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A GENETIC ALGORITHM FOR SIZING OPTIMIZATION OF GEOMETRICALLY NON-LINEAR DOME STRUCTURES

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Abstract. *In this paper a genetic algorithm is proposed to solve the weight minimization problem of a dome considering sizing design variables. The dome is assembled from standard modules composed of ten bars which are grouped into seven distinct design variables. Previous work done by the authors considered the geometrically non-linear behavior of the domes and in this paper loads due the self-weight are included in the analyses during the optimization process. In this way, the load conditions is the sum of the static and the self-weight loads, observing that the self-weight loads vary along the optimization process since the cross-sectional areas of the bars change. In addition, cardinality constraints are introduced to define alternative member groupings of the bars by using a special genetic encoding. This type of optimization can be considered an interesting feature because it allows the designer to automate the search for the best solutions for a given maximum number of distinct cross-sectional areas. Several comparisons are performed using a 120-bar dome as test-bed including linear and non-linear analysis, with and without the consideration of the self-weight loads. Finally a trade-off curve is provided in order to give the designer several options of optimized structures.*

Keywords: *Structural optimization, non-linear analysis, Genetic algorithms*

1 INTRODUCTION

Several types of structures such as domes and frames naturally present a geometrically non-linear behavior, particularly when subject to certain load conditions. Sometimes these structures may present instability which can be avoided by choosing an adequate sizing of cross-sectional areas, configuration, supports, etc.. When the optimum configuration of these structures is searched by means of optimization algorithms the designer needs to carefully consider the effects of geometrically non-linear behavior.

In general, to define the optimum structural configuration, preliminary engineering analysis is usually made of a few conventional solutions and the engineer only chooses the sizes of the members to satisfy the applicable codes, mainly considering economic aspects. The normal and buckling stresses arising from the axial forces in the members, and the displacements at the nodes, are the values that determine the sizes of the members and the final cost of the structure. When displacements are significant a more rigorous –non-linear– analysis is required including the interaction between axial forces and bending moments in members with a high slenderness coefficient in order to check the stability of the structure (Ebenau et. al., 2005), (Kameshi & Saka, 2007).

There are many papers in the literature with respect to structural optimization considering geometrically non-linearities (Saka & Kameshi, 1997), (Pyrz, 1990), (Suleman & Sedaghati, 2005), (Saka, 2007), (Hrinda & Nguyen,2008), (Degertekin et al., 2008). When the real structural behavior deviates from that predicted by the linear model, a non-linear analysis is required to determine if the design derived from the optimized linear model is in fact safe and/or if it is still possible to reduce weight maintaining the required safety margin. On the other hand, considering non-linear models it is possible to increase the total cost of the optimized model with safety.

In this paper, a genetic algorithm (GA) is used to minimize the weight of domes considering both geometrically linear and non-linear static behaviors. The domes are assembled from standard modules composed of 10 members which are grouped into seven distinct profiles. The set of design variables to be considered in the experiments conducted in this paper includes sizing design variables (cross-sectional areas of the bars) and a shape design variable corresponding to the number of standard modules. In addition, cardinality constraints are introduced to define alternative member groupings (Barbosa et. al, 2008), of the standard modules.

It is then clear that such a grouping procedure affects the final results and that its effectiveness depends crucially on the designer's skill in allocating members/variables to a group. As a result, it would be advantageous for the designer to be able to: 1) limit the number of different design parameters (such as cross-sectional areas) in order to (a) achieve economies of bulk purchasing/fabrication, and (b) simplify construction, 2) leave to the optimizer algorithm the task of deciding how to group members and/or design variables, and 3) achieve the best possible solution within the available computational budget. Objectives 1) and 2) can be achieved by introducing a cardinality constraint as shown in (Barbosa et. al, 2008). A cardinality constraint arises naturally when the designer, faced with the task of selecting from a large set of commercial profiles, wishes to employ a reduced number of distinct profiles. Objective number 3) can only be attained by a careful formulation of the optimization problem on the part of the designer. Two co-authors of this paper presented in (Lemonge et. al, 2011a) a study of design optimization of geometrically non-linear truss structures considering cardinality constraints. This type of evolution of the whole structural configuration is an attractive feature because it allows the designer to automate the search for the global configuration of the dome finding the best number of standard modules simultaneously with the shape and sizing design variables for these modules. Several analyses are performed using a 120-bar dome as test-bed and are inspired by the experiments published in (Lemonge et. al, 2010).

This paper is organized as follows: the structural optimization problem is presented in Section 2. The geometrically non-linear approach is summarized in Section 3. The genetic algorithm and the constraint handling technique are discussed in Section 4. A special encoding enforcing the cardinality constraint is presented in Section 5. Computational experiments are presented and discussions concerning the results found are presented in Section 6, and, finally, conclusions are presented in Section 7.

2 THE STRUCTURAL OPTIMIZATION PROBLEM

The standard structural sizing optimization problem reads: find the set of areas $x = \{A_1, A_2, \ldots, A_N\}$ which minimizes the volume of the structure

$$f(x) = \sum_{i=1}^{N} A_i l_i, \tag{1}$$

where l_i is the length of the i-th member of the frame and N is the number of members. When shape design variables are considered the values of l_i change and the weight depends not only on the values of A_i, but also on the joint coordinates of the structure. The problem is usually subject to inequality constraints $g_p(x) \leq 0,\ p = 1, 2, \ldots, \bar{p}$ and sometimes equality constraints $h_q(x) = 0,\ q = 1, 2, \ldots, \bar{q}$. Also, the variables are usually subject to bounds $x_i^L \leq x_i \leq x_i^U$ but this type of constraint is trivially enforced in a GA and does not require further consideration here.

Since geometrically non-linear analysis is considered in some experiments presented in this paper, the compression bars must to be loaded up to their critical values whereas the tension bars up to the yielding stress σ_y. The critical stresses are the same as those considered in the experiments presented in (Saka & Ulker, 1991) and written as:

$$\begin{aligned} &\text{for}\lambda > C_c \qquad \sigma_{cr} = \pi^2 E/\lambda^2, \\ &\text{for}\lambda < C_c \qquad \sigma_{cr} = \sigma_y \left(1 - 0.5\lambda^2/C_c^2\right) \end{aligned}$$

where λ is the slenderness ratio of the bar and $C_c = \sqrt{(2\pi^2/\sigma_y)}$

Also, in this paper, the displacement constraints are considered and written in the normalized expression as:

$$\frac{|d_j|}{d_{max}} - 1 \leq 0, \quad j = 1, 2, \ldots, \bar{p} \tag{2}$$

where d_j is the displacement at the j-th global degree of freedom, and d_{max} is the maximum allowable displacement.

3 THE GEOMETRICALLY NON-LINEAR APPROACH

Although the material of the structures discussed in this paper presents a linear elastic behavior, geometrical non-linearity needs to be considered in the analysis. In order to provide an exact structural analysis, the equilibrium equation in each joint of the structure must be written considering the final (unknown) geometry of the structure. In these equations non-linear terms involving strain and displacement must be considered and the overall matrix equilibrium equation can be written as:

$$[K_T]\{u\} = \{P^*\} \tag{3}$$

where

$$[K_T] = [K_E] + [K_G] \tag{4}$$

and $[K_T]$ is the tangent stiffness matrix of the structure, $[K_E]$ is the linear elastic stiffness matrix, and $[K_G]$ is the geometric stiffness matrix. $\{P^*\}$ is the vector of unbalanced loads. To solve the non-linear equation (3) an iterative scheme is required and the Newton-Raphson Method is adopted. Newtons's Method can be summarized as follows:

1. Perform the linear analysis of the structure and obtain the displacements for the first load step;
2. Update the joint coordinates of the structure considering the displacements obtained in the previous step;
3. Evaluate the internal member actions;
4. Evaluate, at each node, the resultant of the internal member actions in the global axes;
5. Evaluate the unbalanced load vector, which is the difference between the member actions obtained in the previous step and the load applied at the structure in this load step;
6. Assemble the stiffness matrix and check if it is positive-definite. During the non-linear analysis it is necessary to check the determinant of the matrix $[K_T]$ to identify if at any load increment it becomes non-positive-definite. Otherwise, the structure is analysed considering the unbalanced load vector so that new incremental displacements are obtained;
7. Update the nodal coordinates;
8. Repeat steps 3 to 7 until the maximum number of iterations per load step is reached.

It is important to note that in the non-linear procedure used in this paper, when loss of stability occurs the displacements and internal actions in the members of the previous load step are amplified by a factor of 100. This candidate solution is thus strongly penalized and consequently has a low position in the rank of the population of the genetic algorithm.

4 THE GA AND THE PENALTY SCHEME

A *rank-based* selection scheme is adopted in the binary-coded GA used here where the selection scheme operates on the current population sorted according to the values of the fitness function where better solutions have higher rank. A form of elitism is used where the best individual is always copied into the next generation along with one copy where one randomly chosen bit has been changed.

Standard one-point, two-point, and uniform crossover operators are applied with probabilities $p_c^1 = 0.16$, $p_c^2 = 0.32$, and $p_c^u = 0.32$, respectively. Mutation was applied bit-wise to the offspring with rate $p_m = 0.03$.

The adaptive penalty method introduced by Barbosa & Lemonge (Barbosa & Lemonge, 2002), and applied to structural optimization problems in (Barbosa & Lemonge, 2000), (Barbosa & Lemonge, 2008), (Lemonge et al., 2011b), are adopted here to enforce all the mechanical constraints considered in the numerical experiments.

5 CARDINALITY CONSTRAINTS

An additional constraint may be considered with respect to the desire of the designer to link the members of the structure in groups. For example, in order to explore architectonic effects, a designer may require that at most four distinct profiles are to be used in the whole structure, leading to the need of arranging all bars in only four groups.

The members in each group present the same design characteristics, i.e., same material and same cross-sectional properties. By linking the bars in a reduced number of groups (Barbosa & Lemonge, 2008), a gain in the cost of fabrication, storing, and checking is achieved. An intuitive and coherent member grouping consisting of seven different cross-sectional areas is adopted in the 120-bar dome to be analysed in this paper. The question arising is: is that the best member grouping? or even, how will the designer choose a good member grouping?

Engineering design practice, in conjunction with the designer's expertise will provide some basic rules that guide the definition of the bars to be linked in a certain group. However, the task of discovering the best member grouping can be extremely complex due the high number of unknown feasible solutions, each one with distinct member groupings.

We are particularly interested here in the additional constraint requiring that no more than m ($m \leq N$) different cross-sectional areas should be used, that is,

$$A_i \in C_m = \{S_1, S_2, \ldots, S_m\}, \quad i = 1, 2, \ldots, N$$

where the areas $S_j, j = 1, 2, \ldots, m$ are unknown but belong to a larger ($M > m$) given set $S = \{A_1, A_2, \ldots, A_M\}$. The standard structural optimization problem (without the cardinality constraint) is recovered when $m = N$.

Although the introduction of cardinality constraints in structural optimization problems is clearly desirable, few papers can be found in the literature dealing with problems that include such constraints or where automatic variable linking is performed. Some of them are: (Biedermann & Grierson, 1997), (Galante, 1996), (Shea et al., 1997), (Togan & Daloglu, 2006).

A very important decision concerning a GA is that of the *encoding* of the variables. The special encoding used in this paper in order to handle the member grouping task ensures that all candidate solutions automatically satisfy the cardinality constraint (Barbosa & Lemonge, 2005), (Barbosa et al., 2008).

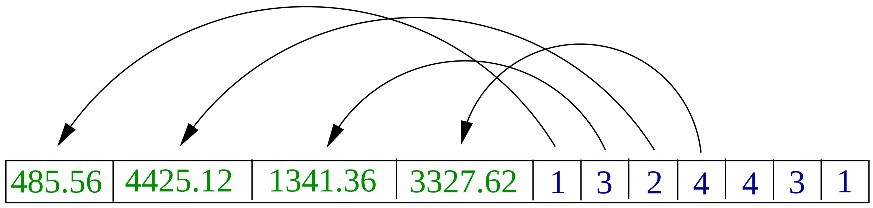
For the continuous case (Barbosa et al., 2008), one specifies the maximum number of different sizes allowed, for instance $m = 4$, and the chromosome structure proposed here is as follows:

size(1) size(2) size(3) size(4) pt(1) pt(2) ... pt(7), with `size(i)`$\in [A^i_{min}, A^i_{max}]$, where again 7 design variables/cross-sectional areas are assumed. Analogously to the discrete case, the area of the `i`-th bar in the structure is given by

$$\texttt{area}(\texttt{i}) = \texttt{size}(\texttt{pt}(\texttt{i})).$$

Now the variable `size(i)` is continuous and has lower (A^i_{min}) and upper (A^i_{max}) bounds specified by the designer, and the variable `pt` is again a pointer to one of the $m = 4$ sizes listed at the left end of the chromosome. The chromosome exemplified in Figure 1 corresponds to the following values of the design variables: $A_1 = A_7 =$ 485.56 mm^2, $A_2 = A_6 =$ 1341.36 mm^2, $A_3 =$ 4425.12 mm^2, and $A_4 = A_5 =$ 3327.62 mm^2.

Figure 1: Example of chromosome for the continuous variable case. The third design variable has its area defined by the third pointer value (2) which indicates the second continuous area value (1341.36 mm^2). For convenience, the values are shown in decimal representation; as explained in the text, a binary representation is used in the experiments.



6 COMPUTATIONAL EXPERIMENTS

The dome depicted in Figure 2 is a 120-bar three-dimensional truss previously discussed in the References (Saka & Ulker, 1991), (Ebenau et al., 2005), (Capriles et al., 2007), (Lemonge et al., 2010), (Lemonge et al., 2011a). One can note that the dome is made up of standard modules, as depicted in Figure 3, containing 10 members grouped in seven distinct profiles. Thus, the dome shown in Figure 2 presents 12 standard modules leading to a total of 120 bars. This dome is the baseline structure of a set of experiments conducted in this section. The dome is subjected to a downward vertical equal to 60 kN at node 1, 30 kN at nodes 2 to 14 and 10 kN at nodes 15 to 37. The displacements are limited to 1 cm, in the x, y and z directions. The density of the material is 7860 kg/m^3 and Young's modulus is equal to 21000 kN/cm^2 and the yielding stress is σ_y = 240 N/mm^2. The cross-sectional areas are the sizing design variables and 200 mm^2 is the minimum value for them.

A set of 28 distinct experiments are conducted in order to demonstrate different options available to the designer by means of appropriate encodings. The differences among them is the linear or non-linear analysis as well as the maximum number of distinct cross-sectional areas (cardinality constraint m), presented in the optimized solution. The limits for the sizing design variables are 200 mm$^2 \leq A_i \leq$ 14000 mm^2.

The population size for all experiments is equal to 80 evolved with a maximum number of generations equal to 40 for m = 1, 80 for m = 2 and m = 3, 100 for m = 4, 200 for m = 5 and m = 6 and, finally, 240 for m = 7. The case where $m = 7$ means no cardinality constraints and the chromosome contains only 7 sizing design variable without the left part represented in Figure 1. Also, 10 independent runs were set for each experiment. The number of load steps and iterations in the the Newton Raphson Method are set equal to 8.

Figure 2: 120-bar truss dome.

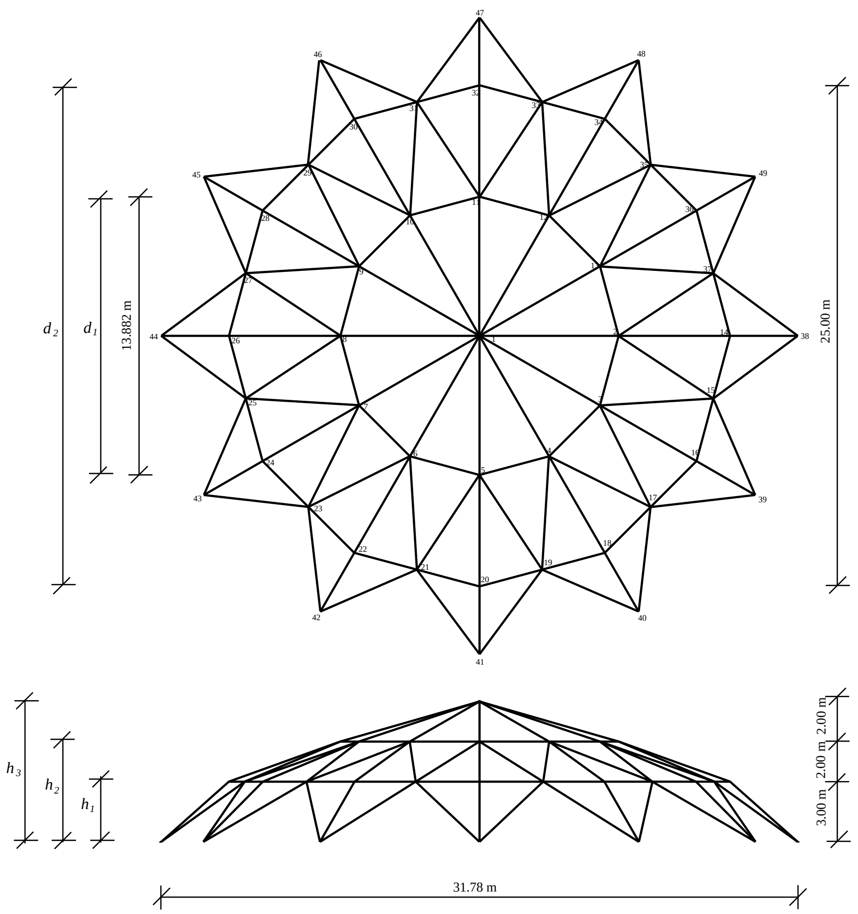


Figure 3: The standard module.

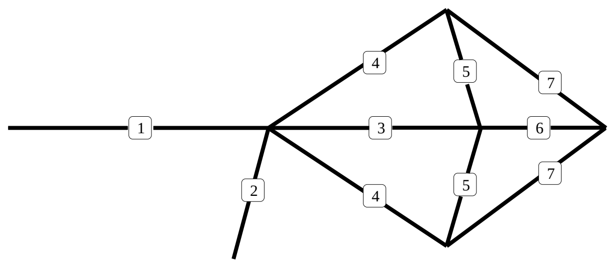


Table 1 presents the best final solutions for the optimized domes for the 28 experiments. In this Table "LI" and "NL" mean Linear and Non-linear Analysis, respectively, and the superscript dl means dead loads. It is easy to observe the heavier weights found when m = 1 was set against lighter weights found when m = 7 was set, as expected.

Tables 2 to 5 present the final sizing design variables (mm^2), considering linear and non-linear analysis with and without dead loads. Just one comparison is possible with the results encountered in the literature. This comparison is provided in Tables 6, linear , and 7, non-linear analysis. The results presented by Saka & Ulker in (Saka & Ulker, 1991) provide only cases without member grouping and dead loads. Observing these comparisons the experiments analysed in this paper found better results than those presented in (Saka & Ulker, 1991). Considering linear analysis the final weights of 7283.83 kg and 7837.67 kg (dl) are better than 8511 kg. In the same way, considering non-linear analysis, 5747.88 kg and 6014.88 kg (dl) are better than 7587 kg.

As expected, for both linear and non-linear analysis the final weights considering dead loads are greater than those without dead loads. On the other hand, it is very important to observe that results considering non-linear analysis, except for m = 1 and m = 2 (dl), achieved the best results. The trade-off curve between final weight and number of distinct cross-sectional areas is displayed in Figure 4.

Table 1: Final weights in kg.

m	LI	LI^{dl}	NL	NL^{dl}
1	13244.44	14726.57	13486.95	15032.08
2	8266.46	8939.79	6946.17	9581.94
3	7692.54	8201.20	6189.96	6603.04
4	7470.34	8036.94	6137.86	6319.49
5	7453.47	7984.02	5973.47	6292.84
6	7400.05	7915.56	5914.50	6280.29
7	7283.83	7837.67	5747.79	6014.88

Table 2: Final weights in kg - Linear analysis without dead loads.

m	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
1	2692.46	2692.46	2692.46	2692.46	2692.46	2692.46	2692.46	13244.44
2	821.46	3620.46	3620.46	821.46	3620.46	821.46	821.46	8266.46
3	808.31	5879.23	2515.78	808.31	2515.78	808.31	808.31	7692.54
4	770.56	5590.91	1929.46	770.56	3053.15	770.56	770.56	7470.34
5	715.90	5861.11	1489.96	715.90	3040.87	1489.96	715.90	7453.47
6	762.25	5500.11	1661.65	762.25	3118.94	990.45	762.25	7400.05
7	582.55	4834.56	1975.85	851.56	3049.15	1213.06	668.65	7283.83

Table 3: Final weights in kg - Linear analysis with dead loads.

m	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
1	2993.76	2993.76	2993.76	2993.76	2993.76	2993.76	2993.76	14726.57
2	895.86	3898.45	3898.45	895.86	3898.45	895.86	895.86	8939.79
3	738.75	4430.24	1805.20	738.75	4430.24	1805.20	738.75	8201.20
4	773.00	5103.08	1621.57	773.00	3919.89	1621.57	773.00	8036.94
5	659.33	5044.72	1877.07	732.98	3733.20	1877.07	732.98	7984.02
6	651.45	5517.26	2168.86	980.46	3389.55	980.46	651.45	7915.56
7	589.06	5169.85	1925.66	855.76	3370.65	1364.96	847.27	7837.76

Table 4: Final weights in kg - Non Linear analysis without dead loads.

m	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
1	2741.75	2741.75	2741.75	2741.75	2741.75	2741.75	2741.75	13486.95
2	458.04	3566.64	3566.64	458.04	3566.64	458.04	458.04	6946.17
3	442.10	3579.96	1481.17	442.10	3579.96	1481.17	442.10	6189.96
4	454.58	3649.77	1325.29	454.58	3649.77	1325.29	454.58	6137.86
5	448.86	4305.66	1293.85	448.86	3099.25	1293.85	448.86	5973.53
6	454.38	3986.51	1834.19	454.38	2734.81	1190.53	454.38	5914.50
7	419.15	3912.85	1795.71	552.38	2482.75	948.48	494.06	5747.79

Table 5: Final weights in kg - Non Linear analysis with dead loads.

m	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
1	3055.86	3055.86	3055.86	3055.86	3055.86	3055.86	3055.86	13486.95
2	1393.58	4253.91	1393.58	1393.58	4253.91	1393.58	1393.58	9581.94
3	458.62	3951.15	1494.01	458.62	3951.15	1494.01	458.62	6603.04
4	485.56	4425.12	1341.36	485.56	3327.62	1341.36	485.56	6310.49
5	484.90	4391.06	1547.35	484.90	2991.54	1547.35	484.90	6292.84
6	484.27	4458.43	1486.96	484.27	3033.65	1486.96	484.27	6280.29
7	418.75	4028.66	1429.16	723.06	2834.96	918.07	515.39	6014.88

Table 6: Comparison of results considering linear analysis with and without dead loads.

Ref.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
(Saka & Ulker, 1991)	1666	4484	2489	966	2193	1659	1174	8511
This Study	582.55	4834.56	1975.85	851.56	3049.15	1213.06	668.65	7283.83
This study ^{dl}	589.06	5169.85	1925.66	855.76	3370.65	1364.96	847.27	7837.67

Table 7: Comparison of results considering non-linear analysis with and without dead loads.

Ref.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	W
(Saka & Ulker, 1991)	1750	4556	2545	844	2230	1596	390	7587
This Study	419.15	3912.85	1795.71	552.38	2482.75	948.48	494.06	5747.79
This study ^{dl}	418.75	4028.66	1429.16	723.06	2834.96	918.07	515.39	6014.88

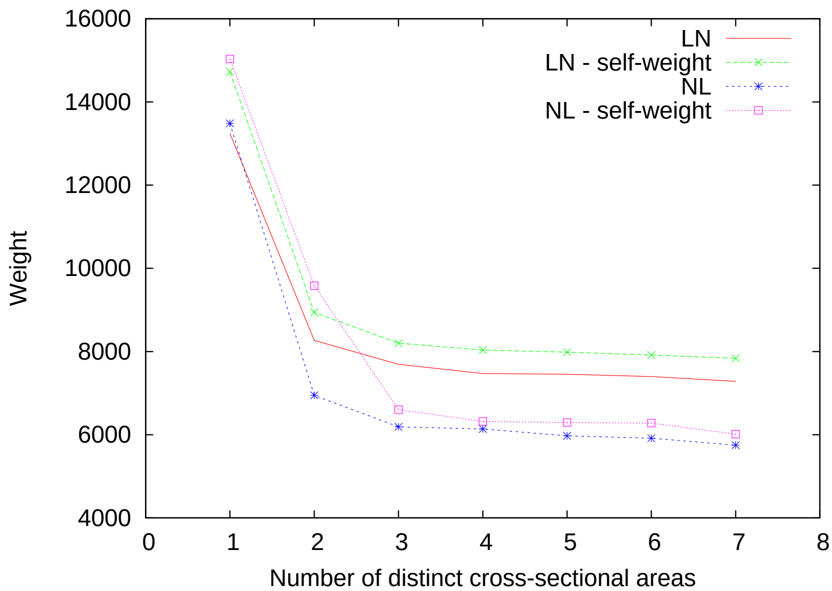


Figure 4: Trade-off between Weight and Number of distinct cross-sectional areas.

7 CONCLUSIONS

In this paper, a 120-bar dome made up of standard modules was used as a test-bed problem to discover optimal solutions considering sizing design variables. The GA searches for the optimum areas of the cross-sectional areas of the bars (continuous).

The results are interesting since the consideration of dead loads in the analysis introduces more realism into the structural optimization problems. In addition, the use of a special encoding to reach the best member groupings is an attractive tool relieving the designer of the task of discovering these solutions.

Ii also identifies the importance of respecting results that consider both linear and non-linear analysis. The best solutions were found considering non-linear analysis, but it often counter-intuitive and thus not considered by the designer.

New experiments will be conducted by introducing new shape design variables such as the number of standard modules, the heights and the inner diameters of rings of the dome, as presented in (Lemonge et. al, 2010). Also, in a real world problem of dome structural optimization, as well as in other structures, additional design requirements may arise.

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