FE PREDICTION OF BEARING CAPACITY OF SHALLOW FOUNDATION ON ROCK MASS BY USING AN ELASTO-PLASTIC HOEK-BROWN CONSTITUTIVE MODEL

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Abstract. *This paper presents a numerical simulation of bearing capacity problem of shallow foundations on rock mass. Using the finite element method (FEM), the study analyzes the problem considering plane strain and axisymmetric conditions. The rock mass is modeled as an associative elastic-plastic Hoek-Brown material. The ultimate bearing capacity numerically obtained is compared to solutions obtained by using limit equilibrium and limit analysis. Good agreement is observed between the limit equilibrium and numerical results for purely cohesive material. Results obtained by a parametric study, varying the Geological Strength Index (GSI) highlight the importance of accounting for the impact of the geological condition of the rock mass on the bearing capacity.*

Keywords: FEM, Bearing Capacity, Shallow Foundation, Hoek-Brown, Rock Mass

1 INTRODUCTION

This paper presents the finite element method (FEM) in a non-linear elasto-plastic analysis of shallow foundations on rock mass. In the continuous mechanical context, analyses of mechanical equilibrium's problems on rock masses have been limited to hard rock or non-fractured rock mass. The number of geotechnical works carried out on fractured rock masses is increasing. Because of this, a need has arisen to use a constitutive model that takes into account the geological condition of a rock mass. Such a model should be able to provide more realistic results when using a displacement formulation of FEM based on a continuous mechanical context.

To represent the rock mass stress-strain-strength relationship, this study adopts a non-linear, elastic perfectly plastic model based on Hoek-Brown's failure criterion (Hoek et al., 2002). The Hoek-Brown's failure criterion was initially developed for hard rock (Hoek and Brown, 1980). Beginning early in the 1980's, researchers have published several versions to include the influence of rock masses' geological condition on the failure parameter (Hoek et al., 1992, 1995 and 2002; Hoek and Brown, 1988).

To investigate the influence of a shallow foundation's bearing capacity, the study adopts and uses a general formulation for plane and axisymmetric strain. The study also conducts, using the code ANLOG – *Non Linear Analysis of Geotechnical Problems* (Nogueira, 2010), a parametric study considering the different geological conditions of a rock mass (Simão, 2014, 2015). The results show the importance of taking into account the geological condition of the rock mass on the bearing capacity.

2 MECHANICAL PROBLEM FORMULATION

When adopting an elasto-plastic stress-strain-strength relationship (Teixeira et al., 2012), the displacement-based equilibrium equation of a mechanical problem, in static condition, leads to a non-linear equation system. During a given equilibrium path, the variations in the displacement, strain, and stress fields depend on the stress and strain levels and their history through the equilibrium path.

Thus starting from a given equilibrium configuration *n*, where the stress and strain states are known, a new equilibrium configuration *n+1* can be obtained by considering an incremental-iterative procedure (Crisfield, 1991). In this strategy, the problem solution is obtained by updating the nodal displacement vector ($\hat{\mathbf{U}}$) as below:

$$\hat{\mathbf{U}}_{n+1} = \hat{\mathbf{U}}_n + \Delta\hat{\mathbf{U}}^k \tag{1}$$

where,

$$\Delta\hat{\mathbf{U}}^k = \Delta\hat{\mathbf{U}}_n^0 + \sum_{k=1}^{iter} \delta\Delta\hat{\mathbf{U}}^k \tag{2}$$

is the incremental displacement vector, which is obtained by correcting the predicted incremental displacement ($\Delta\hat{\mathbf{U}}_n^0$) by successive iteration (*iter*) until reaching a new equilibrium configuration *n+1*. The predicted incremental displacement vector ($\Delta\hat{\mathbf{U}}_n^0$) can be obtained as follows:

$$\Delta\hat{\mathbf{U}}_n^0 = \left[\mathbf{K}_{ep}\right]^{-1}\Delta\lambda\mathbf{F}_{ext} \tag{3}$$

while the iterative incremental displacement ($\delta\Delta\hat{\mathbf{U}}$) can be obtained thus:

$$\delta\Delta\hat{\mathbf{U}}^k = \left[\mathbf{K}_{ep}\right]^{-1}\boldsymbol{\psi}^k \tag{4}$$

$\Delta\lambda$ is the increment of load factor, which can be automatically defined starting from the initial trial provided by the user (Nogueira, 1998; Oliveira, 2006; Teixeira, 2011; Simão, 2014). $\mathbf{K}_{ep}$ is the global stiffness matrix that represents the global arrangement of the element stiffness matrix $\mathbf{K}_{ep}^e$ defined by:

$$\mathbf{K}_{ep}^e = \int_{V_e} \mathbf{B}^T\mathbf{D}_{ep}\mathbf{B}dV_e \tag{5}$$

where, $\mathbf{D}_{ep}$ is the elasto-plastic constitutive matrix which depends on the stress-strain-strength relationship. According to the Modified Newton-Raphson iterative scheme the global stiffness matrix is kept constant during the iterative cycles. On each iterative cycle, the unbalanced force vector ($\boldsymbol{\psi}$) is evaluated as follows:

$$\boldsymbol{\psi}^k = \mathbf{F}_{ext}{}^{n+1} - \mathbf{F}_{int}{}^k \tag{6}$$

where $\mathbf{F}_{ext}$ represents the applied external force at the end of a given load step and $\mathbf{F}_{int}$ is the internal force vector that represents the global arrangement of the element internal force vector $\mathbf{F}_{int}^e$ equivalent to the stress state $\boldsymbol{\sigma}$ in a given element that is defined as:

$$\mathbf{F}_{int}^e = \int_{V_e} \mathbf{B}^T\boldsymbol{\sigma}dV_e \tag{7}$$

where $\mathbf{B}$ is the cinematic matrix, which depends on the strain-displacement relationship.

According to the Modified Newton-Raphson iterative scheme, the global stiffness matrix and applied external force are kept constant during the iterative cycles while the internal force vector is evaluated at each iterative cycle according to the stress state reached at this iterative cycle, $\boldsymbol{\sigma}^k$. At the end of each iterative cycle, the solution's convergence is verified by using a criterion that relates the Euclidian norm of the unbalanced force vector with the Euclidian norm of the external force vector.

The Newton-Raphson iterative scheme involves the stress state evaluation at each iterative cycle. Then, in each element, the stress vector $\boldsymbol{\sigma}^k$ is obtained as follows:

$$\boldsymbol{\sigma}^k = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}^k \tag{8}$$

Where

$$\Delta\boldsymbol{\sigma}^k = \mathbf{D}_{ep}\Delta\boldsymbol{\varepsilon}^k \tag{9}$$

$$\Delta\boldsymbol{\varepsilon}^k = -\mathbf{B}\Delta\hat{\mathbf{u}}^k \tag{10}$$

and, $\Delta\hat{\mathbf{u}}^k$ is the incremental displacement vector at the element level and updated at the current iterative cycle. The negative signal indicates positive stress in compression.

By adopting a linear elastic and perfectly plastic constitutive model with associated plasticity, the elasto-plastic constitutive matrix can be written as:

$$\mathbf{D}_{ep} = \mathbf{D}_{e} - \mathbf{D}_{e}{}^{T} \frac{\left(\mathbf{a}^{T}\mathbf{a}\right)}{\left(\mathbf{a}^{T}\mathbf{D}_{e}\mathbf{a}\right)} \mathbf{D}_{e} \tag{11}$$

where, $\mathbf{D}_e$ is the elastic constitutive matrix which depends on Young's modulus (E) and Poisson's coefficient (ν). The vector **a** is the gradient of the yield function (F). As no hardening occurs in a perfectly plastic constitutive model and the yield concept merges with the failure concept, so the yield function for a generalized Hoek-Brown model in terms of stress invariants can be written as (Simão, 2014):

$$F(I_1, I_{2D}, \theta) = \sigma_{ci}\left[\frac{\sqrt{I_{2D}}\,(2\cos\theta)}{\sigma_{ci}}\right]^{1/a} + \sqrt{I_{2D}}\, m_b \left(\cos\theta + \frac{1}{\sqrt{3}}\operatorname{sen}\theta\right) - \frac{m_b I_1}{3} - \sigma_{ci} s = 0 \tag{12}$$

where, I_1 is the first invariant of the stress tensor; I_{2D} is the second invariant of the deviator stress tensor, and θ is the Lode angle that depends on the second and the third (I_{3D}) invariant of deviator stress tensor (Oliveira, 2006). According to Hoek et al (2002) σ_{ci} is the uniaxial compressive strength of intact rock, and m_b, a and s are constants defined as:

$$m_b = m_i e^{(GSI-100)/(28-14D)} \tag{13}$$

$$s = e^{(GSI-100)/(9-3D)} \tag{14}$$

$$a = 0.5 + \left(e^{-GSI/15} - e^{-20/3}\right)/6 \tag{15}$$

where, m_i is a constant for intact rock; D is a disturbance coefficient which varies from 0.0 for undisturbed in situ rock mass to 1.0 for very disturbed rock mass, and GSI is the Geological Strength Index, which takes into account the geological condition of the rock mass (Hoek et al., 2002).

In theory, the iterative scheme satisfies, at each increment and for a selected tolerance, the global equilibrium equations, the compatibility and boundary conditions, and the constitutive equations. However, the constitutive equation integration is not trivial, for even if the incremental strain magnitude on each iterative cycle is known, the way it varies across the incremental path remains unknown. It is necessary, then, to use an accurate stress integration algorithm.

It is possible, by adopting the forward Euler stress integration scheme, as suggested by Sloan et al (2001), to increase the precision of the stress calculation, Eq. (9). According to these authors the increment of stress can be divided into two parcels: elastic, $\Delta\boldsymbol{\sigma}_e$, and elastic plastic, $\Delta\boldsymbol{\sigma}_{ep}$, such as:

$$\Delta\boldsymbol{\sigma}^{k} = \Delta\boldsymbol{\sigma}_e + \Delta\boldsymbol{\sigma}_{ep} = \alpha\mathbf{D}_e\Delta\boldsymbol{\varepsilon} + \Delta\boldsymbol{\sigma}_{ep} \tag{16}$$

where, α is a scalar that varies from 0 to unity and is obtained by solving iteratively:

$$\left|F(\boldsymbol{\sigma}_n + \alpha\mathbf{D}_e\Delta\boldsymbol{\varepsilon})\right| \leq FTOL \tag{17}$$

where FTOL is the tolerance suggested as 10^{-5} (Owen and Hinton, 1980). When $\alpha = 0$ the strain increment generates an elastic plastic stress increment. When $\alpha = 1$ the strain increment generates a purely elastic stress increment. Once defined, the α scalar, the stress state upon the yield surface, is updated as such:

$$\boldsymbol{\sigma}_{int} = \boldsymbol{\sigma}_n + \Delta\boldsymbol{\sigma}_e \tag{18}$$

The increment of elastic plastic stress is obtained by:

$$\boldsymbol{\sigma}^j = \boldsymbol{\sigma}^{j-1} + d\boldsymbol{\sigma}_{ep}{}^j \tag{19}$$

$$d\boldsymbol{\sigma}_{ep}{}^j = \mathbf{D}_{ep}(\boldsymbol{\sigma}^j) d\boldsymbol{\varepsilon}_{ep}{}^j \tag{20}$$

$$d\boldsymbol{\varepsilon}_{ep}{}^j = \Delta T^j \Delta\boldsymbol{\varepsilon}_{ep} \tag{21}$$

$$\Delta\boldsymbol{\varepsilon}_{ep} = (1-\alpha)\Delta\boldsymbol{\varepsilon} \tag{22}$$

where ΔT^j is a scalar known as an increment of pseudo-time (T); it varies from zero to unity and is evaluated while taking into account the local error committed during the stress integration. This error cannot exceed a STOL tolerance. The first trial is conducted by adopting $\Delta T = 1$. The procedure is controlled by pseudo-time T ($0 \leq T \leq 1$) at the end of each sub increment, $\Sigma\Delta T = T = 1$. The stress state is updated at the end of each sub increment starting from stress state upon the yield function ($\boldsymbol{\sigma}^0 = \boldsymbol{\sigma}_{int}$). The procedure described in this paper was implemented into ANLOG (Nogueira, 2010; Simão, 2014).

3 BEARING CAPACITY PROBLEM

The analyses presented in this study involve a bearing capacity determination of rigid (smooth and rough) shallow foundation, without embedment and resting in a weightless and undisturbed rock mass. The problem is analyzed under plane (strip footing) and axisymmetric (circular footing) strain conditions and is modeled as rigid foundation using displacement controls (Fig. 1).

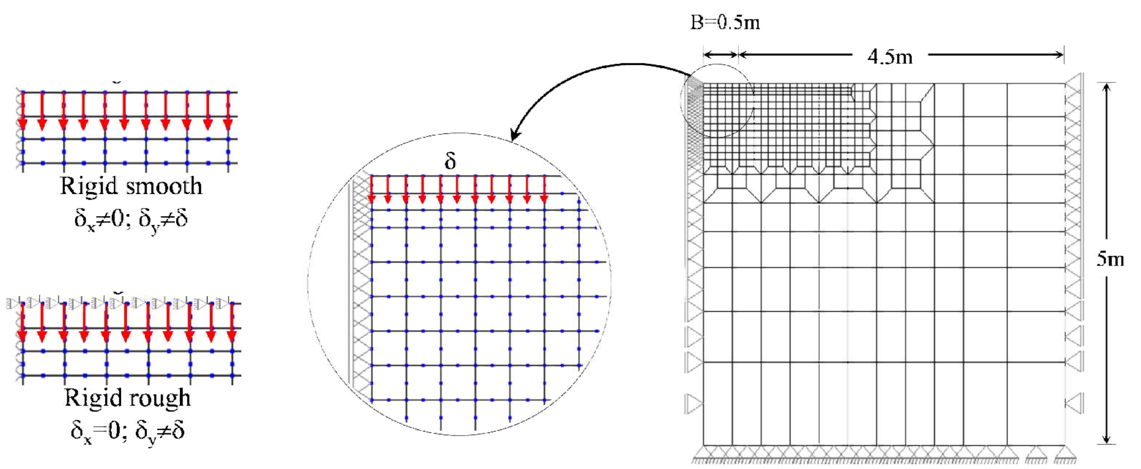

Figure 1. Rigid shallow foundation (427 quadrilateral quadratic elements and 1350 nodal points)

The rock mass is considered as elastic perfectly plastic material described by an associative Hoek-Brown model. The results obtained by ANLOG are presented in terms of the κ-factor defined as (Simão, 2014):

$$\kappa = \frac{Q/A}{\sigma_{ref}} \tag{23}$$

where Q is the reaction force obtained by the sum of the vertical force component of the internal force, equivalent to the stress state of the element right below the footing (Potts and Zdravkovic, 2001; Nogueira et al, 2008;); A is the footing area, and σ_{ref} is a reference stress, which depends on the constitutive model. For the Modified Mohr-Coulomb model, the reference stress is the cohesion (c). For the Hoek-Brown model, the reference stress is the uniaxial compressive strength (σ_c) of intact rock. The bearing capacity factor (N_c) is defined as the ultimate κ-factor. Results of this study are compared with results of studies that utilize equilibrium limit and limit analysis theories.

A tolerance of 0.01% was adopted for the incremental-iterative Modified Newton Raphson procedure with automatic increments. The increment of the load factor, $\Delta\lambda$, varies from 1% (its maximal value) to 0.001% (its minimal value). Related to the stress integration scheme the following tolerance was adopted: FTOL = 10^{-6} and STOL = 10^{-2}.

Figure 2 presents the load-displacement curve in terms of the κ-factor and relative displacement (δ/B), for a rigid-smooth strip footing (plane strain condition) resting on a purely cohesive rock mass. The properties of the material are also presented in Fig. 2. Good agreement can be observed between the ultimate κ-factor (κ_{ult}), provided by both the Modified Morh-Coulomb model (MC) (Oliveira, 2006) and Hoek-Brown model (HB) (Simão, 2014), and the classical solution (N_c = 5.14) provided by limit equilibrium theory (Terzaghi, 1943).

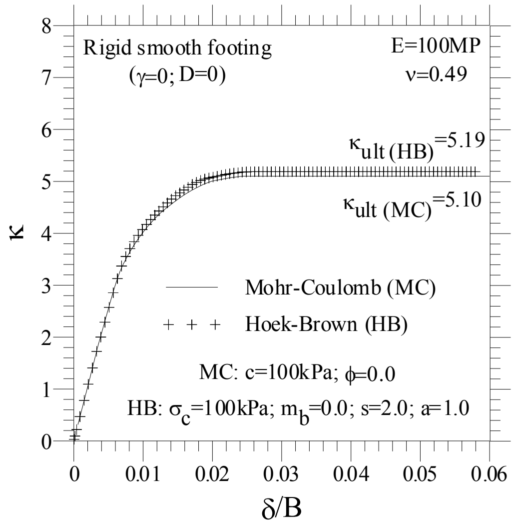

Figure 2. Load-displacement curve – Rigid smooth - Strip footing MC versus HB model

Figure 3 presents the load-displacement curve in terms of κ-factor and relative displacement (δ/B) for a rigid and smooth strip (plane strain condition) and circular (axisymmetric strain) footing resting on a purely cohesive rock mass. Also presented in this figure are the properties of the material, considering the Hoek-Brown model. For strip footing

(N_c = 5.14) and circular footing (N_c = 6.20), good agreement can be observed between the numerical solution (κ_{ult}) provided by ANLOG and the classical solution provided by limit equilibrium theory (Terzaghi, 1943).

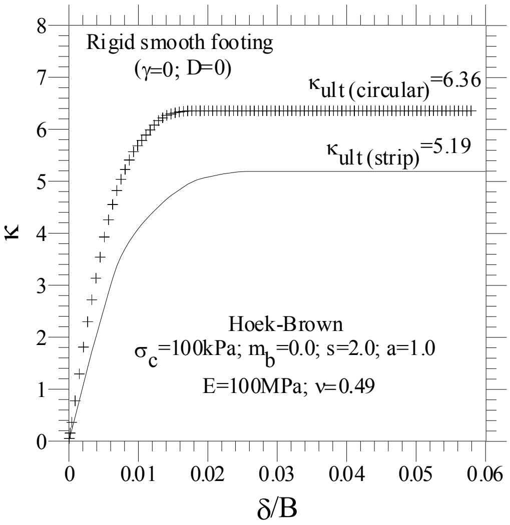

Figure 3. Load-displacement curve – Rigid smooth - HB model - strip versus circular footing

Figure 4 presents the results in terms of κ-factor and relative displacement (δ/B), considering the Hoek-Brown model, for a rigid and rough strip footing. Three different rock masses (named Phyllite, Sandstone, and Basalt), and three different Geological Strength Index (GSI) are used in order to observe the influence of the intact rock mass parameters (m_i and σ_{ci}) and the influence of rock mass quality on the bearing capacity of a shallow foundation. Table 1 presents the material properties adopted and Table 2 present the Hoek-Brown parameters (Hoek et al., 2002) according to Eqs. (13) through (15).

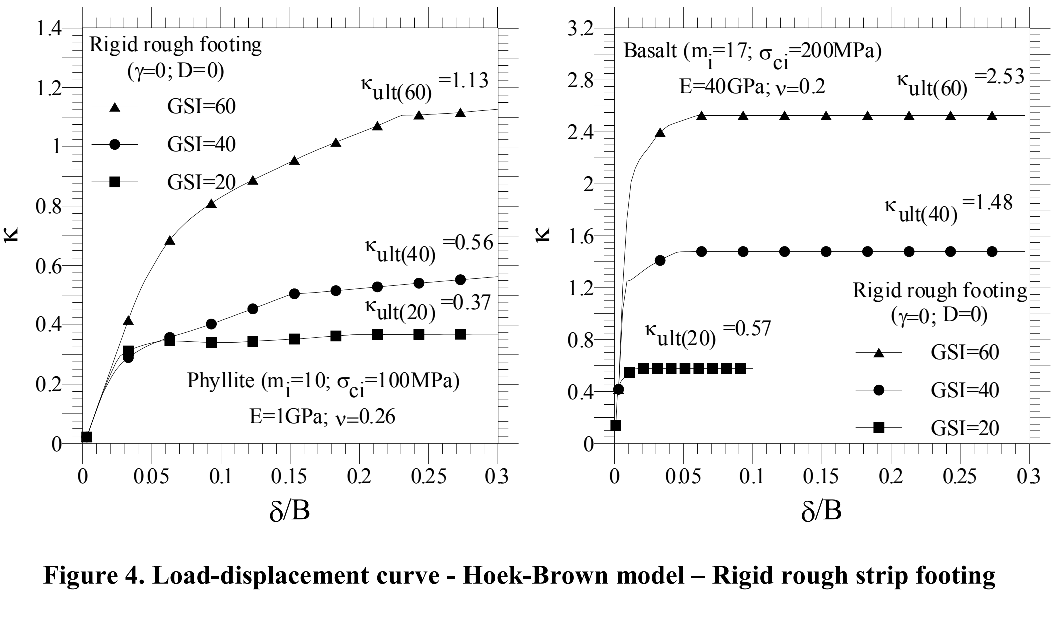

Figure 4. Load-displacement curve - Hoek-Brown model – Rigid rough strip footing

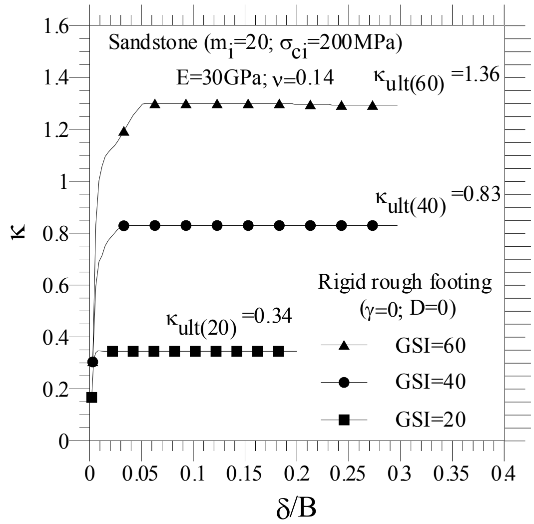

Figure 4. Load-displacement curve - Hoek-Brown model – Rigid rough strip footing (cont.)

Table 1. Material properties (Hoek and Brown, 1997)

Material	E(GPa)	ν	σ_{ci}(MPa)	m_i
Phyllite	10	0.26	100	10
Basalt	40	0.20	200	17
Sandstone	30	0.14	150	20

Table 2. Hoek-Brown parameters' model

GSI	m_b			s	a
	m_i=10	m_i=17	m_i=20		
20	0.5743	0.9764	1.1487	0.0001	0.5437
40	1.1732	1.9944	2.3464	0.0013	0.5114
60	2.3965	4.0741	4.7930	0.0117	0.5028

Table 3 summarizes the bearing capacity factor in terms of the ultimate κ-factor (κ_{ult}) for a rigid and rough strip footing. As was expected, as higher the GSI, higher the bearing capacity factor. A comparison of the numerical results obtained in this work and those obtained by lower bound solution of the limit analysis (Kulhawy and Carter, 1992) and by employing FEM in conjunction with the upper and lower bound limit theorems of classical plasticity (Merifield et al., 2006) reveal a wide variance—a finding that calls for more research. The results presented by Saada et al. (2008), in the framework of the kinematic approach of limit analysis theory, agree with those presented by Merifield et al. (2006).

Table 3. Bearing capacity factor - κ_{ult} - (D=0; γ=0) – strip footing

m_i	GSI	this work	Kulhawy and Carter (1992)	Merifield et al. (2006)	Saada et al. (2008)
10	20	0.37	0.06	0.21	0.18
	40	0.56	0.23	0.66	0.67
	60	1.13	0.62	1.60	1.60
17	20	0.57	0.09	0.34	0.27
	40	1.48	0.28	1.00	1.02
	60	2.53	0.77	2.35	2.36
20	20	0.34	0.09	0.39	-
	40	0.83	0.31	1.49	-
	60	1.36	0.83	2.67	-

4 CONCLUSIONS

When it comes to defining the bearing capacity of shallow foundations on rock mass, these results demonstrate the importance of numerical simulations by FEM. By stress-strain-strength analysis, it is possible to predict the equilibrium path as a function of displacement by using constitutive parameters that take into account geological conditions. For a wide variety of material configurations, the computer program ANLOG can easily perform parametric studies. Such studies could provide support for defining the requirements that ought to be applied to foundation projects.

Based on the results obtained in the present study, it may be concluded that:

- The use of forward Euler with an automatic sub increment stress integration scheme plays an important role in providing an accurate numerical answer at the global level.
- For purely cohesive material, the Hoek-Brown model provides good agreement with the results presented by Terzaghi (1943), based on limit equilibrium theory, for the bearing capacity of a shallow foundation both in plane and axisymmetric strains.
- For granular materials, it was not possible to determine the equivalence between the Hoek-Brown parameters' model and the Mohr-Coulomb parameters' model (c and ϕ).
- Defining the bearing capacity of shallow foundation on rock mass by adopting the elasto-plastic Hoek-Brown model has the great advantage of making it possible, through the geological strength index (GSI), to include in the analysis the geological condition of the rock mass.
- For a rigid, rough, strip footing resting on a weightless and undisturbed rock mass the numerical results show a direct relationship between the ultimate κ-factor and the GSI.

High GSI leads to the highest bearing capacity factor independent of the uniaxial compressive strength of the intact rock.
- The numerical results found in the literature by Kulhawy and Carter (1992) and Merifield et al. (2006) diverge from one another indicating the need to continue researching this subject.

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