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EXPERIMENTAL AND NUMERICA DETERMINATION OF THE AXIAL LOAD IN STRUCTURAL TIE-RODS WITH COMPLEX BOUNDARY CONDITIONS

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Abstract. *Monitoring of the in-exercise axial load in structural tie-rods is more often necessary in order to monitor the health of a part or of an entire ancient building. In recent works we proposed a non-invasive method devoted to the estimation of the axial load in these slender elements that is based on the measure of the vibrational behavior and the comparison of natural frequencies with a numerical model. In this work we add to this method a complementary set of parameters to take into account non-conventional boundary conditions at the extremities of the tie-rods that usually occur in real cases, i.e. an elastic bed at the rod/wall interface. In comparison with fixed-end or pinned conditions, the elastic bed condition better approximates the experimental behavior and, many cases, make the estimation more conservative.*

Keywords: *numerical algorithm, tie-rods, Winkler bed.*

1 INTRODUCTION

Tie-rods are structural elements used in a wide range of civil constructions. One of purposes they serve is to provide support for masonry arches and vaults in ancient buildings (e.g. churches, castles, etc.), which are known to lurch and founder course of time. Tie-rods are subjected to axial tension and, thus, help the building resist occurring lateral loads exerted by structural elements, walls and facades.

Indeed, over the years deformation of masonry walls and some displacements in the building may cause significant changes in the axial loads of tie-rods. In the extremes, this can lead to either failures in tie-rods structural integrity, or to "laziness" of tie-rods that stop carrying out their duty when they loose loads. That is why regular monitoring of tie-rods condition is of a great importance.

The purpose of the presented work is to determine axial loads acting on tie-rods with a complex boundary condition at the rod's extremities that takes onto account an elastic bed at the tie-rod/wall interface. The analytical solution of the problem, and an automated optimization algorithm is here presented.

The approach is tested on the fifteenth century "Casa Romei" located in Ferrara, Emilia-Romagna (Italy), in order to evaluate functionality and reliability of the method.

2 STATE OF THE ART

Since there is no possibility for a direct in-situ measurement of the force acting on the tie-rod, multiple attempts have been made during last years to develop techniques for an indirect evaluation of the load. Previous studies (Blasi, 1988, 1994, Sorace, 1996) report a method that combined static and dynamic force identification. Tie-rods were modelled as simply supported Euler beams with rotational springs of similar stiffness added on each edge. The stiffness of the spring and the force were the two unknowns obtained from the system of equations, built with a static equation for deflection and a dynamic equation for natural frequencies.

Another study (Briccoli Bati, 2001) introduced a static approach for force identification. The experiment consisted of measuring three vertical displacements and strains variations at three sections of the tie-rod under a concentrated load; Lagomarsino *et al.* (Lagomarsino, 2005) introduced an algorithm to identify the axial tensile force in ancient tie-rods by using the first three natural frequencies. The tie-rod was modelled as an Euler beam of uniform cross-section, neglecting the shear deformation and rotary inertia, and was assumed to be simply supported at the ends with additional rotational springs.

Recently, Maes *et al.* (Maes, 2013) introduced a method that enables definition of axial loads in slender beams with unknown boundary conditions, taking into account affects of rotational inertia of the beam and masses of sensors. However, it requires data from five or more sensors along the length of the beam to determine all the introduced unknown of the inverted problem.

A similar technique of the axial force identification was developed by Li *et al.* (Li, 2013) focusing on studies of Euler–Bernoulli beams and takes into account bending stiffness effects. Rebecchi *et al.* (Rebecchi, 2013) established an analytical method of processing experimental data from five instrumented sections of a prismatic slender beam, which showed satisfying results in estimation of the axial load. The method does not require any exact value of effective length of the beam, but neglects both rotary inertia and shear deformations effects in the solution for beam vibrations. For cases of similar beams, Tullini *et al.* (Tullini, 2008, 2012, 2013) proposed a static method of axial force identification. The analytical algorithm makes use of any set of experimental data represented by flexural displacements or curvatures measured at five cross-sections of the beam subjected to an additional concentrated lateral load. Again, Gentilini *et al.* (Gentilini, 2013) developed a procedure that combines dynamical testing with FEM simulations using added masses. The method was tested out for tie-rods of various length and load intensity, showing reliable results. Livingston *et al.* (Livingston, 1995) identified the tensile force in prismatic beams of uniform section by using modal data and assuming rotational and vertical springs at each end of the beam. Shear deformation and rotary inertia were neglected (according to the Euler beam model).

The present authors, in their previous studies (Garziera, 2011), experienced a method consisting in taking into account several natural frequencies that, in conjunction with a numerical model, permit the calculation of the axial load in ancient tie-rods “affected” by discontinuities, added masses, irregular section, and complex boundary conditions. The method has been applied with success to many case studies.

3 MODELING OF THE SYSTEM

The technique we present in this study is based on an identification method that allows us to know the dynamical behaviour of a tie-rod in a simple way, and to compare it with a numerical model. In particular, the first natural frequencies of the tie-rods are experimentally identified by measuring the frequency response functions (FRFs) with instrumented hammer excitation; four to six natural frequencies can be easily identified with a simple test. Then, a numerical model is developed for tie-rod which is axially loaded and, moreover, can contain other singularities as added masses, elastic constraints, changing in the cross section, and so on. All these singularities are very common to meet in real cases, which, in most of cases, involve ancient hand-made rods. However, the main unknown we wish to determine with accuracy is the axial tension that causes potentially dangerous stress states. One has to remind that the rupture of a tie-rod can be very hazardous for a part of (ancient) building.

The part of the tie-rod inserted inside the masonry wall is critical, because many uncertainties of real cases, see for example the tie-rod inserted into the column of Figure 1. As a first approximation, it can be assumed a perfect encastre condition, and in many previous cases it worked good. But in some cases, when the matching between experimental and numerical results is difficult (especially for higher frequencies), one tries to change the conditions at the extremities. Actually, the interface between tie-rod and wall is difficult to predict or to model.

Our idea is to insert an elastic foundation, i.e. Winkler-type bed of the type of elastic foundation usually modelled for pipes or beams in contact with the (elastic) ground. The stiffness of the foundation is then another unknown parameter, together with the length of the rod inside the masonry wall. The authors have previously tested the present technique over different buildings and situations. The novelty introduced in the present work is an optimization algorithm with respect to the elastic bed, of which the analytical solution for Bernoulli-Euler beam is also introduced in the following.

Figure 1. Tie-rod inserting in a masonry column.

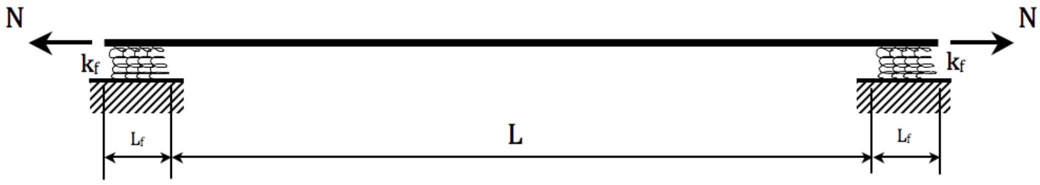

Figure 2. Scheme of the model of tie-rod.

3.1 Analytical closed solution

With reference to Figure 2, the model of tie-rod can be divided in three sections, of length L_f, L and again L_f, respectively. The length L_f identifies the portion inserted into the wall, L is the measured length ("free" length), on the edges the free-end boundary conditions apply, and between the sections, correspondingly, the congruence conditions.

We will use the Lagrangian of the system to obtain the equations of motion. Assuming the hypotheses of Bernoulli-Euler beam theory, we choose to neglect the shear deformation and rotational inertia, because in particular slender rods (for which the relation between the linear dimensions of cross-sections to the length is a very small number) are subject of this study. Thus, the energy functionals for the system can be represented as follows:

- Kinetic energy

$$T = \frac{1}{2}\int_0^L \left[\rho A \left(\frac{\partial w}{\partial t} \right)^2 + \rho I \left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 \right] dx \quad (1)$$

- Potential energy of elastic strain

$$U = \frac{1}{2}\int_0^L \left[EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right] dx \quad (2)$$

- Potential energy associated with the axial load

$$V_F = \frac{1}{2}\int_0^L \left[F \left(\frac{\partial w}{\partial x} \right)^2 \right] dx \quad (3)$$

- Potential energy associated by the elastic foundation

$$V_W = \frac{1}{2}\int_0^L k_f w^2 \left[H\left(l_f - x\right) + H\left(x - L + l_f\right) \right] dx \quad . \tag{4}$$

Introducing the Heaviside function *H(x)* allows us to write down a single equation for the whole tie-rod, taking into account different conditions for its parts.

Lagrangian of the system is equal to the difference between the overall kinetic and potential energy:

$$L\left(w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial t}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial x \partial t} \right) = T - \Pi \quad . \tag{5}$$

Applying the Hamilton's principle, we arrive to the Lagrange equation of motion

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = 0 \tag{6}$$

where q_k identifies the generalized coordinates (degrees of freedom). Finally, the equation of natural vibrations of the rod is obtained:

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} - F\frac{\partial^2 w}{\partial x^2} + k_f w\left[H\left(l_f - x\right) + H\left(x - L + l_f\right) \right] = 0 \tag{7}$$

As stated above, the edges of tie-rods (where $x = 0$ and $x = \mathrm{L}$) are free and between the parts I – II and II – III eight conditions of continuity emerge, which provides twelve conditions that can be expressed in form (8).

$$x = 0 : \begin{cases} Q_I = 0 \\ M_I = 0 \end{cases} \qquad x = L : \begin{cases} Q_{III} = 0 \\ M_{III} = 0 \end{cases}$$

$$x = L_f : \begin{cases} w_I = w_{II} \\ \dfrac{\partial w_I}{\partial x} = -\dfrac{\partial w_{II}}{\partial x} \\ Q_I = Q_{II} \\ M_I = M_{II} \end{cases} \qquad x = L - L_f : \begin{cases} w_{II} = w_{III} \\ \dfrac{\partial w_{II}}{\partial x} = -\dfrac{\partial w_{III}}{\partial x} \\ Q_{II} = Q_{III} \\ M_{II} = M_{III} \end{cases} \tag{8}$$

where the shearing force and the bending moment are introduced by expressions (9):

$$Q = EI\frac{\partial^3 w}{\partial x^3} - F\frac{\partial w}{\partial x}$$
$$M = EI\frac{\partial^2 w}{\partial x^2} \quad . \tag{9}$$

For each part in which a tie-rod is divided during the solution, a separate function w is used: $w_I(x,t)$, $w_{II}(x,t)$, $w_{III}(x,t)$. However, it is clear that the parts are vibrating with the same time frequency. The differential equation of motion is solved by means of the Fourier method.

$$w_i\left(x,t\right)=W_i(x)T(t) \tag{10}$$

For the time function we obtain:

$$T\left(t\right)=A\cos\omega t+B\sin\omega t \tag{11}$$

The form equations will appear different for parts I, III and II.

$$\begin{aligned}&\frac{\partial^4 w_{II}}{\partial x^4}-\frac{F}{\rho Ac^2}\frac{\partial^2 w_{II}}{\partial x^2}-\frac{\omega^2}{c^2}w_{II}=0\\&\frac{\partial^4 w_{I,III}}{\partial x^4}-\frac{F}{\rho Ac^2}\frac{\partial^2 w_{I,III}}{\partial x^2}+\left(\frac{k_f}{\rho Ac^2}-\frac{\omega^2}{c^2}\right)w_{I,III}=0\end{aligned} \tag{12}$$

These equations are solved separately, however, the general solution is of the following form:

$$w_{I,II,III}\left(x\right)=Ce^{sx} \tag{13}$$

Further, the coefficients s are expressed from the equations (14) obtained throughout substitution of the general solution (13) into differential equations (12):

$$\begin{aligned}&s_{II}^4-\frac{F}{\rho Ac^2}s_{II}^2-\frac{\omega^2}{c^2}=0\\&s_{I,III}^4-\frac{F}{\rho Ac^2}s_{I,III}^2+\left(\frac{k_f}{\rho Ac^2}-\frac{\omega^2}{c^2}\right)=0\end{aligned} \tag{14}$$

Each of the equations gives correspondingly four solutions for s: four complex roots $s_{1,2} = \pm k_1 \pm ik_2$ for the first equation, and two complex and two real roots $s_3 = \pm k_3$ and $s_4 = \pm k_4$ for the second. Taking this into account we write the form functions for the natural modes of the tie-rod, Eq. (15):

$$\begin{aligned}&w_I\left(x\right)=C_1e^{s_1x}+C_2e^{-s_1x}+C_3e^{s_2x}+C_4e^{-s_2x}\\&w_{II}\left(x\right)=C_5e^{s_3x}+C_6e^{-s_3x}+C_7e^{s_4x}+C_8e^{-s_4x}\\&w_{III}\left(x\right)=C_9e^{s_1x}+C_{10}e^{-s_1x}+C_{11}e^{s_2x}+C_{12}e^{-s_2x}\end{aligned} \tag{15}$$

Substituting the expressions (15) into twelve boundary and congruence conditions (8), we thus obtain the algebraic system of twelve equations containing the same quantity of unknowns $C_1,\dots, C_{12}$. The determinant of the system (16), expressed in Eq. (17), is supposed to be equal to zero in order to provide a non-trivial solution, which results in a characteristic equation of the eigenvalue problem, which further can be solved by means of the Newton-Raphson method.

$$\begin{aligned}&\left[A\right]\left\{C\right\}=\left\{0\right\}\\&\det\left(A\right)=0\end{aligned} \tag{16}$$

3.2 Numerical approach

Analytical solution presented in the previous section is not of easily solution; in particular, numerical solution of eq. (16) can be instable. Hence, an alternative is the solution via numerical modeling for example with finite element method. As shown elsewhere (Garziera, 2011), in order to determine the axial load tie-rods can be modelled in FEM software using 3D beam elements (we used Simulia ABAQUS), which allows taking into account irregular cross sections, added masses and other elastic supports. As a first step a pretension load *N*, which is the main unknown of the problem, is applied to the beam and as a second step modal analysis is performed. Since the value of the pretension load is the subject of search, it has to be tuned to make the results of modal analysis in FEM reach a good agreement with experimentally defined frequencies. Optimization criteria we have chosen for the problem is represented by the residual error between experimentally defined frequencies and those obtained from FEM analysis. The error *E* can be calculated according to the following formula:

$$E = \sqrt{\sum_{k=1}^{n} p_k^2 (f_k^{\exp} - f_k^{FEM})^2} \tag{18}$$

in which the weights p_k are arbitrary given to each frequency *k*.

Since the extremities of tie-rods are anchored into the masonry walls in a way that hardly leaves a possibility to be certain about the end constraints, as a first try boundary conditions for the edges can be modelled as plain fixed and/or simply supported, and then the elastic bed condition can be apply. Effectively, the real condition on the edges of tie-rods are unknown and may vary from one rod to another, because they are fixed inside masonry walls in a way that is hard to determine using non-destructive techniques. However, certain attempts can be made in order to develop a generalized parametric model suitable for numerical determination of loads in tie-rods. Nevertheless, manual tuning of the load is not considered to be an option in this case, because there is no certainty about the boundary conditions at the beam edges.

This problem is here overcome by means of a parametric, FE model and an optimization tool for computing. In FE model the measured length of the tie-rod is represented by a certain number of beam elements, and boundary conditions are assumed to be an elastic bed, thus representing a general condition of translational and rotational stiffness acting for a length L_f. The sketch of the assumed model is the same displayed in Figure 2. An arbitrary number of springs are then introduced as “spring elements”: stiffness of each spring element together with the sought load can be chosen as optimization parameters in the problem stated. The optimization process is then addressed to the seek of minima of multi-parameter function. The program requires a set of experimentally obtained frequencies, ranges of stiffness and force with sizes of steps for each and also a set of weight coefficients for frequencies, that define importance of the corresponding frequency for the analysis.

In the present work, a combinations of parameters form a grid which measures are chosen *a priori*, and the program launches ABAQUS input file for each node of this grid and then extracts and filters natural frequencies. The optimal values of stiffness of the elastic bed and of axial force deliver minimum value of the error function.

4 A CASE STUDY

The technique we proposed has been applied to the tie-rods installed in “Casa Romei”, an ancient palace in Ferrara, Italy (see Figures 3 and 4). Tie-rods have been placed in this building in different times along its existence and differ in dimensions and cross-section shapes.

First, measurements of geometrical characteristics and of natural frequencies were performed for each tie-rod of the building. Several natural frequencies of tie-rods were experimentally obtained: as an example, a frequency response function (FRF) for a ground floor tie-rod is shown in Figure 5.

For the further analysis, first six natural frequencies were identified in each tie-rod. Six eigen modes are providing an overconstrained problem. Besides that, identification of higher modes might appear inaccurate due to larger possible measurement errors.

Material constants are assumed to be the following: E = 210 GPa, ν = 0.3, ρ = 7850 kg/m^3.

Figure 3. Inner yard of Casa Romei and detail of the ground floor tie-rods.

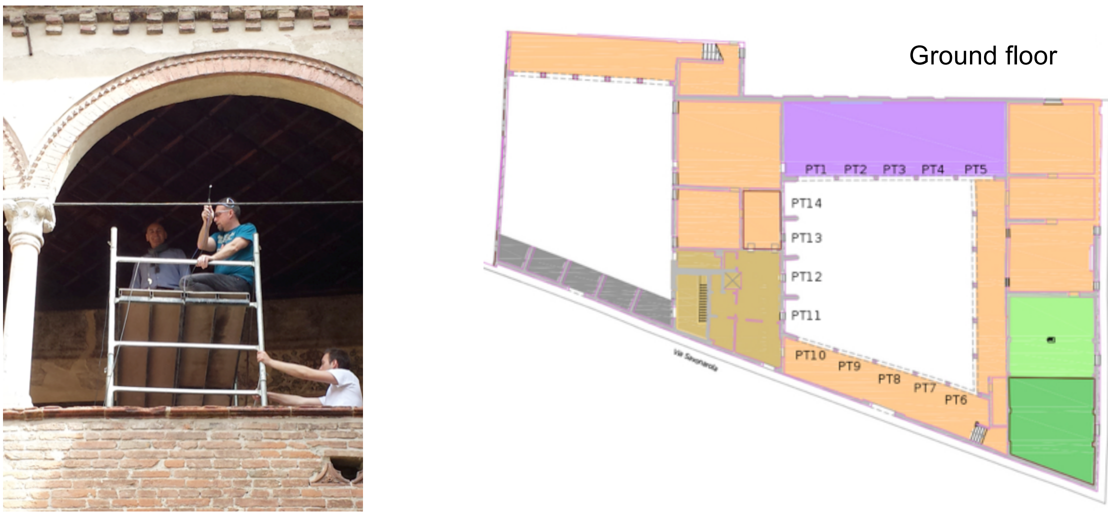

Figure 4. Our team during hammer excitation of a tie-rod; map of ground floor.

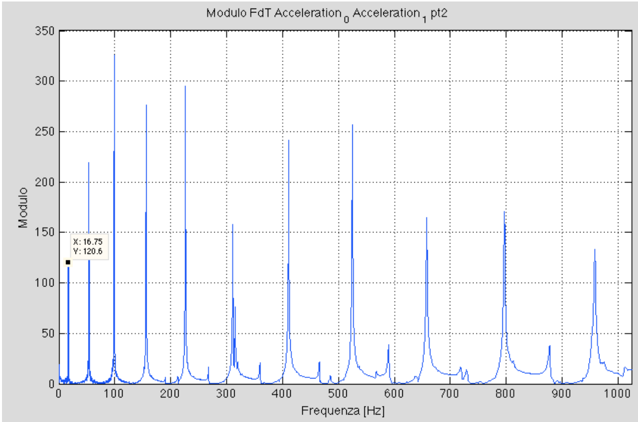

Figure 5. Example of experimental FRF of a tie-rod.

4.1 Elaboration of results

In Tab. 1 the experimental acquisitions about cross section, free length L and natural frequencies of the tested tie-rods are reported. Numeration of the rods follows the scheme of Figure 4. Some of the experimental frequencies miss, due to the position of the accelerometer over a modal node; in these cases, in the optimization process a higher number of frequencies is considered.

The optimization process started from a "reasonable" value of the load, and from an infinite stiffness of the elastic bed at the constraints (encastre condition), and follow a refinement process of the parameters along what that we called "zooming technique", see (Garziera, 2011). Operatively, the algorithm defines a matrix of values and refines grid of parameters to be solved just where local minima of the error as defined in Eq. (18) are found. In this way, we are able to determine almost automatically a value of the axial load that, for an encastre condition, minimizes the error between experimental and numerical natural frequencies of a tie-rod. Then, the second step to be done is to vary the stiffness of the spring bed, and optimize their value again to minimize the error. It is worth to say that the so-defined minima are not independent; nevertheless, once the first parameter (axial load) is optimized, the influence of the second (stiffness of constraints) on the first parameter is nearly negligible.

A typical graphic scenario of the optimization process is depicted in Figure 6: here, several error values are plotted vs. stiffness and axial load, for the tie-rod named PT4. Here the free length L, and length of the elastic constraint inside the wall (L_f in Figure 2) are kept constant. It is possible to note that many minima of the error function are present in correspondence of several N-K_f couples, but a local minimum is also clearly visible, that corresponds to the wanted values. The grid of stiffness and axial load can be reduced around this local minimum, in order to refine the search of the optimal parameters. In our calculation, a satisfactory variation has been chosen equal to 1%. Table 2 summarizes the results of the optimization process to all the tie-rods obtained by the FE method.

From Table 1, we can notice that most of the tie-rods show a meaningful improvement in the error value when a series of springs are modelled at their extremities: only results of 3 rods on 14 are best fitted by a perfect encastre condition. However, the error value belonging to optimization of these rods is much higher than the error of the other rods. The improvement in the process of considering an elastic bed has been more deeply investigated; for example, for PT4 tie-rod, the error goes from 7 (encastre) down to 0.8 (optimal stiffness), and correspondingly the "optimal" axial load increases from 32 kN to about 39 kN (+22%).

Table 1. Tie-rods characteristics and results of stress analysis.

Tie-rod	Cross section (mm^2)	Length L (m)	Natural frequencies (Hz)					Load (kN)	Stress (MPa)	K_f (MN/m)	Error Eq. (18)
			I	II	III	IV	V				
PT1	52x9	3,178	15,30	-	49,00	-	92,30	29400	62,82	20	0,31
PT2	51x9	3,233	16,80	-	53,80	-	98,80	46900	102,18	0,19	6,64
PT3	52x10	3,228	16,30	-	53,80	-	103,00	37500	72,12	101,2	3,95
PT4	51x10	3,218	16,00	33,50	51,30	71,80	95,00	38700	75,88	1,5	0,77
PT5	53x10	3,188	5,50	-	28,30	-	65,50	1000	1,89	10,7	1,17
PT6	50x20	2,748	21,75	49,00	85,25	131,20	188,50	66500	66,50	45,2	0,48
PT7	50x20	2,768	20,00	45,50	80,00	123,80	179,20	43000	43,00	∞	28,08
PT8	50x20	3,358	14,75	32,75	55,75	85,25	121,20	46300	46,30	24	6,60
PT9	50x20	3,248	15,50	34,25	58,75	90,00	128,00	39365	39,37	∞	28,10
PT10	50x20	3,388	17,75	38,00	63,25	94,75	133,00	70955	70,96	∞	14,41
PT11	50x12	3,440	13,25	29,75	51,25	78,25	111,50	29100	48,50	1980	13,77
PT12	50x12	3,298	14,50	30,25	48,25	69,25	94,00	37200	62,00	1,7	0,48
PT13	50x12	3,140	13,75	-	47,25	-	95,75	28200	47,00	2,2	0,27
PT14	50x12	2,510	19,25	46,50		84,00	134,50	38000	63,33	0,85	5,58

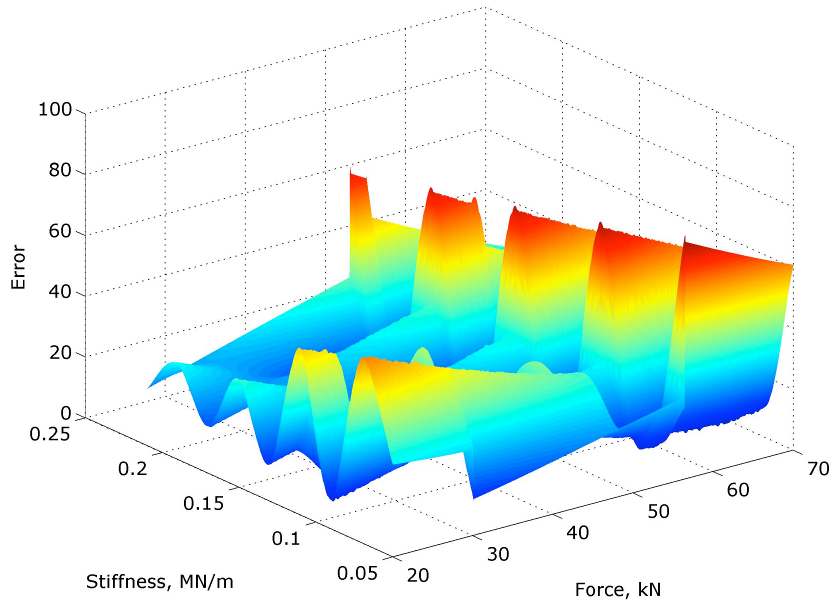

Figure 6. Bi-dimensional error function for tie-rod PT4.

4.2 Effect of weights

The error, or “distance”, between numerical and experimental frequencies is calculated on Eq. (18). The difference (in Hz) between couple of frequencies is multiplied by a weight p_k in order to allow us to attribute higher or lower importance to some frequencies rather than others. The set of weights is then arbitrary chosen, but generally, the first frequencies, or just the first, have bigger importance. A sensitivity analysis has been conducted in order to estimate the influence of the weights set on the final result, that is the axial load closest to reality. In Figure 7 we report a plot in which 3 different error functions corresponding to 3 different weights set, namely $W_1 = \{1,1,1,1,1,1\}$, $W_2 = \{10,1,1,1,1,1\}$, $W_3 = \{4,1,0.5,0.25,0.1,0.05\}$, are shown. As one can notice the surfaces of the error function differ, even if the minimum is reached nearly the same values of load and stiffness.

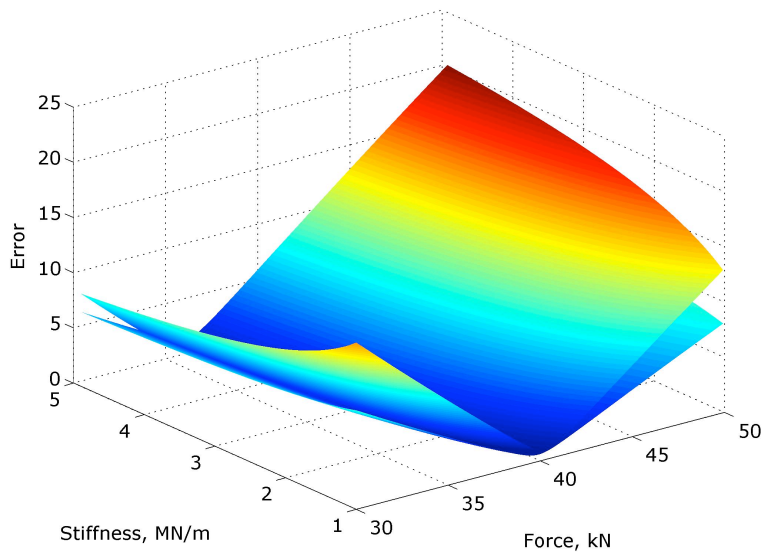

Figure 7. Error surfaces for weights set W_1, W_2, W_3.

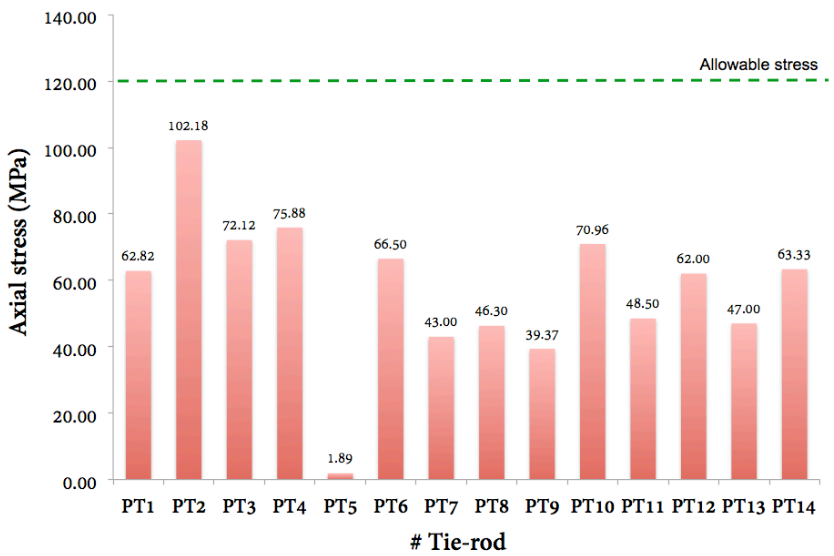

Figure 8. Average axial stress distribution.

As final result, the average stress $\overline{\sigma}_{k,N}$ induced in the tie-rods by the axial load can be calculated by the classical formula:

$$\overline{\sigma}_{k,N} = \frac{N_{k,opt}}{A_{k,\min}} \tag{19}$$

where the optimal load $N_{k,opt}$ and the minimum section $A_{k,min}$ can be considered in order to estimate the rods safety and the load distribution into the building. In Eq. (19) no local effects (screws, fillets, holes, joints, etc.), are considered and must be taken into account, if present, for a correct local integrity asessment. In Figure 8 the stresses so calculated are plotted for the set of considered tie-rods. All tie-rods but PT5 rod works properly well below a reasonable admissible stress for ancient iron (120-140 MPa). Rod PT5 is definitely unloaded: this can be due to a damage at the anchorages and should be further investigated.

5 CONCLUSIONS

In this work an optimization process is shown and discussed in determining the axial load in structural iron tie-rods. From our study the following conclusions can be drawn:

1. the technique is capable in identifying axial load in tie-rods via experimental and numerical methods, making it possible a structural assessment of the overall building stability;

2. the model of tie-rod we used is accurate enough to take into account even unknown boundary conditions;

3. from the simulations, it comes that axial force shifts the set of frequencies (higher the force higher the frequency), while stiffness of the elastic bed into the wall changes the distance between natural frequencies;

4. taking into account an elasticity of the anchorages in the wall generally increases the axial load of 10-25%, hence the assumption of perfect encastre at the extremities is not cautionary;

5. different set of weights bring to different optimal values of parameters, nevertheless the axial load in tie-rods that we are looking for does not change significantly.

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