transient analysis at the evaluated time-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy, or fuzzy random variables.

2.4 Defuzzification

The defuzzification techniques are required to obtain an optimum value that represents the best value of the uncertain parameter (i.e., the robust parameter). Two methods are proposed in this work:

Centroid method. Geometrical center of the fuzzy number.

$$z_{j0} = \frac{1}{r}\sum_{j=1}\left[z_j \mu\left(z_j\right)\right] \Big/ \sum_{j=1}\left[\mu\left(z_j\right)\right]. \tag{7}$$

Level Rank Method. The mean value for the mean at each α-level.

$$z_{j0} = \frac{1}{r}\sum_{k=1}^{r}\frac{z_{j,\alpha_k l} + z_{j,\alpha_k r}}{2}. \tag{8}$$

In this way, the robust parameter is obtained by using two different approaches, as follows:

Methodology 1 (M1). The robust parameter (i.e., defuzzified parameter) is obtained from an additional optimization process on the membership function of the responses $\mu(z)$.

Methodology 2 (M2). The robust parameter is obtained directly from the α-level optimization process. The defuzzification process values is applied on the membership function of the parameters associated with $\mu(z)$. No additional operation is needed after the defuzzification.

Both of the methodologies are represented in the flowchart presented at Figure 3.

3 ROTOR MODELING

The equations of motion are based on the developments described by Lalanne and Ferraris (1998), in which the Rayleigh-Ritz method is used to obtain the model of the rotating machine, as presented by Eq. 9.

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{D} + \Omega\mathbf{G})\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}_{\mathbf{u}}. \tag{9}$$

where **M** is the matrix mass; **D** is the damping matrix; **G** is the gyroscopic matrix; Ω is the rotation speed; **K** is the stiffness matrix, **q** is the displacement vector, and $\mathbf{F_u}$ is the unbalance forces. Is this paper, the damping is modeled by a proportional damping coefficient, $\mathbf{D} = \beta\,\mathbf{K}$.

4 NUMERICAL APPLICATION

A rotor composed by a shaft with 400 mm length and 20 mm diameter (E = 2×10^{11} Pa, ρ = 7800 kg/m^3) containing a rigid disc with 150 mm diameter and 30 mm thickness (ρ = 7800 kg/m^3) is used to perform the numerical simulation. An unbalance mass m_u = 1×10^{-4} kg, placed in the exterior circumference of the disc. The bearing is represented by a single spring, as shown in Fig. 4. The rotor is simply supported at its ends. The damping coefficient is given by β = 1×10^{-5}. The disk is placed at one third of the shaft ($\ell_1 = L/3$) and the bearing is at 2 thirds of the shaft ($\ell_2 = 2\,L/3$).

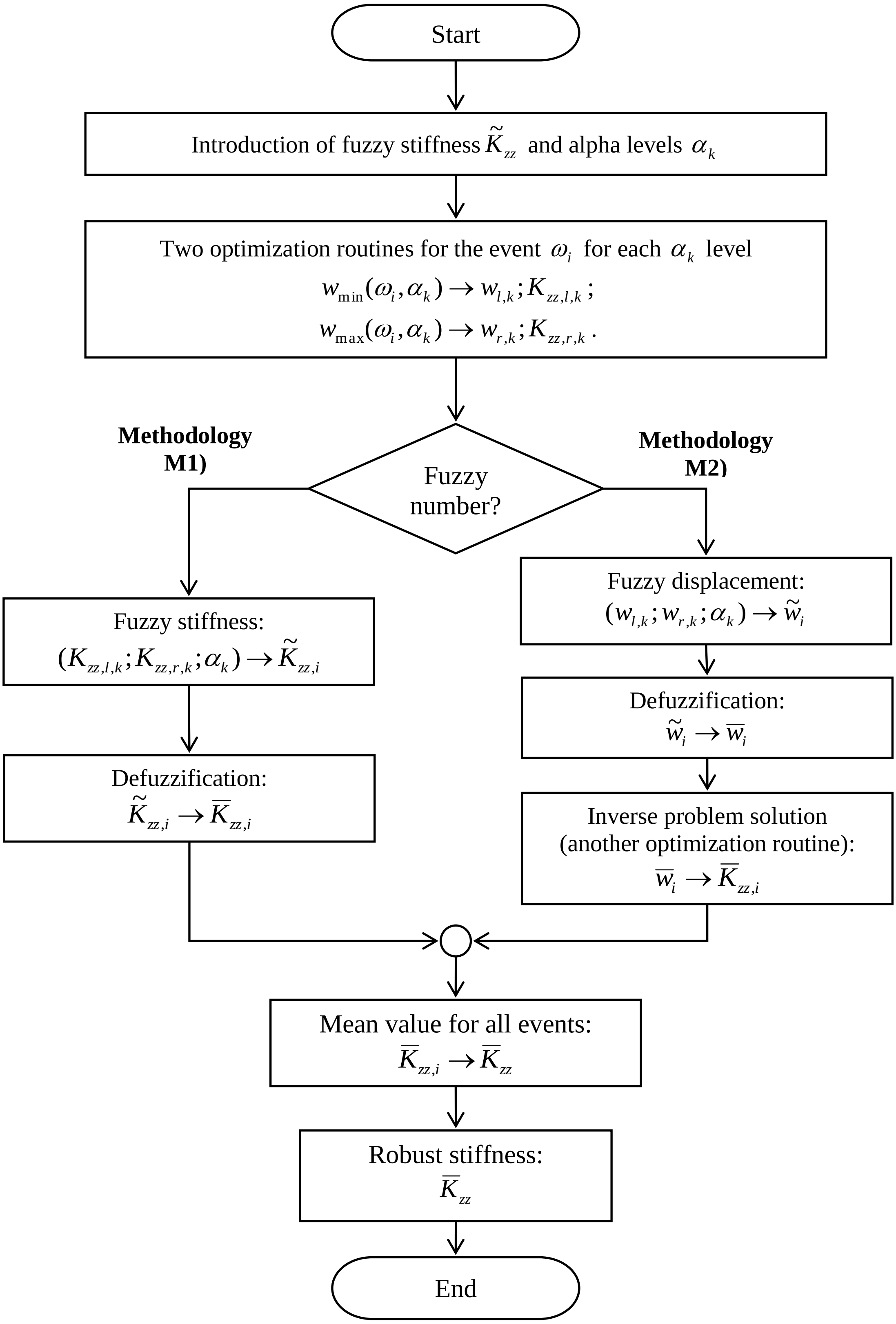


Figure 3. Flowchart of the α-level optimization methodologies.

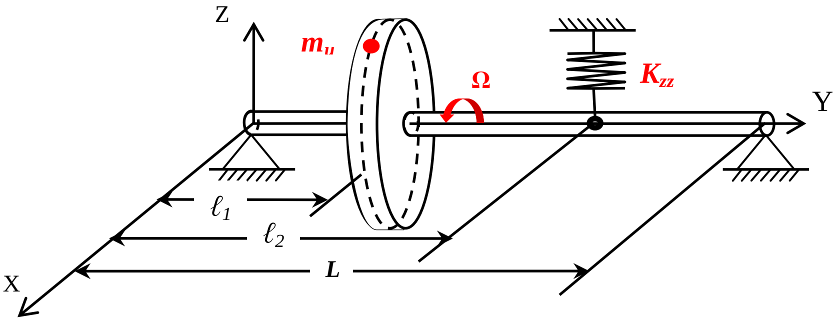


Figure 4. Model of the rotor

The stiffness K_{zz} is adopted as a triangular membership function fuzzy number with 11 α-levels, whose center value is K_{nom} = 5×10^5 N/m. Asymmetry is also added in order to measure the effects of the input on the optimized response, such as shown in Fig. 5. The fitness function of the associated optimization problem is written as the maximum displacement along the Z direction of the disc ($w(\ell_1)$). The upper and lower bounds of the analysis at a given rotation speed are obtained from the Differential Evolution algorithm (Storn and Price, 1995).

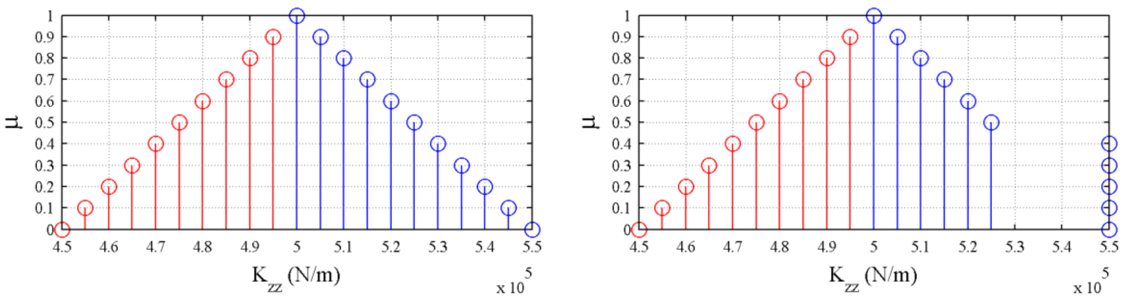


Figure 5. Fuzzy inputs (bearing stiffness)

5 RESULTS

Figure 6 shows the unbalance response of the rotor applying the fuzzy approach to evaluate the uncertainties affecting the stiffness K_{zz} (the limits are shown in Fig. 5). Before the first critical speed (Ω_1 = 2641 rev/min), the largest displacement value is associated with the smallest stiffness (K_{low} = 4.5×10^5 N/m). After the critical speed, the largest stiffness value (K_{high} = 5.5×10^5 N/m) leads to the largest disc displacement. The transition occurs at the critical speed itself and this phenomenon is replicated at the second critical speed (Ω_2 = 3377 rev/min). Another switch occurs between Ω_1 and Ω_2, around Ω=2760 rev/min.

The optimization routine creates a curve where the maximum and the minimum values are obtained for the modeled fuzzy stiffness, as showed in Fig. 7 and Fig. 8. The behavior observed in the response regarding the stiffness limits results in a kind of flat displacement around both critical speeds (as seen in Fig. 7) : the increase of the stiffness causes an increase in both critical speeds and a small associated reduction of the disc displacement. The bandwidth varies with the uncertainty applied to the stiffness (10% variation in this case). The transition associated with the critical speeds is also observed in the fuzzy representation.

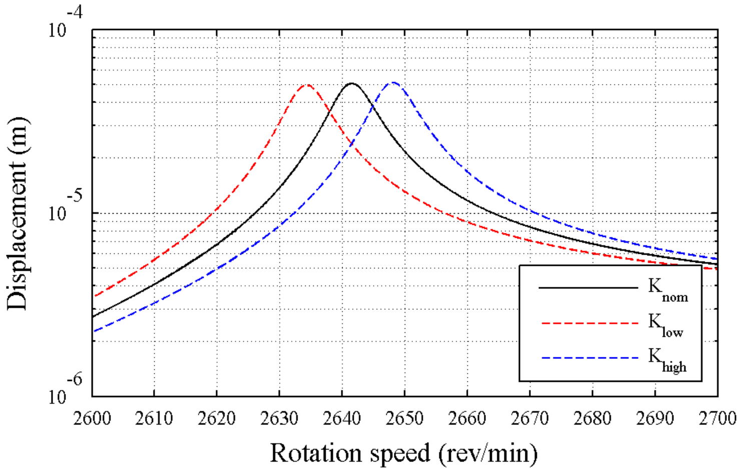


Figure 6. Response to unbalancing for the stiffness limits

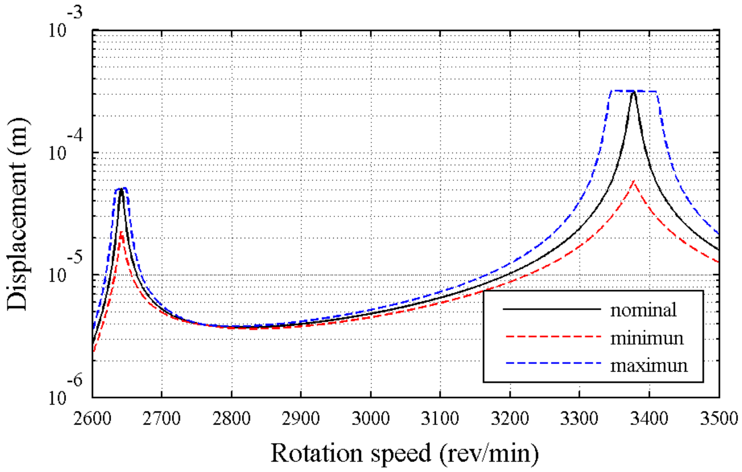


Figure 7. Optimized stiffness values

For both defuzzification methodologies (***M1*** and ***M2***), there is an anomaly associated with the critical speeds as shown in Fig.10.

For the first methodology ***M1***, there is a clear transition between the maximum and minimum stiffness for the optimized displacement, see Fig. 8. When the same approach is applied for the displacement, methodology ***M2***, there is no clear transition in the fuzzy displacement. However, when the second optimization routine is applied, a similar behavior is reported (Fig. 10).

The defuzzificated stiffness is obtained for each rotation speed, methodology, and defuzzification method as follows. All the results are depicted in Fig. 10.

For ***M1***, the most representative stiffness value is the nominal one, except near the critical speeds (see Fig.10). Considering the methodology ***M2***, it can be seen a pronounced stiffness variation around the nominal value. It is worth mentioning that the both tested defuzzification methods result in the same behavior, leading to slightly different robust values. The global mean value of the defuzzificated stiffness obtained for all the 8 configurations are shown in Table 1.

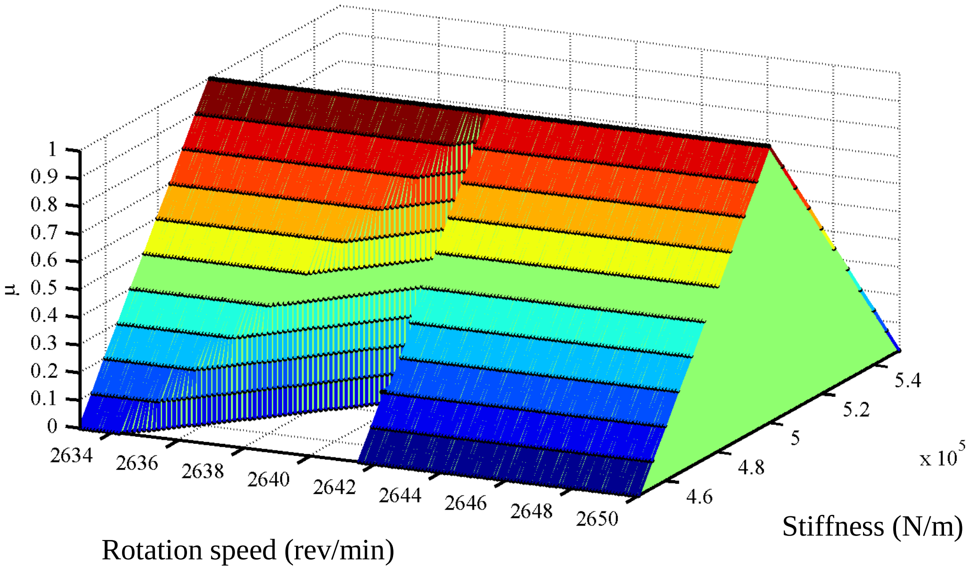


Figure 8.Optimized fuzzy stiffness (methodology M1)

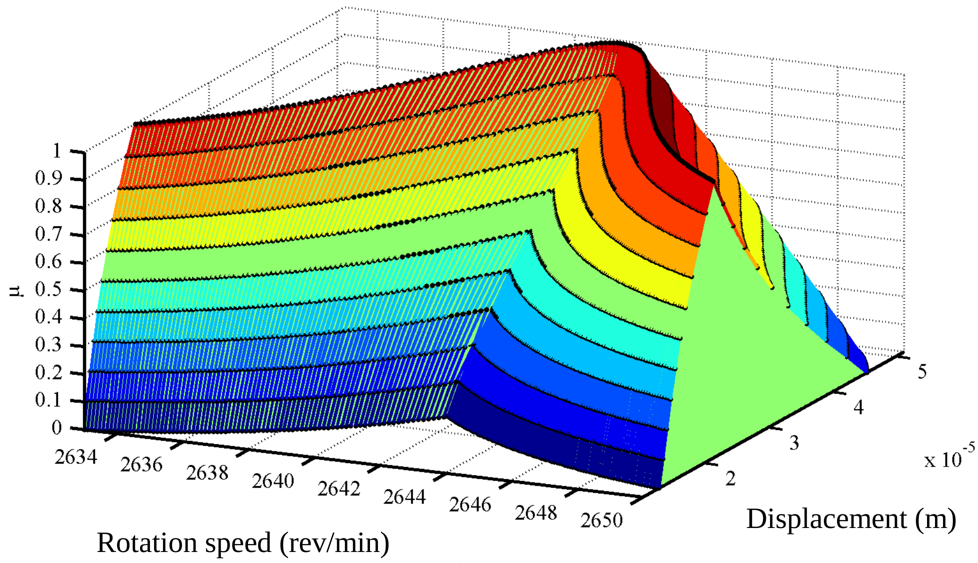


Figure 9. Optimized fuzzy displacement (methodology M2)

Table 1. Defuzzificated stiffness for the 2 defuzzification methods and 2 methodologies

	Centroid		Level rank	
Fuzzy input	***M1***	***M2***	***M1***	***M2***
Triangular	5.0000*	4.9984*	5.0000*	4.9964*
Asymmetric	5.0140*	5.0089*	5.0233*	5.0149*

* $\overline{K}_{zz} \times 10^{-5}$ (N/m) – nominal stiffness.

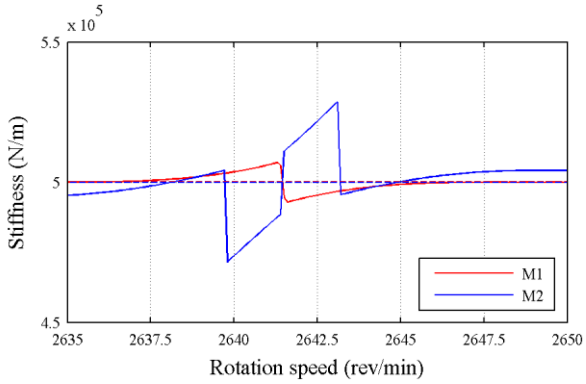


Figure 10. Defuzzificated stiffness for the centroid method, symmetrical input (left) and centroid method, asymmetrical input (right)

Note that the obtained robust parameter corresponds to the nominal value for both the defuzzification methodologies. However, the increase of the asymmetry (see Fig. 5) makes the robust result different from the nominal one.

6 CONCLUSIONS

The influence of the uncertainty parameter on a rotor system was analyzed by the fuzzy logic approach. The robust values were the same for the two defuzzification methods performed. The shape of the input fuzzy parameter shows to be relevant in this type of analysis.

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REFERENCES

Cavalini Jr, A.A., Lara-Molina, F.A., Sales, T. P., Koroishi, E.H., & Steffen Jr, V., 2014. Uncertainty Analysis of a Flexible Rotor Supported by Fluid Film Bearings. *VIII Congresso Nacional de Engenharia Mecânica (CONEM).*

Chowdhury, R. and Adhikari, S., 2012,." Fuzzy parametric uncertainty analysis of linear dynamical systems: A surrogate modeling approach". *Mechanical Systems and Signal Processing*, 32:5–17.

Didier, J., Faverjon, B. and Sinou, J.J., 2011, "Analysing the Dynamic Response of a Rotor System Under Uncertain Parameters Polynomial Chaos Expansion", *Journal of Vibration and Control*, Vol. 18, pp. 712-732.

Ghanem, R.G. and Spanos, P.D., 1991, *Stochastic Finite Elements – A Spectral Approach*, Spring Verlag.

Koroishi, E.H., Cavalini Jr, A.A., de Lima, A.M.G., and Steffen Jr, V., 2012, "Stochastics Modeling of Flexible Rotors*",* *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 34, pp. 597-603.

Lallane, M. and Ferraris, G., 1998, *Rotordynamics Prediction in Engineering*, John Wiley and Sons, Second Edition.

Lara-Molina, F.A., Koroishi, E.H. and Steffen Jr, V., 2014, Fuzzy Stochastic Dynamic Analysis of Flexible Rotors, *Uncertainties 2014*, Rouen, France.

Moens, D. and Hanss, M., 2011, Non-probabilistic Finite Element Analysis for Parametric Uncertainty Treatment in Applied Mechanics: Recent Advances, *Finite Elements in Analysis and Design*, Vol. 47.

Möller, B. and Beer, M., 2004, *Fuzzy Randomness, Uncertainty in Civil Engineering and Computational Mechanics*, Springer-Verlag.

Price, K.V., Storn, R.M. and Lampinen, J.A., 2005, *Differential Evolution a Practical Approach to Global Optimization*, Springer-Verlag.

Waltz, N.P. and Hanss, M., 2013, Fuzzy Arithmetical Analysis of Multibody Systems with Uncertainties, *The archive of mechanical engineering*, Vol. 1, pp. 109-125.

Zadeh, L., 1965, Fuzzy Sets, *Information and Control*, Vol. 8, pp. 338-353.

Zadeh, L., 1978, Fuzzy Sets as Basis for a Theory of Possibility, *Fuzzy Sets and Systems*, Vol. 1, pp. 3-28.