DYNAMIC ANALYSIS OF SHALLOW TRUSSES

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Abstract. *This paper aims to study the nonlinear dynamics of low rise structures such as trusses.*

It is well known that under almost static loading a symmetric planar truss consisting of two deformable rods, also known as von Mises truss, due its geometry, is subjected to snap-through stability. There is dynamic passage of the structure to another remote and stable equilibrium configuration, jumping to the post-critical configuration with large displacements and reversal of concavity.

However, if the load is dynamic, for example, harmonic, the system's response acquires a wealth of possible behaviors, depending on the amplitude and frequency of loading. The response may result in periodic vibrations of several different periods, almost periodic, chaotic etc.

This work intends to make a parametric study, as exhaustive as possible, of the problem of a simple symmetric planar truss with two bars, linear elastic behavior and a concentrated mass in the central node, modeled mathematically by differential equations obtained by the method of Newton. The evaluation of the response and the characterization of its stability will be given by numerical integration of this mathematical model and a geometric study by obtaining time histories and phase plans.

Keywords: *Nonlinear dynamics, Trusses, Dynamics of structures, Stability*

1. INTRODUCTION

This project aims to study the nonlinear dynamics of low rise structures such as trusses.

It is well known that under almost static loading a a symmetric planar truss consisting of two deformable rods, also known as von Mises truss, due its geometry, is subjected to snap-through stability, in which there is dynamic passage of the structure to another remote and stable equilibrium configuration, jumping to the post-critical configuration with large displacements and reversal of concavity.

However, if the load is dynamic, for example, harmonic, the system's response acquires a wealth of possible behaviors, depending on the amplitude and frequency of loading. The response may result in periodic vibrations of several different periods, almost periodic, chaotic etc.

This work intends to make a parametric study, as exhaustive as possible, of the problem of a simple truss with two bars, linear elastic behavior and a concentrated mass in the central node, modeled mathematically by differential equations obtained by the method of Newton. The evaluation of the response and the characterization of its stability will be given by numerical integration of this mathematical model and a geometric study by obtaining time histories and the phase plans.

The symmetric truss of two bars is one of the simplest possible structures. However, its study allows extension to more complex practical problems, such as low rise arches and shells present in large roof structures exited by wind loads and earthquakes. It is also known as Von Mises truss (Bazant and Cedolin, 1991), as displayed in Fig. 1.

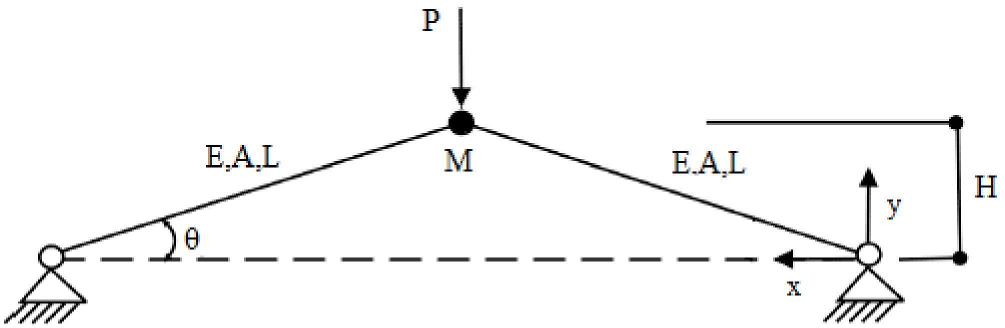

Figure 1. Von Mises truss (Adapted Greco and Vicente, 2009)

The initial length L bars form an initial angle θ to the horizontal direction. They are supported at the ends in order to restrict motions in the plan. In the center articulation, we consider a vertical periodic load,

$$P = Po\sin(\omega t) \tag{1}$$

The system´s mass is concentrated at the intermediate node. We consider only the vertical displacement degree of freedom, resulting in symmetrical motions of the model. The bars are sufficiently rigid so that local instability does not occur (bar buckling).

In complex engineering applications we may have low rise arches and shells. They are commonly found in civil, petroleum and aerospace structures, for example. For large models, the analysis tool should be a dynamic nonlinear finite element formulation, as implemented in ANDROS program, BRASIL (1990); MAZZILLI and BRASIL (1995).

The highly generic and interdisciplinary quality of developments in Nonlinear Dynamic gained in the last few decades has allowed thousands of applications in almost all areas of science and technology, and even beyond. Where modeling and analysis of complex nonlinear phenomena, possibly chaotic, is necessary, its methods can be extremely important. Chaos and nonlinear dynamics have developed into an emergent field with fundamentals in engineering and applied sciences. Applications of analytical models, numerical simulations and experimental research lead to new skills and challenges. It should be kept in mind that actual vibration systems are better described as multi-frequency, time varying, stochastic systems etc. Linear narrow-band models are highly inefficient under these circumstances. Linear models are not able to respond over a wide range of frequencies and amplitudes, suggesting an intrinsic nonlinearity of reality.

2. A SIMPLIFIED ANALYSIS OF THE PROBLEMA

Initially we started with a simpler form of the motion that the mass (m) will perform in the truss

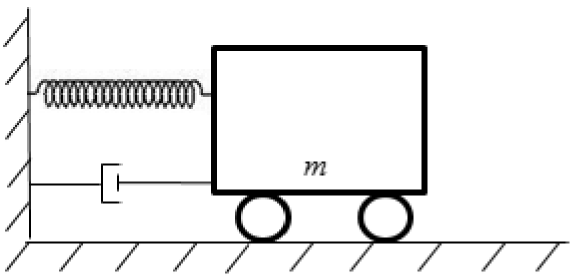

Figure 2. The simplest one degree of freedom model.

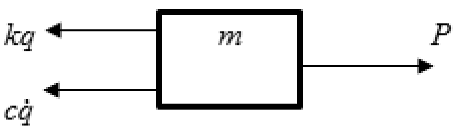

Figure 3. Mass (m) under action of loading P, spring and damping forces.

Quantities that will be used:

L: Length (m)

A: Cross section (m^2)

m : Mass (kg)

N :Normal Force or Axial Force (N)

E :Modulus of Elasticity of a Material $\left(\frac{N}{m^2}\right)$

K :Stiffness $\left(\frac{N}{m}\right)$

Functions:

Force as a function of time: $P = P(t)$

Mass position as a function of time: $q = q(t)$

Axial force as a function of time: $N = N(t)$

Velocity: $\dot{q} = \frac{d}{dt}q$

Acceleration: $\ddot{q} = \frac{d^2}{dt^2}q$

$$m\ddot{q} + c\dot{q} + kq = P \tag{2}$$

$$m\ddot{q} = P - c\dot{q} - kq \tag{3}$$

where,

$$\ddot{q} = \frac{1}{m}[P - c\dot{q} - kq] \tag{4}$$

3. RETURNING TO TRUSS ANALYSIS

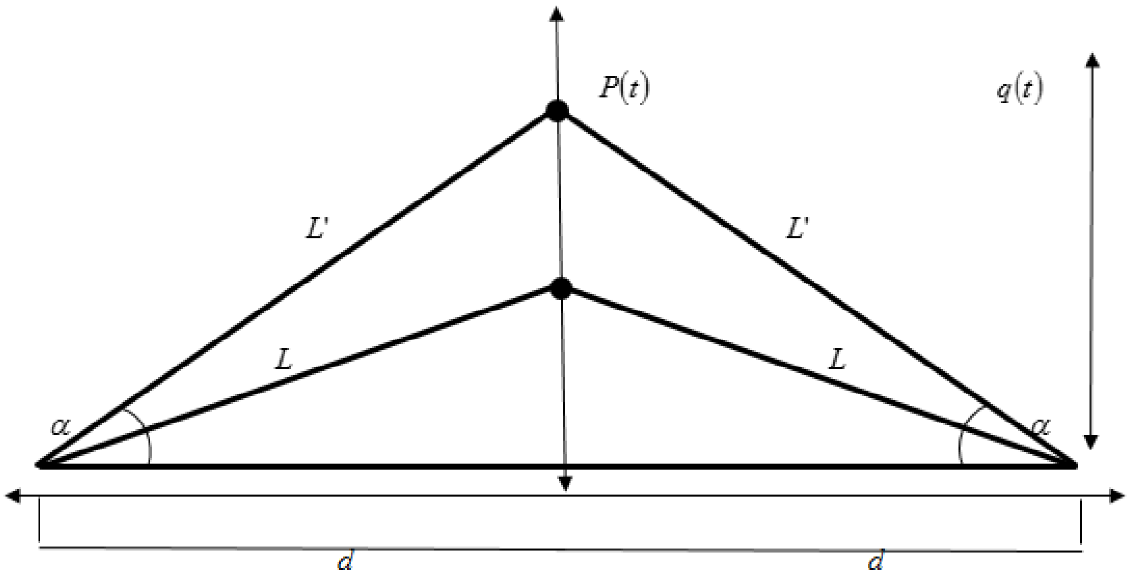

Figure 4. Schematic example of the truss with two nodes fixed and one free node

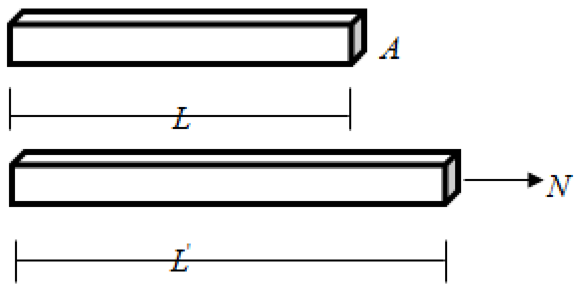

Figure 5. Scheme for linear expansion experienced by the bar length *L*

Length of bar after mass motion:

$$L' = \sqrt{d^2 + q^2} \tag{5}$$

Analyzing bar length variation:

$$\Delta L = L' - L \tag{6}$$

Using the elastic strain concepts we have that the variation in the length is proportional to the applied axial force and the length of the bar and is inversely proportional to the modulus of elasticity of the material and the cross sectional area of the bar. This gives us the following equation:

$$\Delta L = \frac{NL}{EA} \tag{7}$$

Isolating the axial force, we get:

$$N = \frac{EA}{L}\Delta L \tag{8}$$

Knowing that the stiffness is

$$\frac{EA}{L} \tag{9}$$

we get,

$$N = K\Delta L \tag{10}$$

Applying the equilibrium of vertical forces (2nd Newton’s law)

$$m\ddot{q} = P - mg - c\dot{q} - 2Nsen\alpha \tag{11}$$

where,

$$\sin\alpha = \frac{q}{L'} = \frac{q}{\sqrt{d^2+q^2}} \tag{12}$$

therefore:

$$m\ddot{q} = P - mg - c\dot{q} - 2\frac{EA}{L}\frac{(\sqrt{d^2+q^2}-L)}{\sqrt{d^2+q^2}}q \tag{13}$$

Recalling,

$$m\ddot{q} = P - mg - c\dot{q} - 2\frac{EA}{L}\frac{(\sqrt{d^2+q^2}-L)}{\sqrt{d^2+q^2}}q \tag{14}$$

isolating the $\ddot{q}$, we get:

$$\ddot{q} = \frac{1}{m}[P - mg - c\dot{q} - 2\frac{EA}{L}\frac{(\sqrt{d^2+q^2}-L)}{\sqrt{d^2+q^2}}q] \tag{15}$$

For better development of the studied situation, we transform the 2nd order Ordinary Differential Equation (15) in three 1st order state state equations:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = \frac{1}{m}[P - mg - c\dot{q} - 2\frac{EA}{L}\frac{(\sqrt{d^2+q^2}-L)}{\sqrt{d^2+q^2}}q] \\ \dot{y}_3 = 1 \end{cases} \tag{16}$$

4. SIMULATIONS

Programs used to do the simulations in MATLAB

Initial Conditions program

```
function y0=condvdpp
% initial conditions of ODE truss movement
y0(1)=1;
y0(2)=0;
y0(3)=0;
```

Program with derivatives

```
function f=vdpp(y)
% differential equations of first order
% vdpp.m 08/10/2014
P=0;
%p0=10;
%Omega=1;
%P=p0*sin(Omega*y(3));
m=100;
L=0.5;
d=0.4;
g=0;
c=35;
k=200;
f(1,1)=y(2);
f(2,1)=(P-m*g-c*y(2)-2*k*(sqrt(d^2+y(1)^2)-L)/sqrt(d^2+y(1)^2)*y(1))/m;
f(3,1)=1;
```

Main program

```
ne=3;
ti=30; %integration time in seconds
h=0.005; % step
np=ti/h; % steps numbers
%
% sizing matrices
y=zeros(ne,np);
%
% initial conditions
%
y(:,1)=condvdpp;
%
% main loop
%
for n=1:np
    yarg=y(:,n);
    k1=vdpp(yarg);
    yarg=y(:,n)+0.5*h*k1;
    k2=vdpp(yarg);
    yarg=y(:,n)+0.5*h*k2;
    k3=vdpp(yarg);
    yarg=y(:,n)+h*k3;
    k4=vdpp(yarg);
    y(:,n+1)=y(:,n)+h*(k1+2*k2+2*k3+k4)/6;
end
%
% plot
%time * position
plot(y(3,:),y(1,:))
pause
%phase plane
plot(y(1,:),y(2,:))
```

Simulations with P equal zero

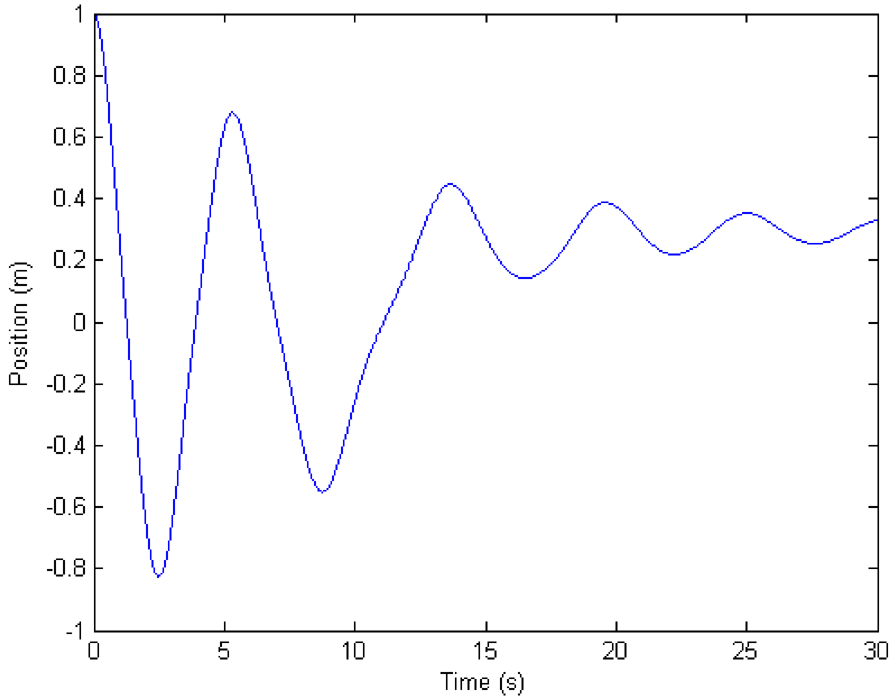

Figure 6. Position – Time history with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=20; k=200

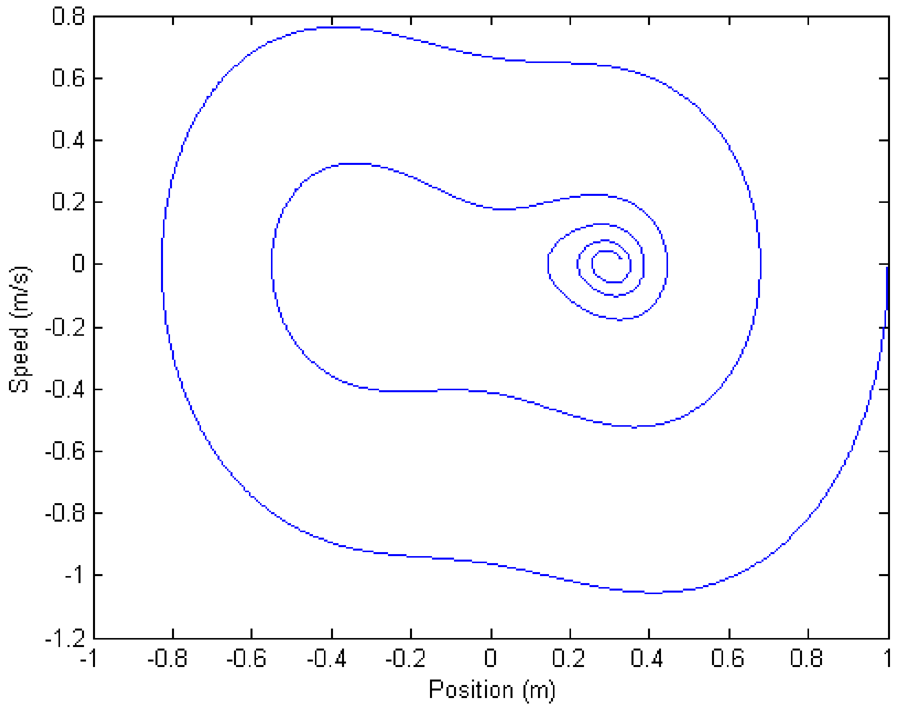

Figure 7. Phase Plane with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=20; k=200

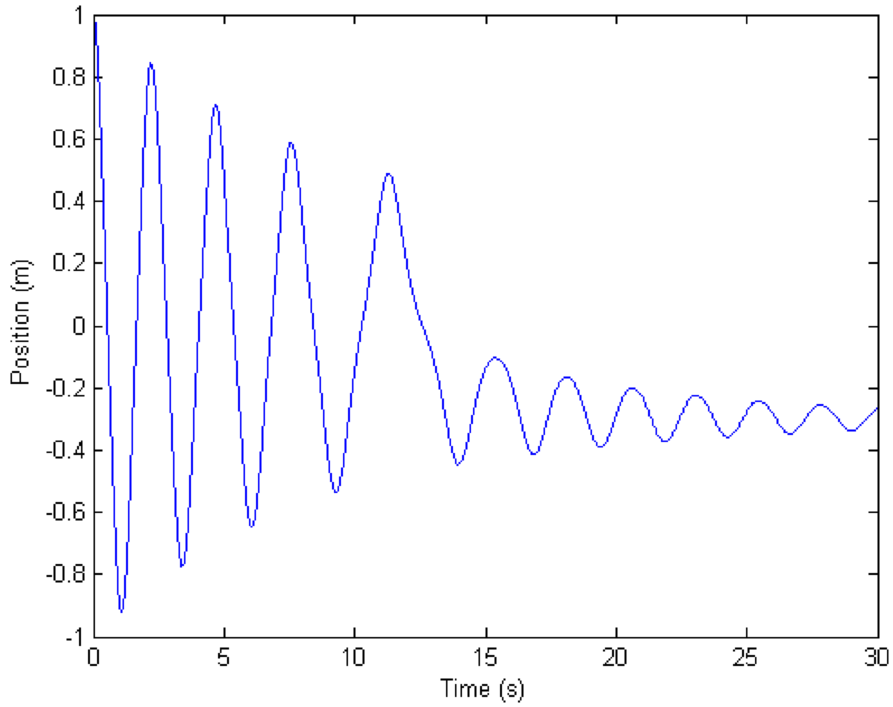

Figure 8. Position – Time history with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=20; k=1000

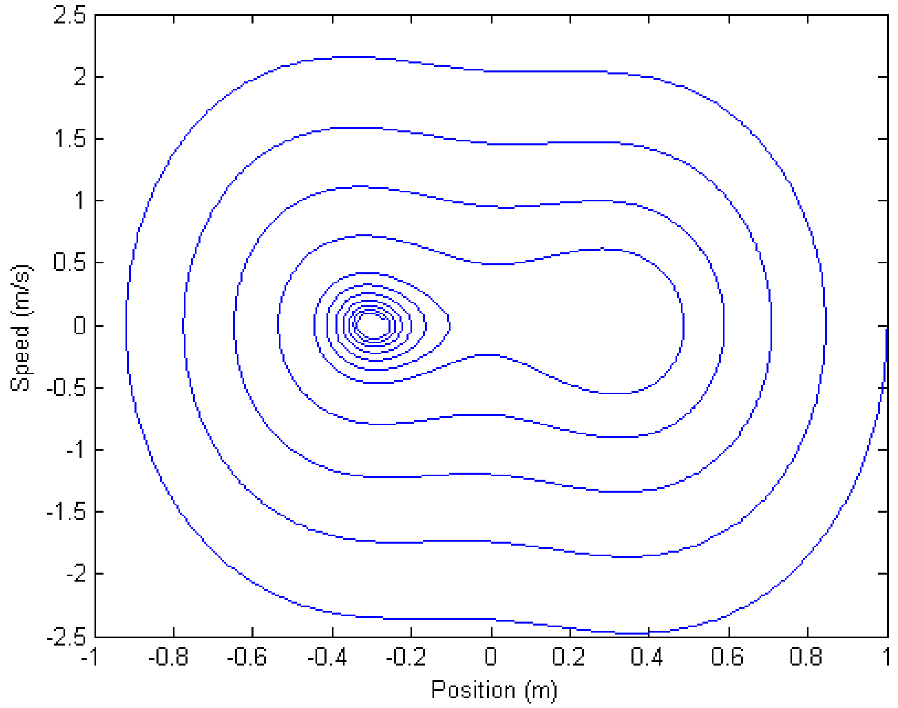

Figure 9. Phase Plane with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=20; k=1000

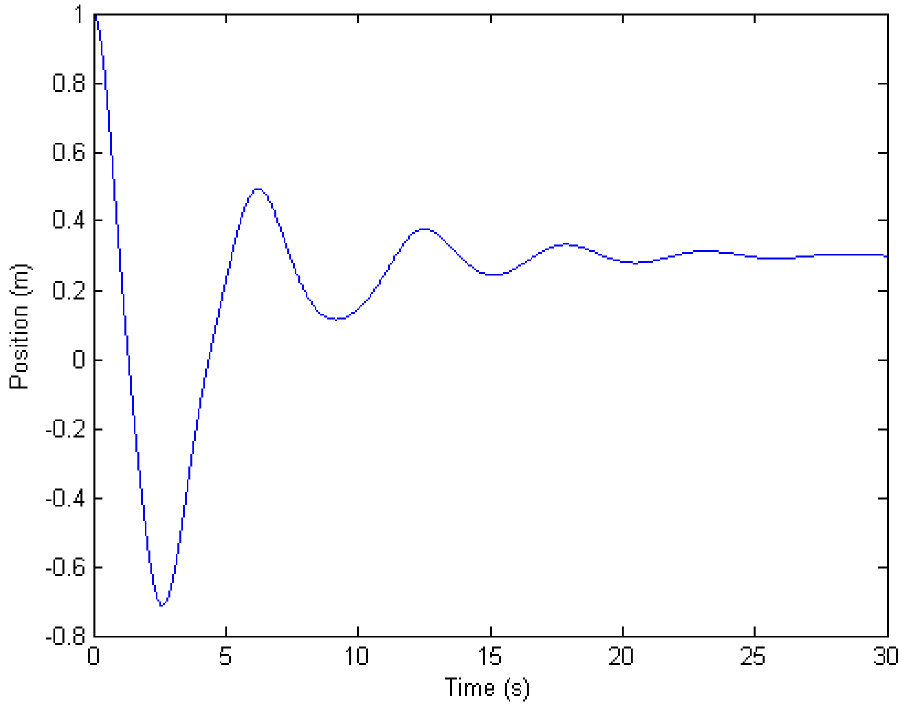

Figure 10. Position – Time history with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=35; k=200

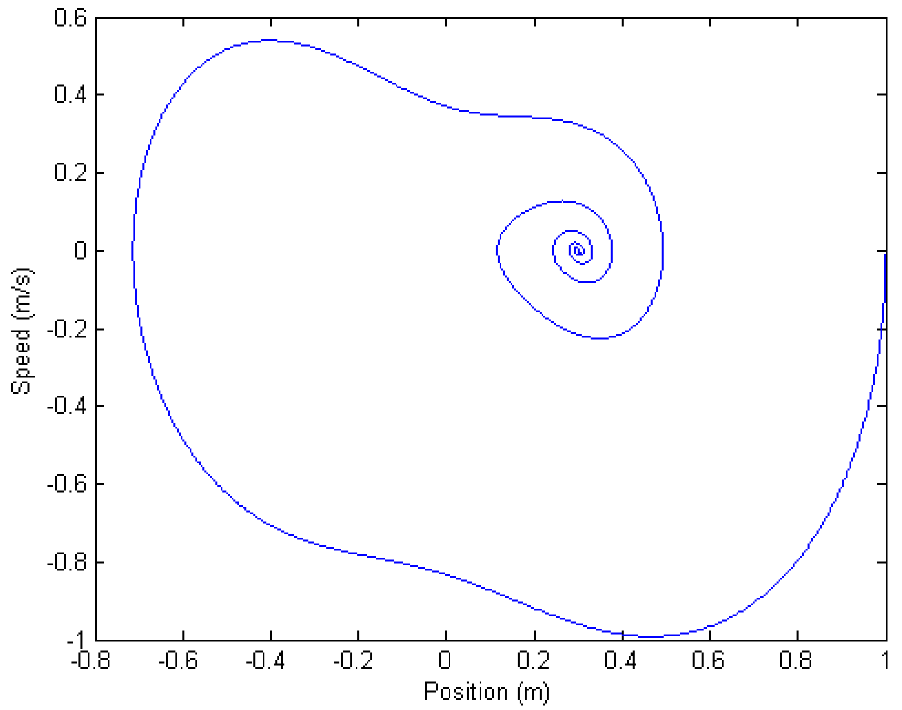

Figure 11. Phase Plane with conditions: P=0; m=100; L=0.5; d=0.4; g=0; c=35; k=200

Simulations with $P = Po\sin(\omega t)$

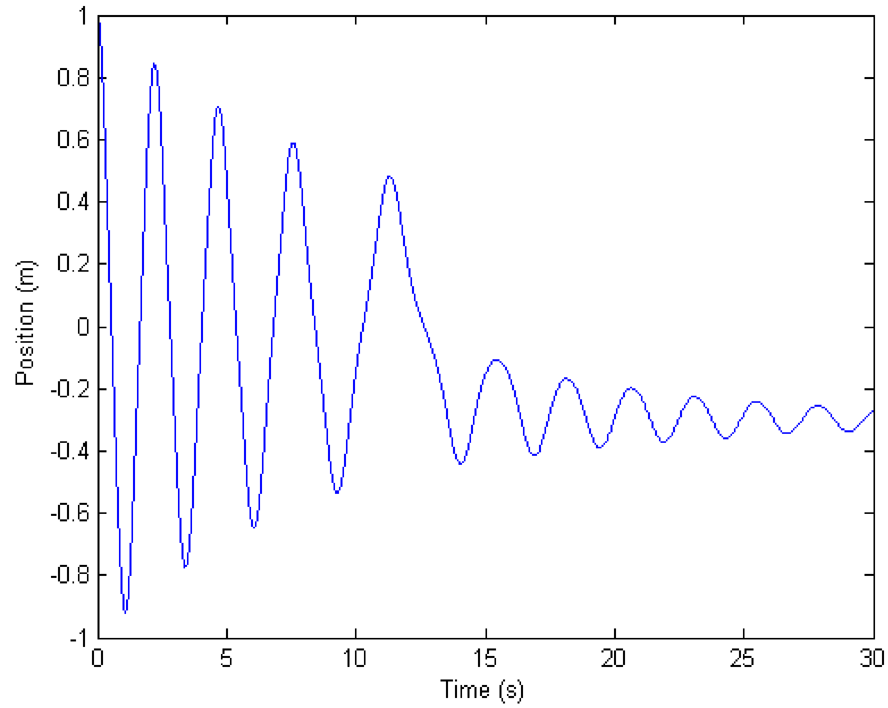

Figure 12. Position – Time history with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=1000

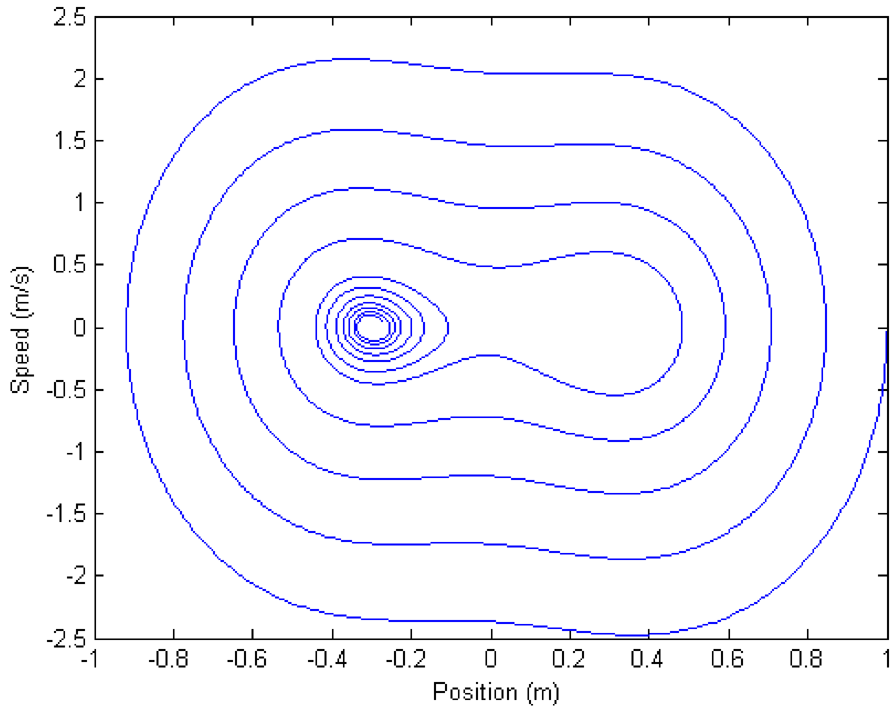

Figure 13. Phase Plane with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=1000

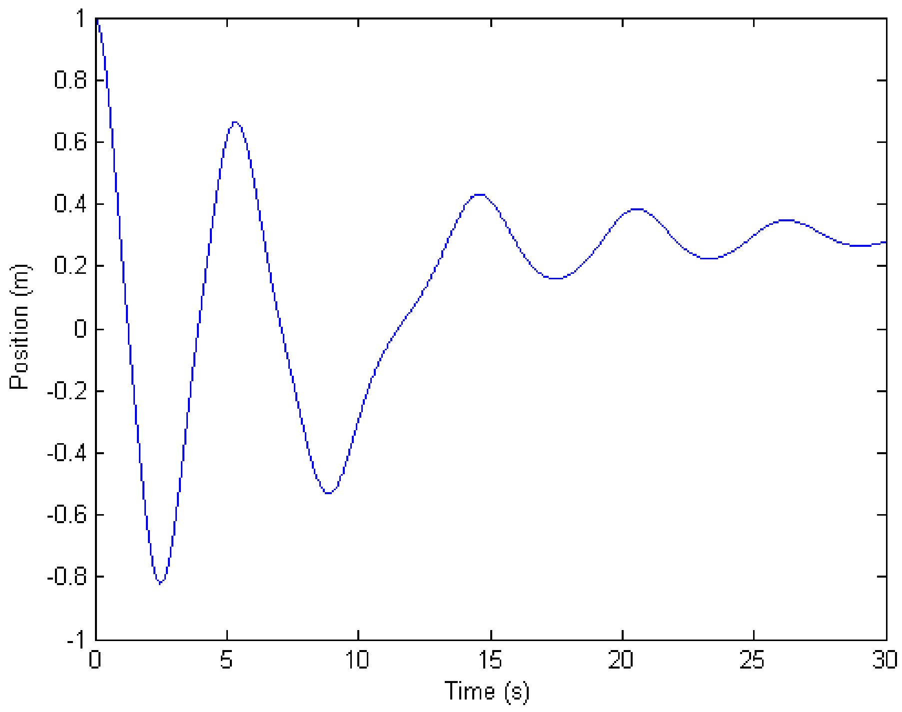

Figure 14. Position – Time history with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

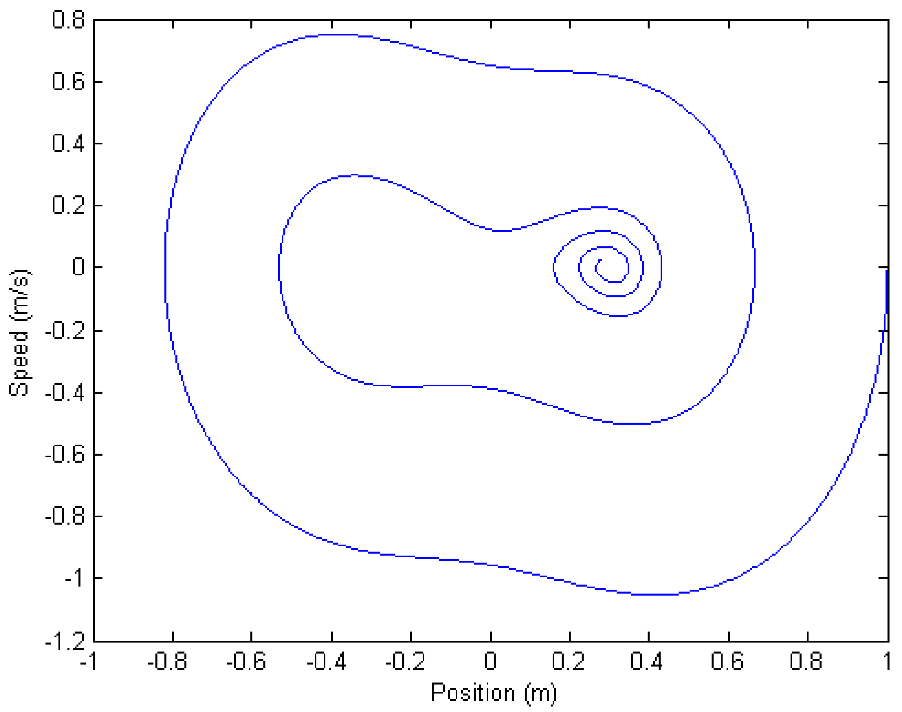

Figure 15. Phase Plane with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

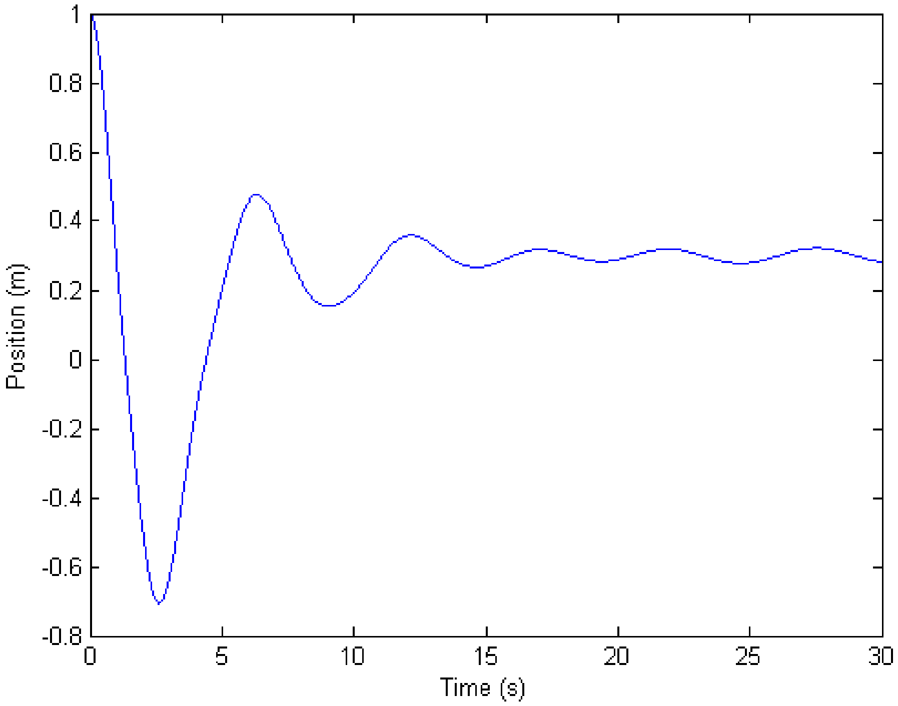

Figure 16. Position – Time history with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3));m=100; L=0.5; d=0.4; g=0; c=35; k=200

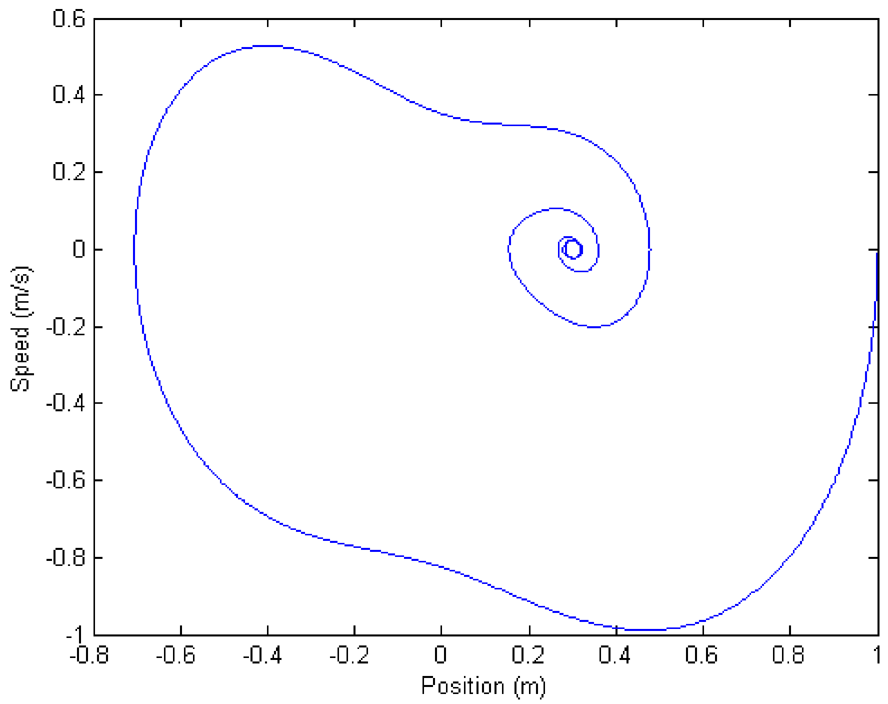

Figure 17. Phase Plane with conditions: p0=1; Omega=1; P=p0*sin(Omega*y(3));m=100; L=0.5; d=0.4; g=0; c=35; k=200

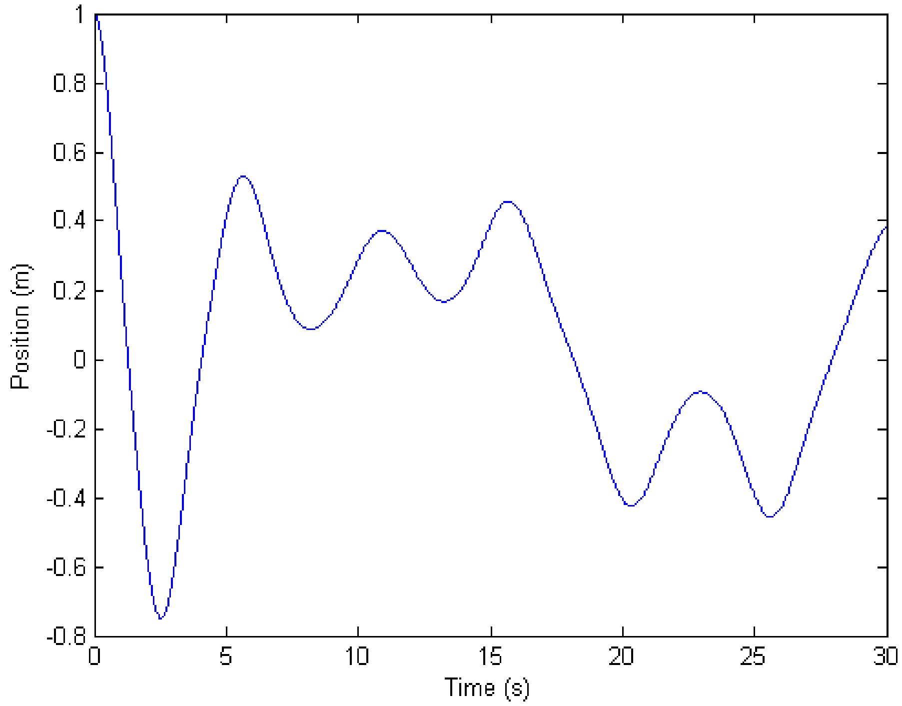

Figure 18. Position – Time history with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

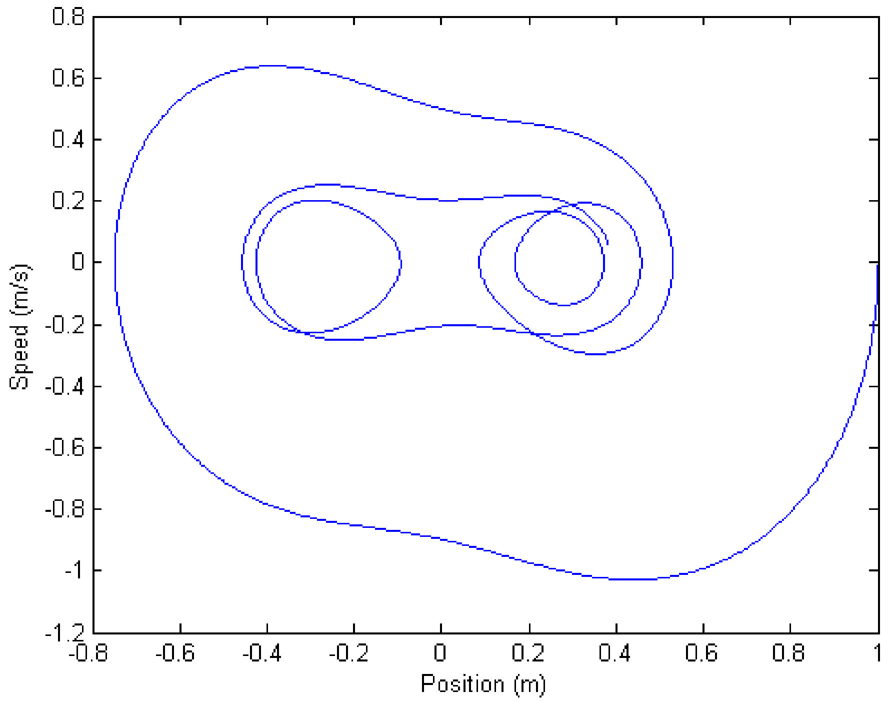

Figure 19. Phase Plane with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

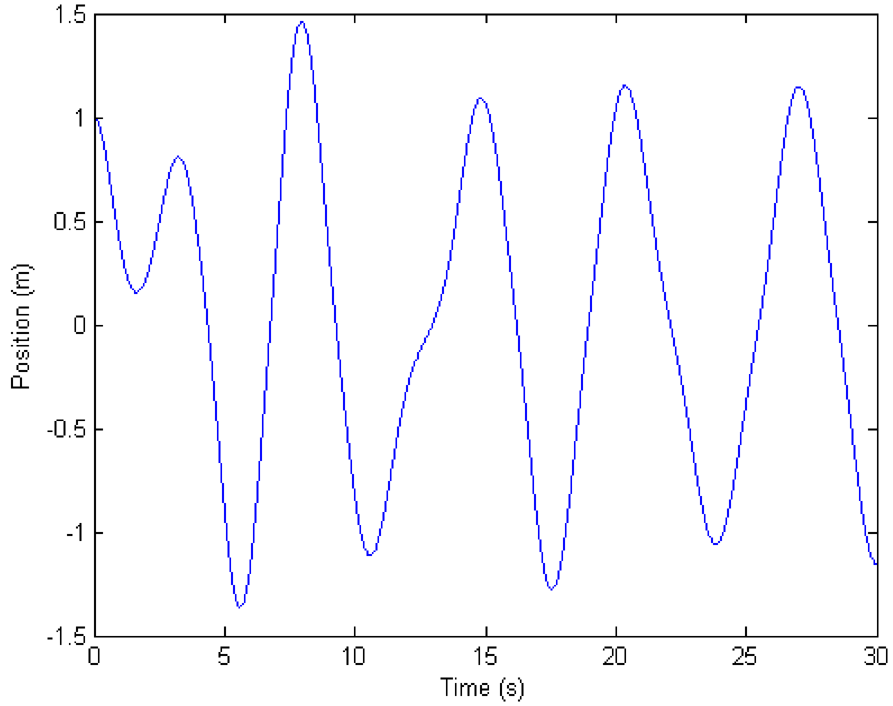

Figure 20. Position – Time history with conditions: p0=100; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

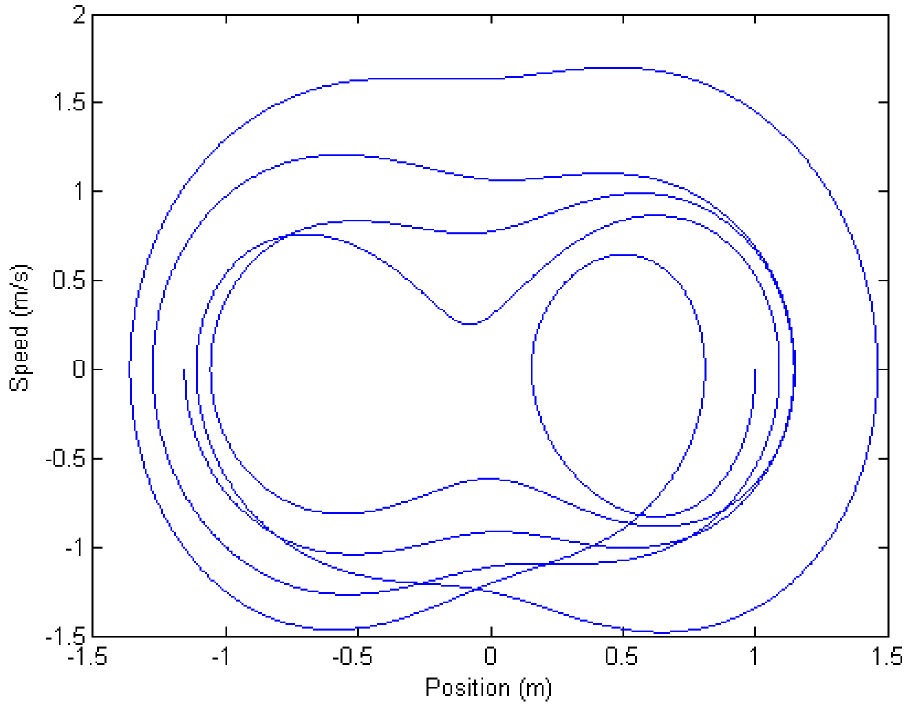

Figure 21. Phase Plane with conditions: p0=100; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=200

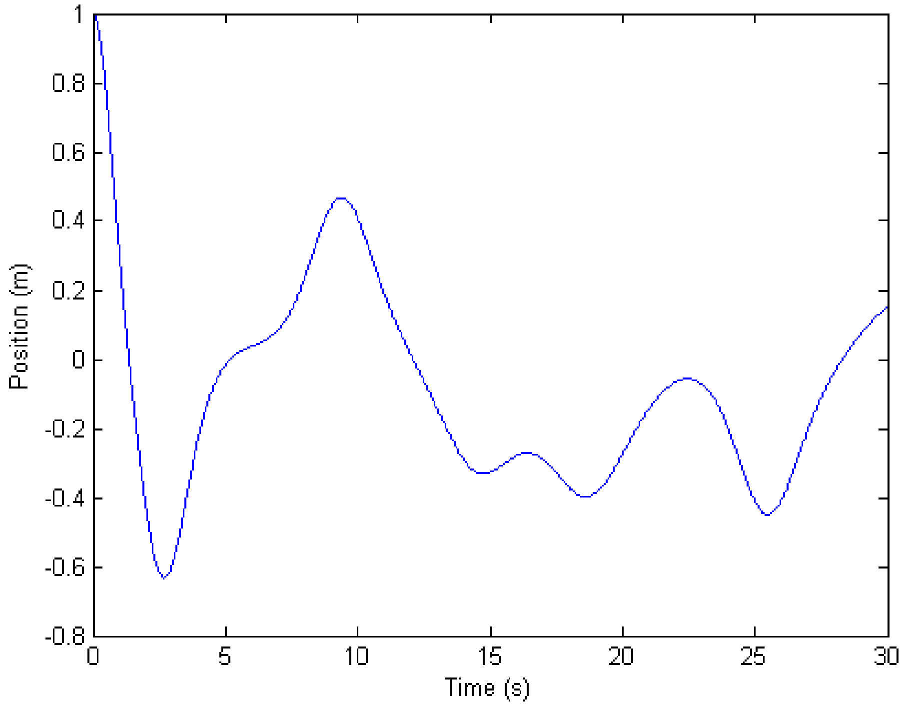

Figure 22. Position – Time histoy with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=35; k=200;

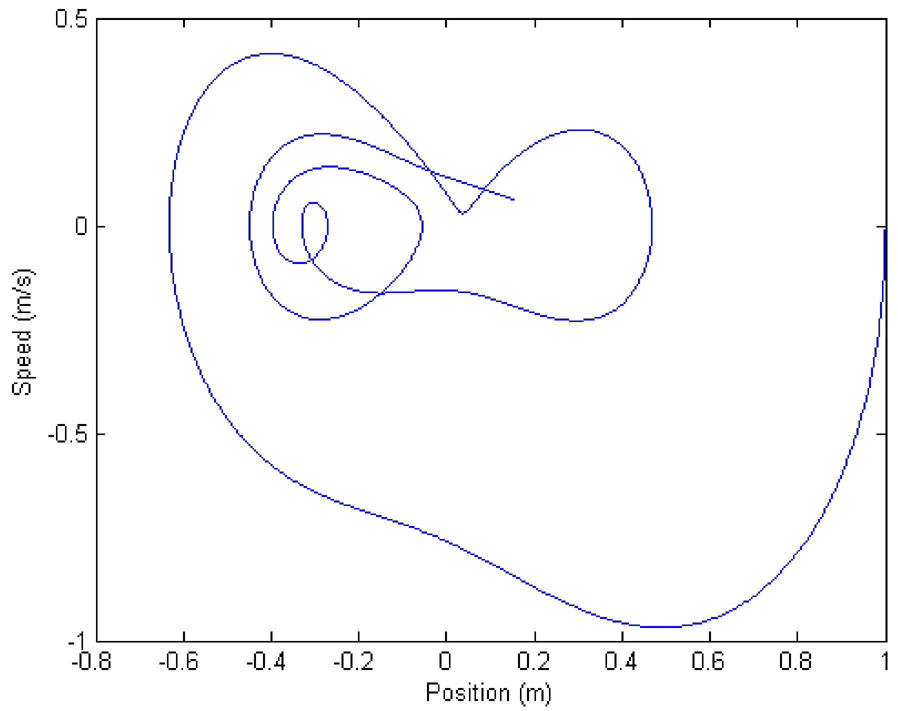

Figure 23. Phase Plane with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=35; k=200

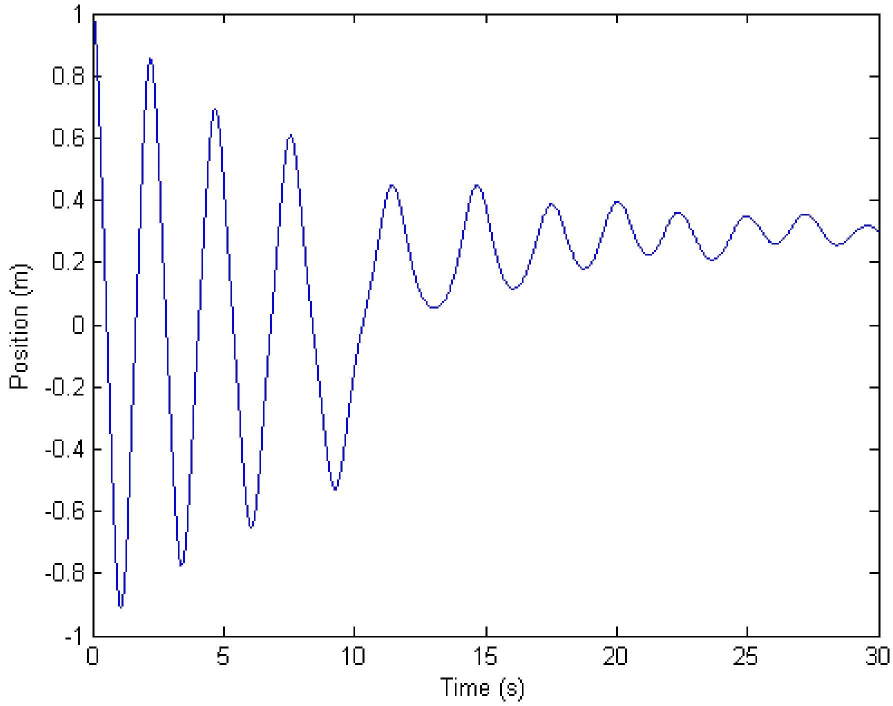

Figure 24. Position – Time history with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=1000

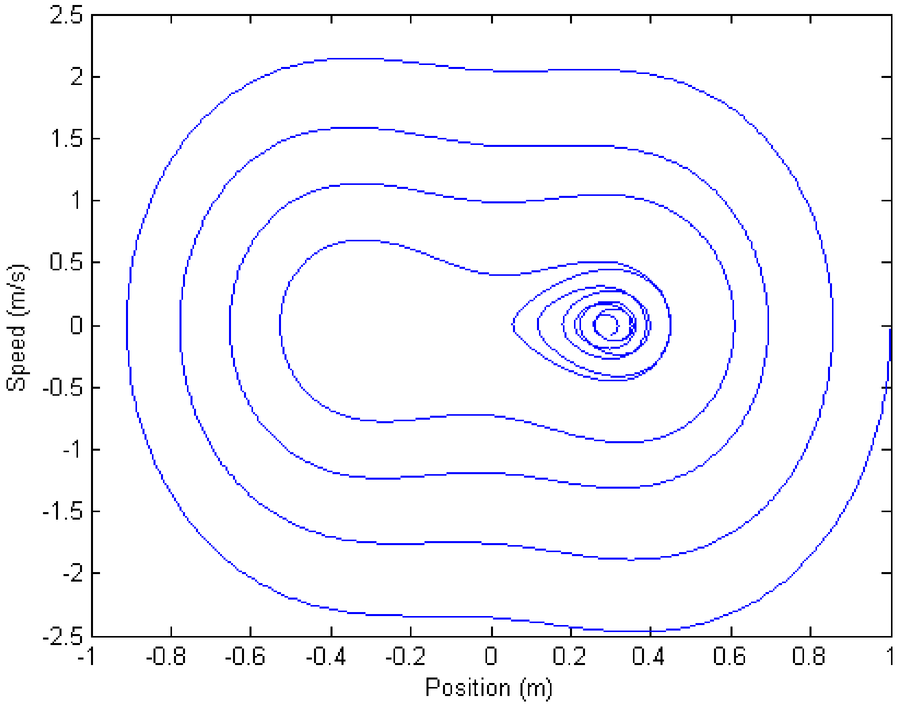

Figure 25. Phase Plane with conditions: p0=10; Omega=1; P=p0*sin(Omega*y(3)); m=100; L=0.5; d=0.4; g=0; c=20; k=1000

5. INITIAL CONCLUSIONS

We presented a initial nonlinear dynamic study of motions of a symmetric two bar planar truss in free and forced vibrations for several parameter settings.

6. ACKNOWLEDGEMENTS

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