OPTIMIZATION UNDER UNCERTAINTY IN MECHANICAL SYSTEMS

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Abstract. *Recently the search for robust mechanical systems has increased significantly. To develop robust systems, the inherent uncertainties must be taken into account and to do so generally an optimization problem with uncertain quantities is formulated. Thus, a reliable methodology that can perform the optimization of a system with uncertain quantities is very important. The current work aims at evaluating the use of fuzzy optimization techniques in the robust design of mechanical systems. The proposed fuzzy robust optimization strategy uses a dual evaluation of the unconstrained cost function by means of the pessimistic and optimistic values of the fuzzy inputs. Thus, by generating the fuzzy response the robust values are obtained from the defuzzification process. To evaluate the performance of procedure proposed, examples from the literature are used to test the methodology conveyed, as follows: the design of a welded cantilever beam and the design of a tension/compression spring were performed considering uncertainties both in the cost functions and in the constraints.*

Keywords: *Fuzzy Optimization, Uncertainties, Robust Optimization*

1 INTRODUCTION

Usually, in the design/operation of mechanical systems a deterministic model is formulated so that the dynamic behavior of the process can be evaluated. These deterministic models considerer the parameters as known and well defined. However, whether by lack of knowledge or due to design/operational fluctuations (e.g. manufacturing errors, temperature variations) or due to damage, uncertainties are inherent aspects of mechanical systems. Thus, the deterministic approach can be regarded as outdated and should be replaced by one that takes into account such uncertainties.

Among the various approaches for modeling uncertain quantities in mechanical systems, the stochastic approach and the fuzzy logic are two of the most widely used techniques. The stochastic methodology based on the probability theory has a longer history of applications in mechanical systems. On the other hand, the intuitive fuzzy logic approach based on fuzzy sets (Zadeh, 1965) and the possibility theory (Zadeh, 1968) has recently gained more attention and relevant applications can be found in the literature (Möller and Beer 2004; Lara-Molina et al. 2014; Cavalini et al. 2015). Both methodologies are well suited for modeling uncertain or inaccurate quantities; however, due to its mathematical simplicity the fuzzy logic seems to be more appropriate for the optimization procedure.

Robustness is a widely used term with different meanings, but in general, it is understood as a value or solution that may not be the best for one particular scenario but is the best one for various scenarios considered. In a robust optimization procedure, the goal is to find a solution that is the least sensitive as possible to the model uncertainties. The pioneer of robust optimization is G. Taguchi (Taguchi, 1984), who proposed the insertion of noise factors in the cost function thus defining a mean square deviation that should be maximized regarding the design variables.

Most of the known robust optimization approaches are based on stochastic techniques relying on the statistical moments of the problem (Paenke et al. 2006; Beyer and Sendhoff, 2007; Borges et al. 2010). These techniques usually lead to large combinatorial problems, especially if they rely on sampling methodologies such as the Monte Carlo Simulation, which can impose serious disadvantage to the optimization task. Therefore, a simpler mathematical scheme as the fuzzy logic may prove to be more advantageous for robust optimization.

In this way, the present work aims at contributing to the development of fuzzy robust optimization techniques, and for such, a new robust optimization concept is proposed. Based on the pessimistic and optimistic concepts of fuzzy sets, a fuzzy evaluation of the optimization problem is conceived through the evaluation of the pessimistic and optimistic values of the uncertain parameters in the optimization task, thus generating a fuzzy response that is defuzzified to obtain a robust value. This new methodology was applied in the welded cantilever beam design and in the tension/compression spring design, considering uncertainties both in the cost functions and in the constraints.

This paper is organized as follows. First the key concepts of fuzzy logic are discussed, and then the fuzzy robust optimization technique is presented and evaluated for the two design problems considered. Finally, a few remarks are made and further research work is proposed.

2 FUZZY LOGIC

Introduced by Zadeh (1965), the fuzzy set theory formulated the graded membership notion represented by the membership function. In a classical sense, a set is a collection of elements that belong to a definition. Any element either belong or not to the set. In a fuzzy set this *Boolean* notion of membership is replaced by a graded notion, i.e., an element can belong, not belong or partially belong to the set.

As presented by Dubois and Prade (1997) the membership function in a fuzzy set can basically be seen from three different perspectives: a degree of similarity, in which the membership function quantifies how similar an element of the set is compared to a prototype element; a degree of preference, where the membership function quantifies the preference or the feasibility of an element of the set; or a degree of uncertainty, in which the membership function is the degree of possibility that a determined parameter *u* has the value of an specific element *x* of the set.

For modeling an uncertain quantity, the degree of uncertainty perspective is more suitable, while for an optimization procedure the degree of preference interpretation seems more appropriate. Nevertheless in the fuzzy robust optimization procedure conceived in this paper the degree of uncertainty interpretation is used both in the modeling of the uncertain parameters (fuzzy inputs) and in the optimization step.

Through the fuzzy set theory, uncertainties of the model are represented as fuzzy inputs (fuzzy variables) and the model itself becomes a fuzzy function that maps these inputs. Among the several methods for mapping a fuzzy variable the so-called α-level optimization method (Möller and Beer, 2004) is one of the most efficient. In this method, the fuzzy inputs are discretized by means of α-cuts (fixed levels of the membership function) creating a *crisp* subspace (a subspace in the real number domain) that is the search space of an optimization problem. The optimization process is carried out in order to find the minimum and maximum values of the model output for each α-level (i.e., lower and upper limits of the correspondent α-level, respectively; see Fig. 1). The effectiveness of the method is highlly related to the performance of the selected optimization technique.

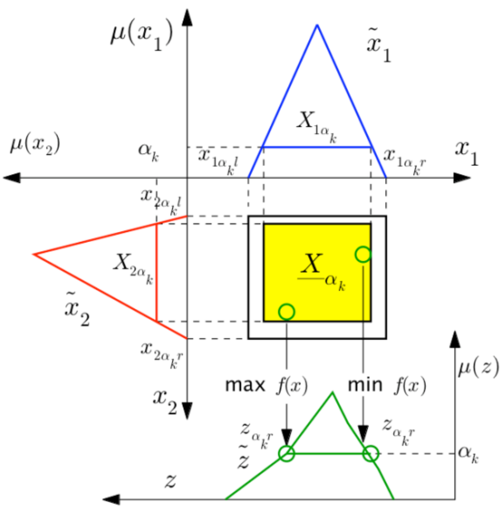

Figure 1. The α-level optimization method.

This mapping generates an associated fuzzy output that, for a decision making process, must be "defuzzified" to a single value (a *crisp* value). The defuzzification procedure is basically the aggregation of a fuzzy information (fuzzy set) into a single information. Different defuzzification techniques exist in the literature wherein the most commonly known and used techniques are the centroid method and the bisection method. In the centroid method, also known as center of area/gravity method, the center of the area (center of gravity) of the fuzzy parameter membership function is calculated and taken as the crisp value; while for the bisection method the chosen value is simply the bisector of the area.

In this contribution the *centroid method* was selected for defuzzification, since this method is the closest to what can be defined as the weighted average of a fuzzy set.

3 FUZZY ROBUST OPTIMIZATION

Considering the uncertain parameters of the optimization problem (found in the cost function and/or in the constraints) as fuzzy inputs, the optimization function becomes a fuzzy function that maps these fuzzy inputs. Following the degree of uncertainty interpretation, these fuzzy inputs represent a set of possible realizations for the uncertain parameters; possible scenarios that the optimization procedure should take into account.

According to the possibility theory, a fuzzy set can be view as the union of the pessimist and optimist values of the uncertain information. Due to this duality of the uncertain values, a dual optimization problem can be conceived: i) an optimization problem where the goal is to find an optimum considering the pessimist values of the uncertain information in a respective α-level, and ii) a second optimization problem dedicated to search an optimum by taking into account the optimist values in the same α-level. Through this dual evaluation procedure a fuzzy optimum will be obtained, that in practice is nothing more than a set of possible optima. However, in a decision making procedure the goal is a single value, an optimum, not a set of possible optima, so a defuzzification procedure should be performed to aggregate the fuzzy information into a single value, the robust optimum. The dual optimization procedure is illustrated in Fig. 2.

4 ROBUST OPTIMIZATION RESULTS

In order to ascertain the efficiency of the proposed methodology, the conceived fuzzy robust optimization procedure was applied in the robust design of a welded cantilever beam and in the design of a tension/compression spring. The formulation and discussion of the welded beam design problem can be found in Rao (1996) and in Lobato and Steffen Jr. (2014). Mathematically, the problem can be formulated as a cost function represented by Eq. (1) as follow:

$$\min f(\boldsymbol{X}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14 + x_2) \tag{1}$$

subject to the constraints set presented in Eq. (2-10).

$$g_1(\boldsymbol{X}) = \tau(\boldsymbol{X}) - \tau_{max} \leq 0 \tag{2}$$

$$g_2(\boldsymbol{X}) = \sigma(\boldsymbol{X}) - \sigma_{max} \leq 0 \tag{3}$$

$$g_3(\boldsymbol{X}) = x_1 - x_4 \leq 0 \tag{4}$$

$$g_4(\boldsymbol{X}) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14 + x_2) - 5 \leq 0 \tag{5}$$

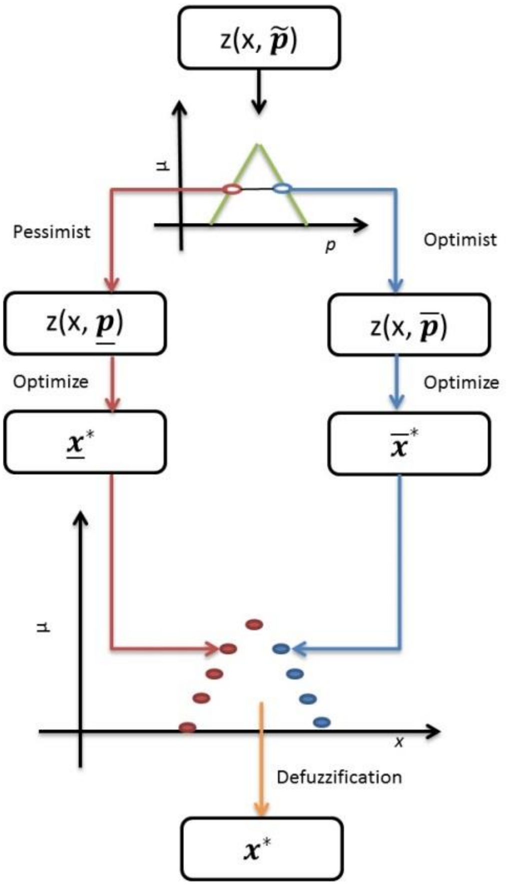

Figure 2. Dual optimization procedure flowchart.

$$g_5(\boldsymbol{X}) = 0.125 - x_1 \leq 0 \tag{6}$$

$$g_6(\boldsymbol{X}) = \delta(\boldsymbol{X}) - \delta_{max} \leq 0 \tag{7}$$

$$g_7(\boldsymbol{X}) = P - P_c(\boldsymbol{X}) \leq 0 \tag{8}$$

$$g_8(\boldsymbol{X}) \text{ to } g_{11}(\boldsymbol{X}): 0.1 \leq x_i \leq 2, \quad i = 1,4 \tag{9}$$

$$g_{12}(\boldsymbol{X}) \text{ to } g_{15}(\boldsymbol{X}): 0.1 \leq x_i \leq 10, \quad i = 2,3 \tag{10}$$

where

$$\tau(\boldsymbol{X}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}, \tau' = \frac{P}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}, M = P\left(L + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}, J = 2\left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right]\right\}, \sigma(\boldsymbol{X}) = \frac{6PL}{x_4x_3^2}, \delta(\boldsymbol{X}) = \frac{4PL^3}{Ex_3^3x_4},$$

$$P_c(\boldsymbol{X}) = \frac{4.013\sqrt{EG(x_3^2x_4^6/36)}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

$P = 6000$ lb, $L = 14$ in., $E = 30\text{x}10^6$ psi, $G = 12\text{x}10^6$ psi, $\tau_{max} = 13600$ psi, $\sigma_{max} = 30000$ psi, and $\delta_{max} = 0.25$ in.

Regarding the welded bean design, the uncertainty scenario considered an uncertainty level of $\pm 5\%$ in the cost function parameters (a_1=1.10471, a_2=0.04811) and in the constraint bounds ($\tau_{\max}$, $\sigma_{\max}$ and $\delta_{\max}$).

Arora (1989) presented the tension/compression spring design problem presented by Eq. (11-18), in which the design variables are the wire diameter (x_1), the mean coil diameter (x_2) and the number of active coils (x_3).

$$\min f(\boldsymbol{X}) = \frac{1}{4}(x_3 + Q)\pi^2 x_2 x_1^2 \frac{\gamma}{g} \tag{11}$$

$$g_1(\boldsymbol{X}) = \frac{8Px_2^3x_3}{x_1^4G} \geq \Delta \tag{12}$$

$$g_2(\boldsymbol{X}) = \frac{8Px_2}{\pi x_1^3}\left[\frac{(4x_2 - x_1)}{4(x_2 - x_1)}\right] \leq \tau \tag{13}$$

$$g_3(\boldsymbol{X}) = \frac{x_1}{2\pi x_3 x_2^2}\sqrt{\frac{Gg}{2\gamma}} \geq \omega_0 \tag{14}$$

$$g_4(\boldsymbol{X}) = x_1 + x_2 \leq D_0 \tag{15}$$

$$g_5(\boldsymbol{X}) \text{ and } g_6(\boldsymbol{X}): 0.05 \leq x_1 \leq 2 \tag{16}$$

$$g_7(\boldsymbol{X}) \text{ and } g_8(\boldsymbol{X}): 0.25 \leq x_2 \leq 1.3 \tag{17}$$

$$g_9(\boldsymbol{X}) \text{ and } g_{10}(\boldsymbol{X}): 2 \leq x_3 \leq 15 \tag{18}$$

where $Q = 2$, $\gamma = 0.285\ \mathrm{lb/in^2}$, $g = 386\ \mathrm{in/s^2}$, P =10 lb, $G = 1.15\text{x}10^7\ \mathrm{lb/in^2}$, Δ= 0.5 in., $\tau = 80000\ \mathrm{lb/in^2}$, $\omega_0 = 100$ Hz, $D_0 = 1.5$ in.

To create an uncertain scenario, an uncertainty level of $\pm 5\%$ was considered for the material density (γ), for the applied load (P), and for the allowable shear stress (τ). Table 1 presents the original and the robust optimum results.

Table 1. Optimization Results

Design Problem	Deterministic Optimum				Robust Optimum			
Welded Beam	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
	0.2374	3.2589	8.0295	0.2606	0.6686	2.2192	7.2814	0.3751
Tension/Compression Spring	x_1	x_2	x_3		x_1	x_2	x_3	
	0.0517	0.3582	11.20		0.0517	0.3563	11.31	

A comparison between the optima results regarding the three different scenarios, namely, nominal (all uncertain parameters assume their nominal values), pessimist (the uncertain parameters assume their lower values), and optimist (uncertain parameters at their upper values) is shown in Table 2.

Figures 3-4 demonstrate the uncertainty analysis of both systems (welded beam and tension/compression spring) considering the deterministic design (deterministic optimum) and the robust design (robust optimum). From this analysis it is clear that for the welded beam design problem the solution proposed by the dual optimization technique is very robust, being practically insensitive to the parameters variation. However, for the tension/compression spring design problem there is virtually no difference between the performance of the deterministic solution and the dual optimization solution, whereas the dual solution is slightly less sensitive than the pessimist set and, therefore, more robust.

Table 2. Optimization Results

Design Problem	Cost Function					
	Pessimist		Nominal		Optimist	
	Det.	Rob.	Det.	Rob.	Det.	Rob.
Welded Beam	$7.46\text{x}10^4$	3.0685	1.9402	3.2275	2.0372	3.3909
Tension/Compression Spring	$8.535\text{x}10^{-5}$	$8.471\text{x}10^{-5}$	$2.305\text{x}10^{-5}$	$2.305\text{x}10^{-5}$	$2.422\text{x}10^{-5}$	$2.422\text{x}10^{-5}$

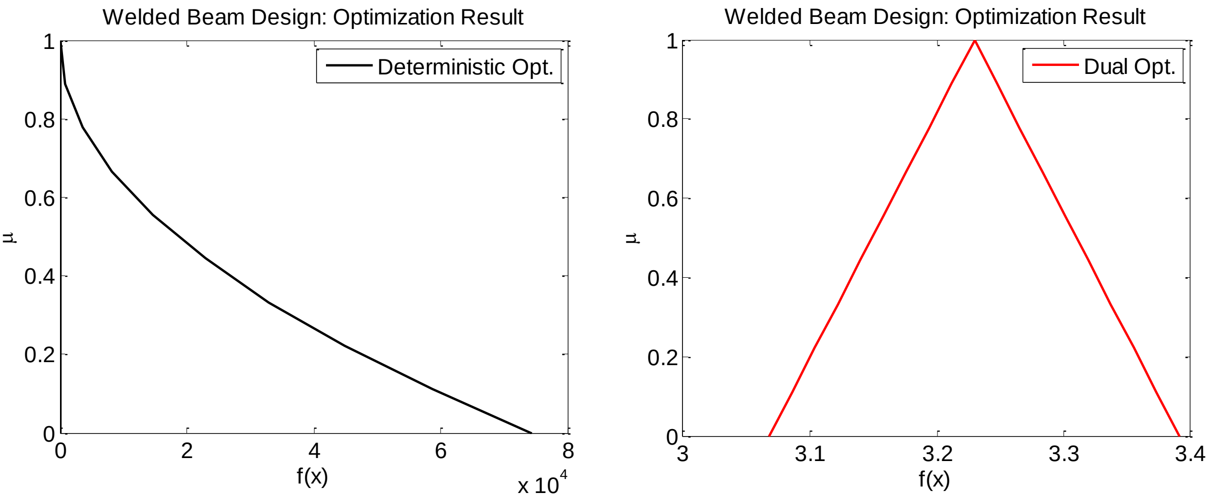

Figure 3. Welded beam uncertainty analysis.

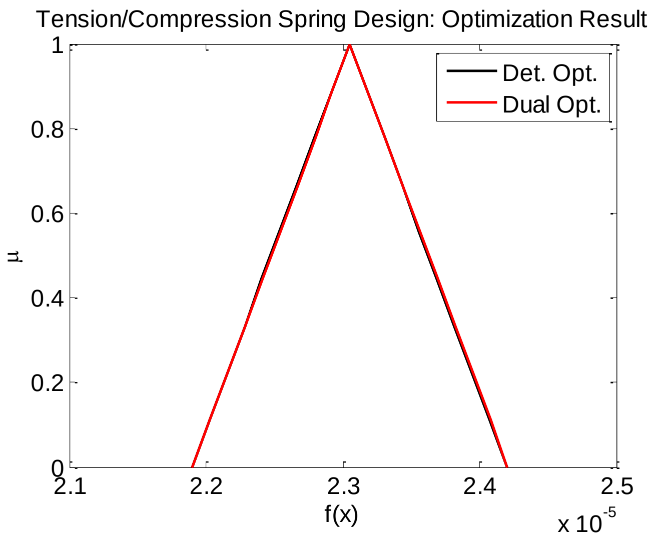

Figure 4. Tension/Compression spring uncertainty analysis.

5 FINAL REMARKS

This paper aimed at contributing to the development of a new and simpler robust optimization approach. To do so, a new robust optimization procedure, namely a dual fuzzy optimization was conceived and tested in the robust design of two simple engineering problems from the literature, i.e., the welded cantilever beam and the tension/compression spring.

Considering a robust optimum as the value that presents better results in as many different scenarios as possible, the results show that the presented procedure is very efficient at finding robust optima. However, depending on the problem features the robustness gain obtained with the procedure solution may not be very large. This indicates that the procedure performance regarding the robustness is problem depending.

Fuzzy robust optimization techniques are very promising for robust optimization due to its simple mathematical formulation, which leads to fast computation. However, depending on the problem features it still imposes a large number of evaluations of the cost function; consequently, a new goal for the development of fuzzy robust optimization procedures should be the reduction of cost function evaluations, which in general are very time consuming.

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