SIMPLIFIED STRUCTURAL DYNAMIC ANALYSIS INVOLVING NONLINEAR BOUNDARY EFFECTS WITH APPLICATION ON ENGINEERING PROBLEMS

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Abstract. *This paper aims at apply a simplified approach to solve one-dimensional dynamic structural systems under nonlinear physical boundary field, with practical application on current engineering problems. The mathematical model simplifies the solution of the coupled differential equation system by the use of explicit interactive method. The cases analyzed involve physical nonlinearities on the boundary field defined by polynomial force-displacement behavior. The problems of interest are in the field of soil structure interaction and fluid structure interaction. Examples of application presented along the paper are pile driving by pilling method, launch and penetration of torpedo piles, problems involving static and dynamic friction and even problems involving drag resistance forces. The results are compared with bibliographic references when available; the authors have no intention to discuss in deep the peculiarities of each engineering problem, aiming at merely mathematically represent the physics involved in those specific problems on a simplified and practical approach.*

Keywords: *Nonlinear Analysis, Structural Dynamics*

1 INTRODUCTION

Simulation of engineering problems in many fields of application involves the good representation of dynamic behavior of mechanical systems. On the field of structural dynamics, in which the bodies are submitted to dynamic excitations or self-induced dynamic response, fully analytical mathematical results based on the closed differential equation solution are not feasible, being necessary the application of numerical technics for differential equations resolution. The most accurate definition of physical aspects, main parameters and numerical mathematical differential equation solution technique are the pillars for the good representation.

Problems which involve energy dissipation, even when associated with rigid body motion, e.g. the freefall of a torpedo pile when launched by an AHTS (Anchor Handling Tug Supply Vessels), can be solved through numerical simulation of the associated dynamical differential equation problem.

The advantages of numerical computational simplified method presented along this paper is that can be easily implemented on programming languages or, in the current case, in a friendly mathematical solution software, with the possibility to be implemented on free-to-program commercial packages intended to engineering problems simulation.

Despite of the fact the method is called simplified along this paper, the routine has been designed to solve fully coupled dynamic differential equation systems. However, as the examples provided along this paper present basically unidimensional behavior, the coupled equation system becomes decoupled, with explicit solution.

2 SIMULATION OF DYNAMIC STRUCTURAL RESPONSE

In this work the authors have the aim to provide a simple approach to simulate current engineering problems based on the dynamic differential equation problem, see Eq. (1) through the use of the Newmark (1959) discrete solution routine and an incremental process to account for the nonlinear boundary condition. M, C and K are respectively Mass, Damping and Stiffness Matrices, being x'', x' and x the nodal acceleration, velocity and displacement vectors. F(t) is nodal time forces vector.

$$M.x'' + C.x' + K.x = F(t) \tag{1}$$

Physical and geometrical properties of the structural elements are considered within the linear range since the boundary behavior is highly nonlinear in comparison to the previous effects within the cases analyzed. Figure 1 shows the polynomial approximation of boundary conditions which can be simulated by the algorithm, it ranges from basically linear with no yield point, passing by bi-linear, bi-linear with derating and highly nonlinear stress-strain. As can be seen, the basic assumption considers the boundary condition nonlinearity arising from stress-strain or force-displacement relationship, which contemplates a series of engineering problems as for example soil-structure interaction, hydrostatic, simulation of static and dynamic friction and other practical problems.

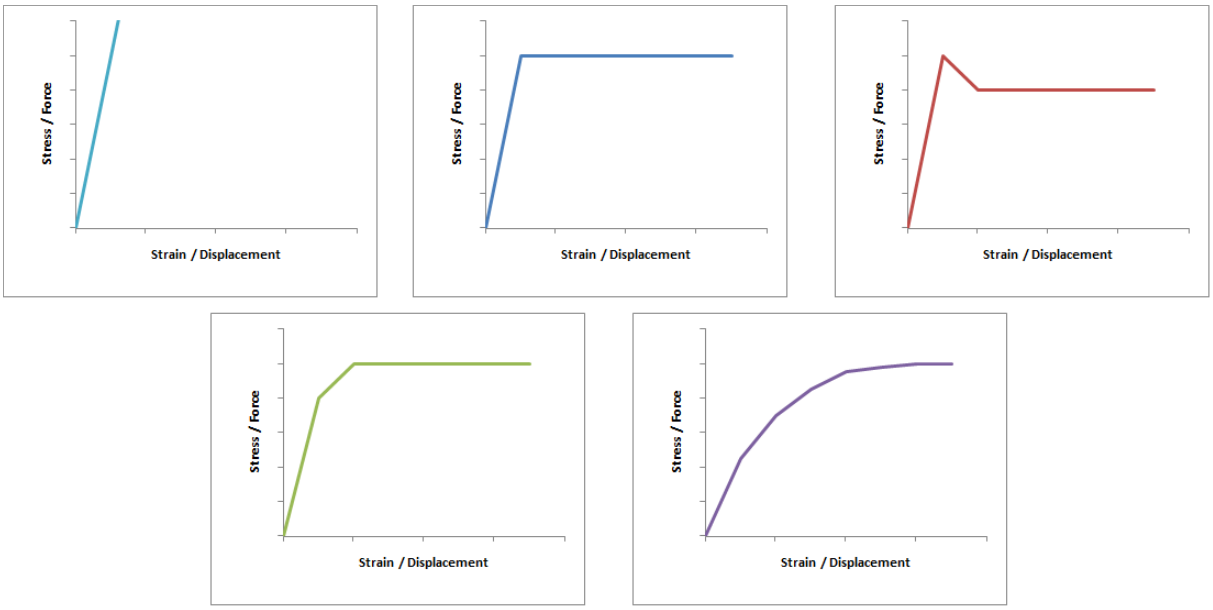

Figure 1. Boundary Conditions Behavior

Nonlinear boundary effects are accounted for in the dynamic system when imposing constrains into the stiffness matrix. Thus, dynamic system becomes dependent of the boundary responses, which in turn are dependent of imposed forces, see Eq. (2). Numerically the solution of this problem seeks for the parameter which solves the convergence problem in equilibrium position at each step. The stiffness matrix with its imposed boundary conditions becomes force parameter dependent at each time step.

$$M.x'' + C.x' + K(F).x = F(t) \tag{2}$$

3 PHYSICAL BOUNDARY NONLINEARITY INTRODUCTION ON THE DIFFERENTIAL EQUATION

3.1 Modification in the Stiffness Matrix

The induction of nonlinear effects in the dynamic system is simulated by introduction of the force dependent parameter in the element stiffness matrix, forcing that analysis be performed in an interactive way, requiring model equilibrium at each time step. Aiming at facilitate the equilibrium solution, a direct incremental process has been chosen to solve the model equilibrium, which makes process less robust when compared to root-finding interactive methods, however, the time consumption rate at simple problems as the ones presented in this work has no real impact, in special when treating on de-coupled explicit problems.

When imposing constrains into the stiffness matrix, traditional Dirichlet boundary conditions are not applied, instead, the algorithm forces the system to converge at each time step to the equilibrium configuration, dependent of the stress-strain (or force-displacement) curve chosen for the boundary at a given degree of freedom.

3.2 Modification in the Solution Algorithm

The advantage of the method presented in this paper is that the time consumption for equilibrium completion is low, being the algorithm very simple, written in few lines and, applicable to a range of practical engineering problems. Newmark-β (1959) solution for complete systems is presented in Eq. (3), the components being set based on the implementation by displacements.

$$\left(\frac{1}{\beta.\Delta t^2}.M+\frac{\gamma}{\beta.\Delta t}.C+K\right).d_{n+1}=F_{n+1}+M.\left[\frac{1}{\beta.\Delta t^2}.d_n-\frac{1}{\beta.\Delta t}.v_n-\left(\frac{1}{2.\beta}-1\right).a_n\right]+$$
$$+C.\left[\frac{\gamma}{\beta.\Delta t}.d_n-\left(1-\frac{\gamma}{\beta}\right).v_n-\left(1-\frac{\gamma}{2.\beta}\right).\Delta t.a_n\right]$$

$$a_{n+1}=\frac{1}{\beta.\Delta t^2}.(d_{n+1}-d_n)-\frac{1}{\beta.\Delta t}.v_n-\left(\frac{1}{2.\beta}-1\right).a_n \tag{3}$$

$$v_{n+1}=\frac{\gamma}{\beta.\Delta t}.(d_{n+1}-d_n)-\left(1-\frac{\gamma}{\beta}\right).v_n-\left(1-\frac{\gamma}{2.\beta}\right).\Delta t.a_n$$

The direct solution of the problem does not require interactive processes, however, when the nonlinear boundary conditions are introduced into the system, the terms become time-force-displacement dependent, and the interactive process is required to solve the system. The resultant system may be summarized as per Eq. (4). The system can be solved according to the flowchart shown in Fig. 2.

$$\left[\frac{1}{\beta.\Delta t^2}.M+\frac{\gamma}{\beta.\Delta t}.C+K(F_n)\right].d_{n+1}=F_{n+1}+M.\left[\frac{1}{\beta.\Delta t^2}.d_n-\frac{\gamma}{\beta.\Delta t}.v_n-\left(\frac{1}{2.\beta}-1\right).a_n\right]+$$
$$+C.\left[\frac{\gamma}{\beta.\Delta t}.d_n-\left(1-\frac{\gamma}{\beta}\right).v_n-\left(1-\frac{\gamma}{2.\beta}\right).\Delta t.a_n\right] \tag{4}$$

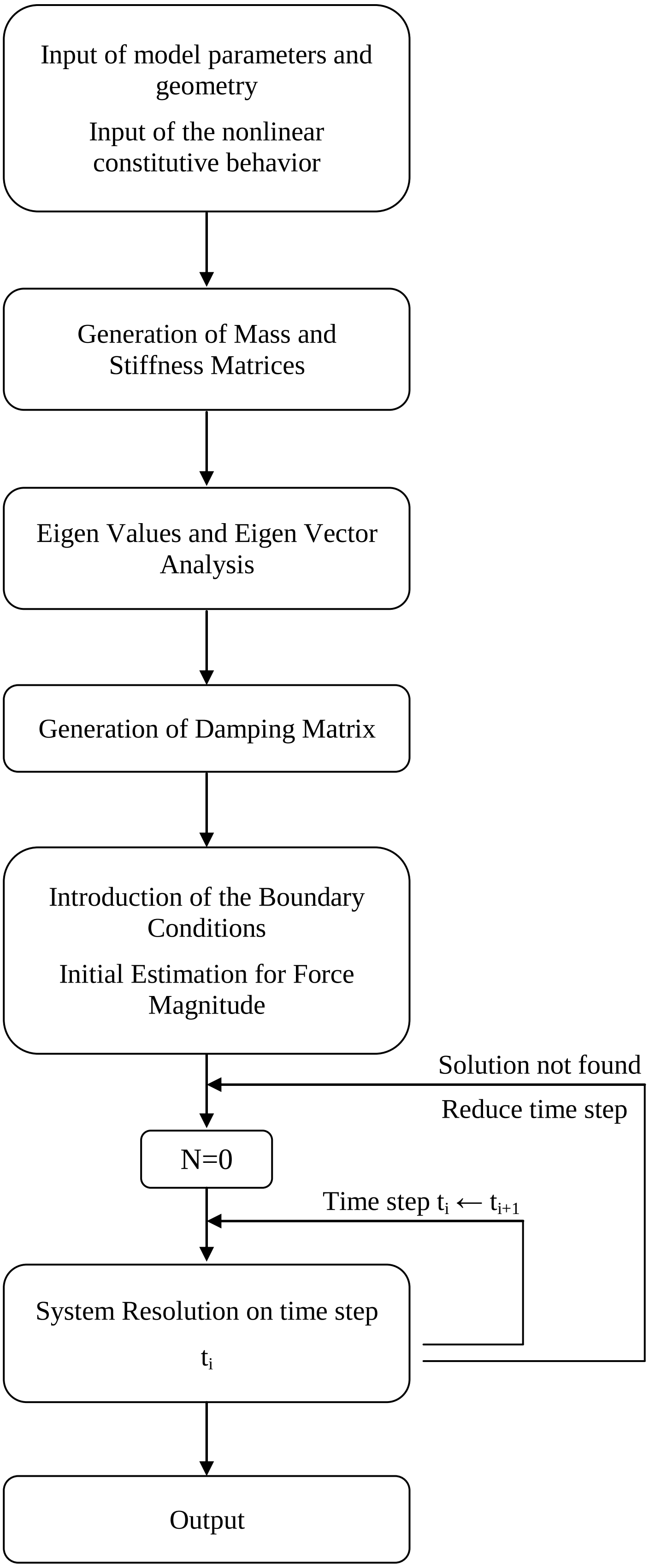

Figure 2. Dynamic Analysis Algorithm

4 PACTICAL EXAMPLES OF THE APPLICATION OF THE ROUTINE

In order to present in practical terms the application of simplified methodology described above, a computational routine was developed for the computation of response parameters. The first example comprises a pile driving by hammer method, in this example the soil parameters are kept constant along depth. The second one is similar to the first one, however, it has been adopted the soil rigidity rapidly increases with depth. For the last example, a torpedo file is simulated penetrating in the soil after a free-fall stage.

4.1 Application on Pile Driving by Hammer

The dynamics in this process is related to the impact energy imposed to the pile by the free fall of the hammer over the pile head. An illustration is presented on Fig. 3. When the hammer hits the pile head, part of the energy is lost through noise dissipation but the great part is transferred to the pile. The shock wave is propagated throughout the pile into the soil and reflected. The damping arises from the pile structure and soil dissipation.

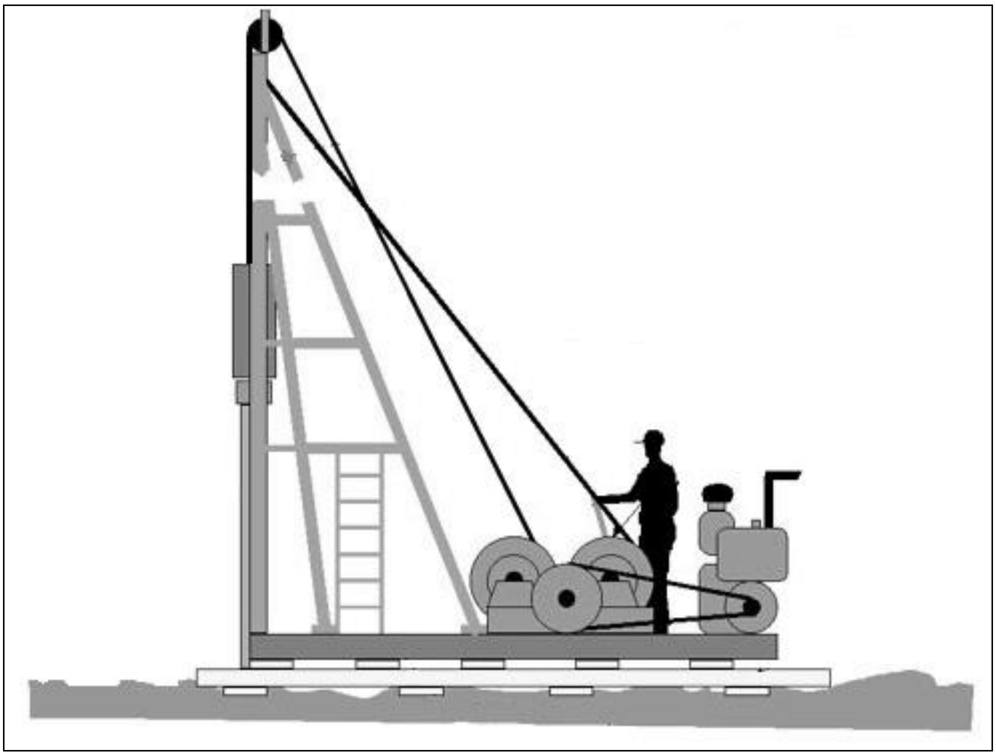

Figure 3. Illustration of the Pile Driving by Hammer

Comparison with references indicate good agreement to the reality when evaluating parameter results. Computational costs for obtaining displacement, velocity and acceleration results can be considered irrelevant. Figure 3 illustrates the penetration vs. time parameter. Other parameters such as displacement along the pile, internal stresses, and strains in soil contour can be obtained directly, however disregarded in this study since they are not a focus of the current work. The hammer hits the pile on a constant energy through time, on constant intervals of 3.0 sec. in the case analyzed. These input parameters, e.g. energy intensity and hit time interval, can be set to vary along the analysis in order to better adjust to real cases. Figure 4 shows a displacement vs. time graph for the nonlinear pile driving result, it can be seen that on vertical direction the pile base is free to move, which occurs in the wave reflection.

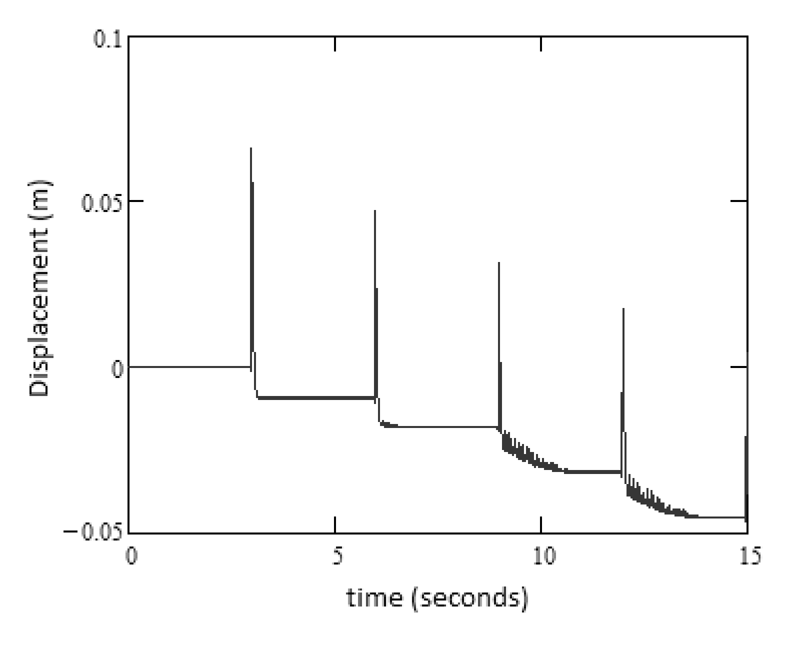

Figure 4. Displacement vs. Time in Pile Driving by Hammer Simulation

Considering the case the soil rigidity varies slowly with depth, the analysis shows a basically linear response not affected by the soil parameters as can be seen on Fig. 5. This behavior is observed when the long-term simulation is performed. This simulation has good agreement to the pilling passing through the upper soil layers, however, once the soil stiffness becomes relevant, both the wave reflection and pile residual penetration change drastically due to the change in the conditions; next section covers the results in this case.

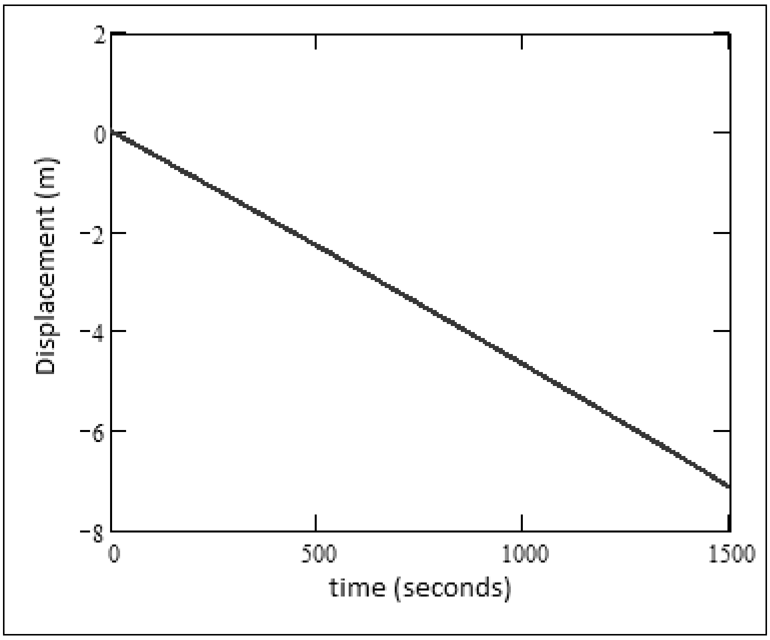

Figure 5. Displacement vs. Time in Pile Driving by Hammer Simulation – long-term simulation

4.2 Application on Pile Driving by Hammer with High Nonlinear Soil Response

It is important to remark this paper has no intention to discuss pipe-soil interaction curves, differences between cohesive and non-cohesive soils, layers constitution, and other aspects of the soil mechanics discipline. Following this premise, the routine may allow the introduction of depth dependent soil rigidity, with linear or nonlinear increase along depth. The result of a pile driving in terms of displacement vs. time close to the target depth can be found on Fig. 6. This result shows both the variation in the shock wave when the soil stiffness increases and the decrease in the residual penetration. When observing the results, one can observe the impact the soil stiffness brings to the shock wave, which in turn has the damping reduced, leading to a higher energy transferred to the soil, making the residual penetration increase in turn.

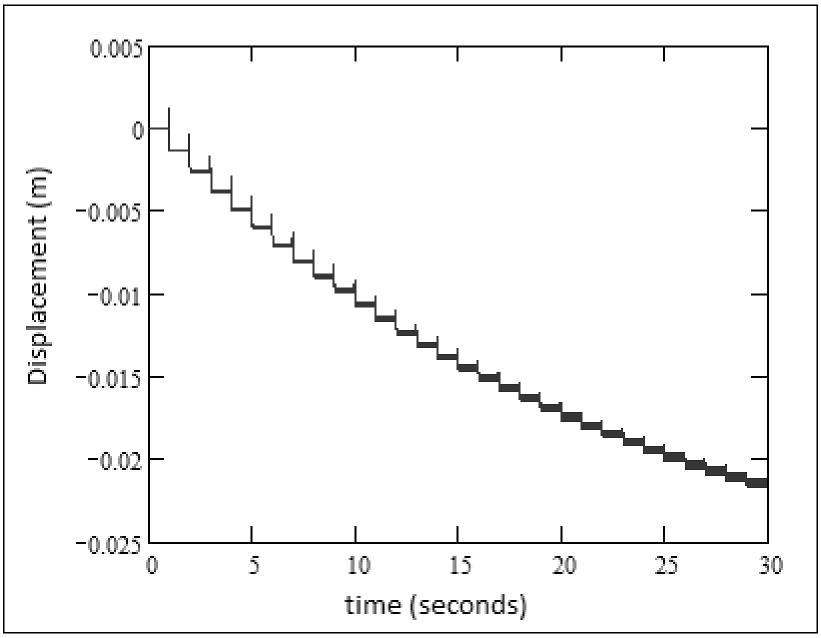

Figure 6. Displacement, Nodal Results of Pile Driving by Hammer

4.3 Application on the Simulation of Torpedo Pile Freefall and Drivability

One of the most interesting practical problems in the line this paper has been drawn is the simulation of torpedo piles drivability after free fall. The torpedo piles are usually launched from a distance to seabed in order to maximize the impact energy when arriving to the seabed, these torpedo piles are also launched in such a way the angle of penetration be appropriate to increase resistance when the mooring lines are loaded.

The problem involves two phases, the first one related to freefall of the torpedo pile. The second one related to the pile drivability where the target depth is the main parameter. For the freefall, ignoring the influence of sea current, the drag resistance of the surrounding fluid is one of the effects considered. A comparison is presented in Fig. 7 in order to illustrate the responses when the drag is considered proportional to the square velocity and the case there is no surrounding fluid resistance. It is noted that the vertical velocity converges asymptotically to the terminal velocity in the free fall conditions in seawater.

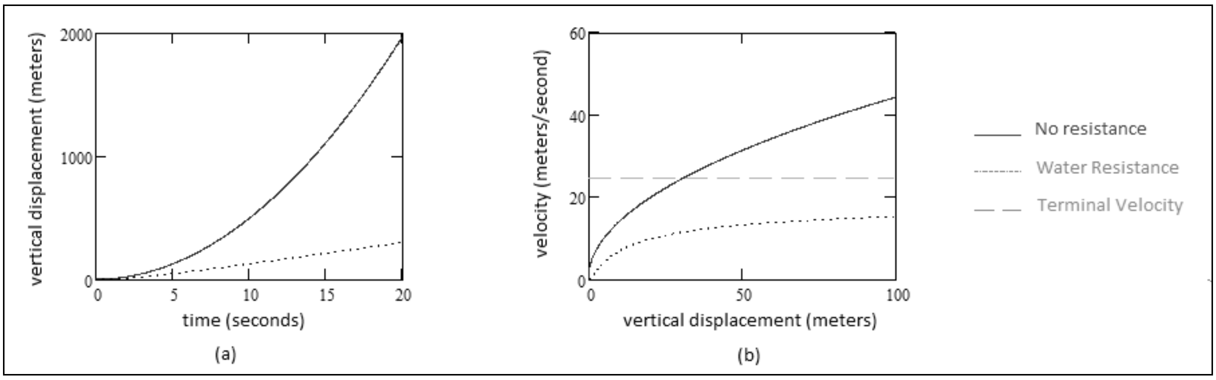

Figure 7. (a) Vertical Displacement vs. time for a circular pile 0.5 meters diameter in vacuum and seawater; (b) Vertical Velocity vs. Vertical Displacement for a circular pile 0.5 meters diameter in vacuum and seawater

The problem of maximize the energy is such that distance between seabed and launching quote leads to the maximum kinetic energy, thus the maximum velocity. The designer may adopt to identify through derivate the velocity function the position in which the velocity increase rate (acceleration) minimize. In the Fig. 8 one can consider around 100 meters the position the velocity increase rate minimizes.

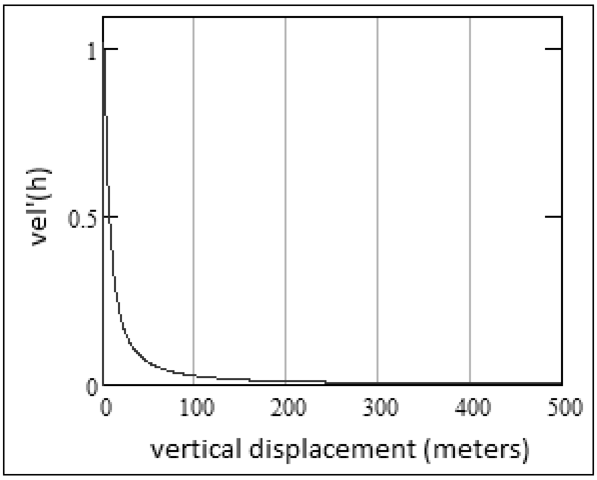

Figure 8. Derivate of the Velocity Function on Vertical Displacement parameter

Effects of lateral loads during launch, as the effect of sea current waves (at low depths) may influence on the torpedo pile inclination. The pile must also be design in such a way it does not destabilize when in free fall. In this paper, the parameter associated with the ultimate depth of torpedo pile is considered for evaluation.

The results are compared with references and show good agreement, especially in terms of velocity on impact moment and final depth of the torpedo pile, key parameters in the design of these systems. Considering the simplified approach of the analysis, it can be said that the results represent an alternative approach to preliminary studies. Figure 9 shows results for drivability and final coordinate for pile embedment after performing the nonlinear dynamic analysis.

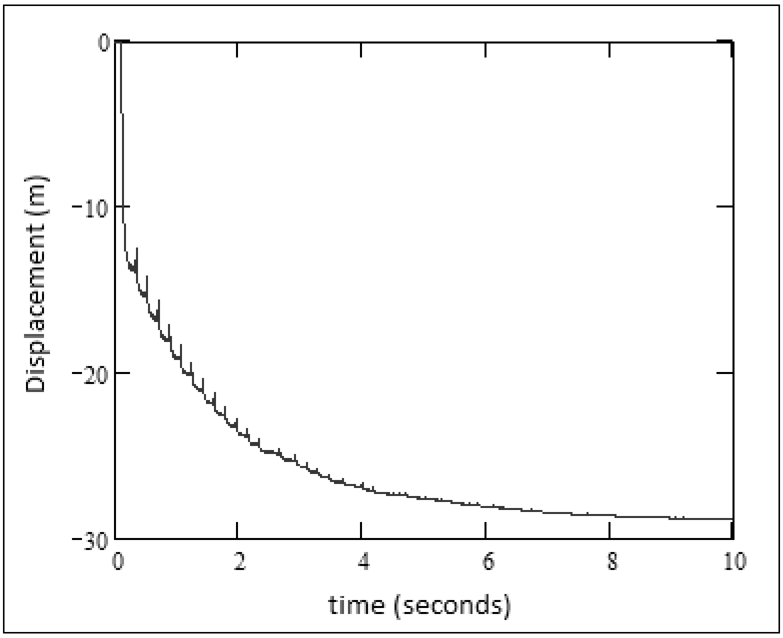

Figure 9. Displacement, Torpedo Pile Driving Results

5 SUMMARY AND CONCLUSIONS

Along this paper the authors presented a simplified methodology for analysis of dynamic structural systems under nonlinear boundary effects, this approach has no intention to replace the current computer simulation algorithms which account for physical and geometric nonlinear characteristic of the system. This work has the practical goal of presenting to engineers a first assessment mechanism when a fast response to a physical nonlinear boundary condition characteristic problem is needed, represented by a range of problems in the field of civil, naval and even mechanical engineering.

For simulations that involve complex issues, which cannot be simplified in models with few degrees of freedom or several grades of nonlinearities, it is recommended the application of computational tools to simulate effectively imposed nonlinearities.

In this work we chose three examples to demonstrate the application of the algorithm, showing its feasibility, in which in all cases the results were in good agreement with academic references, which in turn considered more complex analysis and a larger number of degrees of freedom.

REFERENCES

Danziger, B. R. (1991). Análise Dinâmica de Cravação de Estacas. *D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil*.

Jacob, B. P. (1990). Estratégias Computacionais para a Análise Não-linear Dinâmica de Estruturas Complacentes para Águas Profundas. *D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil*.

Kunitaki, D. M. (2006). Tratamento de Incertezas no Comportamento Dinâmico de Estacas Torpedo para Ancoragem de Sistemas Flutuantes na Explotação de Petróleo Offshore. *D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil*.

Morison, J. R. (1950). The Force exerted by Surface Waves on Piles. *Petroleum Transactions, AIME, v. 189*, 149-154.

Newmark N. M. (1959). A Method of Computation for Structural Dynamics. *Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers*, 67-95.

Poulos S. J. (1971). *The Stress-Strain Curves of Soils.* Winchester, Massachusetts: Geothecnical Engineers, Inc.