STATISTICAL UNCERTAINTY ANALYSIS IN TIME-DOMAIN FATIGUE ASSESSMENT OF STEEL RISERS

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Abstract. *In this work we propose two estimators for the variance of the short-term fatigue damage with the intent of mitigating the computational cost involved in obtaining a better description of the statistics of this variable, what is required for employing a reliability-based design approach for riser fatigue designs. These estimators are based on a variation of the original non-parametric bootstrap (Efron and Tibshirani, 1994), however they work with blocks of data, instead of discrete values, in order to better account for the autocorrelation of the stress time series. These versatile estimators can be applied in time-domain fatigue analyses to assess the variance of the short-term (and, consequently, long-term) fatigue damage using a single stress time series.*

The riser configuration named Riser Suspended and Anchored by Chains, which has an analytical stress response solution, is used to test the performance of the proposed estimators and compare them to the estimator proposed by Vasconcelos Neto (2009) and Steinkjer et al. (2010). A Monte Carlo sampling estimator and Low's analytical method (Low, 2012) are used as benchmarks.

Keywords: *Statistical uncertainty, stochastic fatigue, bootstrap for correlated data, steel risers.*

1 INTRODUCTION

With the expansion of the offshore oil exploration activities in Brazil's coastal areas to ultra-deep and pre-salt water depths, the structural safety of novel riser concepts have become a great concern, especially in terms of their long-term fatigue behavior. Risers are complex structures and proper models for determining their long-term structural behavior are difficult to obtain. Even when they are available, these models are often computationally intensive and, consequently, time consuming.

For this reason, fatigue designs are often based on single-point estimates, factored by the use of a safety factor. However, this methodology is arguably generic and tend result in over-conservative (and over-costly) designs. To mitigate these problems, the DNV-RP-F204 (2010) suggests the use of a reliability-based designs for special cases and novel designs where limited (or no) experience exists. Nonetheless, in order to employ such methods we are required to obtain information about the uncertainties involved in the fatigue damage assessment, such as those related to Palmgren-Miner's rule, S-N curves and uncertainties of statistical nature, the latter being the focus of the present work.

The current literature presents a few alternative estimators to assess the uncertainty in short-term fatigue damage estimates based on limited information. The methods proposed by Low (2012), and Vasconcelos Neto (2009) and Steinkjer et al. (2010) are of especial relevance. Low's method is an analytical solution that is very accurate to estimate the variance of the fatigue damage for Gaussian narrowband processes, however, real-world scenarios usually present stress spectra that are not Gaussian nor narrowband, therefore the method has limited applicability. The estimator proposed by the other two authors is based on a non-parametric bootstrap scheme, and has the interesting property of being able to estimate the variance of the short-term fatigue damage based on a single realization of the stress process. The idea behind a bootstrap estimator is using data from a single short-term stress time series, called *parent sample*. The parent sample for this estimator consists in a sequence of stress cycles, identified by a cycle-counting procedure. Instead of sampling from the population, the bootstrap resamples with replacement from the parent sample, creating a large number of *bootstrap replicates*, and use them to approximate the parameter of interest. Nonetheless, the random sampling ignores the underlying autocorrelation between the stress cycles, making this estimator unreliable in certain cases. Due to this characteristic, this method is here called *uncorrelated-cycles bootstrap* (UCB) estimator.

Considering the aforementioned, this work seeks to develop an estimator for the variance of the short-term fatigue damage that requires the least possible information, accounts for the autocorrelation of the stress process and has wide applicability. Thus, two estimators are proposed here.

The first of these estimators is based on a variation of the traditional bootstrap method. The difference with this proposed method is that it resamples blocks of data, instead of discrete values, in order to better account for the autocorrelation of the process. This method is here called cycle-counting block bootstrap estimator (CCBB).

The second proposed estimator is the product of a simplified manner of facing the block data which is used in the CCBB. Instead of using a bootstrap estimator and putting together several bootstrap replicates, this approach treats each block of data as a shorter realization of the whole process, thus, it is here called Subsampling estimator.

In this paper the accuracy and efficiency of both estimators are tested in the fatigue analysis of a riser anchored by chains.

2 METHODOLOGY

2.1 Statistical uncertainty in fatigue analysis

Estimating the fatigue behavior of a steel riser for complex stress processes is a time-consuming task and, consequently, we are usually forced to rely on limited information, such as a single short-term stress time series. Estimates of the long-term fatigue damage based on short-term simulations will always contain both aleatory and epistemic uncertainties. Since separating these two sources of uncertainty is difficult, the sum of the aleatory and the epistemic uncertainties will be simply referred to by the generic (and shorter) term fatigue damage *statistical uncertainty*.

The fatigue damage is a random variable which presents an usually unknown level of statistical uncertainty. In order to go a little bit further in this subject, let us consider a sequence **S**=$(s_1, s_2, \ldots, s_n)$ of stress ranges identified from a stress time history. Since the stress time history consists of a random process, all s_i in **S** are in fact random variables identically distributed. Due to the own nature of a stochastic process, the stress ranges are also correlated or, in other words, they cannot be considered statistically independent. Thus, using the linear damage accumulation rule, the expected value of the short-term fatigue damage D_{st} can be obtained as

$$\mathbb{E}[D_{st}] = \mathbb{E}\left[\frac{\sum_{i=1}^{N} s_i^m}{K}\right] = \frac{1}{K}\sum_{i=1}^{N}\mathbb{E}[s_i]^m = \frac{N}{K}\,\mathbb{E}[S^m] = \frac{N}{K}\int_0^{\infty} s^m f_S(s)ds \tag{1}$$

where N is the expected number of stress cycles in the short-term period T_{st}, K and m are fatigue parameters, and $f_S(s)$ is the probability distribution of the stress ranges. The variance of D_{st} can be obtained as (Ang and Tang, 1975)

$$Var(D_{st}) = Var\Big(\frac{\sum_{i=1}^{N} s_i^m}{K}\Big) = \frac{1}{K^2}\Big(\sum_{i=1}^{N} Var(s_i^m) + \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{N}}_{i\neq j} Cov(s_i^m, s_j^m)\Big)$$

$$Var(D_{st}) = \frac{1}{K^2}\Big(N\cdot Var(S^m) + 2\sum_{k=1}^{N} Cov(s_k^m, s_{k+1}^m)\Big) \tag{2}$$

where Cov($\cdot$) stands for the covariance and the term $Var(S^m)$ can be computed as

$$Var(S^m) = \mathbb{E}[S^{2m}] - (\mathbb{E}[S^m])^2 = \int_0^{\infty} s^{2m} f_S(s)ds - \Big(\int_0^{\infty} s^m f_S(s)ds\Big)^2 \tag{3}$$

The second term in Equation 2 accounts for the statistical dependence between all stress ranges in **S**. When the stress cycles are statistically independent this term is null, however this is not a common situation in practical engineering. On the other hand, it is very difficult to be addressed (Madsen et al., 2006). One special case for which an analytical solution exists are Gaussian narrowband stress processes (Low, 2012).

In the particular case of marine structures, the long-term fatigue damage accounts for various short-term contributions by considering the sea states as statistically independent. A common method for estimating the long-term fatigue damage considers that the number of stress cycles increases linearly, thus, the expected fatigue damage for the long term (usually chosen as one year to facilitate computing the expected fatigue life) can be computed from the fatigue damage calculated for the short term T_{st} period as

$$\mathbb{E}[D_{lt}] = 2920\frac{T_{lt}}{T_{st}}\sum_{i=1}^{N_{Hs}}\sum_{j=1}^{N_{Tz}}\mathbb{E}[D_{st}^{i,j}\gamma_{i,j}] \tag{4}$$

and its variance is given by

$$Var(D_{lt}) = \left(\frac{T_{lt}}{T_{st}}\right)^2\sum_{i=1}^{N_{Hs}}\sum_{j=1}^{N_{Tz}}(\gamma_{i,j})^2 Var(D_{st}^{i,j}) \tag{5}$$

where $\gamma_{i,j}$ and $Var(D_{st}^{i,j})$ are, respectively, the probability of occurrence and variance of the sea state with significant wave height $Hs = hs_i$ and zero up-crossing period $Tz = tz_j$, and 2920 refers to the number of 3-h sea states in a year. Even though the variance is arguably the most intuitive measure of uncertainty, it may be difficult to interpret the results in terms of it. Considering this, throughout this paper, the uncertainty is measure by the coefficient of variation (CoV) of the results, which is a normalized measure of uncertainty. The CoV is given by

$$CoV(D_{lt}) = \sqrt{Var(D_{lt})}/\,\mathbb{E}[D_{lt}] \tag{6}$$

The probability distribution of the fatigue damage, either for short-term or for long-term, is very difficult to be defined. Nonetheless, for practical applications it has been considered that the Normal distribution is a good approximation (Madsen et al., 2006). The Normal distribution depends only on the mean value and the variance of the variable, thus, the previous parameters computed above are enough to define the fatigue damage statistics. However, even when $f_S(s)$ is known, the evaluation of its variance is not easy because the stress cycles are not statistically independent.

In the case of the design of marine risers, to account for the uncertainties (Miner's rule, S-N curves, and others of statistical nature) in fatigue damage estimates, the DNV-OS-F201 (2010) suggests the use of a *design fatigue factor* (DFF). The DFF is a function of the *safety class*, which varies depending on the conditions in which the riser works. In the case of underwater sections of a riser the necessary safety class is usually assumed *high*, thus $DFF = 10$.

However, the DFF methodology is only recommended for traditional riser concepts, known to have adequate reliability. For novel designs, for example, the DNV-RP-F204 (2010) suggests replacing the use of a generic DFF with risk-based fatigue safety factors. In order to use this alternative, it is extremely necessary to obtain the fatigue damage statistical description as presented above. Thus, in this work, we investigate a sort methods to obtain the statistics of the fatigue damage trying always to use the minimum data as possible.

2.2 The cycle-counting block bootstrap estimator

Obtaining a second-order description of the fatigue damage may prove to be a very difficult task since it requires knowledge about the probability distribution of the stress cycle ranges. For the special case of Gaussian narrowband process, it is known that this PDF follows a Rayleigh distribution. However, in a general scenario, the PDF of the stress cycles ranges is not known and we are required to rely on simulation-based estimators to determine the variance of the fatigue damage. In this context, the most solid way of doing it is employing a *Monte Carlo sampling estimator* (MCSE). This random sample estimating procedure consists in drawing several different estimates of the fatigue damage, obtained from several independent realizations of the stress process, in order to obtain a representation of its intrinsic randomness. The independent realizations of the stress process can be obtained as the result of various numerical structural analysis simulations or by generating various stress time series from a given stress spectrum. However, a Monte Carlo sampling is usually is too expensive. In the specific case of fatigue analysis of risers, obtaining several independent short-term stress time series would require too much time and computational effort. Nonetheless, due to its accuracy, the Monte Carlo sampling estimator is used as a benchmark here and the alternative estimators will have their quality assessed by how well they can represent the MCSE results.

Considering the limitations of the applicability of the MCSE, alternative methods for obtaining second-order descriptions of random variables employing less information have been developed. Among those, the bootstrap method poses itself as a powerful alternative. The bootstrap is a method for estimating the distribution and statistics of an estimator by resampling from a data sample (non-parametric sampling) or a model estimated (parametric sampling) from data, which was formally proposed by Efron (1979) for the first time. Under conditions that hold a wide variety of applications, the bootstrap provides approximations of distributions of statistics, coverage probabilities of confidence intervals, and rejection probabilities of tests that are at least as accurate as the approximations of first-order asymptotic distributions theory (Härdle et al., 2003).

The idea behind the bootstrap method is using data from a single sample, called *parent sample* and, instead of sampling from the population, resample with replacement from the sample itself, creating a large number of *bootstrap replicates* of the parent sample. The efficiency of the bootstrap estimator to represent its equivalent in the population is a function of the parameter being estimated, the parent sample size, N, the number of replicates, R, and, more important, of how well the population statistics are preserved in the sample at hand.

A bootstrap estimator offers a by-pass to a full Monte Carlo simulation, where several realization of the stress process would be necessary. However, a known limitation of the traditional bootstrap method is that it requires the data in the parent sample to be uncorrelated, whereas any underlying correlation structure in the data will be lost due to the random resampling.

With that in mind, several alternative resampling schemes have been proposed for different authors. Among those, the block bootstrap (BB) is the best comprehended and most widely used technique to make the bootstrap resampling possible for correlated data (Härdle et al., 2003). Its most important perk is the fact that it is applicable to virtually every situation with few adjustments.

The only actual difference in the BB is that blocks of data are resampled, in opposition to a discrete data resampling of the traditional bootstrap. In the context of fatigue analysis, the blocks of data are sections of a stress time series, as shown in Figure 1. This modification seeks to partially capture the correlation structure of the parent sample.

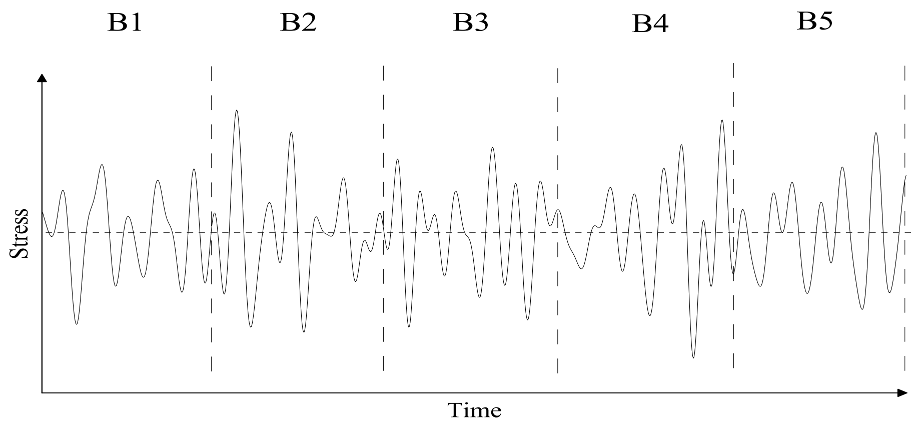

Figure 1: Stress time series divided into blocks of data.

Nonetheless, applying the bootstrap resampling method to the blocks defined in Figure 1 is not optimal, since if we rearrange the blocks, the connecting points will not match most times. As long as we will apply the Rainflow Cycle Counting (RFC) algorithm to identify the stress ranges of the bootstrap replicates, if the blocks boundaries are not perfectly matched, the RFC will be mislead. Even though we apply smoothing techniques, they do not eliminate all mismatches in the connecting points, what may lead to significant errors in the fatigue damage estimates due to the exponential relation between the stress ranges and the fatigue damage.

This particular requirement to the proper application of the RFC implicates in the development of a new block resampling approach, and variation of the original block bootstrap method. This method was thought to eliminate the problem aforementioned, allowing a proper cycle counting with the RFC, hence the name *cycle-counting block bootstrap* (CCBB) estimator. This is a very intuitive method but, to the knowledge of the authors, this approach is not formally employed in the current literature.

The CCBB basic idea starts by defining an initial block with length ℓ_0 and then, if the ending point of this first block is not the series mean, we extend the block until the next mean upcrossing point. For instance, if the first block ended in a point p_1 with $p_1 \neq mean$, the block is extended until a point $p_1^{\mu+} = mean$, and its initial length is added of this extension. We then move on to the second block, setting its starting point to $p_1^{\mu+} + 1$ and its last point to $p_2 = p_1^{\mu+} + 1 + \ell_0$. Now, p_2 is compared to the mean if it differs from it the same correction is applied.

This procedure eliminates the problems with the mismatching connecting points, and also significantly simplifies the bootstrap resampling procedure. Since the blocks are considered independent of one another and end in a closed cycle, we can simply apply the RFC to each block separately and determine their respective fatigue damage, obtaining a series of b independent damage values $d_i^*\{i = 1, \ldots, b\}$. Our parent sample thus become a simple collection of b values d_i^*. This way, the RFC is used a single time for each block (and not R times), what makes the process dramatically faster.

A caveat here is that robust and automated methods for defining an optimal block length are difficult to obtain. Even though some effort has been put in order to develop such methods, the current literature only presents a handful alternatives with very specific applications. In this cases, the optimal block length is usually tied to the (single-side) correlation length, M, of the process, since it is a measure of the statistical dependence between terms in a time series. Thus, throughout this work, the initial block length, ℓ_0, is defined as the $\ell_0 = 2 \times M$.

Lastly, we need to determine three factors to mitigate possible biases caused by the process of dividing the parent series into blocks. Note that all of them can be determined using a single parent series as well.

Since the separation into blocks discontinues the cycles in the connection points, it is expected that the sum of the damage of each block to be smaller than the fatigue damage calculated by applying the RFC to the whole parent series, D_{RFC}. To avoid such biases, we use a *correction factor*, φ_c, given by

$$\varphi_c = \frac{D_{RFC}}{\sum_{i=1}^{b} d_i^*} \tag{7}$$

which is used to compute the corrected damage corresponding to each block in the parent series, i.e., $d_i = \varphi_c d_i^*$.

The second of these factors is due to the fact that the *extended* block length is random, thus, the bootstrap replicates might use more or less information than the parent series. To address this problem a *proportionality factor* is calculated by dividing the size of the parent series by the sum of the sizes of each block used to compose the bootstrap sample, T^*, such as

$$\varphi_p = T/T^* = T/\sum_{i=1}^{b} \ell_i \tag{8}$$

Finally, in order to compare the results obtained when different series lengths are used, we normalize the damage calculated for periods different than 3 hours (usually defined as the duration of short-term sea states) for a period of such length. To do so, we calculate an *extrapolation factor*, φ_e, which assumes that the damage increases linearly. This factor is given by

$$\varphi_e = \frac{10800}{T} \tag{9}$$

These last two factors are applied to the fatigue damage calculated for each bootstrap estimate.

We can then apply this simple, but fundamental, modifications and factorizations to the traditional block bootstrap to obtain a second-order approximation of the distribution of the short-term fatigue damage by generating R bootstrap replicates and determining their average and variance.

2.3 The Subsampling estimator

The aforementioned method is based on the traditional block bootstrap approach and presents itself as an easily implementable alternative to estimate the distribution of the short-term fatigue damage using a single stress time series. Nonetheless, if all the conditions to the application of the CCBB hold, we can offer another alternative method to estimate the uncertainty in the short-term fatigue damage.

Consider we have followed the CCBB's steps up until determining the corrected block damage. We have a single parent series (only one sea-state), which was divided into b blocks. The present method proposes facing each block as a small sample, a small time series on its own. Thus, using μ_d and $Var(d)$ to represent the average and variance of the d_i values, and D to represent the total damage induced by the parent series, and recalling that these values are assumed to be independent of one another, the expected total damage for the entire series may be approximated as

$$\mathbb{E}[D_{st}] = \mathbb{E}[\sum_{i=1}^{b} d_i] = \sum_{i=1}^{b} \mathbb{E}[d_i] = b \cdot \mu_d \tag{10}$$

and finally, the variance of the total damage can be written as

$$Var(D_{st}) = Var\Big(\sum_{i=1}^{b} d_i\Big) = \sum_{i=1}^{b} Var(d_i) = b \cdot Var(d) \tag{11}$$

This semi-analytical estimator can be interpreted as a subsampling scheme, hence its name, and it is the ultimate product of this work in terms of simplification and computational efficiency.

3 MODEL DESCRIPTION AND CASE STUDY

In this work we evaluated the uncertainty in the long-term fatigue analysis of the system known as Riser Suspended and Anchored by Chains (RSAA, from the acronym in Portuguese). The RSAA configuration is composed of a vertical steel riser connected at its top end to a FPSO and at its lower end to a flexible riser and to a mooring chain cable, as shown in Figure 2. The stress along the steel riser, except at the very top and very lower ends, is mainly due to axial tension, i.e., bending stresses are not important (Lemos, 2012).

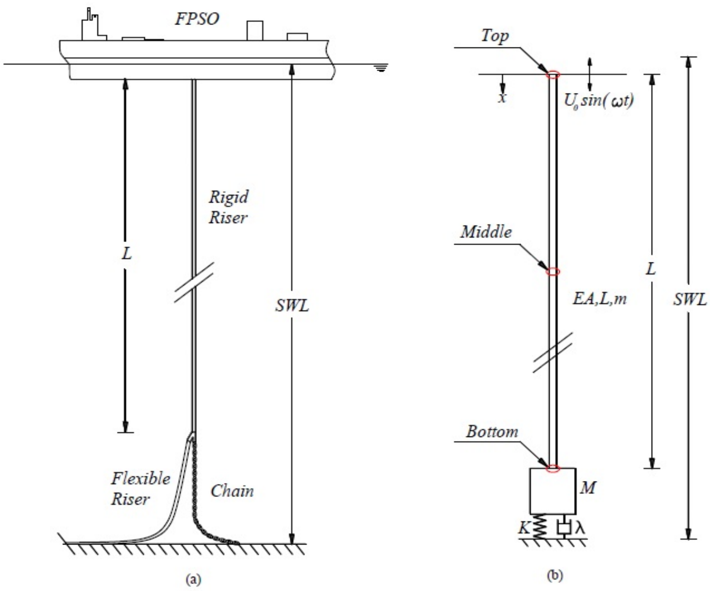

Figure 2: RSSA schematic model. a) configuration; b) structural model (Monsalve Giraldo, 2014).

Due to its virtually linear behavior, it is possible to analytically obtain the stress spectrum for the RSAA, as presented by Pereira (2011). For this reason, the RSAA was chosen to test the proposed methods, as well as the UCB and Low's estimator in regard to their capacity of representing the expected CoV obtained from a dense Monte Carlo simulation.

We here focused our efforts on studying the fatigue behavior of the rigid tube section of the RSAA. Since the point at which the riser is connected to the vessel is, by far, the most critical in terms of fatigue damage, it was chosen as our analysis point.

The long-term fatigue damage was evaluated considering a simplified scatter diagram containing 10 sea-states, $Ns = 10$. The main descriptors of these waves (significant wave height, zero up-crossing period and annual occurrence) are shown in Table 1, while wind and current loads were not considered. For all waves the incidence direction was East and the relative angle between the ship and the wave 90°. We here assumed that the sea surface elevation is represented by a Pierson-Moskowitz spectrum.

Table 1: Wave parameters for the RSAA

#	Hs(m)	Tz(s)	γ
1	0.683	3.95	9.15E-3
2	1.348	4.85	4.99E-1
3	2.013	5.75	3.78E-1
4	2.678	6.65	9.43E-2
5	3.343	7.55	1.60E-2
6	4.008	8.45	2.41E-3
7	4.672	9.35	3.43E-4
8	5.338	10.25	4.89E-5
9	6.002	11.15	7.13E-6
10	6.668	12.05	1.02E-6

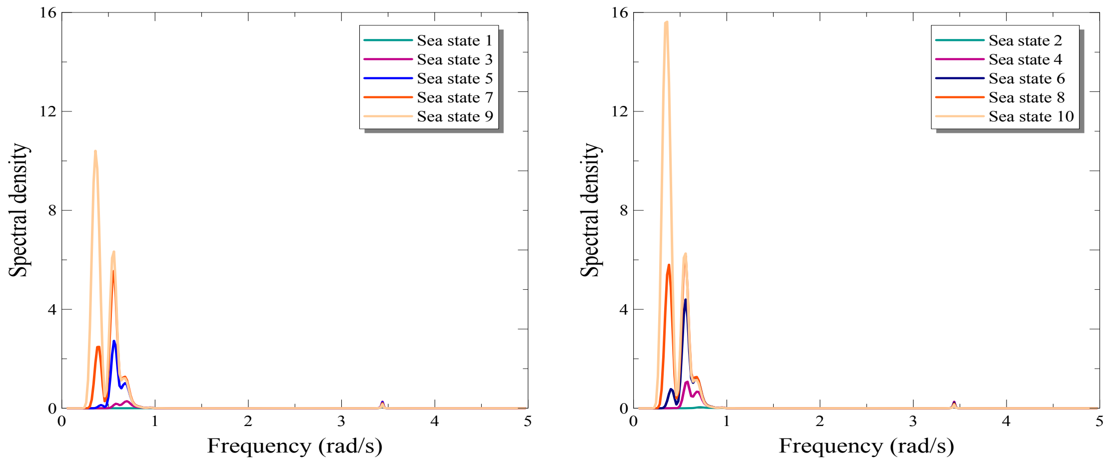

Figure 3: Stress spectrum related to the odd (left) and even (right) sea states in Table 1.

In this study the short-term stress time series lengths considered were 1800s, 2700s, 3600s, 5400s, 7200s and 10800s and the short-term fatigue damage for a stress time-series with length T for any sea-state is $Dst(T)$. Then, the long term fatigue damage for all sea states, $D(T)$, was obtained by adding up the damages corresponding to each sea-state multiplied by their respective annual occurrence, γ, and extrapolating it over an year, using Equation 4.

Furthermore, the fatigue analyses were run considering two values of the fatigue parameter m, namely m=3 and m=6. Also, since our results will be shown in terms of the coefficient of variation of the fatigue damage, they are independent of the parameter K.

To obtain a CoV estimate using the MCSE several independent realizations of the stress process were ran, say $X_i(t)$. From each of those, an expected fatigue damage $D(X_i(t))$ was determined and the mean, $\mu(D)$, and standard deviation, $\sigma(D)$, of these realizations was used to determine the coefficient of variation of the fatigue damage as $CoV(D) = \sigma(D)/\mu(D)$.

Since the other three estimators, namely UCB, CCBB and Subsampling are strongly dependent on the parent stress time series in which they are based on, a single point estimate using them may not reflect the actual accuracy and precision of the estimator itself. In order to evaluate the true properties of the mentioned estimators, we needed to eliminate the parent-sample dependency. A robust method to do this is by running several independent realizations of the stress process, estimating the CoV for each of those and determining the mean and variance of these estimates. This way we will be properly evaluating the estimators, and not their point estimates.

With that in mind, for each realization $X_i(t)$, the UCB and CCBB estimators and their respective bootstrap resampling schemes were used to generate R replicates of $X_i(t)$ and determine the expected value and variance of the fatigue damage from this set of replicates. On the other hand, the Subsampling estimator does not make use of replicates, but similarly, it yields an expected value and variance estimates for the short-term fatigue damage for each $X_i(t)$.

This way, once we had a set of N realization of the stochastic stress process $X(t)$, we applied the MCSE to them and obtained a single CoV estimate, while for the other three estimators we managed to obtain a set of N CoV estimates and from those, we determined the confidence interval for the CoV of the short-term damage. Figure 4 presents an schematic panel highlighting the differences between these estimators.

Lastly, since several methods for numerically simulating stochastic processes by spectral decomposition are present in the literature, it was opted here to employ the method that uses fixed representative frequencies, equal to the mean frequency of the interval, and Rayleigh-distributed harmonic amplitudes, since it is known to be the most robust of those (Costa, 2015). Additionally, the stress spectra were divide into Nω=2000 intervals in order to obtain the desirable representation of the variance of the stress process.

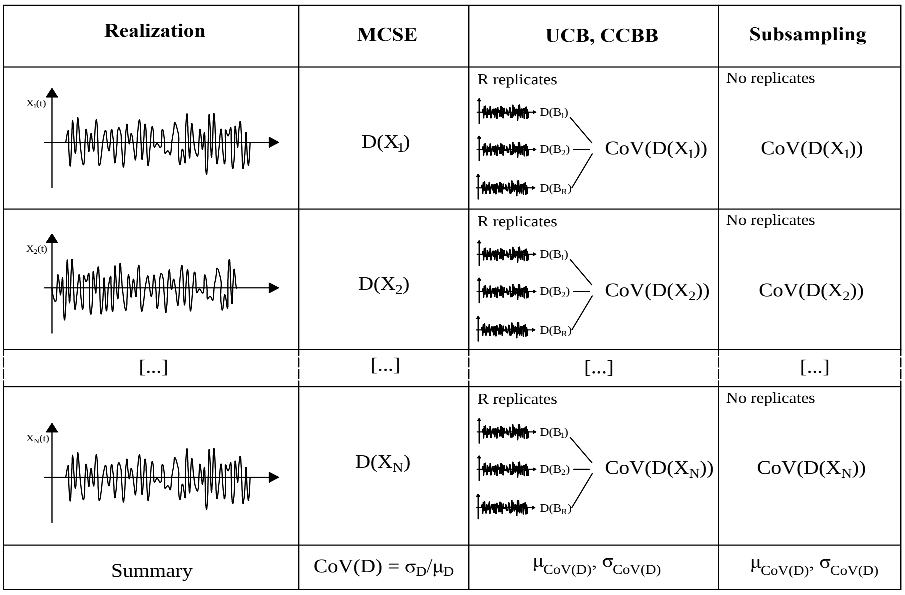

Figure 4: CoV estimation using the simulation-based estimators.

4 RESULTS AND DISCUSSIONS

4.1 The benchmarks

The benchmarks for this example were the MCSE and Low's theoretical estimator. The MCSE employed 500 different realizations of each sea-state, for each series length studied. This way, 500 estimates of the long-term fatigue damage for each series length were obtained, and, from these results a mean, a variance and a CoV were determined. The CoVs of the long-term fatigue damage obtained with the MCSE and Low's estimator are shown in Figure 5

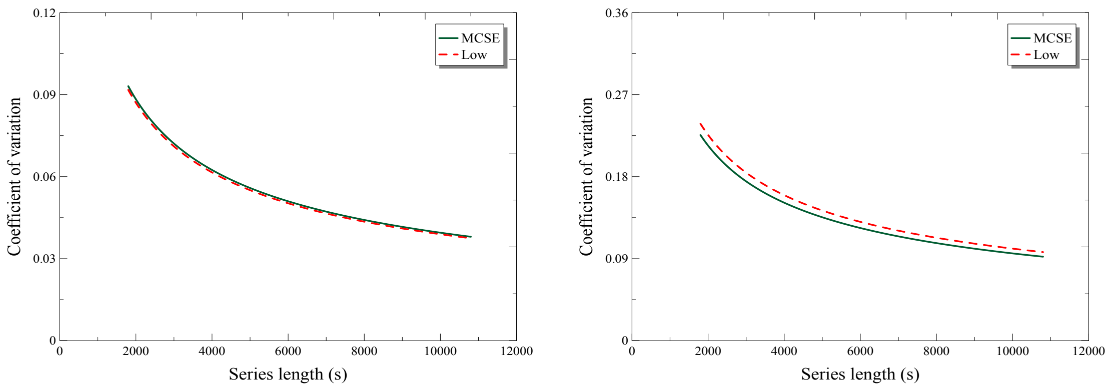

Figure 5: Results for the MCSE with different simulation methods for m=3 (left) and m=6 (right).

4.2 The UCB estimator applied to the RSAA

Once the benchmarks were defined, it was possible to test how well the other three estimators perform when applied to the same case study. In order to properly evaluate the estimator properties, 100 different parent stress time series were ran, their respective CoVs estimated and delimited the 95% Normal confidence interval. In an optimal scenario, the expected value should be as close as possible to the target results (MCSE and Low's estimator) and the confidence interval as narrow as possible. The first of these methods to be tested was the UCB estimator.

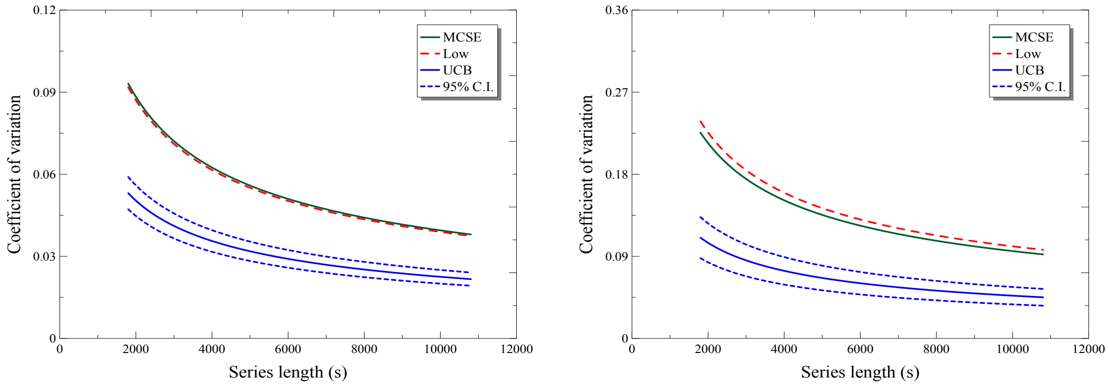

Figure 6: Results for the UCB estimator for the RSAA for m=3 (left) and m=6 (right).

We could see from these results that the UCB estimator did not achieve the desirable results, presenting a strong negative bias for both S-N curves exponents m. Although presenting a small variance, the expected CoV for this estimator was much smaller than the target value.

4.3 The cycle-counting block bootstrap estimator applied to the RSAA

In order to employ the CCBB, an initial block length needed to be defined. As previously discussed, it was assumed that $\ell = 2 \times M$. The correlation length, M, was obtained by visual inspection of the correlograms for the spectra shown in Figurs 3. Since 10 sea-states were used, it was opted to use a single ℓ large enough to cover the correlation length of all 10 spectra. This value was defined to be ℓ=200s, whereas the longest correlation length was close to M=100s.

Using a block length of ℓ=200s and R=10.000 replicates the CCBB was ran for 100 different parent stress time series. The results for this estimator are shown in Figure 7.

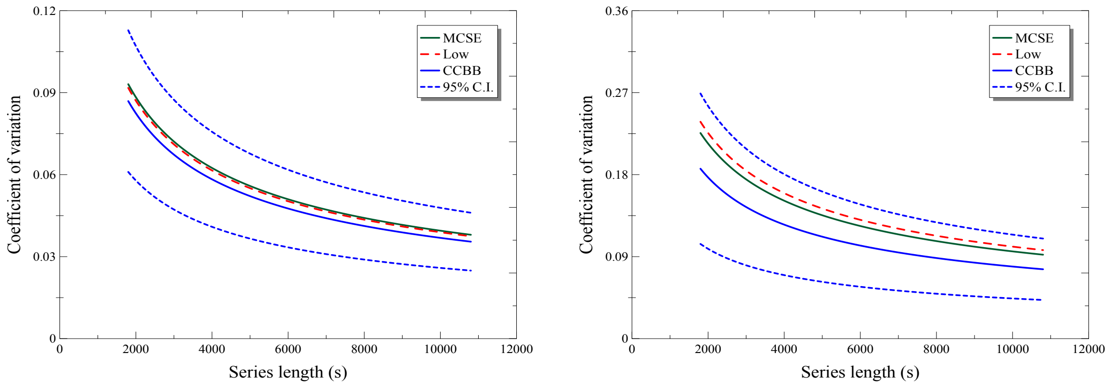

Figure 7: Results for the CCBB estimator for the RSAA for m=3 (left) and m=6 (right).

The results show that the expected fatigue damage CoV for the CCBB is, in general, smaller than that obtained with the MCSE and Low's method. For m=3 the results are very close to the targeted ones, but when the exponent m is doubled, a negative bias is more pronounced. For both m, the CCBB confidence interval was capable of covering the target value for all series length, nonetheless, it is very wide, showing the method still lacks in precision in its CoV estimates.

4.4 The Subsampling estimator applied to the RSAA

For the Subsampling estimator, the same block length determined for the CCBB was used, and similarly to what was shown for the other estimators, Figure 8 compares its expected results for 100 independent parent series to those yielded by the MCSE and Low's method, as well as, its 95% Normal confidence interval.

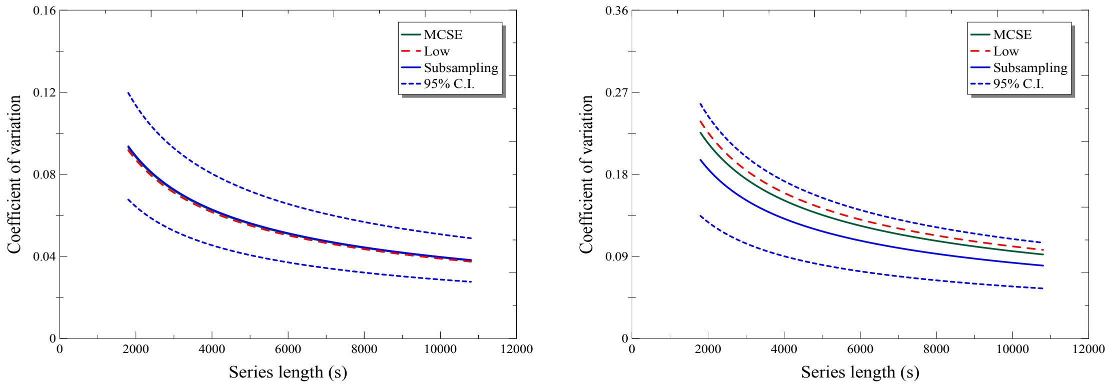

Figure 8: Results for the Subsampling estimator for the RSAA for m=3 (left) and m=6 (right).

The Subsampling presented the best results for all cases, behaving as an unbiased estimator for m=3 and presenting a small negative bias for m=6. On the other hand, its confidence interval, specially for the latter value of m is quite wider when compared to the UCB and CCBB.

5 CONCLUSIONS

We saw that the proposed estimators proved to be superior to the UCB estimator in terms of variance estimation, putting themselves as a stronger option for estimating the statistical uncertainty in long-term fatigue damage analyzes using a single realization of the stress process. To put that in perspective, Figure 9 plots the results for the three estimators. The results for the Low's method and for the MCSE are also plotted for comparison purposes.

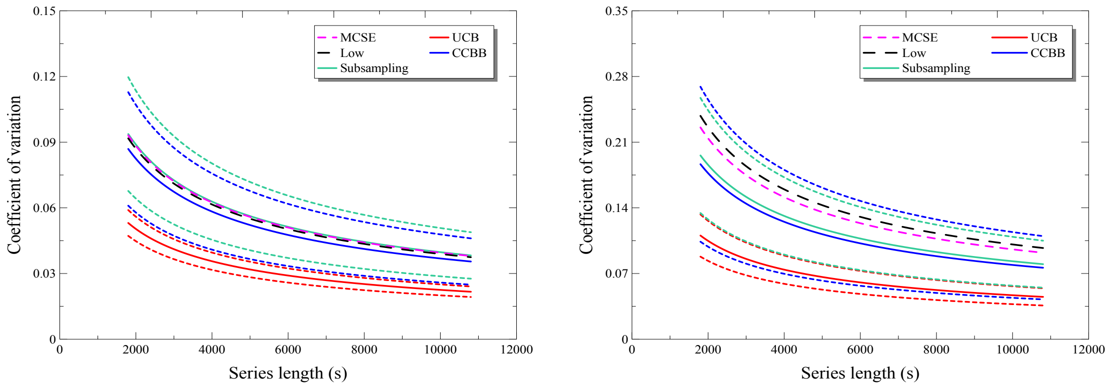

Figure 9: Comparison of the confidence intervals for the estimators based on a single realization.

It is easy to see that the block-based estimators have a great potential, since the behave arguably as unbiased estimators for m=3 and presented considerably better results for m=6, when compared to the UCB estimator. The proposed methods outperformed the UCB estimator in terms of representing the variance of the stress process, they do not have the limitations intrinsic to Low's method, and present themselves as interesting tools to contour the necessity of running expensive Monte Carlo simulations for code calibrations, for example. Since these methods can drawn CoV estimates from a single realization of the stress process, it is possible to estimate the process' uncertainty without running a full Monte Carlo study. On the other hand, their major limitation is that they also presented large variability, denoted by a wide confidence interval. This shows us that the limited information employed in usual fatigue analysis (stress time series lengths shorter than 3-h) is not enough to drawn reliable estimates of the whole process uncertainty.

Considering these results were obtained with a preliminary version of both proposed estimators, future investments in order to improve the performance of both methods are completely justifiable and have the potential of obtaining a robust estimator for the variance of the long-term fatigue damage based on very limited information.

6 REFERENCES

Ang, A. and Tang, W. *Probabilistic concepts in engineering planning and design–Volume I: Basic principles*. New York, NY: John Wiley & Sons, 1975.

Costa, R. C. d. Statistical Uncertainty Analysis in Time-Domain Fatigue Assessment of Steel Risers. Master's thesis, Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 2015.

DNV-OS-F201. Dnv-os-f201 - dynamic risers. *Hovik, Norway*, 2010.

DNV-RP-F204. Recommended practice dnv-rp-f204, riser fatigue. *Hovik, Norway*, 2010.

Efron, B. Bootstrap methods: another look at the jackknife. *The annals of Statistics*, pages 1–26, 1979.

Efron, B. and Tibshirani, R. J. *An introduction to the bootstrap*, volume 57. CRC press, 1994.

Härdle, W., Horowitz, J., and Kreiss, J.-P. Bootstrap methods for time series. *International Statistical Review*, 71(2):435–459, 2003.

Lemos, H. B. L. Análise Estocástica da Resposta Analítica de Riser Vertical Ancorado por Amarras. Master's thesis, Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 2012.

Low, Y. M. Variance of the fatigue damage due to a gaussian narrowband process. *Structural Safety*, 34(1):381 – 389, 2012. ISSN 0167-4730.

Madsen, H. O., Krenk, S., and Lind, N. C. *Methods of structural safety*. Courier Corporation, 2006.

Monsalve Giraldo, J. S. Efficient Methods for Probabilistic Fatigue Analysis of Marine Structures. Master's thesis, Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 2014.

Pereira, V. M. d. S. Análise da Resposta Dinâmica do Tubo Vertical do Sistema de Riser Suspenso e Ancorado por Amarras (RSAA). Master's thesis, Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 2011.

Steinkjer, O., Sodahl, N., and Grytoyr, G. Methodology for time domain fatigue life assessment of risers and umbilicals. *ASME 2010 29th International Conference on Ocean, Offshore and Arctic Engineering*, pages 177–184, 2010.

Vasconcelos Neto, A. G. Análise de incertezas no cálculo do dano de estruturas marítimas pela técnica bootstrap. Master's thesis, Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro, Rio de Janeiro, RJ, Brazil, 2009.