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ROBUSTNESS ANALYSIS OF A DYNAMIC VIBRATION ABSORBER APPLIED TO A HELICOPTER FOR THE MITIGATION OF INSTABILITIES

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Abstract. *This paper presents a robustness analysis of the use of dynamic vibration absorbers (DVA) for the mitigation of instabilities due to the ground resonance phenomenon in helicopters. The DVA, located in the fuselage, alters the dynamic behavior of the helicopter and, consequently, its stability characteristics when it is on the ground. It has been proven by the authors in earlier works that the DVA helps to reduce the instabilities due to ground resonance. The purpose of this work is to use the lifting procedure in the uncertain linear time period (LTP) model of the system (helicopter+DVA) and transform it into an augmented linear time invariant (LTI) in order to allow the application of* μ*-analysis tools. The lifted model calculated by the* μ*-analysis is compared with the Floquet Theory in a parametric configuration for a range of rotor angular rate. The helicopter model has four blades and the parametric uncertainties taken into account are the dynamic characteristics (stiffness and damping) of one blade.*

Keywords: *Ground Resonance, Floquet's Theory, Vibration Absorbers, Robustness Analysis.*

1 INTRODUCTION

When helicopters with hinged and hingless blades are on the ground they are susceptible to the ground resonance phenomenon, in which the poorly damped cyclic lag modes couple with the fuselage modes, leading the aircraft to oscillations that may cause severe damage in the structure (Coleman and Feingold, 1957; Wang and Chopra, 1992; Gandhi, 1997). One of the solutions to mitigate those oscillations was proposed by (Sanches *et al*, 2014), in which a spring-mass vibration system located in the fuselage of a helicopter act as dynamic vibration absorber DVA.

Nonetheless, dynamic systems are susceptible to failure and aging of its components that may appear randomly, compromising its nominal operation. In the case of the DVA, it may change its natural frequency, which can change the system (helicopter+DVA) dynamics in a way that it may be catastrophic. Therefore, it is important to perform a robustness analysis to assess how the systems handles those effects. The purpose of this paper is to perform the robustness analysis using μ-analysis and compare it to Floquet's theory (Sanches *et al*, 2012).

2 MECHANICAL MODEL

In order to perform the robustness analysis, the helicopter studied in this paper is the model presented in (Sanches *et al*, 2014). The fuselage is modeled as a rigid body, and contains one translational movement along its longitudinal direction. The fuselage center of mass (point *O*) and the origin of the inertial frame $x(t)$ are coincident when the helicopter is at its equilibrium position.

One rigid rotor hub and an assembly of $N_b = 4$ blades comprise the rotor head system. The blades have mass m_b and moment of inertia I_{zb} around z-axis located at its center of mass. For each *k*th blade, there is an in-plane lead-lag motion defined by $\varphi_k(t)$ and an azimuth angle defined as $\varsigma_k = 2\pi(k-1)/N_b$, with respect to the x-axis. The rotor has a revolving speed of Ω. The rotational reference frame (*x*, *y*, *z*) is parallel to the inertial frame and their origins are coincident.

Fuselage and rotor head are joined by a rigid shaft. Since the helicopter is on the ground, the aerodynamic forces are not taken into account. The DVA has mass m_a, stiffness k_a, and frequency $\omega_a = \sqrt{k_a/m_a}$. It has a translational movement $x_a(t)$ only along the longitudinal direction with respect to the fuselage. Figure 1 shows the model scheme.

2.1 Equations of motion

In order to find the equations of motion for the model, the Lagrange's equations are applied using the expressions of the kinetic and potential energies and the virtual work of the external forces and moments applied to the dynamic system. The expressions for the kinetic and potential energies for the helicopter with hinged blades are given in (Sanches *et al*, 2012), however the additional effects of the DVA must be accounted and are shown in (Sanches *et al*, 2014). By using that, it can be found that the equation of motion in the matrix form is given as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{G}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}_{\mathbf{ext}} \tag{1}$$

where, $\mathbf{u}(t)=\{x(t) \quad x_a(t) \quad \phi_1(t) \quad \phi_2(t) \quad \phi_3(t) \quad \phi_4(t)\}^T$.

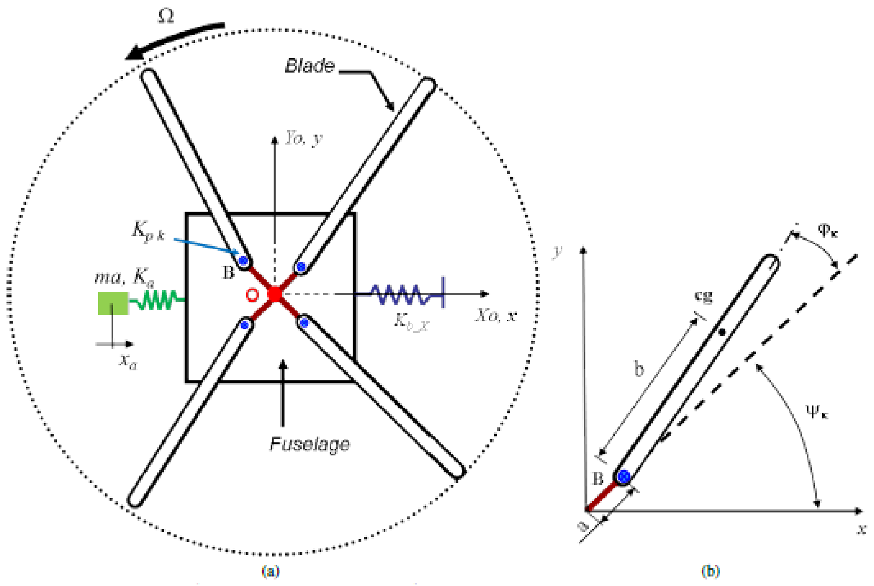

Figure 1. Scheme of the mechanical system: a) general view and b) side view

In Eq.(1), $\mathbf{F_{ext}}$ is a zero column vector since all blades are assumed to have same inertial and geometrical properties. Also, $\mathbf{M}$, $\mathbf{G}$, and $\mathbf{K}$ are time-dependent and represent the system mass, damping, and stiffness matrix, respectively, which are shown in (Sanches *et al*, 2014). All the data for the helicopter as well as the DVA was determined in preceding works done by the authors and are also shown in (Sanches *et al*, 2014).

3 STABILITY ROBUSTNESS ANALYSIS

3.1 General background

The system described in Section 2 is a continuous linear time periodic (LTP) dynamic system. In order to apply the μ-analysis methods, one need first to cast the LTP system into the form of a Linear Fractional Transformation (LFT). The lifting procedure is applied to provide a LFT that leads to uncertainty structures with highly-repeated parameters.

To reduce the size of the uncertainty block, a first-order hold discretization method with 30 elements is used. This discretization method is best suited for this analysis, as seen in (Sanches *et al*, 2012). In order to perform the parametric robustness analysis, one must consider the following LTP system $\delta(\Delta)$:

$$
\begin{array}{l}
r(t) \rightarrow \boxed{\begin{array}{l} \dot{w}(t) = \mathbf{A}(t)w(t) + \mathbf{B}(t)r(t) \\ y(t) = \mathbf{C}(t)w(t) + \mathbf{D}(t)r(t) \end{array}} \rightarrow y(t) \\
r(t) \leftarrow \boxed{r(t) = \Delta y(t)} \leftarrow y(t)
\end{array}
\tag{2}
$$

where $v(t) \in \mathbb{R}^n$ represents the state variables, and $r(t)$ and $y(t)$ are inputs and outputs for the LFT.

Also, the matrices $\mathbf{A}(t)$, $\mathbf{B}(t)$, $\mathbf{C}(t)$, and $\mathbf{D}(t)$ are *T*-periodic and continuous. The uncertainties are represented in the system by the Δ diagonal matrix which comprises bounded real unknown parameters:

$$\Delta = \text{diag}\left[\delta_1, \delta_2, \ldots, \delta_k\right] \tag{3}$$

When $\Delta = 0$ the system is considered nominally stable. The primary purpose of the parametric robustness analysis is to find the smallest parameters of uncertainties which destabilize the closed-loop system $\delta(\Delta)$. The Floquet's theory can be applied to perform this analysis, however it is a very CPU time-consuming, which is why the μ-analysis, a much faster technique, is used on the LFT model.

The lifting procedure integrates over one period *T* a *T*-periodic (LTI piecewise) system equivalent to the input-output transfer described in Eq.(2). In order to do that, firstly the LTP system is approximated by a *T*-periodic switching LTI piecewise system. After that, a discretization method is applied to obtain a periodic-time system. The third step consists to integrate the discrete system, which gives a discrete-time-invariant LFT system. Lastly, this discrete LFT system is converted to a continuous-time LFT system by using the TUSTIN transformation in order to apply μ-analysis tools. The complete procedure is shown in (Sanches *et al*, 2012).

3.2 Ground Resonance Parametric Analysis

As discussed before, the robustness analysis of helicopters under structured uncertainties is important in order to predict the smallest allowable perturbation that The uncertainties introduced in the dynamic system in Eq.(1) are related to blade hinge stiffness, and are presented as a function of in-plane lead-lag resonance frequency squared. For this paper, only uncertainties in the 4th blade are accounted for the robustness analysis. By assuming $\bar{\omega}_{b4}$ as the nominal frequency, the modified frequency ω_{b4} is given as:

$$\omega_{b4}^2 = \left(1 + \delta_4\right)\bar{\omega}_{b4}^2 \tag{4}$$

In Fig.2, the rotor stability is verified according to angular rate Ω and uncertainty δ_4 on the 4th blade hinge stiffness (as mentioned before), computed with Floquet's Theory (blue "Δ" beginning point and green "O" ending point) and μ-analysis (red "x"). The robustness analysis is performed in the range of $3\text{Hz} \le \Omega \le 6\text{Hz}$. It is important to note that the μ-analysis is only performed in the range in which the system is nominally stable, hence in Fig.2 there are no uncertainties found between $4.42\text{Hz} \le \Omega \le 5.20\text{Hz}$ and the Floquet´s theory also does not predict uncertainties in this range. As seen in (Sanches *et al*, 2014), this is the unstable region with the DVA attached to the fuselage.

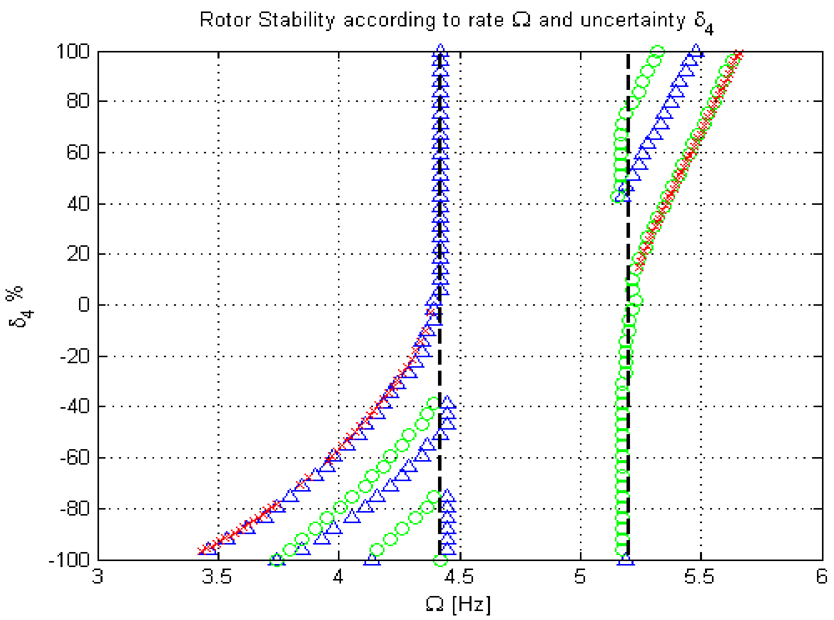

Figure 2. Rotor stability according to rate Ω and uncertainty δ_4

4 CONCLUSIONS

It can be seen from Fig. 2 that the μ-analysis has given the same results as the Floquet's theory for the robustness of the system. This means that it is a powerful tool and it can be used to perform robustness analysis for different parametric configurations, for example taking into account more than one blade at a time.

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